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First Algebra and Probability

Revision on factorizing the algebraic expressions

1 Taking out the H.C.F. : $ab + ac = a(b + c)$

- $6x^2y + 10xy^2 = 2xy(3x + 5y)$
- $2a(x + y) - b(x + y) = (x + y)(2a - b)$

Notice that : The H.C.F. is an algebraic term $(x + y)$

2 Difference between two squares : $a^2 - b^2 = (a + b)(a - b)$

- $x^2 - 9 = (x + 3)(x - 3)$
- $16x^4 - 81 = (4x^2 + 9)(4x^2 - 9) = (4x^2 + 9)(2x + 3)(2x - 3)$

Notice that : We continue factorizing until factorization is complete.

- $2x^3 - 72x = 2x(x^2 - 36) = 2x(x + 6)(x - 6)$

Notice that : We firstly take out the H.C.F.

3
- Sum of two cubes : $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- Difference between two cubes : $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

- $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$
- $3x^4y - 81xy^4 = 3xy(x^3 - 27y^3) = 3xy(x - 3y)(x^2 + 3xy + 9y^2)$

4 Trinomial in the form : $x^2 + bx + c$

- $x^2 + 7x + 12 = (x + 3)(x + 4)$ Their sum = 7, Their product = 12
- $x^2 + x - 12 = (x + 4)(x - 3)$ Their sum = 1, Their product = −12

5 Trinomial in the form : $a x^2 + b x + c$

- $6x^2 + 7x + 2 = (2x + 1)(3x + 2)$
- $36x^3 - 84x^2 - 15x = 3x(12x^2 - 28x - 5)$
  $= 3x(6x + 1)(2x - 5)$

Scissors method

$(2x + 1)$ $(3x + 2)$ | $(6x + 1)$ $(2x - 5)$

6 Perfect square trinomial :

- $a^2 + 2ab + b^2 = (a + b)^2$
- $a^2 - 2ab + b^2 = (a - b)^2$

- $x^2 + 10x + 25 = (x + 5)^2$
- $9x^2 - 24xy + 16y^2 = (3x - 4y)^2$
- $4x^2 - 10x + 25$ is not a perfect square trinomial because : the middle term $\neq \pm 2 \times \sqrt{4x^2} \times \sqrt{25}$

In the perfect square trinomial :

- Each of the first and third terms are perfect square
- The middle term = $\pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Third term}}$

7 Factorizing by grouping :

- $ax + ay + bx + by = a(x + y) + b(x + y)$
  $= (x + y)(a + b)$

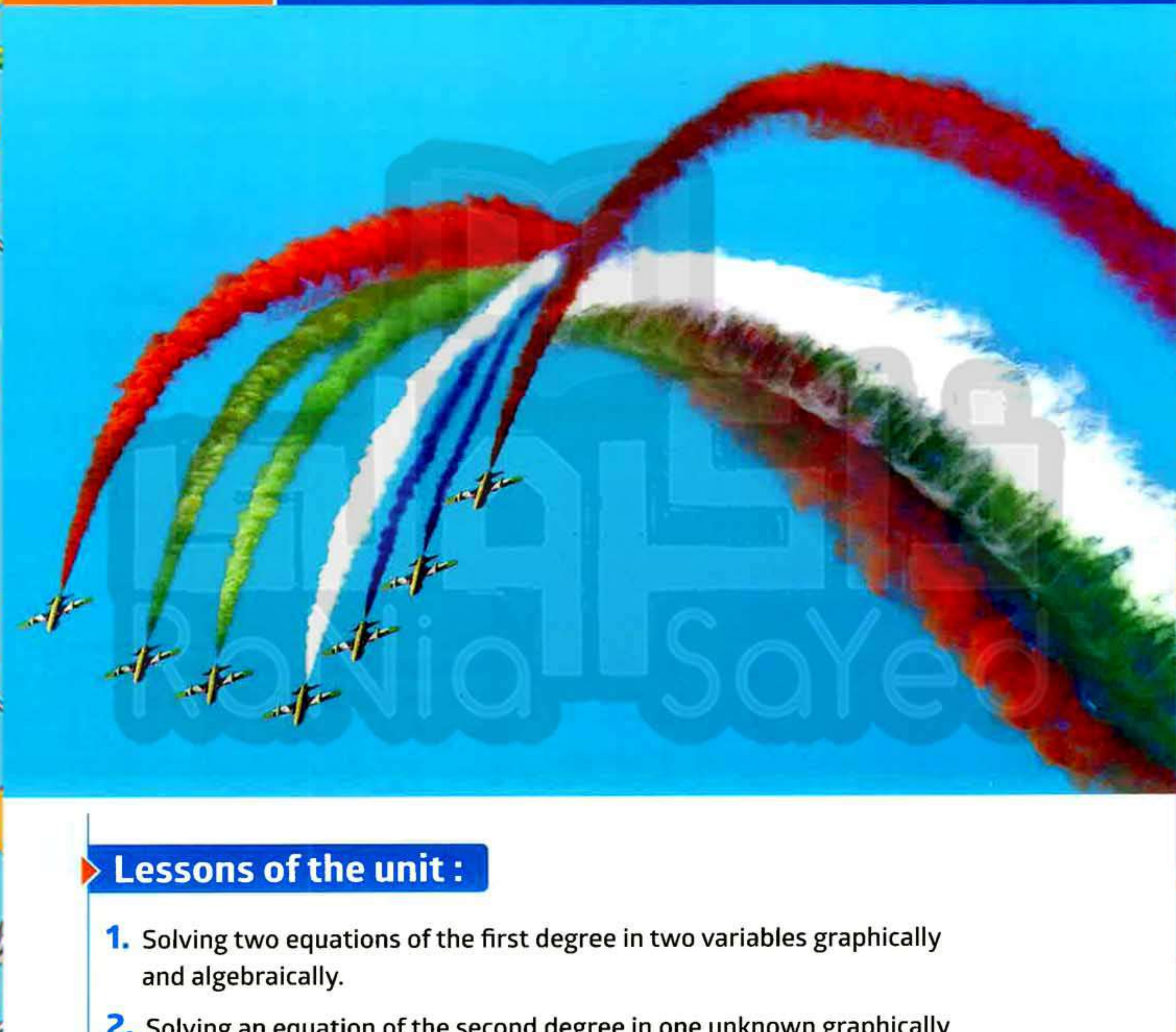
Notice that : We divided the expression into two expressions each of them consists of two terms.

- $x^2 - 2xy + y^2 - 9 = (x^2 - 2xy + y^2) - 9$
  $= (x - y)^2 - (3)^2$
  $= (x - y + 3)(x - y - 3)$

Notice that : We divided the expression into perfect square trinomial and perfect square monomial.

UNIT 1

Equations



Lessons of the unit :

1. Solving two equations of the first degree in two variables graphically and algebraically.
2. Solving an equation of the second degree in one unknown graphically and algebraically.
3. Solving two equations in two variables , one of them is of the first degree and the other is of the second degree.

Unit Objectives :

By the end of this unit, student should be able to :

- Remember what have been studied on factorization of algebraic expressions.
- Solve two equations of the first degree in two variables graphically.
- Solve two equations of the first degree in two variables algebraically by substituting method and omitting method.
- Determine the number of solutions of any two equations of the first degree in two variables.
- Solve word problems that will lead to two equations of the first degree in two variables.
- Solve an equation of the second degree in one unknown graphically.
- Solve an equation of the second degree in one unknown by using the general rule (general formula).
- Determine the number of solutions of an equation of the second degree in one unknown.
- Solve word problems that will lead to an equation of the second degree in one unknown.
- Solve two equations in two variables , one of them is of the first degree and the other is of the second degree.
- Solve word problems that will lead to two equations in two variables , one of them is of the first degree and the other is of the second degree.
- Use the calculator to solve equations.

LESSON **1**

Solving two equations of the first degree in two variables graphically and algebraically

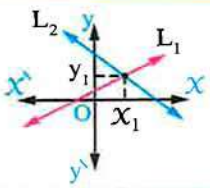
First Solving two equations of the first degree in two variables graphically

- The meaning of solving two equations graphically is finding the ordered pair or ordered pairs which satisfy the two equations simultaneously.
- Since the set of solution of the equation of the first degree in two variables in $\mathbb{R} \times \mathbb{R}$ is represented graphically by a straight line ,

then to solve the two equations graphically, we do as follows :

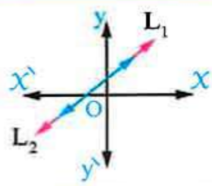
In the Cartesian plane draw the two straight lines which represent the two equations to be L_1 and L_2 , then the S.S. is the point of intersection of the two straight lines L_1 and L_2 , then we have three cases.

1 L_1 and L_2 **intersect** at the point (x_1 , y_1)



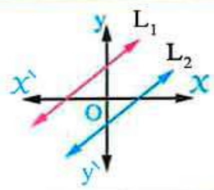
- There is a unique solution (x_1 , y_1)
- The S.S. = $\{(x_1 , y_1)\}$

2 L_1 and L_2 are **coincident**



- There is an infinite number of solutions

3 L_1 and L_2 are **parallel**



- There is no solution
- The S.S. = $\varnothing$

The following examples show each case of the previous cases.

Example 1

$L_1 : 2x - y = 5$

$L_2 : x + 3y + 1 = 0$

$\because L_1 : y = 2x - 5$

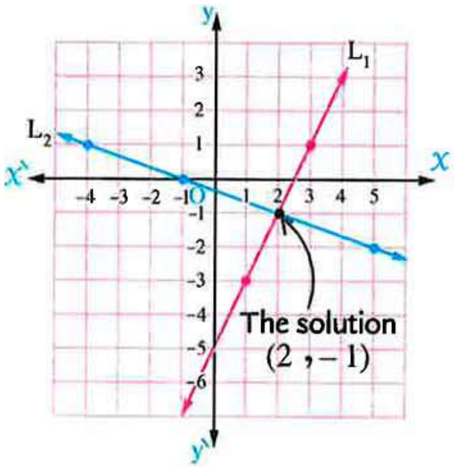
$\therefore$

x	1	2	3
y	−3	−1	1

$\because L_2 : x = -3y - 1$

$\therefore$

x	−1	−4	5
y	0	1	−2



The solution set in $\mathbb{R}^2 = \{(2, -1)\}$

Example 2

$L_1 : y = 2x - 4$

$L_2 : 4x = 2y + 8$

$\because L_1 : y = 2x - 4$

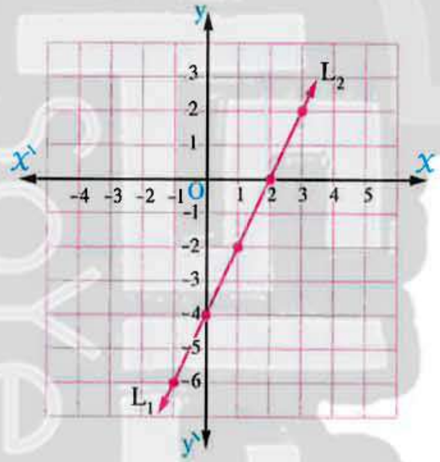
$\therefore$

x	0	1	−1
y	−4	−2	−6

$\because L_2 : x = \frac{2y + 8}{4} = \frac{1}{2} y + 2$

$\therefore$

x	2	3	1
y	0	2	−2



The solution set in $\mathbb{R}^2$

$= \{(x, y) : y = 2x - 4, (x, y) \in \mathbb{R}^2\}$

Example 3

$L_1 : y = 2x - 2$

$L_2 : 2y - 4x - 2 = 0$

$\because L_1 : y = 2x - 2$

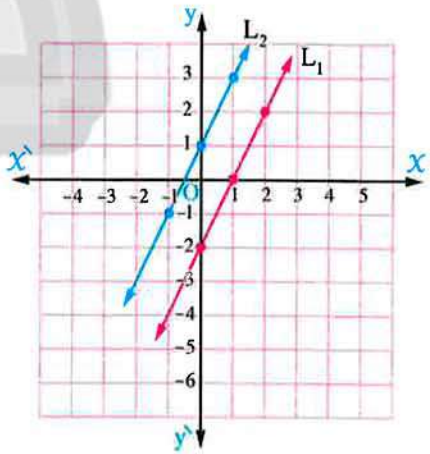
$\therefore$

x	2	1	0
y	2	0	−2

$\because L_2 : y = 2x + 1$

$\therefore$

x	0	1	−1
y	1	3	−1



The solution set in $\mathbb{R}^2 = \varnothing$

Remarks on the previous examples

- **In example 1 :** You can check the truth of the solution that the point (2 , – 1) satisfies each of the two equations simultaneously by substituting by $x = 2$ and $y = -1$ in each of the two equations , then we shall find that :
  the left hand side = the right hand side in each equation.
- **In example 2 :** We notice that the two equations are two different forms of the same equation (Dividing the second equation by 2 , we find that $2x = y + 4$ **i.e.** $y = 2x - 4$ which is the same first equation)
- **In example 3 :** Putting the first equation in the form : $y - 2x = -2$
  , and putting the second equation in the form : $y - 2x = 1$
  We find that they are contrary because it is impossible to get a value for the variable x and another for the variable y which make the expression $(y - 2x)$ equal -2 and 1 in the same time , that explain why the S.S. is $\varnothing$

Determining the number of solutions without graphing

You can determine the number of solutions of any two equations of the first degree in two variables without graphing as follows :

Find the slopes of the two straight lines m_1 and m_2

If

- $m_1 \neq m_2$: Then the two straight lines intersect at one point , and then the number of solutions = 1
- $m_1 = m_2$: Then find the points of intersection of the two straight lines with y-axis

  If

  - The two points are equal: Then the two straight lines are coincident , and then the number of solutions is an infinite number.
  - The two points are different: Then the two straight lines are parallel , and then the number of solutions = 0

Example 4 Find the number of solutions of each two pairs of the following equations :

1. $x + 2y = 1$, $2x + 3y = 12$
2. $4x - y + 7 = 0$, $2y - 8x = 14$
3. $2x - 3y = 6$, $y = \frac{2}{3}x + 3$

Solution

1. $\because m_1 = -\frac{1}{2}$, $m_2 = \frac{-2}{3}$

$\therefore m_1 \neq m_2$ $\therefore$ The two straight lines intersect at one point.

$\therefore$ The number of solutions = 1

2. $\because m_1 = \frac{-\text{ the coefficient of } x}{\text{the coefficient of } y} = \frac{-4}{-1} = 4$

, $m_2 = \frac{-\text{ the coefficient of } x}{\text{the coefficient of } y} = \frac{-(-8)}{2} = 4$

$\therefore m_1 = m_2$

, the two straight lines intersect with y-axis at the same point $(0 , 7)$

$\therefore$ The two straight lines are coincident

$\therefore$ The number of solutions = an infinite number of solutions.

Note that : We can get the point of intersecting with y-axis, by putting $x = 0$ to get the corresponding value of y

3. $\because m_1 = \frac{-\text{ the coefficient of } x}{\text{the coefficient of } y} = \frac{-2}{-3} = \frac{2}{3}$

, $m_2 = \text{the coefficient of } x = \frac{2}{3}$ $\therefore m_1 = m_2$

$\because$ The first straight line intersects y-axis at the point $(0 , -2)$

and the second straight line intersects y-axis at the point $(0 , 3)$

$\therefore$ The two straight lines are parallel because of the equality of the two slopes and the two points of intersection with y-axis are different.

$\therefore$ The number of solutions = zero

TRY 1 by yourself

Find graphically in $\mathbb{R} \times \mathbb{R}$ the solution set of the two following equations :

$y = x + 2$, $y + 2x = 8$

Final answers of try by yourself questions are at the end of each lesson to check your answer.

Second — Solving two equations of the first degree in two variables algebraically

This method depends on removing one of the two variables to get an equation of the first degree in one variable , then we get the value of this variable by solving this equation.

Then we substitute by this value in any of the two equations to get the value of the other variable which we have removed before.

For that purpose, we follow one of the two methods :

1. Substituting method.
2. Omitting method.

In the following , we will explain each of the two methods.

1 Substituting method

The following example shows how to use the substituting method to solve two equations of the first degree in two variables algebraically.

Example 5 **Find by using the substituting method the solution set of the following equations in $\mathbb{R} \times \mathbb{R}$:**

$2x - y = 5 \quad , \quad x + 3y + 1 = 0$

Solution

To use the substituting method, we do the following steps :

1. We get one of the two variables in terms of the other variable from one of the two equations , by putting this variable in one hand side of the equation in condition that its coefficient = 1

From the first equation :

$\because 2x - y = 5 \qquad \therefore y = 2x - 5$

2. Substituting by the value of y in the other equation we get an equation of the first degree in one variable (x) and by solving it we get the value of x

Substituting by $y = 2x - 5$ in the other equation :

$\therefore x + 3(2x - 5) + 1 = 0 \qquad \therefore x + 6x - 15 + 1 = 0$

$\therefore 7x - 14 = 0 \qquad \therefore 7x = 14 \qquad \therefore \boxed{x = 2}$

3. Substituting by the value of x in the equation which we got in the first step we get the value of y

$\therefore y = 2 \times 2 - 5 \qquad \therefore \boxed{y = -1} \qquad \therefore$ The S.S. $= \{(2 , -1)\}$

Remark

In the previous example :

We can solve the problem by getting the variable x in terms of y from the second equation as follows :

- From the second equation :

$\because x + 3y + 1 = 0$ $\therefore x = -3y - 1$ (1)

- Substituting by the value of x in the first equation :

$\therefore 2(-3y - 1) - y = 5$ $\therefore -6y - 2 - y = 5$

$\therefore -7y = 7$ $\therefore \boxed{y = -1}$

- Substituting in equation (1) :

$\therefore x = -3 \times (-1) - 1$ $\therefore \boxed{x = 2}$

$\therefore$ The S.S. $= \{(2, -1)\}$ which is the same result which we got before.

TRY 2 by yourself

Find algebraically in $\mathbb{R} \times \mathbb{R}$ using substituting method the S.S. of the two equations :

$x + y = 4$, $2x - y = 5$

2 Omitting method

The following example shows how to use the omitting method to solve two equations of the first degree in two variables algebraically.

Example 6 **By using the omitting method , find the solution set of the following two equations in $\mathbb{R} \times \mathbb{R}$**

$2x - y = 5$, $x + 3y + 1 = 0$

Solution

To use the omitting method, we do the following steps :

1. We write each of the two equations in the form : $a x + b y = c$

$\therefore 2x - y = 5$ (1)

$, x + 3y = -1$ (2)

2. We make the coefficient of one of the two variables "x or y" in one of the two equations the additive inverse of the coefficient of the same variable in the second equation.

∴ Multiplying the two sides of equation (2) by -2 to make the coefficient of x in equation (2) the additive inverse of the coefficient of x in equation (1)

∴ $-2x-6y=2$ (3)

3 We add the two equations (3) and (1) to get an equation of the first degree in one variable (y) and by solving it we get the value of y

Adding the two equations (3) and (1) : ∴ $-7y=7$ ∴ $\boxed{y=-1}$

4 We substitute by the value of y in one of the two equations to get an equation of the first degree in the one variable (x) and by solving it we get the value of x

∴ Substituting by $y=-1$ in equation (2) we find : $x+3\times(-1)=-1$

∴ $x-3=-1$ ∴ $\boxed{x=2}$

∴ The S.S. $=\{(2,-1)\}$

Note that : The solution set is the same which we got before by using substituting method in example 5

Remark

In the previous example :

We can solve by making the two coefficients of y in the two equations, one of them is the additive inverse of the other as follows :

Multiplying the two sides of equation (1) by 3

∴ $6x-3y=15$

, ∵ $x+3y=-1$

By adding : ∴ $7x=14$ ∴ $\boxed{x=2}$

Substituting by $x=2$ in equation (1) we get : $2\times2-y=5$

∴ $4-y=5$ ∴ $\boxed{y=-1}$

∴ The S.S. $=\{(2,-1)\}$ which is the same result which we got before.

TRY 3 by yourself

Find algebraically in $\mathbb{R}\times\mathbb{R}$ using omitting method the S.S. of the two equations : $x+y=3$, $2x-3y=1$

Third Solving two simultaneous equations of the first degree in two variables using the calculator

You can use the calculator to check the truth of the solution of solving the two equations which we solved before graphically and algebraically by substituting method or omitting method which they are $2x - y = 5$, $x + 3y + 1 = 0$ as follows :

1. Put each equation in the form : $a x + b y = c$

   $\therefore 2x - y = 5$, $x + 3y = -1$

2. Press the key MODE and from the (menu) choose (EQN) by pressing the key of the digit written in front of it.

3. Choose the equation which is in the form : $a_n x + b_n y = c_n$ by pressing the key of the digit written in front of it.

4. Insert the coefficients of x , y and the absolute term with their signs in the order of the first equation , then their corresponding coefficients in the second equation using the inserting key = in each time.

5. Press = to appear the value of x which is 2
   , then press again = to appear the value of y which is -1
   , then the S.S. $= \{(2 , -1)\}$ which is the same result which we got before.

TRY 4 by yourself

Using the calculator, find in $\mathbb{R} \times \mathbb{R}$ the S.S. of the two simultaneous equations :

$$2x + y = 0 \quad , \quad x + 2y - 3 = 0$$

Applications on solving two equations of the first degree in two variables

In this kind of verbal problems, the solution takes the following steps :

1. Let one of the two unknown be x and the other be y
2. From the given data in the problem, form two equations of the first degree in x and y
3. Solve the two equations algebraically or graphically to get the values of x and y
   It is preferable to solve them algebraically.

Example 7 The sum of two rational numbers is 14 ,
if twice the greater exceeds three times the smaller by 3
Find the two numbers.

Solution

Let the greater number be x and the smaller one be y

$\because$ Their sum = 14 $\quad \therefore x + y = 14 \quad (1)$

$\because$ Twice the greater exceeds three times the smaller by 3

$\therefore$ 2 × the greater number − 3 × the smaller number = 3

$\therefore 2x - 3y = 3 \quad (2)$

Multiply the two sides of equation (1) by 3 :

$\therefore 3x + 3y = 42 \quad (3)$

Adding (2) and (3) : $\therefore 5x = 45 \quad \therefore \boxed{x = 9}$

Substituting in (1) : $\therefore 9 + y = 14 \quad \therefore \boxed{y = 5}$

$\therefore$ The greater number = 9 and the smaller number = 5

Example 8 The length of a rectangle is more than its width by 5 cm. ,
and twice its length added to three times its width equals 45 cm.
Find each of the length and the width of the rectangle.

Solution

Let the length be x cm. and the width be y cm.

$\therefore y = x - 5 \quad (1)$

$, 2x + 3y = 45 \quad (2)$

Substituting from (1) in (2) :

$\therefore 2x + 3(x - 5) = 45 \quad \therefore 2x + 3x - 15 = 45$

$\therefore 5x = 60 \quad \therefore \boxed{x = 12}$

Substituting in (1) : $\quad \therefore y = 12 - 5 \quad \therefore \boxed{y = 7}$

$\therefore$ The length = 12 cm. and the width = 7 cm.

Example 9 A 2-digit number , its tens digit is twice its units digit. If the two digits are reversed , the resulting number decreases the original number by 27. **Find the original number.**

Solution Let the units digit be x and the tens digit be y

$\because$ The tens digit is twice the units digit. $\therefore y = 2x$ (1)

The original number $= x + 10y$

If the two digits are reversed.

i.e. The units digit becomes y and the tens digit becomes x ,

then the resulting number $= y + 10x$

The following table shows that :

	Units	**Tens**	**The number**
The original number	x	y	$x + 10y$
The resulting number	y	x	$y + 10x$

$\because$ The resulting number decreases the original number by 27

$\therefore$ The original number – the resulting number = 27

$\therefore (x + 10y) - (y + 10x) = 27$

$\therefore x + 10y - y - 10x = 27$

$\therefore 9y - 9x = 27$ $\therefore y - x = 3$ (2)

Substituting with the value of $y = 2x$ from equation (1) in equation (2) :

$\therefore 2x - x = 3$ $\therefore \boxed{x = 3}$

Substituting by $x = 3$ in equation (1) :

$\therefore y = 2 \times 3$ $\therefore \boxed{y = 6}$

$\therefore$ The units digit = 3 , the tens digit = 6 $\therefore$ The original number = 63

Example 10 Two years ago, the age of a man was four times the age of his son. After 3 years from now, the age of the man will be three times the age of his son. **Find the age of each of them now.**

Solution The following table shows the ages of the man and his son now, two years ago and after 3 years from now.

	Man's age	his son's age
Now	x	y
2 years ago	$x-2$	$y-2$
After 3 years from now	$x+3$	$y+3$

$\because$ Two years ago, the man's age = four times the son's age.

$\therefore x-2=4(y-2)$ $\qquad \therefore x-2=4y-8$

$\therefore x-4y=-6$ $\qquad (1)$

$\because$ After 3 years from now, the man's age = 3 times the son's age.

$\therefore x+3=3(y+3)$ $\qquad \therefore x+3=3y+9$

$\therefore x-3y=6$ $\qquad (2)$

Subtracting equation (1) from equation (2) : $\qquad \therefore \boxed{y=12}$

Substituting in equation (1) : $\therefore x-48=-6$ $\qquad \therefore \boxed{x=42}$

$\therefore$ The man's age now = 42 years and the son's age now = 12 years.

TRY 5 by yourself

The sum of two numbers = 12 and twice one of them is more than the other by 3

Find the two numbers.

Answers *of try by yourself*

1. Draw by yourself, the S.S. = {(2, 4)}
2. The S.S. = {(3, 1)}
3. The S.S. = {(2, 1)}
4. The S.S. = {(−1, 2)}
5. The two numbers are : 5, 7

At the end of each lesson, you will find the final answers of try by yourself questions in the same form.

LESSON 2

Solving an equation of the second degree in one unknown graphically and algebraically

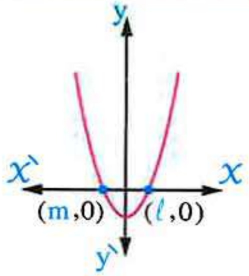
First Solving an equation of the second degree in one unknown graphically

To solve an equation of the second degree in one unknown graphically , we do the following steps :

1. Put the equation in the form : $a X^2 + b X + c = 0$
2. Assume that : $f(X) = a X^2 + b X + c$, draw the curve of the function f
3. Determine the points of intersection of the function curve and X-axis , then the X-coordinates of these points of intersection are the solutions of the equation

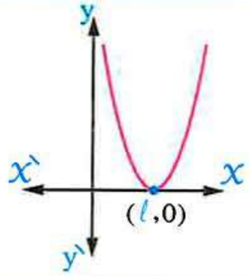
According to that , we find three cases :

1. The curve intersects X-axis at **two points**



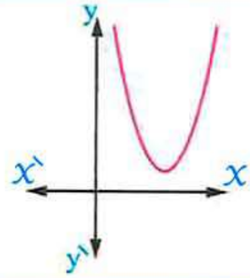
- There are **two solutions** in $\mathbb{R}$
- The S.S. = $\{\ell , m\}$

2. The curve touches X-axis at **one point**



- There is **a unique solution** in $\mathbb{R}$
- The S.S. = $\{\ell\}$

3. The curve **does not intersect** X-axis



- There is **no solution** in $\mathbb{R}$
- The S.S. = $\varnothing$

The following examples show the previous cases :

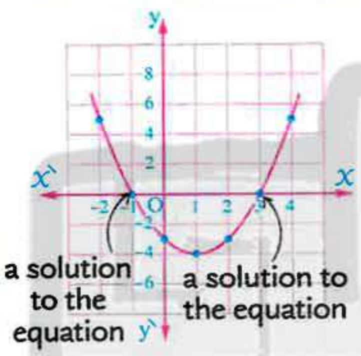
Example 1

Find graphically in $\mathbb{R}$ the S.S. of the equation : $x^2 - 2x - 3 = 0$ on the interval $[-2 , 4]$

Solution

Let $f(x) = x^2 - 2x - 3$

x	−2	−1	0	1	2	3	4
y	5	0	−3	−4	−3	0	5



From the graph , the S.S. $= \{3 , -1\}$

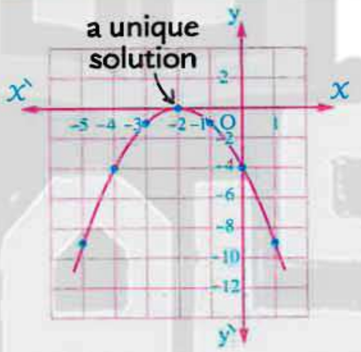
Example 2

Find graphically in $\mathbb{R}$ the S.S. of the equation : $-x^2 - 4x - 4 = 0$ on the interval $[-5 , 1]$

Solution

Let $f(x) = -x^2 - 4x - 4$

x	−5	−4	−3	−2	−1	0	1
y	−9	−4	−1	0	−1	−4	−9



From the graph , the S.S. $= \{-2\}$

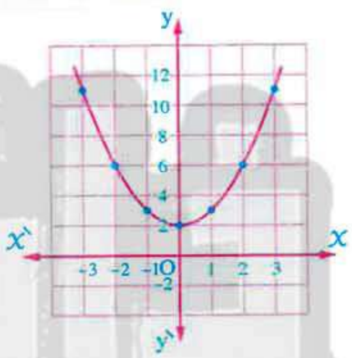
Example 3

Find graphically in $\mathbb{R}$ the S.S. of the equation : $x^2 + 2 = 0$ on the interval $[-3 , 3]$

Solution

Let $f(x) = x^2 + 2$

x	−3	−2	−1	0	1	2	3
y	11	6	3	2	3	6	11



From the graph , the S.S. $= \varnothing$

Remarks on the previous examples

- **In example 1 :** * The vertex of the curve is : $(1 , -4)$
  * The minimum value $= -4$
  * The equation of the axis of symmetry of the curve is : $x = 1$
- **In example 2 :** * The vertex of the curve is : $(-2 , 0)$
  * The maximum value $= 0$
  * The equation of the axis of symmetry of the curve is : $x = -2$
- **In example 3 :** * The vertex of the curve is : $(0 , 2)$
  * The minimum value $= 2$
  * The equation of the axis of symmetry of the curve is : $x = 0$

TRY 1 *by yourself*

Graph the function $f : f(x) = x^2 + 2x - 3$ on the interval $[-4 , 2]$

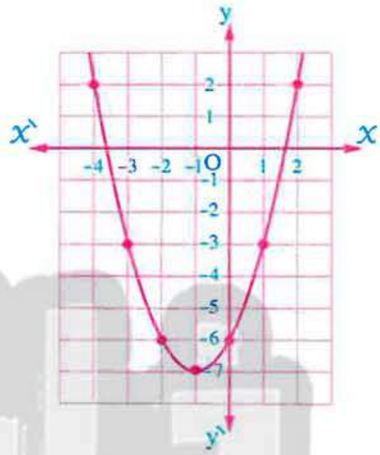
From the graph , find the S.S. of the equation : $x^2 + 2x - 3 = 0$

Example 4 **Graph the function $f : f(x) = x^2 + 2x - 6$ taking $x \in [-4 , 2]$ from the graph , find the two roots of the equation : $x^2 + 2x - 6 = 0$**

Solution

$\because f(x) = x^2 + 2x - 6$

x	-4	-3	-2	-1	0	1	2
y	2	-3	-6	-7	-6	-3	2



From the graph , the approximated values of the two roots of the equation :

$x^2 + 2x - 6 = 0$ are 1.6 and -3.6

Remark

If you substituted $x = 1.6$ in the equation : $x^2 + 2x - 6 = 0$ it will not be satisfied $[(1.6)^2 + 2 \times 1.6 - 6 = -0.24 \neq 0]$ this means that 1.6 is not the actual root for the equation but an approximated value for it , also -3.6 is an approximated value for the other root.

Generally , using the graph to find the two roots of an equation of second degree in one unknown does not always give accurate values for the two roots.

Second — Solving an equation of the second degree in one unknown using the general rule (general formula)

In the previous example : using the graph to find the two roots of the equation : $x^2 + 2x - 6 = 0$ gave approximated values for them so , it's better to solve the equation using the general formula as the following :

The general rule (general formula) for solving an equation of the second degree in one unknown :

If $a x^2 + b x + c = 0$ where a , b and c are real numbers , $a \neq 0$

, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

i.e. The solution set of the equation $= \left\{ \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} , \dfrac{-b - \sqrt{b^2 - 4ac}}{2a} \right\}$

- and here is how to use the general formula to solve the equation : $x^2 + 2x - 6 = 0$

$\because$ $a = 1$, $b = 2$, $c = -6$

, $\because x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \dfrac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-6)}}{2 \times 1} = \dfrac{-2 \pm \sqrt{4 + 24}}{2}$

$= \dfrac{-2 \pm \sqrt{28}}{2} = \dfrac{-2 \pm 2\sqrt{7}}{2}$

$\therefore x = \dfrac{-2 + 2\sqrt{7}}{2} = -1 + \sqrt{7}$ or $x = \dfrac{-2 - 2\sqrt{7}}{2} = -1 - \sqrt{7}$

and these are the actual values of the two roots without approximation

, so the S.S. of the equation in $\mathbb{R}$ is : $\{-1 + \sqrt{7} , -1 - \sqrt{7}\}$

and we can find approximated values for each of the two roots as :

$x = -1 + \sqrt{7} \simeq 1.646$ to the nearest 3 decimal places

$\simeq 1.6$ to the nearest 1 decimal place

, $x = -1 - \sqrt{7} \simeq -3.646$ to the nearest 3 decimal places

$\simeq -3.6$ to the nearest 1 decimal place

Example 5 Find in $\mathbb{R}$ the S.S. of each of the following equations :

1 $x^2 - 5x - 6 = 0$ 2 $8x(x-1) = -2$ 3 $\frac{5}{x^2} - \frac{4}{x} = -1$

Solution

1 $\because$ $\boxed{a = 1 \ , \ b = -5 \ , \ c = -6}$

$, \because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times (-6)}}{2 \times 1}$

$= \frac{5 \pm \sqrt{25 + 24}}{2} = \frac{5 \pm \sqrt{49}}{2}$

$= \frac{5 \pm 7}{2}$

$\therefore x = \frac{5+7}{2} = 6$ or $x = \frac{5-7}{2} = -1$

$\therefore$ The S.S. $= \{6 , -1\}$

Another solution using factorization :

$\because x^2 - 5x - 6 = 0$

$\therefore (x-6)(x+1) = 0$

$\therefore x - 6 = 0 \quad \therefore x = 6$

or $x + 1 = 0$

$\therefore x = -1$

$\therefore$ The S.S. $= \{6 , -1\}$

2 Before using the general forumla we put the equation on the form :

$a x^2 + b x + c = 0$

$\because 8x(x-1) = -2 \qquad \therefore 8x^2 - 8x = -2$

$\therefore 8x^2 - 8x + 2 = 0 \qquad \therefore$ $\boxed{a = 8 \ , \ b = -8 \ , \ c = 2}$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 8 \times 2}}{2 \times 8}$

$= \frac{8 \pm \sqrt{64 - 64}}{16} = \frac{8 \pm \sqrt{0}}{16} = \frac{8}{16} = \frac{1}{2}$

$\therefore$ The S.S. $= \{\frac{1}{2}\}$

3 $\because \frac{5}{x^2} - \frac{4}{x} = -1$ (multiplying both sides by x^2)

$\therefore 5 - 4x = -x^2 \qquad \therefore x^2 - 4x + 5 = 0$

$\therefore$ $\boxed{a = 1 \ , \ b = -4 \ , \ c = 5}$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 5}}{2 \times 1} = \frac{4 \pm \sqrt{-4}}{2}$

$, \because \sqrt{-4} \notin \mathbb{R}$

$\therefore$ The equation : $x^2 - 4x + 5 = 0$ has no real solutions

$\therefore$ The S.S. $= \varnothing$

UNIT 1

Remarks on the previous example

- **In 1** : The value of : $b^2 - 4ac = 49 > 0$ and the equation had two solutions which are : 6 and -1
  Generally if : $\boxed{b^2 - 4ac > 0}$, then the equation has **two different solutions** in $\mathbb{R}$
- **In 2** : The value of : $b^2 - 4ac = 0$ and the equation had one solution which is : $\frac{1}{2}$
  Generally if : $\boxed{b^2 - 4ac = 0}$, then the equation has **a unique solution** in $\mathbb{R}$
- **In 3** : The value of : $b^2 - 4ac = -4 < 0$ and the equation had no real solutions
  Generally if : $\boxed{b^2 - 4ac < 0}$, then the equation has **no real solutions** in $\mathbb{R}$

TRY 2 by yourself

Find in $\mathbb{R}$ the S.S. of each of the following two equations :

1. $2x^2 + 7x - 4 = 0$
2. $x^2 - 2x - 1 = 0$ to the nearest one decimal digit.

Third Solving an equation of the second degree in one unknown using the calculator

The second degree equation in one unknown as : $x^2 + 2x = 6$ could be solved by using calculator (the type supports solving equations) as follows :

1. Put the equation in the form : $a x^2 + b x + c = 0$
   $\because x^2 + 2x = 6 \qquad \therefore x^2 + 2x - 6 = 0$
2. Click the button MODE and from the menu select (EQN) by pressing the opposite key of it.
3. Choose the equation which is in the form : $a x^2 + b x + c = 0$ by pressing the opposite key of it.
4. Insert the coefficients of x^2 , x and the absolute term with their signs respectively by using the key of inserting =
5. Press = , then the first value of x will be : $-1 + \sqrt{7}$, then press the key = again , then we shall get the second value of x which will be : $-1 - \sqrt{7}$

TRY 3 by yourself

Using the calculator , find the S.S. of each of the following two equations in $\mathbb{R}$:

1. $x^2 - 9x + 18 = 0$
2. $x(x - 4) = 3$

An application on solving an equation of second degree in one unknown

Example 6 In a javelin , the pathway of the spear to one of the players follows the relation $y = -0.008\ x^2 + 0.56\ x + 1.2$ where x represents the horizontal distance which the spear covers from the point of projection , and y represents the height of the spear from the floor surface in metres.

Find the horizontal distance at which the spear falls to the nearest hundredth.

Solution

- To find the horizontal distance after which the spear falls , starting from the point of projection we put y = zero in the given relation , then we get a quadratic equation of the second degree as follows :

$-0.008\ x^2 + 0.56\ x + 1.2 = 0$ and by solving it , we get the required distance

$\therefore a = -0.008 \ , \ b = 0.56 \ , \ c = 1.2$

illustration figure

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0.56 \pm \sqrt{(0.56)^2 - 4 \times (-0.008) \times 1.2}}{2 \times (-0.008)}$$

$$\therefore x = \frac{-0.56 + \sqrt{0.352}}{-0.016}$$

$\simeq -2.08$ (refused because the distance should be positive)

$$\text{or } x = \frac{-0.56 - \sqrt{0.352}}{-0.016} \simeq 72.08$$

i.e. The spear will fall at a distance 72.08 metres from the point of projection.

Answers of try by yourself

1 Draw by yourself , the S.S. = $\{-3 , 1\}$

2 (1) The S.S. = $\{\frac{1}{2} , -4\}$ (2) The S.S. = $\{-0.4 , 2.4\}$

3 (1) The S.S. = $\{3 , 6\}$ (2) The S.S. = $\{2 + \sqrt{7} , 2 - \sqrt{7}\}$

LESSON 3

Solving two equations in two variables , one of them is of the first degree and the other is of the second degree

The method of solving two equations in two variables , one of them is of the first degree and the other is of the second degree , depends on the substituting method.

The following example shows the solution steps :

Example 1 **Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations :**

$x - y = 1 \quad , \quad x^2 + y^2 = 13$

Solution

1. From the equation of the first degree we express one of the two variables in terms of the second variable.

$\because x - y = 1 \qquad \therefore x = 1 + y$

2. Substituting in the equation of the second degree we get an equation of the second degree in one variable.

Substituting by $x = 1 + y$ in the second equation

$\therefore (1 + y)^2 + y^2 = 13$

$\therefore 1 + 2y + y^2 + y^2 - 13 = 0$

$\therefore 2y^2 + 2y - 12 = 0$

$\therefore y^2 + y - 6 = 0$

Remember that

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$

3. Solving the result equation by factorization or by general formula we get the value of one of the two variables.

$\because y^2 + y - 6 = 0 \qquad \therefore (y - 2)(y + 3) = 0$

$\therefore$ Either $y - 2 = 0$, then $\boxed{y = 2}$ or $y + 3 = 0$, then $\boxed{y = -3}$

4 Substituting in the equation of the first degree we get the value of the other variable.

At $y = 2$, then $\boxed{x = 3}$

and at $y = -3$, then $\boxed{x = -2}$

$\therefore$ The S.S. $= \{(3, 2), (-2, -3)\}$

Note that :

The elements of the S.S. are ordered pairs.

Example 2 **Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of the following two equations :**

$y - x = 3 \quad , \quad x^2 + x y = 5$

Solution

From the first equation :

$\because y - x = 3 \qquad \therefore y = 3 + x \qquad (1)$

Substituting by "y" in the second equation :

Note that :

In this problem , it is easier to express "y" in terms of "x"

We get : $x^2 + x(3 + x) = 5$

$\therefore x^2 + 3x + x^2 - 5 = 0$

$\therefore 2x^2 + 3x - 5 = 0$

$\therefore (2x + 5)(x - 1) = 0$

$\therefore$ Either $2x + 5 = 0$, then $\boxed{x = -\frac{5}{2}}$ Substituting in (1) : $\therefore \boxed{y = \frac{1}{2}}$

or $x - 1 = 0$, then $\boxed{x = 1}$ Substituting in (1) : $\therefore \boxed{y = 4}$

$\therefore$ The S.S. $= \{(-\frac{5}{2}, \frac{1}{2}), (1, 4)\}$

TRY 1 by yourself

Find in $\mathbb{R} \times \mathbb{R}$ the S.S. for each of the two following pairs of equations :

1. $x - 2y = 0 \quad , \quad x^2 - y^2 = 3$
2. $x + y = 1 \quad , \quad x^2 + x y + y^2 = 3$

UNIT 1

Applications on solving two equations in two variables one of them of first degree and the other of second degree

Example 3 The sum of two real numbers is 7 and the difference between their squares is 7

Find the two numbers.

Solution Let the greatest number be x and the smallest be y

$\because$ Their sum = 7 $\therefore x + y = 7$ (1)

$\because$ The difference between their squares is 7

$\therefore x^2 - y^2 = 7$ (2)

From equation (1) : $\therefore x = 7 - y$ (3)

Substituting in equation (2) : $\therefore (7-y)^2 - y^2 = 7$

$\therefore 49 - 14y + y^2 - y^2 = 7$

$\therefore -14y = 7 - 49$ $\therefore -14y = -42$ $\therefore y = \frac{-42}{-14} = 3$

Substituting in equation (3) : $\therefore x = 7 - 3 = 4$

$\therefore$ The two numbers are 4 and 3

Example 4 The product of two real numbers is 2 , if the greatest is added to twice the smallest the result will be 4

Find the two numbers.

Solution Let the smallest number be x and the greatest number be y

$\therefore x y = 2$ (1)

, $y + 2x = 4$ (2)

From equation (2) : $y = 4 - 2x$ (3)

Substituting in (1) : $\therefore x(4 - 2x) = 2$

$\therefore 4x - 2x^2 = 2$ $\therefore 2x^2 - 4x + 2 = 0$

$\therefore x^2 - 2x + 1 = 0$ $\therefore (x-1)^2 = 0$ $\therefore x = 1$

Substituting in equation (3) : $\therefore y = 2$

$\therefore$ The two numbers are 1 and 2

Example 5 The perimeter of a rectangle is 24 cm. and its area is 20 cm^2.

Find its two dimensions.

Solution

Let the two dimensions of the rectangle be x cm. and y cm.

Remember that

- Perimeter of the rectangle = (length + width) × 2
- Area of the rectangle = length × width
- Perimeter of the square = side length × 4
- Area of the square = side length × itself

$\because$ The perimeter of the rectangle = 24 cm.

$\therefore 2(x + y) = 24$ $\qquad \therefore x + y = 12$ $\qquad$ (1)

$\because$ The area of the rectangle = 20 cm^2. $\qquad \therefore x y = 20$ $\qquad$ (2)

From equation (1) : $x = 12 - y$ $\qquad$ (3)

Substituting in equation (2) : $\qquad \therefore (12 - y)\, y = 20$

$\therefore 12y - y^2 = 20$ $\qquad \therefore y^2 - 12y + 20 = 0$

$\therefore (y - 10)(y - 2) = 0$

$\therefore$ Either $y - 10 = 0$ $\qquad \therefore y = 10$

or $y - 2 = 0$ $\qquad \therefore y = 2$

Substituting by the values of y in equation (3) :

$\therefore$ At $y = 10$, then $x = 2$ $\qquad$ At $y = 2$, then $x = 10$

$\therefore$ The two dimensions of the rectangle are 10 cm. and 2 cm.

TRY 2 *by yourself*

The difference between two positive real numbers is 4 and their product is 12

Find the two numbers.

Answers *of try by yourself*

1 [1] The S.S. = {(−2, −1), (2, 1)}

[2] The S.S. = {(2, −1), (−1, 2)}

2 The two numbers are : 6, 2

UNIT 2

Algebraic fractional functions and the operations on them

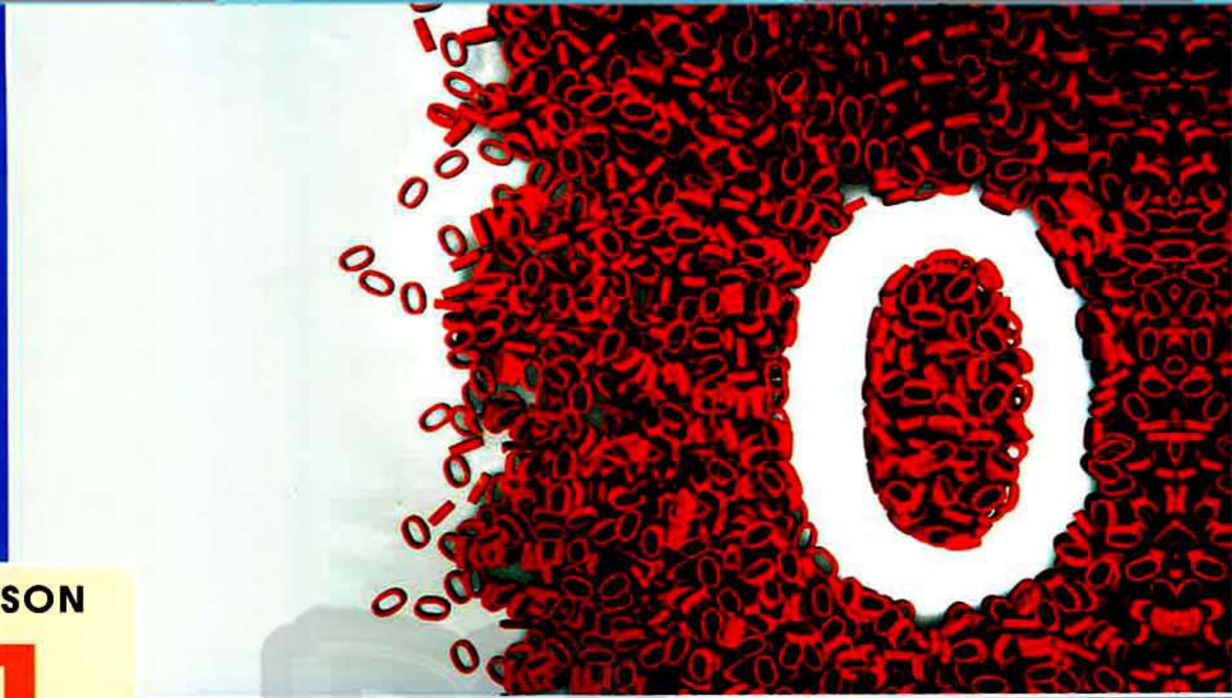
Lessons of the unit :

1. Set of zeroes of a polynomial function.
2. Algebraic fractional function.
3. Equality of two algebraic fractions.
4. Operations on algebraic fractions (Adding and subtracting algebraic fractions).
5. Operations on algebraic fractions (follow)
   (Multiplying and dividing algebraic fractions).

Unit Objectives :

By the end of this unit, student should be able to :

- Recognize the zero of a polynomial function.
- Find the set of zeroes of a polynomial function.
- Recognize the algebraic fractional function.
- Find the domain of the algebraic fractional function.
- Find the common domain of two algebraic fractions or more.
- Reduce the algebraic fraction to the simplest form.
- Prove that two algebraic fractions are equal.
- Add , subtract , multiply and divide algebraic fractions.
- Recognize the additive inverse of the algebraic fraction.
- Recognize the multiplicative inverse of the algebraic fraction.



LESSON
1

Set of zeroes of a polynomial function

Zero of the function f is the value that makes $f(x) = 0$

For example:

If the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = x^3 - 5x^2 + 6x$

Then :

- at $x = 0$, then $f(0) = 0^3 - 5 \times 0^2 + 6 \times 0 = 0$
- at $x = 2$, then $f(2) = 2^3 - 5 \times 2^2 + 6 \times 2 = 8 - 20 + 12 = 0$
- at $x = 3$, then $f(3) = 3^3 - 5 \times 3^2 + 6 \times 3 = 27 - 45 + 18 = 0$

i.e. Each of the numbers 0 , 2 , 3 is called a zero of the function f

Generally

If f is a polynomial function in x , then the set of values of x which makes $f(x) = 0$ is called the set of zeroes of the function f and is denoted by $z(f)$

i.e. $z(f)$ is the solution set of the equation $f(x) = 0$ in $\mathbb{R}$

Notice the difference among f , $f(x)$, $z(f)$:

- f denotes to the function
- $f(x)$ denotes to the rule of the function or the image of x by the function f
- $z(f)$ denotes to the set of zeroes of the function f and it is the solution set of the equation $f(x) = 0$ in $\mathbb{R}$

The following examples show how to get the zeroes of the function :

Example 1 **Find z (k) of each of the polynomial functions defined by the following rules in $\mathbb{R}$:**

1. $k(x) = 2x - 6$
2. $k(x) = x^2 - 3x - 10$
3. $k(x) = 8$
4. $k(x) = 0$

Solution **To get the zeroes of the function k we put $k(x) = 0$ and solve the resultant equation.**

1. Putting $2x - 6 = 0$ $\quad \therefore x = \frac{6}{2}$

   $\therefore x = 3$ $\quad \therefore z(k) = \{3\}$

2. Putting $x^2 - 3x - 10 = 0$

   $\therefore (x - 5)(x + 2) = 0$ $\quad \therefore x = 5$ or $x = -2$

   $\therefore z(k) = \{5, -2\}$

3. $\because k(x) = 8$ $\quad \therefore$ The image of any number by the function k equals 8

   $\therefore$ There is no number x makes $k(x) = 0$

   $\therefore z(k) = \varnothing$

4. $\because k(x) = 0$

   $\therefore$ The image of any number by the function k equals zero

   $\therefore$ All the real numbers are zeroes of this function **i.e.** $z(k) = \mathbb{R}$

Remark

From 3 and 4 in the previous example , we deduce that :

- If $k(x) = a$ where $a \in \mathbb{R}^*$, then $z(k) = \varnothing$
- If $k(x) = 0$, then $z(k) = \mathbb{R}$

Example 2 **Find in $\mathbb{R}$ the set of zeroes of each of the polynomial functions defined by the following rules :**

1. $f(x) = x^2 - 16$
2. $k(x) = x^2 + 49$
3. $g(x) = x^2 - 10x + 25$
4. $f(x) = x^3 + 7x^2 - 18x$
5. $h(x) = x^6 - 64$
6. $f(x) = x^8 - 128x$

Solution

1 Putting $x^2 - 16 = 0$ $\therefore x^2 = 16$

$\therefore x = 4$ or $x = -4$ $\therefore z(f) = \{4, -4\}$

2 Putting $x^2 + 49 = 0$ $\therefore x^2 = -49$

$\therefore x = \pm\sqrt{-49}$, $\because \sqrt{-49} \notin \mathbb{R}$

$\therefore x^2 + 49 = 0$ has no real roots i.e. its solution set in $\mathbb{R} = \varnothing$

$\therefore z(k) = \varnothing$

3 Putting $x^2 - 10x + 25 = 0$ $\therefore (x-5)^2 = 0$

$\therefore x = 5$ $\therefore z(g) = \{5\}$

4 Putting $x^3 + 7x^2 - 18x = 0$ $\therefore x(x^2 + 7x - 18) = 0$

$\therefore x(x-2)(x+9) = 0$ $\therefore x = 0$ or $x = 2$ or $x = -9$

$\therefore z(f) = \{0, 2, -9\}$

5 Putting $x^6 - 64 = 0$ $\therefore (x^3 - 8)(x^3 + 8) = 0$

$\therefore x^3 - 8 = 0$, then $x = 2$ or $x^3 + 8 = 0$, then $x = -2$

$\therefore z(h) = \{2, -2\}$

Another Solution :

Putting $x^6 - 64 = 0$

$\therefore x^6 = 64$

Notice that : $2^6 = 64$, $(-2)^6 = 64$

$\therefore x^6 = 2^6$ or $x^6 = (-2)^6$

, $\because$ The index = the index

$\therefore$ The base = the base

$\therefore x = 2$ or $x = -2$ $\therefore z(h) = \{2, -2\}$

6 Putting $x^8 - 128x = 0$ $\therefore x(x^7 - 128) = 0$ $\therefore x = 0$

or $x^7 - 128 = 0$, then $x^7 = 128$

$\therefore x^7 = 2^7$

, $\because$ The index = the index

$\therefore$ The base = the base

$\therefore x = 2$ $\therefore z(f) = \{0, 2\}$

Example 3 **Find in $\mathbb{R}$ the set of zeroes of each of the polynomial functions defined by the following rules :**

1. $f(x) = x^2 - 2x - 1$
2. $g(x) = x^2 - 3x + 7$

Solution

1. Putting $x^2 - 2x - 1 = 0$

This equation is difficult to be solved by factorization , therefore we shall use the general formula.

$$\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad , \quad a = 1 \quad , \quad b = -2 \quad , \quad c = -1$$

$$\therefore x = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

$$\therefore x = 1 + \sqrt{2} \quad \text{or} \quad x = 1 - \sqrt{2}$$

$$\therefore z(f) = \{1 + \sqrt{2} , 1 - \sqrt{2}\}$$

2. Putting $x^2 - 3x + 7 = 0$, $\because a = 1$, $b = -3$, $c = 7$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 28}}{2} = \frac{3 \pm \sqrt{-19}}{2}$$

$$, \because \sqrt{-19} \notin \mathbb{R}$$

$\therefore$ There is no real roots for the equation $x^2 - 3x + 7 = 0$

$$\therefore z(g) = \varnothing$$

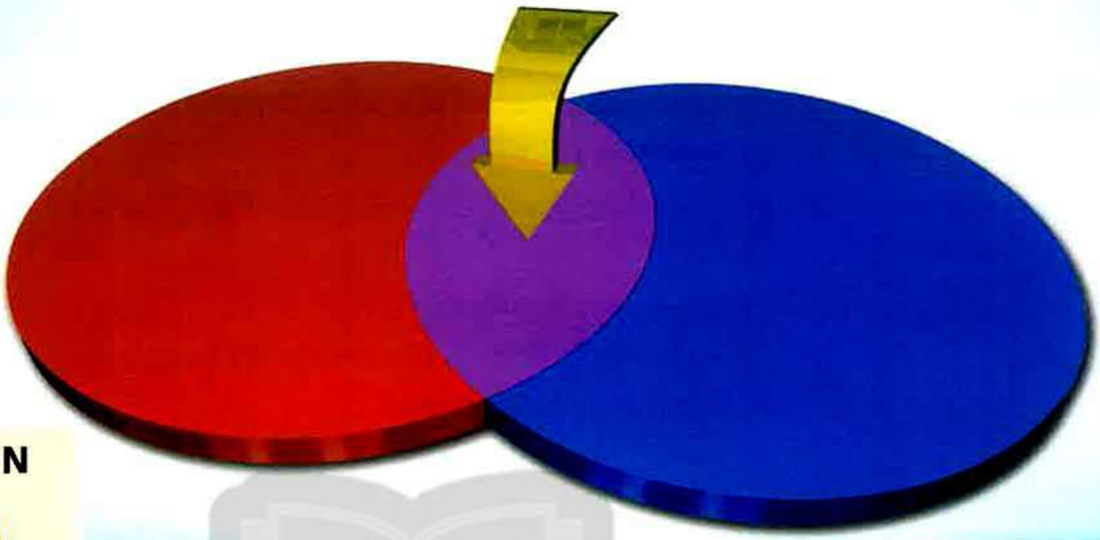
TRY by yourself

Find in $\mathbb{R}$ the set of zeroes of each of the polynomial functions defined by the following rules :

1. $f(x) = x^2 - 2x$
2. $g(x) = x^2 - 81$
3. $h(x) = x^3 + 27$
4. $k(x) = x^2 - 8x + 12$

Answers of try by yourself

1. $z(f) = \{0 , 2\}$
2. $z(g) = \{-9 , 9\}$
3. $z(h) = \{-3\}$
4. $z(k) = \{2 , 6\}$



LESSON
2

Algebraic fractional function

Algebraic fractional function

The algebraic fractional function is a function whose rule is in the form of an algebraic fraction whose numerator and denominator are polynomial functions

Examples for algebraic fractional functions :

- $f : f(x) = \dfrac{x-3}{x+2}$
- $g : g(x) = \dfrac{3x-1}{12x}$
- $n : n(x) = \dfrac{3}{x-4}$
- $k : k(x) = \dfrac{2x+5}{(x-1)(x+4)}$

The domain of the algebraic fractional function

The domain of the algebraic fractional function is all real numbers except the numbers that make the fraction is undefined (**i.e.** except the set of zeroes of the denominator)

i.e. The domain of algebraic fractional function = $\mathbb{R}$ $-$ the set of zeroes of the denominator

For example :

- The domain of $f : f(x) = \dfrac{x-3}{x+2}$ is $\mathbb{R} - \{-2\}$
- The domain of $n : n(x) = \dfrac{3}{x-4}$ is $\mathbb{R} - \{4\}$
- The domain of $g : g(x) = \dfrac{3x-1}{12x}$ is $\mathbb{R} - \{0\}$
- The domain of $k : k(x) = \dfrac{2x+5}{(x-1)(x+4)}$ is $\mathbb{R} - \{1 , -4\}$

Remember that

Dividing by zero is meaningless.

Definition

If p and k are two polynomial functions ,
then the function n where n : $\mathbb{R} - z(k) \longrightarrow \mathbb{R}$, $n(x) = \frac{p(x)}{k(x)}$ where : z (k) is the set of zeroes of the function k , n is called a real algebraic fractional function or briefly it is called an algebraic fraction.

Example 1 **Determine the domain of each of the following algebraic fractional function that are defined by the following rules , then find n (0) , n (– 1) and n (2) :**

1. $n(x) = \frac{x+5}{x^3-8}$
2. $n(x) = \frac{3x+2}{x^3-4x^2-12x}$
3. $n(x) = \frac{x^2+1}{3}$
4. $n(x) = \frac{x+1}{x^2+1}$

Solution **To determine the domain of the algebraic fractional function , we put the denominator = zero to know the set of zeroes of the denominator.**

1 By putting : $x^3 - 8 = 0 \quad \therefore x^3 = 8 \quad \therefore x = \sqrt[3]{8}$

$\therefore x = 2$

$\therefore$ The set of zeroes of the denominator $= \{2\}$

$\therefore$ The domain of n $= \mathbb{R} - \{2\}$

, $\because n(x) = \frac{x+5}{x^3-8} \quad \therefore n(0) = \frac{0+5}{0-8} = -\frac{5}{8}$, $n(-1) = \frac{-1+5}{-1-8} = -\frac{4}{9}$

, n (2) is meaningless because $2 \notin$ the domain of n

2 By putting : $x^3 - 4x^2 - 12x = 0$

$\therefore x(x^2 - 4x - 12) = 0$

$\therefore x(x+2)(x-6) = 0$

$\therefore x = 0$ or $x = -2$ or $x = 6$

$\therefore$ The set of zeroes of the denominator $= \{0 , -2 , 6\}$

$\therefore$ The domain of n $= \mathbb{R} - \{0 , -2 , 6\}$, $\because n(x) = \frac{3x+2}{x^3-4x^2-12x}$

$\therefore$ n (0) is meaningless because $0 \notin$ the domain of n

, $n(-1) = \frac{-3+2}{-1-4+12} = -\frac{1}{7}$, $n(2) = \frac{6+2}{8-16-24} = \frac{8}{-32} = -\frac{1}{4}$

3 $\because$ The denominator of the function n is a constant

$\therefore$ There are no zeroes for the denominator because it equals 3 always.

$\therefore$ The domain of n = $\mathbb{R}$

, $\because n(x) = \frac{x^2+1}{3}$ $\therefore n(0) = \frac{0+1}{3} = \frac{1}{3}$

, $n(-1) = \frac{1+1}{3} = \frac{2}{3}$, $n(2) = \frac{4+1}{3} = \frac{5}{3}$

4 By putting : $x^2 + 1 = 0$ $\therefore x^2 = -1$ (which has no solutions in $\mathbb{R}$)

$\therefore$ There are no zeroes for the denominator

$\therefore$ The domain of n = $\mathbb{R}$

, $\because n(x) = \frac{x+1}{x^2+1}$ $\therefore n(0) = \frac{0+1}{0+1} = 1$

, $n(-1) = \frac{-1+1}{1+1} = \frac{0}{2} = 0$, $n(2) = \frac{2+1}{4+1} = \frac{3}{5}$

Example 2 If the function n : $n(x) = \frac{x+2}{x^2 - a x + 25}$, the domain of n = $\mathbb{R} - \{5\}$

Find the value of a where a $\in \mathbb{R}$

Solution

$\because$ The domain of n = $\mathbb{R} - \{5\}$

$\therefore$ When $x = 5$, then $x^2 - a x + 25 = 0$

$\therefore 25 - 5a + 25 = 0$ $\therefore 5a = 50$ $\therefore a = 10$

Remark

The set of zeroes of the algebraic fractional function is the set of values which makes its numerator equals zero and its denominator does not equal zero.

i.e. The set of zeroes of the algebraic fractional function

= the set of zeroes of the numerator – the set of zeroes of the denominator.

For example:

- If the function n : $n(x) = \frac{x^2+3x}{x^2-9}$, then $n(x) = \frac{x(x+3)}{(x-3)(x+3)}$

i.e. $z(n) = \{0, -3\} - \{3, -3\} = \{0\}$

- If the function n : $n(x) = \frac{3x+6}{x^2+x-2}$, then $n(x) = \frac{3(x+2)}{(x-1)(x+2)}$

i.e. $z(n) = \{-2\} - \{1, -2\} = \varnothing$

TRY by yourself

1 ***Determine the domain of each of the following algebraic fractional functions :***

(1) $f(x) = \frac{3x + 12}{x^2 - 25}$ (2) $r(x) = \frac{x^2 - 9}{x^2 - 2x - 8}$

2 If the domain of the function $f : f(x) = \frac{x + 5}{x^2 - a}$ is $\mathbb{R} - \{4, -4\}$, ***find the value of a***

3 ***Complete :*** If the function $n : n(x) = \frac{x^3 - 5x^2}{x^2 - 25}$, then $z(n) = \cdots\cdots$

The common domain of two algebraic fractions or more

The common domain of two algebraic fractions :

is the set of real numbers that makes the two algebraic fractions identified together (at the same time)

Assume that we have the two algebraic fractions n_1 and n_2 where :

$n_1(x) = \frac{3}{x - 2}$ and $n_2(x) = \frac{5x}{x^2 - 1}$,

then the domain of n_1 (say) $m_1 = \mathbb{R} - \{2\}$ (because n_1 is undefined when $x = 2$)

and the domain of n_2 (say) $m_2 = \mathbb{R} - \{1, -1\}$ (because n_2 is undefined when $x = 1$ or $x = -1$)

According to that :

The common domain of the two fractions n_1 and $n_2 = m_1 \cap m_2$

$= (\mathbb{R} - \{2\}) \cap (\mathbb{R} - \{1, -1\})$

$= \mathbb{R} - \{2, 1, -1\}$

$= \mathbb{R} -$ the set of zeroes of the two denominators

(because n_1 and n_2 are undefined together when $x = 2$ or $x = 1$ or $x = -1$)

We Notice that :

For any value of the variable x that belongs to this common domain , the two fractions n_1 and n_2 are defined together.

Generally

If n_1 and n_2 are two algebraic fractions ,
and the domain of $n_1 = \mathbb{R} - X_1$ (where X_1 is the set of zeroes of the denominator of n_1)
and the domain of $n_2 = \mathbb{R} - X_2$ (where X_2 is the set of zeroes of the denominator of n_2), then :

The common domain of the two fractions n_1 and $n_2 = \mathbb{R} - (X_1 \cup X_2)$
$= \mathbb{R} -$ the set of zeroes of the two denominators of the two fractions.

Then we can generalize the same thing for any number of algebraic fractions :

i.e. The common domain of any number of algebraic fractions

$= \mathbb{R} -$ the set of zeroes of the denominators of these fractions.

Example 3 **Find the common domain of each of the following :**

1 $n_1(x) = \frac{x+3}{x^2-9}$, $n_2(x) = \frac{x^2+5}{x^2-5x+6}$

2 $n_1(x) = \frac{2x}{x+1}$, $n_2(x) = \frac{3}{x^3-1}$, $n_3(x) = \frac{2x+3}{x^2-3x+2}$

Solution

1 $\because n_1(x) = \frac{x+3}{(x-3)(x+3)}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{3, -3\}$

, $\because n_2(x) = \frac{x^2+5}{(x-2)(x-3)}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{2, 3\}$

$\therefore$ The common domain of the two algebraic fractions n_1 and n_2

$= \mathbb{R} - \{3, -3, 2\}$

2 $\because n_1(x) = \frac{2x}{x+1}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{-1\}$

, $\because n_2(x) = \frac{3}{(x-1)(x^2+x+1)}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{1\}$

, $\because n_3(x) = \frac{2x+3}{(x-2)(x-1)}$ $\therefore$ The domain of $n_3 = \mathbb{R} - \{2, 1\}$

$\therefore$ The common domain of the algebraic fractions n_1 , n_2 and n_3

$= \mathbb{R} - \{-1, 1, 2\}$

TRY 2 by yourself

Find the common domain of each of the following :

1 $n_1(x) = \frac{3}{x-5}$, $n_2(x) = \frac{x-1}{x^2-6x+5}$

2 $n_1(x) = \frac{3}{5x}$, $n_2(x) = \frac{x^2}{x^2-2x}$, $n_3(x) = \frac{x-5}{x^2-4}$

Answers *of try by yourself*

1 **1** (1) The domain of $f = \mathbb{R} - \{-5, 5\}$ (2) The domain of $r = \mathbb{R} - \{-2, 4\}$

2 $a = 16$ **3** $\{0\}$

2 **1** $\mathbb{R} - \{5, 1\}$ **2** $\mathbb{R} - \{0, 2, -2\}$

LESSON 3

Equality of two algebraic fractions

Before studying the equality of two algebraic fractions , we will learn how to reduce the algebraic fraction.

Reducing the algebraic fraction

Reducing the algebraic fraction is to put it in the simplest form.

Definition

It is said that the algebraic fraction is in its simplest form if there are no common factors between its numerator and denominator.

For example :

- The following algebraic fractions are in the simplest form :

$$\frac{x-1}{x+1} \ , \ \frac{x^2}{x^2+1} \ , \ \frac{x^2+2x-1}{x^2+5}$$

because , there are no common factors between the numerator and the denominator of each of them.

- The following algebraic fractions are not in the simplest form :

$$\frac{x}{x(x+1)} \ , \ \frac{x^2+1}{x(x^2+1)} \ , \ \frac{x^2(2x-1)}{x^3}$$

because , there is a common factor between the numerator and denominator of each of them.

UNIT 2

How to reduce the algebraic fraction

To reduce the algebraic fraction , we do as follows :

1. Factorize each of the numerator and denominator perfectly.
2. Identify the domain of the algebraic fraction before removing the common factors between the numerator and denominator.
3. Remove the common factors between the numerator and denominator to get the simplest form of the algebraic fraction.

and the following examples will illustrate the previous steps :

Example 1 **Reduce each of the following algebraic fractions and mention the domain of each one :**

1. $n_1(x) = \dfrac{2x+4}{x^2-4}$

2. $n_2(x) = \dfrac{x^3+2x^2-35x}{x^3-25x}$

Solution

1. $\because n_1(x) = \dfrac{2(x+2)}{(x-2)(x+2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{2, -2\}$

Removing $(x+2)$ from the numerator and the denominator :

$\therefore n_1(x) = \dfrac{2}{x-2}$

2. $\because n_2(x) = \dfrac{x(x^2+2x-35)}{x(x^2-25)} = \dfrac{x(x+7)(x-5)}{x(x+5)(x-5)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0, -5, 5\}$

Removing $x(x-5)$ from the numerator and denominator :

$\therefore n_2(x) = \dfrac{x+7}{x+5}$

TRY by yourself 1

Reduce the following two algebraic fractions to the simplest form and mention the domain of each of them :

1. $n_1(x) = \dfrac{x^2-9}{x^2+4x+3}$ 2. $n_2(x) = \dfrac{x^3-8}{x^3+2x^2+4x}$

Equality of two algebraic fractions

If n_1 , n_2 are two algebraic fractions where : $n_1(x) = 3$, $n_2(x) = \frac{3x}{x}$

The question : is $n_2 = n_1$? The answer is : no

because : $n_1(x) = 3$ for all real values of x

but : $n_2(x) = 3$ if $x \neq 0$
, $n_2(x)$ is undefined if $x = 0$

i.e.

$n_2(x) = n_1(x)$ if $x \neq 0$
, $n_2(x) \neq n_1(x)$ if $x = 0$

It is said that the two algebraic fractions n_1 and n_2 are equal (i.e. $n_1 = n_2$) if the two following conditions are satisfied together :

1. The domain of n_1 = the domain of n_2
2. $n_1(x) = n_2(x)$ for each $x \in$ the common domain.

Example 2 **In each of the following : If n_1 and n_2 are two algebraic fractions , is $n_1 = n_2$? Why ?**

1. $n_1(x) = \frac{x^2 - 5x}{x^2 - 7x + 10}$, $n_2(x) = \frac{3x - 15}{3x^2 - 21x + 30}$
2. $n_1(x) = \frac{x^2 + x - 6}{x^2 - 3x + 2}$, $n_2(x) = \frac{x^2 - 2x - 15}{x^2 - 6x + 5}$
3. $n_1(x) = \frac{x^2 - x}{x^3 - 2x^2}$, $n_2(x) = \frac{x^2 - 3x + 2}{x^3 - 4x^2 + 4x}$

Solution

1. $\because n_1(x) = \frac{x(x-5)}{(x-2)(x-5)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{2 , 5\}$, $n_1(x) = \frac{x}{x-2}$ (1)

, $\because n_2(x) = \frac{3(x-5)}{3(x^2 - 7x + 10)} = \frac{3(x-5)}{3(x-2)(x-5)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{2 , 5\}$, $n_2(x) = \frac{1}{x-2}$ (2)

From (1) and (2) : $\therefore n_1 \neq n_2$

Although the domain of n_1 = the domain of n_2
but $n_1(x)$, $n_2(x)$ are not reduced to the same fraction "$n_1(x) \neq n_2(x)$"

2 $\because n_1(x) = \frac{(x+3)(x-2)}{(x-1)(x-2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{1, 2\}$, $n_1(x) = \frac{x+3}{x-1}$ (1)

, $\because n_2(x) = \frac{(x-5)(x+3)}{(x-5)(x-1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{5, 1\}$, $n_2(x) = \frac{x+3}{x-1}$ (2)

From (1) and (2) : $\boxed{\therefore n_1 \neq n_2}$

Although $n_1(x) = n_2(x)$ in the simplest form
but the domain of $n_1 \neq$ the domain of n_2

3 $\because n_1(x) = \frac{x(x-1)}{x^2(x-2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{0, 2\}$, $n_1(x) = \frac{x-1}{x(x-2)}$ (1)

, $\because n_2(x) = \frac{(x-1)(x-2)}{x(x^2-4x+4)} = \frac{(x-1)(x-2)}{x(x-2)^2}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0, 2\}$, $n_2(x) = \frac{x-1}{x(x-2)}$ (2)

From (1) and (2) : $\boxed{\therefore n_1 = n_2}$

because : The domain of n_1 = the domain of n_2 and $n_1(x) = n_2(x)$

TRY 2 *by yourself*

In each of the following , if n_1 and n_2 are two algebraic fractions , is $n_1 = n_2$? Why ?

1 $n_1(x) = \frac{2x^2+4}{x^3+2x}$, $n_2(x) = \frac{4x^2+8}{2x^3+4x}$

2 $n_1(x) = \frac{x^2-2x}{x^2+x-6}$, $n_2(x) = \frac{x^2-3x}{x^2-9}$

Remark

Let n_1 and n_2 be two algebraic fractions where their domains are m_1 and m_2
If we could reduce $n_1(x)$ and $n_2(x)$ to the same fraction , it is said that n_1 and n_2 take the same values in the common domain $m_1 \cap m_2$

Example 3 If $n_1(x) = \dfrac{x^2+3x}{x^2-3x}$, $n_2(x) = \dfrac{x^2+10x+21}{x^2+4x-21}$

Prove that :

$n_1(x) = n_2(x)$ for all the values of x which belong to the common domain and find this domain.

Solution

$\because n_1(x) = \dfrac{x(x+3)}{x(x-3)} = \dfrac{x+3}{x-3}$ where the domain of $n_1 = \mathbb{R} - \{0 , 3\}$

, $n_2(x) = \dfrac{(x+7)(x+3)}{(x+7)(x-3)} = \dfrac{x+3}{x-3}$ where the domain of $n_2 = \mathbb{R} - \{-7 , 3\}$

$\therefore n_1(x) = n_2(x)$ for all the values of x which belong to the common domain of the two functions n_1 and n_2 which is $\mathbb{R} - \{0 , 3 , -7\}$

By another meaning :

$\mathbb{R} - \{0 , 3 , -7\}$ is the common domain in which $n_1 = n_2$

TRY 3 by yourself

If $n_1(x) = \dfrac{3x-6}{x^2-4}$, $n_2(x) = \dfrac{3x+3}{x^2+3x+2}$

Prove that :

$n_1(x) = n_2(x)$ for all the values of x which belong to the common domain and find this domain.

Answers *of try by yourself*

1 (1) $n_1(x) = \dfrac{x-3}{x+1}$, the domain of $n_1 = \mathbb{R} - \{-1 , -3\}$

(2) $n_2(x) = \dfrac{x-2}{x}$, the domain of $n_2 = \mathbb{R} - \{0\}$

2 (1) Yes , because :

The domain of n_1 = the domain of n_2

, $n_1(x) = n_2(x)$

(2) No , because :

The domain of $n_1 \neq$ the domain of n_2

3 Prove by yourself :

, the common domain is : $\mathbb{R} - \{2 , -2 , -1\}$

LESSON 4

Operations on algebraic fractions

Adding and subtracting the algebraic fractions :

Adding and subtracting two algebraic fractions are similar to adding and subtracting two fractional numbers , therefore , it is useful to remember how to add and subtract two fractional numbers.

1 Adding and subtracting two fractions having the same denominator :

- $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$ (where $b \neq 0$)
- $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$ (where $b \neq 0$)

For example:
- $\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5} = \frac{3}{5}$
- $\frac{-2}{7} - \frac{4}{7} = \frac{-2-4}{7} = \frac{-6}{7}$

2 Adding and subtracting two fractions having different denominators :

- $\frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd}$ (where $bd \neq 0$)
- $\frac{a}{b} - \frac{c}{d} = \frac{ad-cb}{bd}$ (where $bd \neq 0$)

For example:
- $\frac{1}{5} + \frac{2}{7} = \frac{1 \times 7 + 2 \times 5}{5 \times 7} = \frac{7+10}{35} = \frac{17}{35}$
- $\frac{1}{4} - \left(\frac{-3}{5}\right) = \frac{1 \times 5 - (-3) \times 4}{4 \times 5} = \frac{5-(-12)}{20} = \frac{5+12}{20} = \frac{17}{20}$

By the same way we can carry out the operations of adding and subtracting two algebraic fractions of the same denominator and those of different denominators as follows :

1 Adding and subtracting two algebraic fractions having the same denominator :

If $x \in$ the common domain of the two algebraic fractions n_1 and n_2 where

$n_1(x) = \dfrac{f(x)}{k(x)}$ and $n_2(x) = \dfrac{p(x)}{k(x)}$, then :

- $n_1(x) + n_2(x) = \dfrac{f(x)}{k(x)} + \dfrac{p(x)}{k(x)} = \dfrac{f(x) + p(x)}{k(x)}$
- $n_1(x) - n_2(x) = \dfrac{f(x)}{k(x)} - \dfrac{p(x)}{k(x)} = \dfrac{f(x) - p(x)}{k(x)}$

For example:

If $n_1(x) = \dfrac{x}{x-2}$ and $n_2(x) = \dfrac{x-1}{x-2}$, then :

- $n_1(x) + n_2(x) = \dfrac{x}{x-2} + \dfrac{x-1}{x-2} = \dfrac{x + x - 1}{x-2} = \dfrac{2x-1}{x-2}$

  where the domain of the sum is $\mathbb{R} - \{2\}$

- $n_1(x) - n_2(x) = \dfrac{x}{x-2} - \dfrac{x-1}{x-2} = \dfrac{x-(x-1)}{x-2} = \dfrac{x - x + 1}{x-2} = \dfrac{1}{x-2}$

  where the domain of the result is $\mathbb{R} - \{2\}$

2 Adding and subtracting two algebraic fractions having different denominators :

If $x \in$ the common domain of the two algebraic fractions n_1 and n_2 where

$n_1(x) = \dfrac{f(x)}{r(x)}$ and $n_2(x) = \dfrac{p(x)}{k(x)}$, then :

- $n_1(x) + n_2(x) = \dfrac{f(x)}{r(x)} + \dfrac{p(x)}{k(x)} = \dfrac{f(x) \times k(x) + p(x) \times r(x)}{r(x) \times k(x)}$
- $n_1(x) - n_2(x) = \dfrac{f(x)}{r(x)} - \dfrac{p(x)}{k(x)} = \dfrac{f(x) \times k(x) - p(x) \times r(x)}{r(x) \times k(x)}$

For example:

If $n_1(x) = \dfrac{5}{x-3}$ and $n_2(x) = \dfrac{3}{x+2}$, then :

- $n_1(x) + n_2(x) = \dfrac{5}{x-3} + \dfrac{3}{x+2} = \dfrac{5(x+2) + 3(x-3)}{(x-3)(x+2)} = \dfrac{5x + 10 + 3x - 9}{(x-3)(x+2)} = \dfrac{8x+1}{(x-3)(x+2)}$

  where the domain of the sum is $\mathbb{R} - \{3, -2\}$
  which is the common domain of the two algebraic fractions n_1 and n_2

- $n_1(x) - n_2(x) = \dfrac{5}{x-3} - \dfrac{3}{x+2} = \dfrac{5(x+2) - 3(x-3)}{(x-3)(x+2)} = \dfrac{5x + 10 - 3x + 9}{(x-3)(x+2)} = \dfrac{2x+19}{(x-3)(x+2)}$

  where the domain of the result is $\mathbb{R} - \{3, -2\}$
  which is the common domain of the two algebraic fractions n_1 and n_2

UNIT 2

The steps of adding or subtracting two algebraic fractions :

1. Arrange the terms of each of the numerator and denominator of each fraction descendingly or ascendingly according to the powers of any symbol in it.
2. Factorize the numerator and the denominator of each fraction if possible.
3. Find the common domain which will be the domain of the result.
4. Reduce each fraction separately to make the operations of addition or subtraction easier.
5. Unify the denominators.
6. Perform the operations of addition or subtraction of the terms of the numerators.
7. Put the final result in the simplest form if possible.

Example 1 **In each of the following , find n (x) in the simplest form showing the domain of n :**

1 $n(x) = \frac{x^2+3x}{x^2+4x+3} + \frac{x-5}{x^2-4x-5}$ **2** $n(x) = \frac{x-1}{x^2-x} - \frac{x-3}{x^2+6-5x}$

Solution

1 $\because n(x) = \frac{x^2+3x}{x^2+4x+3} + \frac{x-5}{x^2-4x-5}$

$\therefore n(x) = \frac{x(x+3)}{(x+3)(x+1)} + \frac{x-5}{(x-5)(x+1)}$ (Factorization)

$\therefore$ The domain of $n = \mathbb{R} - \{-3, -1, 5\}$ (Finding the common domain)

$\therefore n(x) = \frac{x}{x+1} + \frac{1}{x+1}$ (Reducing each fraction separately)

$\therefore n(x) = \frac{x+1}{x+1} = 1$ (Addition operation and simplifying the result)

2 $\because n(x) = \frac{x-1}{x^2-x} - \frac{x-3}{x^2+6-5x}$

$\therefore n(x) = \frac{x-1}{x^2-x} - \frac{x-3}{x^2-5x+6}$ (Ordering)

$\therefore n(x) = \frac{x-1}{x(x-1)} - \frac{x-3}{(x-2)(x-3)}$ (Factorization)

$\therefore$ The domain of $n = \mathbb{R} - \{0, 1, 2, 3\}$ (Finding the common domain)

$\therefore n(x) = \frac{1}{x} - \frac{1}{x-2}$ (Reducing each fraction separately)

$= \frac{x-2-x}{x(x-2)}$ (Unifying the denominators)

$= \frac{-2}{x(x-2)}$ (Subtraction operation)

Example 2 **Find n ($\mathcal{X}$) in its simplest form showing the domain of n :**

1. $n(\mathcal{X}) = \dfrac{10\mathcal{X} - 10}{2\mathcal{X}^2 - 2\mathcal{X} - 12} + \dfrac{\mathcal{X}^2 - 2\mathcal{X} - 15}{\mathcal{X}^2 - 9}$

2. $n(\mathcal{X}) = \dfrac{\mathcal{X} + 1}{\mathcal{X}^2 - 2\mathcal{X} - 3} - \dfrac{4\mathcal{X} - 7}{2\mathcal{X}^2 - 7\mathcal{X} + 3}$

Solution

1. $\because n(\mathcal{X}) = \dfrac{10(\mathcal{X} - 1)}{2(\mathcal{X}^2 - \mathcal{X} - 6)} + \dfrac{\mathcal{X}^2 - 2\mathcal{X} - 15}{\mathcal{X}^2 - 9}$

$$= \frac{10(\mathcal{X} - 1)}{2(\mathcal{X} - 3)(\mathcal{X} + 2)} + \frac{(\mathcal{X} - 5)(\mathcal{X} + 3)}{(\mathcal{X} - 3)(\mathcal{X} + 3)}$$

$\therefore$ The domain of n $= \mathbb{R} - \{3, -3, -2\}$

$$\therefore n(\mathcal{X}) = \frac{5(\mathcal{X} - 1)}{(\mathcal{X} - 3)(\mathcal{X} + 2)} + \frac{\mathcal{X} - 5}{\mathcal{X} - 3}$$

$\because$ L.C.M of the two denominators $= (\mathcal{X} - 3)(\mathcal{X} + 2)$

$$\therefore n(\mathcal{X}) = \frac{5(\mathcal{X} - 1) + (\mathcal{X} - 5)(\mathcal{X} + 2)}{(\mathcal{X} - 3)(\mathcal{X} + 2)} = \frac{5\mathcal{X} - 5 + \mathcal{X}^2 - 3\mathcal{X} - 10}{(\mathcal{X} - 3)(\mathcal{X} + 2)}$$

$$= \frac{\mathcal{X}^2 + 2\mathcal{X} - 15}{(\mathcal{X} - 3)(\mathcal{X} + 2)} = \frac{(\mathcal{X} + 5)(\mathcal{X} - 3)}{(\mathcal{X} - 3)(\mathcal{X} + 2)} = \frac{\mathcal{X} + 5}{\mathcal{X} + 2}$$

2. $\because n(\mathcal{X}) = \dfrac{\mathcal{X} + 1}{(\mathcal{X} - 3)(\mathcal{X} + 1)} - \dfrac{4\mathcal{X} - 7}{(2\mathcal{X} - 1)(\mathcal{X} - 3)}$

$\therefore$ The domain of n $= \mathbb{R} - \{3, -1, \frac{1}{2}\}$

$$\therefore n(\mathcal{X}) = \frac{1}{\mathcal{X} - 3} - \frac{4\mathcal{X} - 7}{(2\mathcal{X} - 1)(\mathcal{X} - 3)}$$

$\because$ L.C.M of the denominators is $(\mathcal{X} - 3)(2\mathcal{X} - 1)$

$$\therefore n(\mathcal{X}) = \frac{(2\mathcal{X} - 1) - (4\mathcal{X} - 7)}{(\mathcal{X} - 3)(2\mathcal{X} - 1)} = \frac{2\mathcal{X} - 1 - 4\mathcal{X} + 7}{(\mathcal{X} - 3)(2\mathcal{X} - 1)}$$

$$= \frac{-2\mathcal{X} + 6}{(\mathcal{X} - 3)(2\mathcal{X} - 1)} = \frac{-2(\mathcal{X} - 3)}{(\mathcal{X} - 3)(2\mathcal{X} - 1)} = \frac{-2}{2\mathcal{X} - 1}$$

TRY 1 by yourself

In each of the following , find n ($\mathcal{X}$) in the simplest form showing the domain of n :

1. $n(\mathcal{X}) = \dfrac{\mathcal{X} - 3}{\mathcal{X}^2 - 2\mathcal{X} - 3} + \dfrac{\mathcal{X}^2 - \mathcal{X}}{\mathcal{X}^2 - 1}$

2. $n(\mathcal{X}) = \dfrac{\mathcal{X} + 4}{\mathcal{X}^2 + \mathcal{X} - 12} - \dfrac{1}{\mathcal{X}^2 - 5\mathcal{X} + 6}$

The properties of the operations of the addition and subtraction of the algebraic fractions :

1 Commutation. 2 Association.

3 Zero is the additive neutral (additive identity) of any algebraic fraction.

4 The additive inverse of any algebraic fraction is available.

i.e. The additive inverse of the algebraic fraction : $\frac{g(x)}{k(x)}$ is $-\frac{g(x)}{k(x)}$, $\frac{-g(x)}{k(x)}$ or $\frac{g(x)}{-k(x)}$

The domain of the algebraic fraction is the same domain of its additive inverse.

Note that : Subtraction operation of algebraic fractions has no property of the previous properties.

Example 3 If n is an algebraic fraction where : $n(x) = \frac{x^2 + 2x}{x^2 - 4}$

Find in the simplest form the additive inverse of n showing its domain.

Solution

$\because n(x) = \frac{x^2 + 2x}{x^2 - 4}$ $\therefore n(x) = \frac{x(x+2)}{(x-2)(x+2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -2\}$, $n(x) = \frac{x}{x-2}$

$\therefore$ The additive inverse of the fraction n is : $-\frac{x}{x-2}$, $\frac{-x}{x-2}$ or $\frac{x}{2-x}$

Its domain = the domain of $n = \mathbb{R} - \{2, -2\}$

Example 4 **Find n (x) in the simplest form showing the domain of n if :**

$n(x) = \frac{2x + 4}{x^2 - 4} + \frac{x}{2x - x^2}$, then find n (1) and n (– 2)

Solution

$\because n(x) = \frac{2(x+2)}{(x-2)(x+2)} + \frac{x}{-x(x-2)}$

$= \frac{2(x+2)}{(x-2)(x+2)} - \frac{x}{x(x-2)}$ (Notice the change of the sign)

$\therefore$ The domain of $n = \mathbb{R} - \{2, -2, 0\}$

, $n(x) = \frac{2}{x-2} - \frac{1}{x-2} = \frac{1}{x-2}$

$\therefore n(1) = \frac{1}{1-2} = -1$, n (– 2) is undefined because $-2 \notin$ the domain of n

TRY 2 by yourself

Find n (x) in the simplest form showing the domain of n where :

$n(x) = \frac{3x - 15}{x^2 - 8x + 15} - \frac{x^2 - 3x - 18}{9 - x^2}$

Answers of try by yourself

1 [1] $n(x) = 1$, the domain of $n = \mathbb{R} - \{-1, 3, 1\}$ [2] $n(x) = \frac{1}{x-2}$, the domain of $n = \mathbb{R} - \{-4, 3, 2\}$

2 $n(x) = 1$, the domain of $n = \mathbb{R} - \{5, 3, -3\}$

LESSON
5

Operations on algebraic fractions (Follow)

Multiplying and dividing the algebraic fractions :

1 Multiplying the algebraic fractions :

Multiplying two algebraic fractions is similar to multiplying two fractional numbers , therefore it is better to remember together how to multiply two fractional numbers.

Remember that

$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ (where $bd \neq 0$)

For example:

- $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$
- $\frac{\cancel{6}^{3}}{\cancel{8}_{4}} \times \frac{1}{5} = \frac{3}{4} \times \frac{1}{5} = \frac{3 \times 1}{4 \times 5} = \frac{3}{20}$
- $\frac{\cancel{3}^{1}}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{9}_{3}} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

- **The following shows how to multiply two algebraic fractions :**

Multiplying two algebraic fractions

If $X \in$ the common domain of the two algebraic fractions n_1 and n_2 where :

$n_1(X) = \frac{f(X)}{r(X)}$, $n_2(X) = \frac{p(X)}{k(X)}$

, then : $n_1(X) \times n_2(X) = \frac{f(X)}{r(X)} \times \frac{p(X)}{k(X)} = \frac{f(X) \times p(X)}{r(X) \times k(X)}$

For example:

If $n_1(x) = \frac{2}{x}$, $n_2(x) = \frac{x}{x-1}$,

then $n_1(x) \times n_2(x) = \frac{2}{x} \times \frac{x}{x-1}$

$= \frac{2 \times \cancel{x}}{\cancel{x}(x-1)}$

where the domain of the product $= \mathbb{R} - \{0 , 1\}$

, $n_1(x) \times n_2(x) = \frac{2}{x-1}$

Notice that :

The domain of the product is the common domain of the two algebraic fractions before reduction.

The steps of multiplying the algebraic fractions :

1. Arrange the terms of each of the numerator and the denominator of each fraction alone descendingly or ascendingly according to the powers of any symbol in it.
2. Factorize the numerator and the denominator of each fraction alone if it is possible.
3. Find the common domain.
4. Remove the common factors between the numerator and the denominator of each fraction and between the numerator of a fraction and the denominator of another fraction.
5. Perform the operation of multiplication and put the result in the simplest form.

Example 1 **Find n (x) in the simplest form showing the domain of n :**

$n(x) = \frac{x^2 + 5x}{x^2 + x - 6} \times \frac{x^2 - 7x + 10}{x^2 - 25}$

Solution

$\because n(x) = \frac{x(x+5)}{(x+3)(x-2)} \times \frac{(x-5)(x-2)}{(x-5)(x+5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3 , 2 , 5 , -5\}$

By removing the common factors : $\therefore n(x) = \frac{x}{x+3}$

TRY 1 by yourself

Find n (x) in the simplest form showing the domain of n :

$n(x) = \frac{x^2 - x}{x^2 + 2x} \times \frac{x^2 - 4}{x^2 - 1}$

The properties of the operation of multiplying the algebraic fractions :

1. Commutation.
2. Association.
3. One is the multiplicative neutral (the multiplicative identity).
4. Existing the multiplicative inverses.

The multiplicative inverse of the algebraic fraction :

If n is an algebraic fraction where $n(x) = \frac{p(x)}{k(x)} \neq 0$

, then n has a multiplicative inverse which is the algebraic fraction n^{-1} where $n^{-1}(x) = \frac{k(x)}{p(x)}$

and the domain of n^{-1} is $\mathbb{R}$ – the set of zeroes of each of the numerator and the denominator of any of the two fractions.

For example:

If $n(x) = \frac{x+1}{x-5}$, then $n^{-1}(x) = \frac{x-5}{x+1}$

where the domain of $n = \mathbb{R} - \{5\}$

and the domain of $n^{-1} = \mathbb{R} - \{5, -1\}$

Note that :

$n(x)$ and $n^{-1}(x)$ each of them is the reciprocal of the other

i.e. the numerator of each of them is a denominator for the other.

Example 2 If $n(x) = \frac{x^3 - 4x^2 - 5x}{x^2 - 25}$

1. **Find :** $n^{-1}(x)$ and state the domain of n^{-1}
2. **Find :** $n^{-1}(-1)$
3. If $n^{-1}(x) = \frac{1}{3}$ **, find the value of x**

Solution

1. $\because n(x) = \frac{x(x^2 - 4x - 5)}{(x-5)(x+5)} = \frac{x(x-5)(x+1)}{(x-5)(x+5)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, 5, -1, -5\}$

$, n^{-1}(x) = \frac{(x-5)(x+5)}{x(x-5)(x+1)}$

$= \frac{x+5}{x(x+1)}$

2. $n^{-1}(-1)$ is undefined because $-1 \notin$ the domain of n^{-1}

3 $\because n^{-1}(x) = \frac{1}{3}$ $\therefore \frac{x+5}{x(x+1)} = \frac{1}{3}$

$\therefore x(x+1) = 3(x+5)$ $\therefore x^2 + x = 3x + 15$

$\therefore x^2 + x - 3x - 15 = 0$

$\therefore x^2 - 2x - 15 = 0$ $\therefore (x-5)(x+3) = 0$

$\therefore x = 5$ refused because $5 \notin$ the domain of n^{-1}

or $x = -3$

TRY 2 by yourself

Complete the following :

1. If $n(x) = \frac{x-8}{x}$, then the domain of n^{-1} is ....
2. If $n(x) = \frac{x-5}{x+3}$, then $n^{-1}(4) =$ ....
3. If $n(x) = \frac{2x-4}{x+2}$, then $n^{-1}(-2) =$ ....

2 Dividing an algebraic fraction by another :

Dividing two algebraic fractions is similar to dividing two fractional numbers, therefore it is better to remember together how to divide two fractional numbers.

Remember that

If $\frac{a}{b}$ and $\frac{c}{d}$ are two fractional numbers , $b \neq 0$ and $\frac{c}{d} \neq 0$

, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times$ the multiplicative inverse of the number $\frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ (where $bd \neq 0$)

For example:

- $\frac{1}{4} \div \frac{3}{5} = \frac{1}{4} \times \frac{5}{3} = \frac{5}{12}$
- $\frac{5}{8} \div \frac{-15}{4} = \frac{\cancel{5}^{1}}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{-\cancel{15}_{3}} = \frac{1}{2} \times \frac{1}{-3} = -\frac{1}{6}$

Regarding that the multiplicative inverses of the algebraic fractions exist , then the operation of division is possible and it is defined as follows :

Dividing an algebraic fraction by another :

If n_1 and n_2 are two algebraic fractions where : $n_1(x) = \frac{f(x)}{r(x)}$, $n_2(x) = \frac{p(x)}{k(x)}$

, then : $n_1(x) \div n_2(x) = n_1(x) \times n_2^{-1}(x) = \frac{f(x)}{r(x)} \times \frac{k(x)}{p(x)}$

where the domain of $n_1 \div n_2$ = the common domain of each of n_1 and n_2^{-1}

$= \mathbb{R} -$ the set of zeroes of denominator of n_1 or denominator of n_2 or numerator of n_2

$= \mathbb{R} - (z(r) \cup z(p) \cup z(k))$

For example:

If $n_1(x) = \frac{x}{x-1}$, $n_2(x) = \frac{2x}{x-1}$,

then $n_1(x) \div n_2(x) = \frac{x}{x-1} \div \frac{2x}{x-1} = \frac{\cancel{x}}{\cancel{x-1}} \times \frac{\cancel{x-1}}{2\cancel{x}} = \frac{1}{2}$ where $x \notin \{1, 0\}$

Example 3 Find n (x) in the simplest form showing the domain of n :

$$n(x) = \frac{x^2 - 7x + 10}{x^2 - 4x - 5} \div \frac{x^3 - 8}{x^2 + 2x + 4}$$

, then find n (2) and n (3) if it is possible.

Solution

$\because n(x) = \frac{(x-2)(x-5)}{(x-5)(x+1)} \div \frac{(x-2)(x^2+2x+4)}{x^2+2x+4}$

$\therefore$ The domain of $n = \mathbb{R} - \{5, -1, 2\}$

, $n(x) = \frac{x-2}{x+1} \times \frac{1}{x-2} = \frac{1}{x+1}$

, n (2) is undefined because $2 \notin$ the domain of n

, $n(3) = \frac{1}{3+1} = \frac{1}{4}$

TRY 3 *by yourself*

Find n (x) in the simplest form showing the domain of n :

$$n(x) = \frac{x^3 - 8}{x^2 + x - 6} \div \frac{x^2 + 2x + 4}{3x + 9}$$

Answers *of try by yourself*

1. $n(x) = \frac{x-2}{x+1}$, the domain of $n = \mathbb{R} - \{0, -2, 1, -1\}$
2. (1) $\mathbb{R} - \{0, 8\}$ (2) -7
   (3) Undefined , because : $-2 \notin$ the domain of n^{-1}
3. $n(x) = 3$, the domain of $n = \mathbb{R} - \{2, -3\}$

UNIT 3

Probability

Lessons of the unit :

1. Operations on events : Intersection and union of two events.
2. Operations on events (follow) :
   Complementary event and the difference between two events.

Unit Objectives :

By the end of this unit, student should be able to :

- Remember what have been studied on calculating the probability.
- Calculate the probability of occurring two events together in the same sample space (Intersection of two events)
- Recognize the mutually exclusive events.
- Calculate the probability of occurring one of two events at least in the same sample space (Union of two events)
- Recognize the complementary event.
- Calculate the probability of non occurrence of an event (Complementary event)
- Calculate the probability of occurrence of an event and non occurrence of another event in the same sample space (Difference between two events)

LESSON 1

Operations on events

Before studying the operations on events , we shall remember some main concepts which we have studied before in probability.

1 The random experiment :

It is an experiment in which we can specify all its possible outcomes before performing it , but we cannot determine which outcome will occur certainly.

2 The sample space (S) :

It is the set of all possible outcomes of a random experiment.

3 The event :

It is a subset of the sample space.

4 The probability of occurrence of the event :

- It is said that an event occurred if the outcome of the random experiment is an element of this event.
- We can calculate the probability of an event (say A) from the relation :

$$P(A) = \frac{\text{The number of elements of the event A}}{\text{The number of elements of the sample spaces}} = \frac{n(A)}{n(S)}$$

For example:

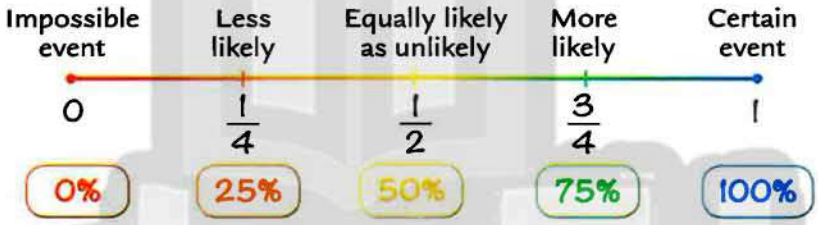
In the experiment of rolling a fair die once and observing the number appears on the upper face , if S is the sample space of the experiment and A is the event of getting an even number , then : $S = \{1 , 2 , 3 , 4 , 5 , 6\}$, $n(S) = 6$, $A = \{2 , 4 , 6\}$, $n(A) = 3$

, then $P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$ (**i.e.** The probability of occurring the event $A = \frac{1}{2}$)

Remarks

- $0 \le$ the probability of any event ≤ 1
- Probability can be written as a fraction or percentage.

The following figure shows the possibility of occurring an event due to the value of its probability.



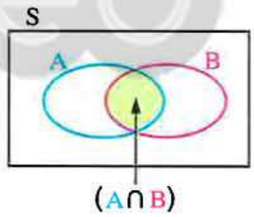
Operations on events

Since the event is a subset of the sample space (S) , then we can carry out on events the same operations which we carry out on sets such as intersection , union , complementary , the difference regarding that the universal set of these events is the sample space. Also we can represent these events by Venn diagrams.

1 Intersection of two events

For any two events A and B of a sample space S :

The event of occurring the two events A and B together = $A \cap B$, then :



The probability of occurring the two events A and B together

$= P(A \cap B) = \frac{n(A \cap B)}{n(S)}$

Example 1 **In the experiment of rolling a fair die once and observing the number appears on the upper face , if A is the event of getting an even number , B is the event of getting a prime number and C is the event of getting an odd prime number , find using Venn diagram :**

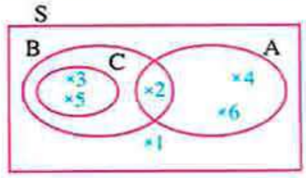
1. The probability of occurring the two events A and B together.
2. The probability of occurring the two events B and C together.
3. The probability of occurring the two events A and C together.

Solution

The sample space (S) = $\{1, 2, 3, 4, 5, 6\}$

, n (S) = 6 , A = $\{2, 4, 6\}$, B = $\{2, 3, 5\}$

, C = $\{3, 5\}$



1 $\because$ The event of occurring A and B together = $A \cap B = \{2\}$

$\therefore n(A \cap B) = 1$

$\therefore$ The probability of occurring A and B together = $P(A \cap B)$

$$= \frac{n(A \cap B)}{n(S)} = \frac{1}{6}$$

2 $\because$ The event of occurring B and C together = $B \cap C = \{3, 5\}$

$\therefore n(B \cap C) = 2$

$\therefore$ The probability of occurring B and C together = $P(B \cap C)$

$$= \frac{n(B \cap C)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

3 $\because$ The event of occurring A and C together = $A \cap C = \varnothing$

Because A and C are two separate sets or distant sets

$\therefore n(A \cap C) = \text{zero}$

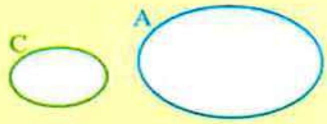
$\therefore$ The probability of occurring A and C together = $P(A \cap C)$

$$= \frac{n(A \cap C)}{n(S)} = \frac{\text{zero}}{6} = \text{zero}$$

Remarks

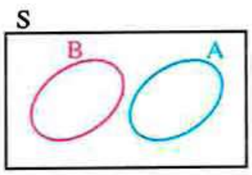
From the previous example we notice that :

1. $C \subset B$ therefore $B \cap C = C$, then we deduce that :
   The probability of occurring the two events B and C together
   = the probability of occurring the event C
   i.e. $P(B \cap C) = P(C) = \frac{n(C)}{n(S)}$



2. $A \cap C = \varnothing$ therefore it is said that the two events A and C are two mutually exclusive events

Mutually exclusive events

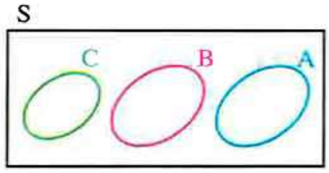


- It is said that the two events A and B are mutually exclusive if $A \cap B = \varnothing$, then $P(A \cap B) = 0$

i.e. The probability of their occurring together = the probability of the impossible event = 0

- It is said that some events are mutually exclusive if every pair of them is mutually exclusive.

For example: If $A \cap B = \varnothing$, $B \cap C = \varnothing$, $A \cap C = \varnothing$, then the events A , B and C are mutually exclusive.



TRY 1 by yourself

In an experiment of drawing a card randomly from 9 identical cards numbered from 1 to 9 , if A is the event that the drawn card is numbered by an even number and B is the event that the drawn card is numbered by a number less than 7

Find the probability of occurring the two events A and B together.

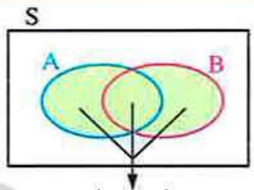
2 Union of two events

For any two events A and B of a sample space (S) :

The event of occurring the event A or the event B or both of them

(i.e. One of them at least occurs) = $A \cup B$, then :

The probability of occurring the event A or the event B or both of them



$(A \cup B)$

i.e. The probability of occurring one of them at least $= P(A \cup B) = \dfrac{n(A \cup B)}{n(S)}$

Example 2 **In the experiment of drawing one card randomly from 10 identical cards mixed very well and numbered from 1 to 10 , if A is the event that the drawn card carries an even number , B is the event that the drawn card carries a prime number and C is the event that the drawn card carries a number divisible by 4 , find using Venn diagram :**

1. The probability of occurring the event A or B
2. The probability of occurring the event B or C
3. The probability of occurring the event A or C

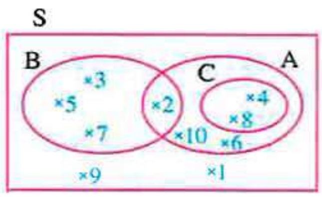
Solution

$\because S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$\therefore n(S) = 10$

$, A = \{2, 4, 6, 8, 10\}$

$, B = \{2, 3, 5, 7\}$, $C = \{4, 8\}$



1 $\because$ The event of occurring the event A or B

$= A \cup B = \{2, 3, 4, 5, 6, 7, 8, 10\}$

$\therefore n(A \cup B) = 8$

$\therefore$ The probability of occurring the event A or B

$= P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{8}{10} = \frac{4}{5}$

2 $\because$ The event of occurring the event B or C $= B \cup C = \{2, 3, 4, 5, 7, 8\}$

$\therefore n(B \cup C) = 6$

$\therefore$ The probability of occurring B or C $= P(B \cup C) = \frac{n(B \cup C)}{n(S)} = \frac{6}{10} = \frac{3}{5}$

3 $\because$ The event of occurring the event A or C $= A \cup C = \{2, 4, 6, 8, 10\}$

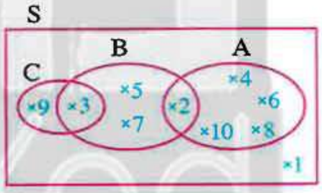
$\therefore n(A \cup C) = 5$

$\therefore$ The probability of occurring A or C $= P(A \cup C) = \frac{n(A \cup C)}{n(S)} = \frac{5}{10} = \frac{1}{2}$

Notice that : $C \subset A$ **i.e.** $A \cup C = A$

, so we can say that : $P(A \cup C) = P(A) = \frac{n(A)}{n(S)}$

Example 3 **In the opposite Venn diagram :**
If A , B and C are three events from the sample space S of a random experiment , find :



1 $P(A \cup B)$, $P(A) + P(B) - P(A \cap B)$

What do you notice ?

2 $P(A \cup C)$, $P(A) + P(C)$ What do you notice ?

Solution

$\because S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $\qquad \therefore n(S) = 10$

$, \because A = \{2, 4, 6, 8, 10\}$ $\qquad \therefore n(A) = 5$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{5}{10} = \frac{1}{2}$

$, \because B = \{2, 3, 5, 7\}$ $\qquad \therefore n(B) = 4$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{4}{10} = \frac{2}{5}$

$, \because C = \{3, 9\}$ $\qquad \therefore n(C) = 2$

$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{2}{10} = \frac{1}{5}$

1 $\because A \cup B = \{2, 3, 4, 5, 6, 7, 8, 10\}$ $\therefore n(A \cup B) = 8$

$\therefore P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{8}{10} = \frac{4}{5}$ (1)

$, \because A \cap B = \{2\}$ $\therefore n(A \cap B) = 1$

$\therefore P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{10}$

$\therefore P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{2}{5} - \frac{1}{10} = \frac{8}{10} = \frac{4}{5}$ (2)

From (1) and (2) we notice that : $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

2 $\because A \cup C = \{2, 3, 4, 6, 8, 9, 10\}$ $\therefore n(A \cup C) = 7$

$\therefore P(A \cup C) = \frac{n(A \cup C)}{n(S)} = \frac{7}{10}$ (1)

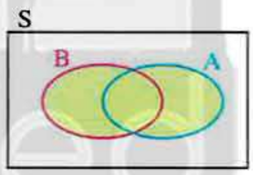
$, \because P(A) + P(C) = \frac{1}{2} + \frac{1}{5} = \frac{7}{10}$ (2)

From (1) and (2) we notice that : $\boxed{P(A \cup C) = P(A) + P(C)}$

• From the previous example we can deduce the following rule :

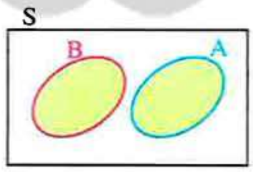
Rule :

• For any two events from the sample space S of a random experiment :



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

• If A and B are two mutually exclusive events , then :
$P(A \cap B) =$ zero , then :



$$P(A \cup B) = P(A) + P(B)$$

Example 4 **If A and B are two events from the sample space S , P (A) = 0.3 and P (B) = 0.2 find :**

1 $P(A \cup B)$ if $P(A \cap B) = 0.1$

2 $P(A \cup B)$ if A and B are two mutually exclusive events.

3 $P(A \cap B)$ if $P(A \cup B) = 0.3$

Solution

1 $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.2 - 0.1 = 0.4$

2 $\because$ A and B are two mutually exclusive events

$\therefore P(A \cup B) = P(A) + P(B) = 0.3 + 0.2 = 0.5$

3 $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore 0.3 = 0.3 + 0.2 - P(A \cap B)$ $\qquad$ $\therefore 0.3 = 0.5 - P(A \cap B)$

$\therefore P(A \cap B) = 0.5 - 0.3 = 0.2$

TRY 2 by yourself

If A and B are two events from the sample space of a random experiment ,

$P(A) = 0.8$, $P(B) = 0.7$, $P(A \cap B) = 0.6$

Find the probability of occurring the event A or B

Answers *of try by yourself*

1 $\frac{1}{3}$ $\qquad$ **2** 0.9

LESSON
2

Operations on events (Follow)

3 The complementary event

If A is an event of the sample space S ($A \subset S$) then :
the complementary event of A which is denoted by $\grave{A}$ is the event of non occurring A where $A \cup \grave{A} = S$, $A \cap \grave{A} = \varnothing$

, then the probability of non occurrence of the event $A = P(\grave{A}) = \frac{n(\grave{A})}{n(S)}$

For example:

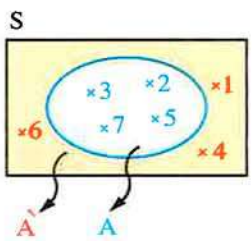
- In the experiment of drawing one card randomly from 7 cards which are identical and numbered by the numbers from 1 to 7 and observing the written number on it.

If A is the event of getting a prime number , then $A = \{2 , 3 , 5 , 7\}$

$\therefore n(A) = 4$

$\therefore \grave{A} = \{1 , 4 , 6\}$ is the complementary event of the event A

It represents the event of non occurring the event A



Since $n(A) = 4$, $n(\grave{A}) = 3$, $n(S) = 7$,

then the probability of occurring the event $A = P(A) = \frac{n(A)}{n(S)} = \frac{4}{7}$

and the probability of non occurring the event $A = P(\grave{A}) = \frac{n(\grave{A})}{n(S)} = \frac{3}{7}$

UNIT 3

Remarks

For any event A of the sample space S it will be :

1. $A \cap \grave{A} = \varnothing$

   i.e. The two events A and $\grave{A}$ are two mutually exclusive events

   i.e. Occurring one of them prevents the occurring of the other , then $P(A \cap \grave{A}) = $ zero

2. $A \cup \grave{A} = S$

   i.e. The union of any event and the complementary event of it = the set of sample space S , then $P(A \cup \grave{A}) = P(A) + P(\grave{A}) = P(S) = 1$

   From that we deduce that :

   $\boxed{P(A) = 1 - P(\grave{A}) \quad , \quad P(\grave{A}) = 1 - P(A)}$

Note that :

$$P(S) = \frac{n(S)}{n(S)} = 1$$

Example 1 If A and B are two events of the sample space of a random experiment , $P(A) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{18}$ and $P(A \cup B) = \frac{4}{9}$ **Find :**

1 The probability of non occurrence the event A

2 $P(\grave{B})$

Solution

1 The probability of non occurrence the event $A = P(\grave{A})$

$\because P(A) = \frac{1}{6}$ $\qquad \therefore P(\grave{A}) = 1 - P(A) = 1 - \frac{1}{6} = \frac{5}{6}$

2 $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \frac{4}{9} = \frac{1}{6} + P(B) - \frac{1}{18}$ $\qquad \therefore P(B) = \frac{4}{9} + \frac{1}{18} - \frac{1}{6} = \frac{6}{18} = \frac{1}{3}$

$\because P(\grave{B}) = 1 - P(B)$ $\qquad \therefore P(\grave{B}) = 1 - \frac{1}{3} = \frac{2}{3}$

Example 2 40 pupils in a school participated in the sports activities in the school. If 25 pupils participated in football team , 10 pupils participated in basketball team , 4 pupils in the two teams together and the rest in other teams. If a pupil is chosen randomly from those pupils.

Find using Venn diagram the probability that :

1 The pupil is participating in football team.

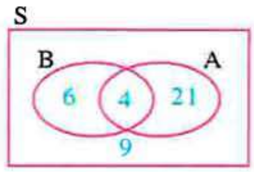
2 The pupil is not participating in football team.

3 The pupil is participating in the two teams together.

4 The pupil is participating in the football team or basketball team.

Solution

Assuming that : A is the event that the pupil is participating in football team , B is the event that the pupil is participating in basketball team and S is the sample space of this experiment.



$\therefore$ n (A) = 25 , n (B) = 10 , n (S) = 40

1 The probability that the pupil is participating in football team

$= P(A) = \frac{n(A)}{n(S)} = \frac{25}{40} = \frac{5}{8}$

2 The probability that the pupil is not participating in football team

$= P(\grave{A}) = 1 - P(A) = 1 - \frac{5}{8} = \frac{3}{8}$

Another solution : $\because$ n (A) = 25 $\quad\therefore$ n ($\grave{A}$) = 40 – 25 = 15

$\therefore$ The probability that the pupil is not participating in football team

$= P(\grave{A}) = \frac{n(\grave{A})}{n(S)} = \frac{15}{40} = \frac{3}{8}$

3 $\because$ The event that the pupil is participating in the two teams together = $A \cap B$

$\therefore$ n ($A \cap B$) = The number of pupils who are participating in the two teams together = 4

$\therefore$ The probability that the pupil is participating in the two teams

$= P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{4}{40} = \frac{1}{10}$

4 $\because$ The event that the pupil is participating in football team or basketball team = $A \cup B$

, $\because$ n ($A \cup B$) = 21 + 6 + 4 = 31

$\therefore$ The probability that the pupil is participating in football team or basketball team $= P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{31}{40}$

Another solution : $\because P(B) = \frac{n(B)}{n(S)} = \frac{10}{40} = \frac{1}{4}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{5}{8} + \frac{1}{4} - \frac{1}{10} = \frac{31}{40}$

TRY 1 *by yourself*

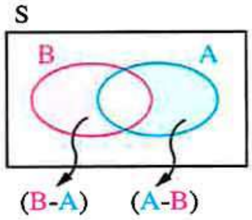
If A and B are two events of the sample space (S) of a random experiment , P (A) = 0.5 , P ($\grave{B}$) = 0.7 and P ($A \cap B$) = 0.1

Find : **1** P (B) **2** P ($A \cup B$)

4 The difference between two events

If A and B are two events of a sample space S then :

- The event of occurrence A and non occurrence B
  (i.e. the event of occurrence A only) = A – B
  , then the probability of occurrence the event A and non occurrence the event B = $P(A-B) = \frac{n(A-B)}{n(S)}$



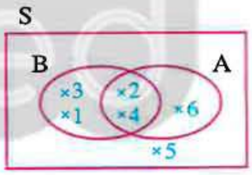
- The event of occurrence B and non occurrence A
  (i.e. the event of occurrence B only) = B – A
  , then the probability of occurrence the event B and non occurrence the event A $= P(B-A) = \frac{n(B-A)}{n(S)}$

Example 3 In the experiment of rolling a fair die once and observing the number on the upper face. If A is the event of getting an even number and B is the event of getting a number less than 5

Find using Venn diagram :

1. The probability of occurring the event A only.
2. The probability of occurring the event B only.

Solution



$\because S = \{1, 2, 3, 4, 5, 6\}$

$\therefore n(S) = 6$

$\because A = \{2, 4, 6\}, B = \{4, 3, 2, 1\}$

1. The event of occurrence A only = the event of occurrence A and non occurrence B = A – B = $\{6\}$

   $\therefore n(A-B) = 1$

   $\therefore$ The probability of occurrence of A only $= P(A-B) = \frac{n(A-B)}{n(S)} = \frac{1}{6}$

2. The event of occurrence B only = the event of occurrence B and non occurrence A = B – A = $\{3, 1\}$

   $\therefore n(B-A) = 2$

   $\therefore$ The probability of occurrence B only $= \frac{2}{6} = \frac{1}{3}$

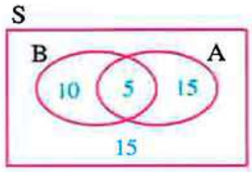
Example 4 A class contains 45 pupils, 20 pupils of them like reading police novels and 15 pupils of them like reading romantic novels and 5 pupils of them like reading the two kinds of novels. If a pupil is chosen randomly from the class.

Calculate the probability that the pupil :

1 likes reading police novels.
2 likes reading police novels only.
3 does not like reading the police novels.
4 likes reading the two kinds together.

Solution

Assuming that : A is the event that the pupil likes reading police novels and B is the event that the pupil likes reading romantic novels and S is the sample space.



$\therefore$ n (A) = 20 , n (B) = 15 , n (S) = 45

1 The probability that the pupil likes reading police novels

$= P(A) = \frac{n(A)}{n(S)} = \frac{20}{45} = \frac{4}{9}$

2 The probability that the pupil likes reading police novels only

$= P(A - B) = \frac{n(A - B)}{n(S)} = \frac{15}{45} = \frac{1}{3}$

3 The probability that the pupil does not like reading the police novels

$= P(\grave{A}) = 1 - P(A) = 1 - \frac{4}{9} = \frac{5}{9}$

4 The probability that the pupil likes reading the two kinds together

$= P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{5}{45} = \frac{1}{9}$

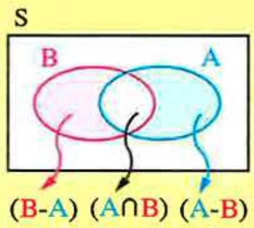
Remarks

If A and B are two events of a sample space (S) of a random experiment , then

- $(A - B) \cup (A \cap B) = A$

i.e. $P(A - B) + P(A \cap B) = P(A)$

and from it : $P(A - B) = P(A) - P(A \cap B)$



Also :

- $(B - A) \cup (A \cap B) = B$ i.e. $P(B - A) + P(A \cap B) = P(B)$

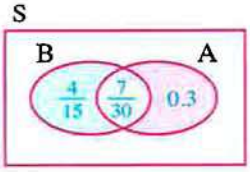
and from it : $P(B - A) = P(B) - P(A \cap B)$

Example 5 If A and B are two events of the sample space (S) of a random experiment where $P(A-B)=0.3$, $P(B-A)=\frac{4}{15}$ and $P(A\cap B)=\frac{7}{30}$ **Find :**

1 The probability of non occurrence of A

2 The probability of occurrence A or B or both of them.

Solution



1 $\because P(A-B)=0.3$, $P(A\cap B)=\frac{7}{30}$

$\therefore P(A)=P(A-B)+P(A\cap B)$

$=0.3+\frac{7}{30}=\frac{16}{30}=\frac{8}{15}$

$\therefore$ The probability of non occurrence $A=P(\acute{A})$

$=1-P(A)=1-\frac{8}{15}=\frac{7}{15}$

2 $\because P(B)=P(B-A)+P(A\cap B)$

$=\frac{4}{15}+\frac{7}{30}=\frac{1}{2}$

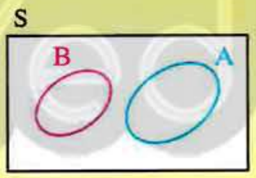
$\therefore$ The probability of occurrence the event A or B or both of them

$=P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{8}{15}+\frac{1}{2}-\frac{7}{30}=\frac{4}{5}$

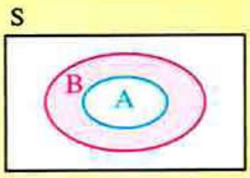
Remarks

1 If A and B are two mutually exclusive events of the sample space (S) , then :



- $A-B=A$ i.e. $P(A-B)=P(A)$
- $B-A=B$ i.e. $P(B-A)=P(B)$

2 If A and B are two events of the sample space (S) and $A\subset B$, then :



- $A-B=\varnothing$
- $P(A-B)=P(\varnothing)=\frac{n(\varnothing)}{n(S)}=$ zero

Example 6 If A and B are two mutually exclusive events of a sample space of a random experiment , $P(A-B)=\frac{1}{2}$ and $P(A\cup B)=\frac{3}{5}$ **Find :**

1 $P(A)$

2 $P(\acute{B})$

3 The probability of non occurrence both of the two events together.

Solution

1 $\because$ A and B are two mutually exclusive events.

$\therefore P(A - B) = P(A) = \frac{1}{2}$

2 $\because$ A and B are two mutually exclusive events.

$\therefore P(A \cup B) = P(A) + P(B)$ $\qquad$ $\therefore \frac{3}{5} = \frac{1}{2} + P(B)$

$\therefore P(B) = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$ $\qquad$ $\therefore P(B') = 1 - P(B) = 1 - \frac{1}{10} = \frac{9}{10}$

3 $\because$ A and B are two mutually exclusive events.

$\therefore P(A \cap B) = \text{zero}$

$\therefore$ The probability of non occurrence both of the two events together

$= P(A \cap B)' = 1 - P(A \cap B) = 1 - 0 = 1$

Example 7 If A and B are two events of the sample space of a random experiment ,

$P(A) = \frac{5}{9}$, $P(B) = \frac{2}{9}$ and $P(A \cap B) = \frac{1}{9}$ **Find :**

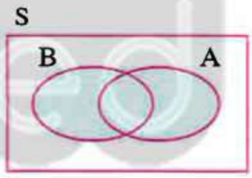
1 The probability of occurrence one of the two events at least.

2 The probability of non occurrence any of the two events.

3 The probability of occurrence one of the two events and non occurrence of the other.

Solution

1 The probability of occurrence one of the two events at least

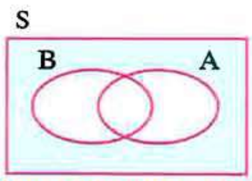


Occurrence one of the two events at least

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{9} + \frac{2}{9} - \frac{1}{9} = \frac{2}{3}$

2 The probability of non occurrence any of the two events



Non occurrence any of the two events

$= P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - \frac{2}{3} = \frac{1}{3}$

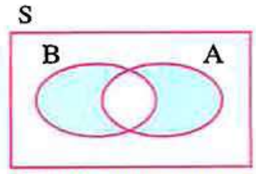
3 The probability of occurrence one of the two events and non occurrence of the other

$= P(A - B) + P(B - A)$

$, \because P\ (A - B) = P(A) - P(A \cap B) = \frac{5}{9} - \frac{1}{9} = \frac{4}{9}$

$, P(B - A) = P(B) - P(A \cap B) = \frac{2}{9} - \frac{1}{9} = \frac{1}{9}$

$\therefore$ The probability of occurrence one of the two events and non occurrence of the other $= \frac{4}{9} + \frac{1}{9} = \frac{5}{9}$



Occurrence one of the two events and non occurrence of the other

TRY 2 *by yourself*

A class contains 40 students. 30 students of them succeeded in mathematics and 24 students succeeded in science and 20 students succeeded in both of the two examinations.

If a student is chosen randomly. ***Find the probability that the chosen student :***

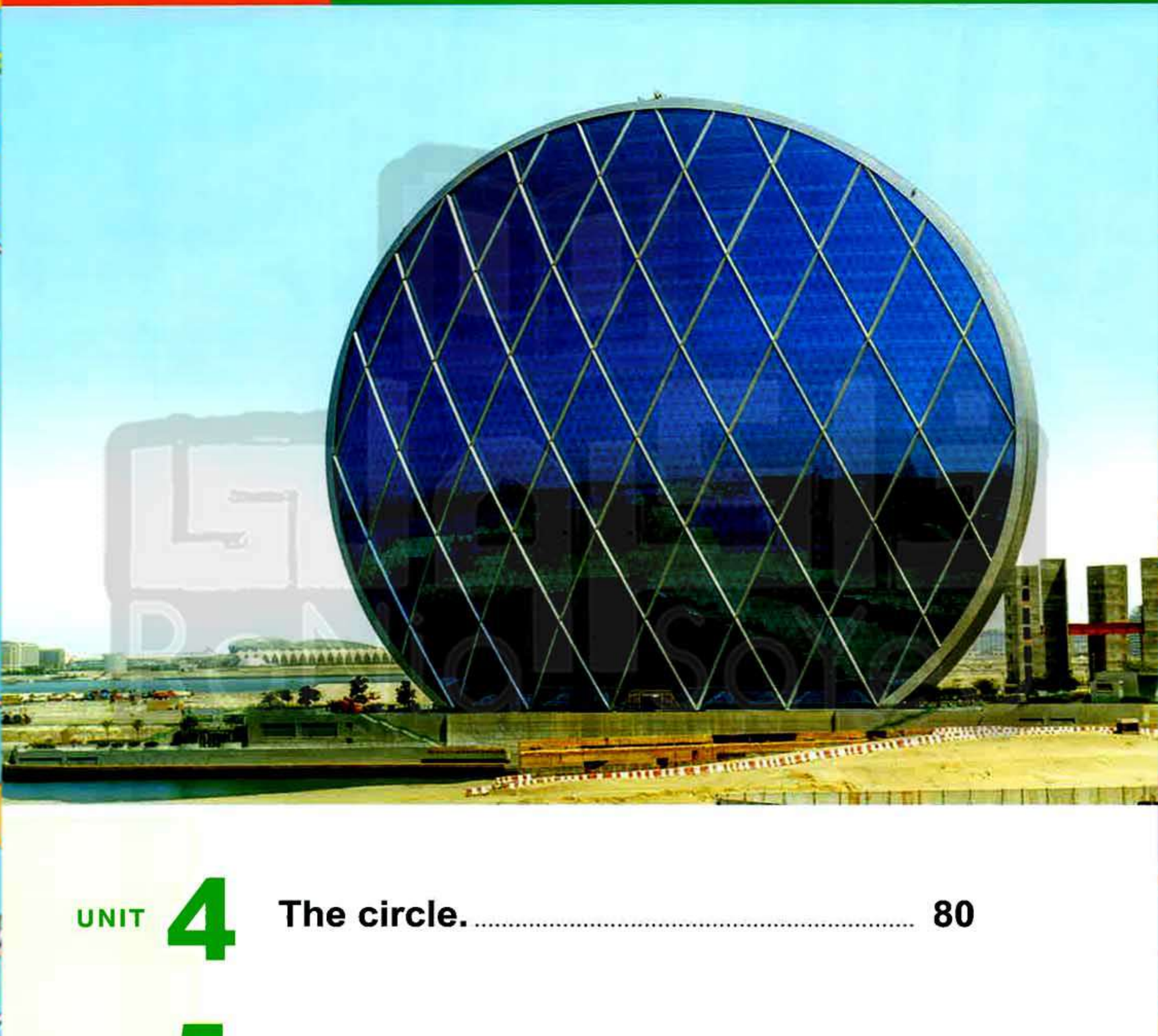
1. Succeeded in mathematics.
2. Succeeded in science only.
3. Succeeded in one of the two examinations at least.

Answers *of try by yourself*

1 [1] 0.3 [2] 0.7

2 [1] $\frac{3}{4}$ [2] $\frac{1}{10}$ [3] $\frac{17}{20}$

Second Geometry



UNIT 4

The circle

Lessons of the unit :

1. Basic definitions and concepts on the circle.
2. Position of a point and a straight line with respect to a circle.
3. Position of a circle with respect to another circle.
4. Identifying the circle.
5. The relation between the chords of a circle and its centre.

Unit Objectives :

By the end of this unit, student should be able to :

- Recognize the circle and basic definitions on it (The radius – the diameter – the chord).
- Calculate the circumference of the circle and its area.
- Recognize the axis of symmetry of the circle and some corollaries related to it.
- Determine the position of a point with respect to a circle.
- Determine the position of a straight line with respect to a circle.
- Determine the position of a circle with respect to another circle.
- Determine the relation between a tangent to a circle and the radius drawn from the point of tangency.
- Determine the relation between the line of centres of two touching circles and the common tangent at the point of tangency.
- Determine the relation between the line of centres of two intersecting circles and the common chord.
- Draw a circle knowing its centre and its radius length.
- Draw a circle passing through a given point.
- Draw a circle passing through two given points.
- Draw a circle passing through three non-collinear points.
- Recognize the circumcircle of a triangle and determine the position of its centre with respect to the triangle.
- Determine the relation between the chords of a circle and its centre.

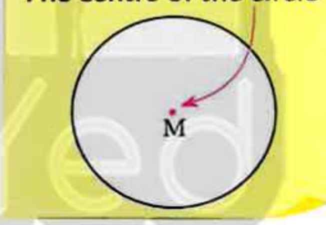
LESSON 1

Basic definitions and concepts on the circle

The circle

It is the set of points of the plane which are at a constant distance from a fixed point in the same plane.

- The fixed point is called "**the centre of the circle**".
- The constant distance is called "the radius length of the circle".
- The circle is usually denoted by its centre , so we say the circle M to mean the circle whose centre is the point M



Partition of the plane by the circle

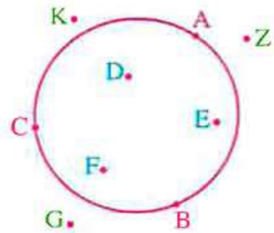
Any circle divides the plane into three sets of points which are :

1. The set of points of the circle.
2. The set of points inside the circle.
3. The set of points outside the circle.

For example :

The drawn circle in the opposite figure divides the plane into :

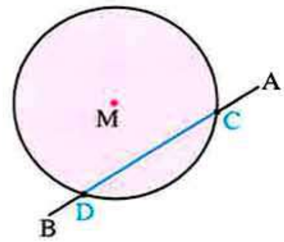
1. The set of points of the circle «on the circle» as : A , B , C , ...
2. The set of points inside the circle as : D , E , F , ...
3. The set of points outside the circle as : Z , K , G , ...



The surface of the circle is : the set of points of the circle $\cup$ the set of points inside it.

So , the surface of the circle differs from the circle.

For example:



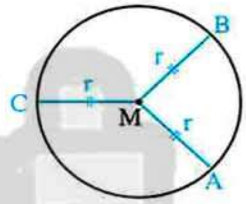
In the opposite figure :

- $\overline{AB} \cap$ the circle = $\{C , D\}$ but $\overline{AB} \cap$ the surface of the circle = $\overline{CD}$
- $M \notin$ the circle but $M \in$ the surface of the circle.

The radius of the circle

It is a line segment whose endpoints are the centre of the circle and any point on the circle.

In the opposite figure :



If the points A , B and C belong to the circle M ,

then $\overline{MA}$, $\overline{MB}$ and $\overline{MC}$ are called radii of the circle M

and $MA = MB = MC = r$ (where r is the radius length of the circle).

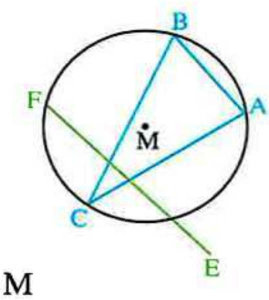
Notice that :

1. Any circle has an infinite number of radii and all of them are equal in length.
2. If two radii of two circles are equal in length , then the two circles are congruent and vice versa.

The chord of the circle

It is a line segment whose endpoints are any two points on the circle.

In the opposite figure :



If A , B and C belong to the circle M ,

then each of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$

is a chord of the circle M

- **Notice that :** $\overline{EF}$ is not a chord of the circle M because $E \notin$ the circle M

UNIT 4

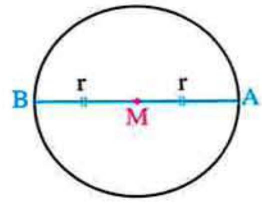
The diameter of the circle

It is a chord passing through the centre of the circle.

In the opposite figure :

If M is a circle , $\overline{AB}$ is a chord of it

, $M \in \overline{AB}$, then $\overline{AB}$ is a diameter of the circle M



Notice that :

1. Any circle has an infinite number of diameters and all of them are equal in length.
2. The diameter of the circle is the longest chord of the circle , and its length = 2 r

In the opposite figure :

AMC is a triangle in which :

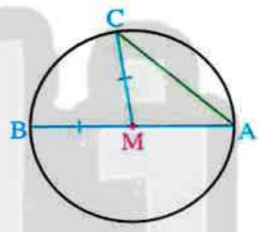
AM + MC > AC (The inequality of the triangle)

$\because$ MC = MB = r

$\therefore$ AM + MB > AC

$\therefore$ AB > AC

i.e. The diameter $\overline{AB}$ is longer than the chord $\overline{AC}$



The circumference of the circle and its area

- The circumference of the circle = $2 \pi r$
- The area of the circle = πr^2

Where r is the radius length of the circle , and π is a constant ratio for any circle , where it represents the ratio between the circumference of the circle and its diameter length and equals 3.14 approximately or $\frac{22}{7}$ approximately.

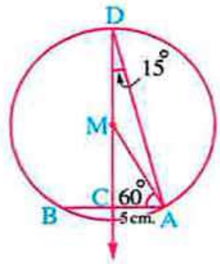
For example: The circle whose radius length is 7 cm. :

1. Its circumference = $2 \pi r \approx 2 \times \frac{22}{7} \times 7 = 44$ cm.
2. Its area = $\pi r^2 \approx \frac{22}{7} \times (7)^2 = 154$ cm^2.

Example 1 **In the opposite figure :**

m (∠ ADM) = 15° , m (∠ MAC) = 60° and AC = 5 cm.

Calculate : The area of the circle M ($\pi \approx 3.14$)



Solution

Given m (∠ ADM) = 15° , m (∠ MAC) = 60° , AC = 5 cm.

R.T.F. The area of the circle M

Proof $\because$ MA = MD = r

$\therefore$ m (∠ DAM) = m (∠ D) = 15°

$\because$ m (∠ AMC) = m (∠ D) + m (∠ DAM) = 15° + 15° = 30°

$\because$ m (∠ MAC) = 60°

$\therefore$ m (∠ ACM) = 90°

$\therefore$ AM = 2 AC = 2 × 5 = 10 cm.

$\therefore$ r = 10 cm.

$\therefore$ The area of the circle M = πr^2

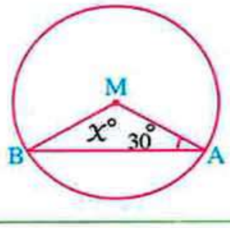
$\approx 3.14 \times (10)^2$

$= 314 \text{ cm}^2$ (The req.)

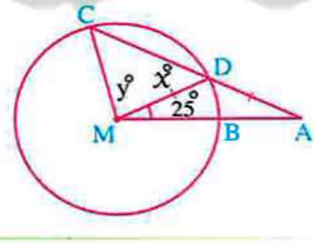
TRY 1 by yourself

In each of the following figures, find the value of the used symbol in measuring where M is the centre of the circle :

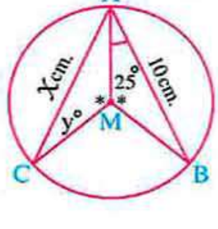
1



2

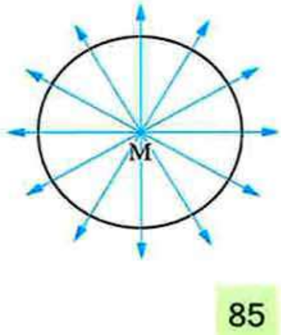


3



Symmetry in the circle

- Any straight line passing through the centre of the circle is an axis of symmetry of it.
- Since the number of these straight lines are infinite , then the circle has an infinite number of axes of symmetry.



UNIT 4

Important corollaries

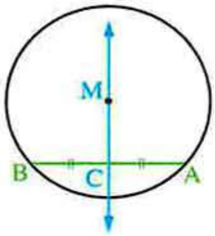
Corollary 1

The straight line passing through the centre of the circle and the midpoint of any chord of it is perpendicular to this chord.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M

and C is the midpoint of $\overline{AB}$, then $\overleftrightarrow{MC} \perp \overline{AB}$

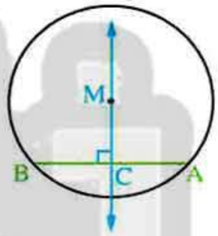


Corollary 2

The straight line passing through the centre of the circle and perpendicular to any chord of it bisects this chord.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M and $\overleftrightarrow{MC} \perp \overline{AB}$, where $C \in \overline{AB}$, then C is the midpoint of $\overline{AB}$



Corollary 3

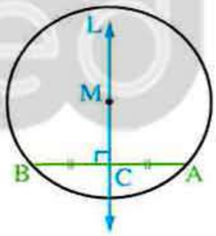
The perpendicular bisector to any chord of a circle passes through the centre of the circle.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M , C is the midpoint of $\overline{AB}$

and the straight line L $\perp$ $\overline{AB}$ from the point C ,

then M $\in$ the straight line L



From the previous, we deduce that :

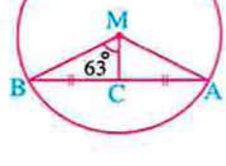
The axis of symmetry of any chord of a circle passes through its centre , so this axis is also an axis of symmetry of the circle.

Example 2 **In the opposite figure :**

If $\overline{AB}$ is a chord of the circle M ,

C is the midpoint of $\overline{AB}$ and m (∠ BMC) = 63°

Find : m (∠ MAB)



Solution

Given $\overline{AB}$ is a chord of the circle M , C is the midpoint of $\overline{AB}$ and m (∠ BMC) = 63°

R.T.F. m (∠ MAB)

Proof ∵ $\overline{AB}$ is a chord of the circle M , C is the midpoint of $\overline{AB}$

∴ $\overline{MC} \perp \overline{AB}$

∴ m (∠ MCB) = 90°

∵ m (∠ BMC) = 63°

∴ m (∠ MBC) = 180° – (90° + 63°) = 27°

∵ MA = MB = r

∴ Δ ABM is an isosceles triangle.

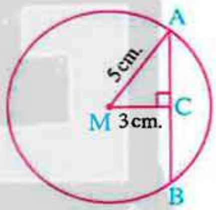
∴ m (∠ MAB) = m (∠ MBA) = 27° (The req.)

Example 3 **In the opposite figure :**

If $\overline{AB}$ is a chord of the circle M whose radius length = 5 cm.

, $\overline{MC} \perp \overline{AB}$ and MC = 3 cm.

Find : The length of $\overline{AB}$



Solution

Given $\overline{AB}$ is a chord of the circle M , $\overline{MC} \perp \overline{AB}$, MC = 3 cm. and MA = 5 cm.

R.T.F. The length of $\overline{AB}$

Proof ∵ $\overline{MC} \perp \overline{AB}$

∴ m (∠ MCA) = 90°

∴ $(AC)^2 = (AM)^2 - (MC)^2 = 25 - 9 = 16$ (Pythagoras' theorem)

∴ AC = 4 cm.

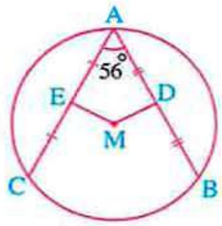
∵ $\overline{MC} \perp \overline{AB}$

∴ C is the midpoint of $\overline{AB}$

∴ AB = 8 cm. (The req.)

Example 4 $\overline{AB}$ and $\overline{AC}$ are two chords of a circle M in two opposite sides of M where m (∠ BAC) = 56° , if D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.
Find : m (∠ DME)

Solution



Given $\overline{AB}$ and $\overline{AC}$ are two chords of the circle M
, m (∠ A) = 56° , D is the midpoint of $\overline{AB}$
and E is the midpoint of $\overline{AC}$

R.T.F. m (∠ DME)

Proof ∵ D is the midpoint of $\overline{AB}$

∴ $\overline{MD} \perp \overline{AB}$

∴ m (∠ MDA) = 90°

∵ E is the midpoint of $\overline{AC}$

∴ $\overline{ME} \perp \overline{AC}$

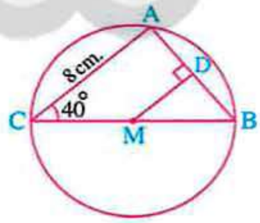
∴ m (∠ MEA) = 90°

∵ The sum of measures of the interior angles of the quadrilateral = 360°

∴ m (∠ DME) = 360° – (90° + 90° + 56°) = 124° (The req.)

TRY 2 by yourself

In the opposite figure :



$\overline{BC}$ is a diameter of the circle M , $\overline{AB}$ is a chord of it ,
$\overline{MD} \perp \overline{AB}$ where $\overline{MD} \cap \overline{AB} = \{D\}$
, m (∠ C) = 40° and AC = 8 cm. ***Find :***

1 m (∠ DMB)   2 The length of $\overline{MD}$

Answers *of try by yourself*

1 [1] x = 120°   [2] x = 50° , y = 80°   [3] x = 10 cm. , y = 25°

2 [1] m (∠ DMB) = 40°   [2] MD = 4 cm.

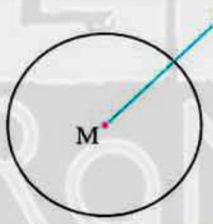
LESSON 2

Position of a point and a straight line with respect to a circle

First Position of a point with respect to a circle

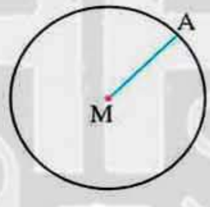
If M is a circle of radius length r and A is a point in its plane , then :

1 A is outside the circle M



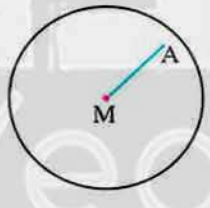
If MA > r

2 A is on the circle M



If MA = r

3 A is inside the circle M



If MA < r

Example 1 **A circle M is with radius length 5 cm. and A is a point in its plane. Complete the following table :**

If	Then	Because
1 MA = 5 cm.	A lies .... the circle M	....
2 MA = 3 cm.	A lies .... the circle M	....
3 MA = 6.5 cm.	A lies .... the circle M	....
4 MA = zero	A lies .... the circle M and A becomes the .... of the circle	....

Solution

1 on , because MA = r
2 inside , because MA < r
3 outside , because MA > r
4 inside , centre , because MA = 0

Example 2 **If M is a circle of radius length 5 cm. and A is a point in the plane of the circle.**
MA = (3 x – 1) cm. , find the values of x when A lies.

1 Outside the circle. 2 Inside the circle. 3 On the circle.

Solution

1 $\because$ A lies outside the circle. $\therefore MA > r$ $\therefore 3x - 1 > 5$
$\therefore 3x > 6$ $\therefore x > 2$ i.e. $x \in]2, \infty[$

2 $\because$ A lies inside the circle. $\therefore MA < r$, $\because MA \geq 0$
$\therefore 0 \leq MA < r$ $\therefore 0 \leq 3x - 1 < 5$ $\therefore 1 \leq 3x < 6$
$\therefore \frac{1}{3} \leq x < 2$ i.e. $x \in [\frac{1}{3}, 2[$

3 $\because$ A lies on the circle. $\therefore MA = r$ $\therefore 3x - 1 = 5$
$\therefore 3x = 6$ $\therefore x = 2$

TRY 1 by yourself

If M is a circle , its diameter length = 12 cm. and A is a point in its plane , complete the following :

1 If MA = 12 cm. , then A lies .... the circle M
2 If MA = 6 cm. , then A lies .... the circle M
3 If MA = 3 cm. , then A lies .... the circle M

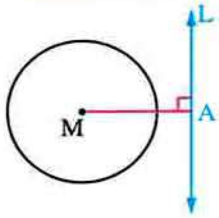
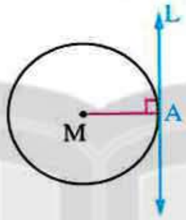
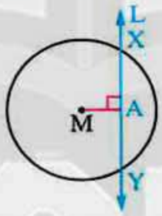
Second Position of a straight line with respect to a circle

If M is a circle with radius length r and L is a straight line in its plane , and we draw $\overrightarrow{MA} \perp L$ to cut it at the point A , then MA is the length of the perpendicular line segment from the centre of the circle to the straight line L

If we compare between MA and r , then we have three probabilities.

1 $MA > r$ 2 $MA = r$ 3 $MA < r$

Each of these probabilities determine a position of the straight line L with respect to the circle M as shown in the following table :

If	Then	The figure	Note that
1 $MA > r$	The straight line L lies outside the circle M		• $L \cap$ the circle $M = \varnothing$ • $L \cap$ the surface of the circle $M = \varnothing$
2 $MA = r$	The straight line L is a tangent to the circle M at A A is called "the point of tangency"		• $L \cap$ the circle $M = \{A\}$ • $L \cap$ the surface of the circle $M = \{A\}$
3 $MA < r$	The straight line L is a secant to the circle M		• $L \cap$ the circle $M = \{X , Y\}$ • $L \cap$ the surface of the circle $M = \overline{XY}$ • $\overline{XY}$ is called the chord of intersection

Example 3 **Let M be a circle of radius length = 5 cm. , $\overline{MA} \perp$ the straight line L where $A \in L$**

Complete the following :

1. If AM = 5 cm. , then the straight line L ....
2. If AM = $5\sqrt{3}$ cm. , then the straight line L ....
3. If AM = $\frac{5}{2}$ cm. , then the straight line L ....

Solution

1. is a tangent to the circle M
2. lies outside the circle M
3. is a secant to the circle M

TRY 2 by yourself

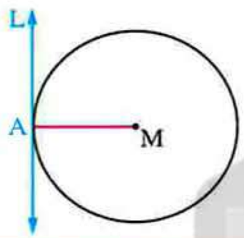
***If* M *is a circle of radius length* r , $\overline{MA} \perp$ *the straight line* L and $A \in L$,**
complete the following :

1. If MA = r , then the straight line L ....
2. If MA = 5 r , then the straight line L ....
3. If MA = $\frac{1}{2}$ r , then the straight line L ....

UNIT 4

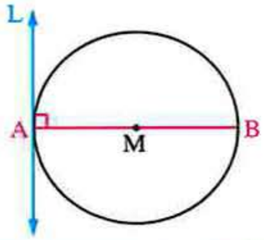
Two important facts

1 The tangent to a circle is perpendicular to the radius drawn from the point of tangency.



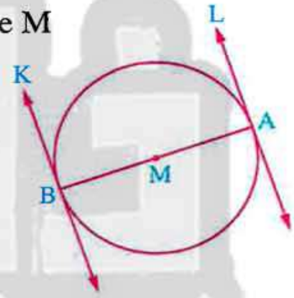
i.e. if the straight line L is a tangent to the circle M at the point A ,
then $\overline{MA} \perp L$

2 The straight line which is perpendicular to the diameter of a circle at one of its endpoints is a tangent to the circle.



i.e. if $\overline{AB}$ is a diameter of the circle M and the straight line $L \perp \overline{AB}$ at the point A ,
then L is a tangent to the circle M at the point A

Example 4 **In the opposite figure :** $\overline{AB}$ is a diameter of the circle M
, the straight line L is a tangent to the circle at A
, and the straight line K is a tangent to the circle at B
Prove that : The straight line L // the straight line K



Solution

Given $\overline{AB}$ is a diameter of the circle M , the two straight lines L and K are two tangents to the circle at A and B respectively.

R.T.P. The straight line L // the straight line K

Proof
$\because$ The straight line L is a tangent to the circle at A
$\therefore$ The straight line $L \perp \overline{MA}$
$\therefore$ The straight line $L \perp \overleftrightarrow{AB}$ (1)
$\because$ The straight line K is a tangent to the circle at B
$\therefore$ The straight line $K \perp \overline{MB}$
$\therefore$ The straight line $K \perp \overleftrightarrow{AB}$ (2)
From (1) and (2) : $\therefore$ The straight line L // the straight line K (Q.E.D.)

From the previous example , we deduce that :

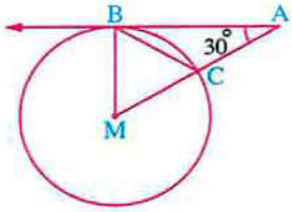
The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.

Example 5 **In the opposite figure :**

$\overrightarrow{AB}$ is a tangent to the circle M at the point B ,

$\overline{AM} \cap$ the circle M $= \{C\}$, m $(\angle A) = 30°$

Prove that : AC = BC



Solution

Given $\overrightarrow{AB}$ is a tangent to the circle M at B , m $(\angle A) = 30°$

R.T.P. AC = BC

Proof $\because$ $\overrightarrow{AB}$ is a tangent to the circle M at B $\therefore \overline{MB} \perp \overrightarrow{AB}$

, $\because$ In ΔAMB : m $(\angle A) = 30°$, m $(\angle ABM) = 90°$

$\therefore$ m $(\angle M) = 180° - (30° + 90°) = 60°$

, $\because$ ΔMBC is isosceles (MB = MC = r)

, $\because$ m $(\angle M) = 60°$ $\therefore$ ΔMBC is equilateral

$\therefore$ m $(\angle MBC) = 60°$

$\therefore$ m $(\angle ABC) =$ m $(\angle ABM) -$ m $(\angle MBC) = 90° - 60° = 30°$

$\therefore$ In ΔABC : m $(\angle A) =$ m $(\angle ABC) = 30°$

$\therefore$ AC = BC (Q.E.D.)

TRY 3 by yourself

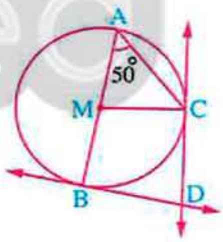
In the opposite figure :

$\overleftrightarrow{BD}$ and $\overleftrightarrow{CD}$ are two tangents to the circle M at B and C where

$\overleftrightarrow{BD} \cap \overleftrightarrow{CD} = \{D\}$, $\overline{BA}$ is a diameter of the circle M

, m $(\angle BAC) = 50°$

Find : m $(\angle BDC)$



Answers *of try by yourself*

1 (1) outside (2) on (3) inside

2 (1) is a tangent to the circle M (2) lies outside the circle M

(3) is a secant to the circle M

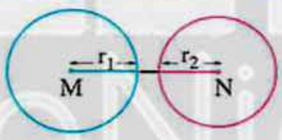
3 m $(\angle BDC) = 80°$

LESSON 3

Position of a circle with respect to another circle

- Let M and N be two circles , their radii lengths are r_1 and r_2 respectively , $r_1 > r_2$, then the straight line passing through the two points M and N is called "the line of centres".
- The two circles M and N takes one of the following six positions :

If $MN > r_1 + r_2$

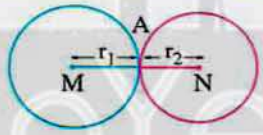


Then the two circles are : Distant

Notice that :

- The circle M $\cap$ the circle N = $\varnothing$
- The surface of circle M $\cap$ the surface of circle N = $\varnothing$

If $MN = r_1 + r_2$

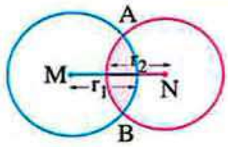


Then the two circles are : Touching externally

Notice that :

- The circle M $\cap$ the circle N = $\{A\}$
- The surface of circle M $\cap$ the surface of circle N = $\{A\}$

If $r_1 - r_2 < MN < r_1 + r_2$

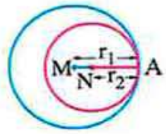


Then the two circles are :
Intersecting

Notice that :

- The circle M $\cap$ the circle N = $\{A , B\}$
- The surface of circle M $\cap$ the surface of circle N = the surface of the shaded part.

If $MN = r_1 - r_2$

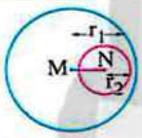


Then the two circles are :
Touching internally

Notice that :

- The circle M $\cap$ the circle N = $\{A\}$
- The surface of circle M $\cap$ the surface of circle N = the surface of circle N

If $MN < r_1 - r_2$



Then the two circles are :
One inside the other
(the circle N is inside the circle M)

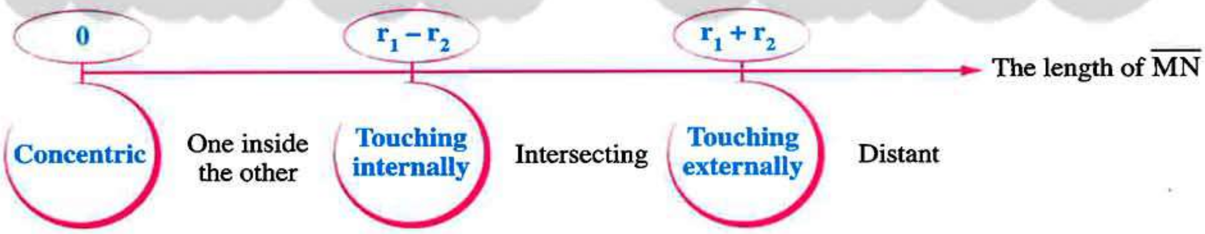
If MN = zero

Then the two circles are :
Concentric

Notice in the two cases that :

- The circle M $\cap$ the circle N = $\varnothing$
- The surface of circle M $\cap$ the surface of circle N = the surface of circle N

Summary



Remarks

From the previous summary , we notice that :

1. If M and N are two distant circles , then : $MN \in]\, r_1 + r_2 , \infty [$
2. If M and N are two intersecting circles , then : $MN \in]\, r_1 - r_2 , r_1 + r_2 [$
3. If M and N (one of them is inside the other) , then : $MN \in]\, 0 , r_1 - r_2 [$

Example 1 **If the radius length of the circle M = 5 cm. , the radius length of the circle N = 3 cm. , determine the position of each of them with respect to the other in each of the following cases :**

1. MN = 2 cm.
2. MN = 8 cm.
3. MN = zero
4. MN = 10 cm.
5. MN = 1 cm.
6. MN = 5 cm.

Solution $\because r_1 = 5$ cm. , $r_2 = 3$ cm. $\therefore$ $\boxed{r_1 + r_2 = 8 \text{ cm.}}$, $\boxed{r_1 - r_2 = 2 \text{ cm.}}$

1. $\because$ MN = 2 cm. $\therefore MN = r_1 - r_2$
   $\therefore$ The two circles are touching internally.
2. $\because$ MN = 8 cm. $\therefore MN = r_1 + r_2$
   $\therefore$ The two circles are touching externally.
3. $\because$ MN = zero. $\therefore$ The two circles are concentric.
4. $\because$ MN = 10 cm. $\therefore MN > r_1 + r_2$
   $\therefore$ The two circles are distant.
5. $\because$ MN = 1 cm. $\therefore MN < r_1 - r_2$
   $\therefore$ One of the two circles is inside the other.
6. $\because$ MN = 5 cm. $\therefore r_1 - r_2 < MN < r_1 + r_2$
   $\therefore$ The two circles are intersecting.

From the previous example , we notice that :

1. As MN = 10 cm.
   i.e. MN $\in]\,8 , \infty\,[$, then the two circles are distant.
2. As MN = 1 cm.
   i.e. MN $\in]\,0 , 2\,[$, then the two circles (one of them is inside the other).
3. As MN = 5 cm.
   i.e. MN $\in]\,2 , 8\,[$, then the two circles are intersecting.

TRY 1 by yourself

Let M and N be two circles , their radii lengths are 4 cm. and 9 cm. respectively. Complete the following :

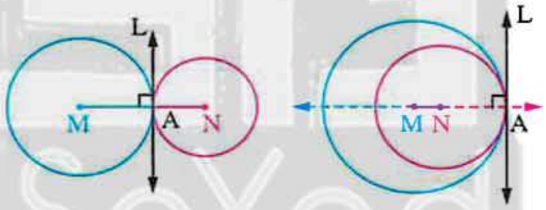
1. If the two circles M and N are touching externally , then : MN ....
2. If the two circles M and N are touching internally , then : MN ....
3. If the two circles M and N are intersecting , then : MN ....
4. If the two circles M and N are concentric , then : MN ....
5. If the two circles M and N are distant , then : MN ....
6. If the two circles M and N are one of them is inside the other , then : MN ....

Corollary 1

The line of centres of two touching circles passes through the point of tangency and is perpendicular to the common tangent at this point.

In the two opposite figures :

If the two circles M and N are touching at A (the point of tangency) , the straight line L is a common tangent to them at A , then $A \in \overleftrightarrow{MN}$ and $\overleftrightarrow{MN} \perp$ the straight line L



Corollary 2

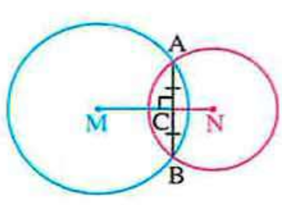
The line of centres of two intersecting circles is perpendicular to the common chord and bisects it.

In the opposite figure :

If M and N are two circles intersecting at A and B ,

then $\overleftrightarrow{MN} \perp \overline{AB}$, $\overleftrightarrow{MN}$ bisects $\overline{AB}$ i.e. AC = BC

This mean that $\overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$

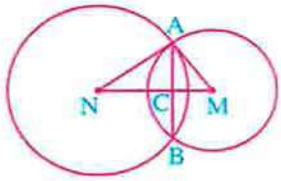


Example 2 **In the opposite figure :**

M and N are two intersecting circles at A and B ,

$\overline{AB} \cap \overline{MN} = \{C\}$ If MA = 6 cm. , NA = 8 cm.

and MN = 10 cm. , **Find :** The length of $\overline{AB}$



Solution

Given Two circles M and N are intersecting at A and B ,

MA = 6 cm. , NA = 8 cm. and MN = 10 cm.

R.T.F. The length of $\overline{AB}$

Proof In Δ AMN : $\because (MA)^2 = 36$, $(NA)^2 = 64$ and $(MN)^2 = 100$

$\therefore (MN)^2 = (MA)^2 + (NA)^2$

$\therefore$ m ($\angle$ MAN) = 90° (converse of Pythagoras' theorem)

$\because$ The common chord $\overline{AB}$ intersects $\overline{MN}$ at C

$\therefore \overline{AB} \perp \overline{MN}$ $\therefore AC = \frac{AM \times AN}{MN} = \frac{6 \times 8}{10} = 4.8$ cm.

$\therefore AB = 4.8 \times 2 = 9.6$ cm. (The req.)

Example 3 **In the opposite figure :**

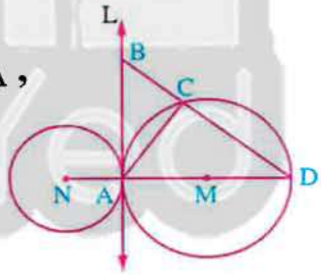
M and N are two circles touching externally at A ,

the straight line L is a common tangent to them at A ,

$\overline{AD}$ is a diameter of the circle M ,

B $\in$ L where AB = MN = 6 cm. ,

$\overline{BD} \cap$ the circle M = $\{C\}$, where BC = 3.6 cm.



1 **Prove that :** m ($\angle$ ACD) = 90°

2 **Find :** The length of $\overline{AN}$

Solution

Given M and N are two circles touching externally at A

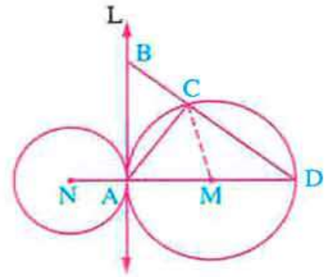
and the straight line L is the common tangent at A

AB = MN = 6 cm. and BC = 3.6 cm.

R.T.P. m ($\angle$ ACD) = 90°

R.T.F. The length of $\overline{AN}$

Construction Draw $\overline{MC}$



Proof

$\because$ CM = DM = MA = r

$\therefore$ Δ ACD in which : CM = $\frac{1}{2}$AD

$\because$ M is the midpoint of $\overline{AD}$

$\therefore$ $\overline{CM}$ is a median of Δ ACD

$\therefore$ m ($\angle$ ACD) = 90° (First req.)

$\because$ The two circles are touching externally at A and the straight line L is the common tangent at A

$\therefore$ $\overleftrightarrow{MN} \perp$ L

$\therefore$ ABD is a triangle in which : m ($\angle$ DAB) = 90° , $\overline{AC} \perp \overline{BD}$

$\therefore$ $(AB)^2$ = BC × BD (Euqlids)

$\therefore$ 36 = 3.6 × BD $\therefore$ BD = $\frac{36}{3.6}$ = 10 cm.

$\because$ $(AD)^2 = (BD)^2 - (AB)^2$ (Pythagoras)

$\therefore$ $(AD)^2$ = 100 – 36 = 64 $\therefore$ AD = 8 cm.

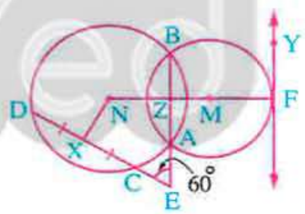
$\therefore$ MA = $\frac{8}{2}$ = 4 cm. , MN = MA + AN

$\because$ MN = 6 cm.

$\therefore$ AN = MN – MA = 6 – 4 = 2 cm. (Second req.)

TRY 2 by yourself

In the opposite figure :



M and N are two intersecting circles at A and B , X is the midpoint of the chord $\overline{CD}$ and $\overrightarrow{DC} \cap \overrightarrow{BA} = \{E\}$ where m ($\angle$ E) = 60°

$\overleftrightarrow{YF}$ touches the circle M at F where $\overrightarrow{NM} \cap \overleftrightarrow{YF} = \{F\}$

1 ***Find :*** m ($\angle$ ZNX)

2 ***Prove that :*** $\overleftrightarrow{YF}$ // $\overline{AB}$

Answers *of try by yourself*

1 1 MN = 13 cm. 2 MN = 5 cm. 3 MN ∈]5 , 13[

4 MN = 0 5 MN ∈]13 , ∞[6 MN ∈]0 , 5[

2 1 m ($\angle$ ZNX) = 120°

2 Prove by yourself [Hint : $\overline{MF} \perp \overleftrightarrow{YF}$, $\overline{MN}$ (line of centres) $\perp$ $\overline{AB}$ (common chord)]

LESSON 4

Identifying the circle

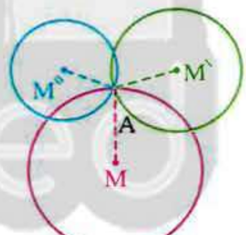
The circle is identified if we know :

1 its centre. 2 its radius length.

In the following, we will study the possibility of identifying (drawing) the circle under certain conditions.

First Drawing a circle passing through a given point

If A is a given point in the plane and the required is drawing a circle passing through the point A



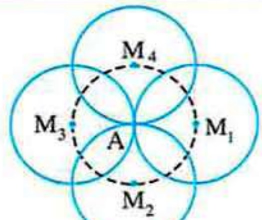
- Assume any other point in the plane as M , then take it as a centre using the compasses , draw a circle with the centre M and radius length = MA , then it will pass through the point A
- **Similarly** , you can draw another circle whose centre is M` and its radius length is M`A , then it will pass through the point A or draw a circle whose centre is M`` and its radius length = M``A , then it will pass through the point A and so on ....

i.e.

You can draw an infinite number of circles passing through a given point.

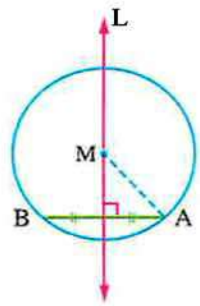
Notice that :

If the circles required to be drawn to pass through A are congruent (their radii are equal in length) , then all the centres of these circles lie on a circle which is congruent to these circles and its centre is the point A as shown in the opposite figure.



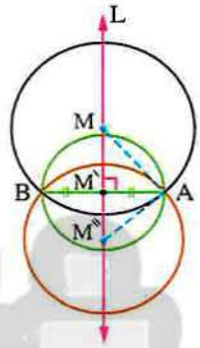
Second — Drawing a circle passing through two given points

If A and B are two given points in the plane and the required is drawing a circle passing through the two points A and B :



- We know that the centre of any circle passing through the two points A and B should be equidistant from A and B

  ∴ The centre of any circle passing through A and B should lie on the axis of symmetry of $\overline{AB}$ which is the straight line that is perpendicular to it from its midpoint , therefore , we draw the straight line L that represents the axis of symmetry of $\overline{AB}$



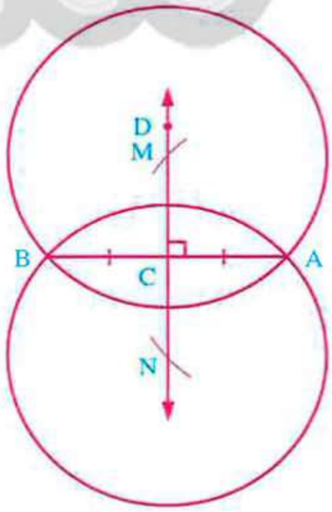
- We take any point on the straight line L as M , then we draw the circle whose centre is M and its radius length = MA (or MB) , then it will pass through the two points A and B
- **Similarly** , we can draw another circle whose centre is M' and its radius length = $M'A$, then it will pass through the two points A and B or we can draw a circle whose centre is M'' and its radius length = $M''A$, then it will pass through the two points A and B

i.e.

There is an infinite number of circles that can be drawn to pass through the two points A and B and all their centres lie on the axis of symmetry of $\overline{AB}$

Example 1 **Using the geometric instruments, draw $\overline{AB}$ with length 3 cm., then draw a circle passing through the two points A and B with radius length 2 cm.**

How many solutions can be obtained ?



Solution

- Draw $\overline{AB}$ such that AB = 3 cm.
- Draw $\overleftrightarrow{CD} \perp \overline{AB}$ from its midpoint C , then $\overleftrightarrow{CD}$ is the axis of symmetry of $\overline{AB}$
- Open the compasses with a length of 2 cm. using A or B as a centre and draw two arcs cutting $\overleftrightarrow{CD}$ at M and N
- By the same opening , use M as a centre and draw a circle and similarly , do the same at N to draw another circle.

So you have two circles with radius length 2 cm. passing through the two points A and B

Remarks

- **If $\overline{AB}$ is a line segment and the required is drawing a circle passing through the two points A and B , then :**

1. If $r > \frac{1}{2} AB$, then we can draw two circles (as shown in the previous example).
2. If $r = \frac{1}{2} AB$, then we can draw one and only one circle (it is the smallest circle) passing through the two points A and B , hence $\overline{AB}$ is a diameter of it and its centre is the midpoint of $\overline{AB}$
3. If $r < \frac{1}{2} AB$, then it is impossible to draw any circle.

- **Any two circles do not intersect at more than two points.**

TRY 1 by yourself

Using the geometric instruments , draw $\overline{XY}$ where XY = 4 cm. , then draw a circle passing through the two points X and Y and its radius length is 2 cm.
How many possible solutions are there ?

Third Drawing a circle passing through three given points

If A , B and C are three points in the plane and the required is drawing a circle passing through the three points A , B and C :

- We know that : In order that the circle passes through the two points A and B , then its centre should lie on the axis of symmetry of $\overline{AB}$, say L_1 , and in order that the circle passes through the two points B and C , then its centre should lie on the axis of symmetry of $\overline{BC}$, say L_2

$\therefore$ The centre of the circle that passes through the three points A , B and C lies on each of L_1 and L_2

Then we must distinguish between two cases :

1 If the points A , B and C are collinear as in figure (1)
, then the two straight lines L_1 and L_2 are parallel not intersecting.

In this case , it is impossible to draw a circle passing through the three points A , B and C

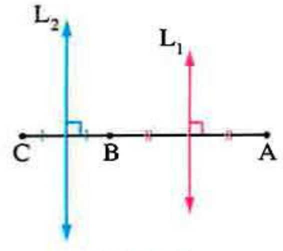


Fig (1)

i.e.
It is impossible to draw a circle passing through three collinear points.

2 If the points A , B and C are not collinear as in figure (2) , then L_1 and L_2 intersect at one point as M , then M is the centre of the required circle which passes through the three points A , B and C , then the radius length of this circle = MA = MB = MC

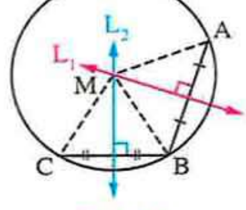


Fig (2)

i.e.

For any three non-collinear points , there is a unique circle can be drawn to pass through them.

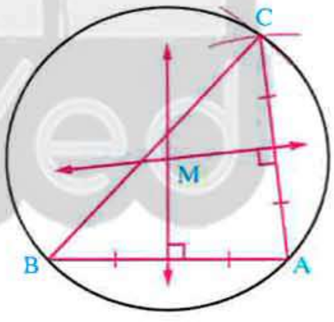
Notice that :

There is a unique circle passing through three points as A , B and C which are not collinear and the centre of this circle is the point of intersection of any two axes of symmetry of the axes of the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$

Example 2 **Using the geometric instruments , draw Δ ABC in which : AB = AC = 3 cm. and BC = 4 cm. , then draw the circle which passes through the points A , B and C**

Solution

- Draw $\overline{AB}$ of length 3 cm.
- Open the compasses with a length of 3 cm. , then use A as a centre and draw an arc , then open the compasses with a length of 4 cm. , then use B as a centre and draw an arc to cut the previous arc at the point C , then draw $\overline{AC}$ and $\overline{BC}$



- Draw the axes of $\overline{AB}$ and $\overline{AC}$ to intersect at M
- Open the compasses with length = AM (or BM or CM) and use M as a centre , then draw the required circle.

TRY 2 by yourself

Using the geometric tools , draw Δ XYZ in which m ($\angle$ X) = 80° , XY = 4 cm. and XZ = 3 cm. , then draw the circle which passes through the points X , Y and Z

UNIT 4

Corollary 1

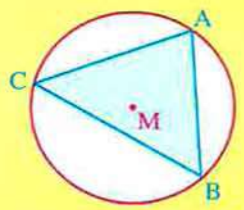
The circle which passes through the vertices of a triangle is called the circumcircle of this triangle.

- The triangle whose vertices lie on a circle is called the inscribed triangle of this circle.

In the opposite figure :

M is the circumcircle of Δ ABC

or Δ ABC is the inscribed triangle of the circle M



Corollary 2

The perpendicular bisectors of the sides of a triangle intersect at a point which is the centre of the circumcircle of the triangle.

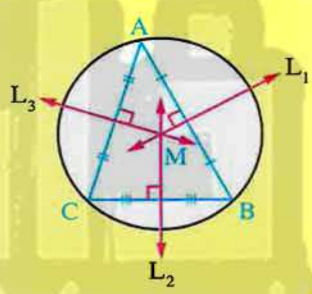
In the opposite figure :

If the straight lines L_1 , L_2 and L_3

are the axes of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively

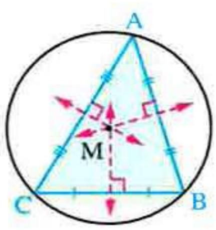
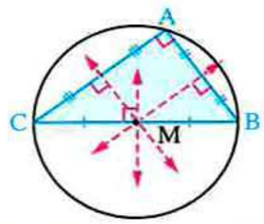
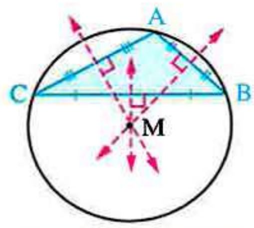
and $L_1 \cap L_2 \cap L_3 = \{M\}$,

then the point M is the centre of the circumcircle of Δ ABC



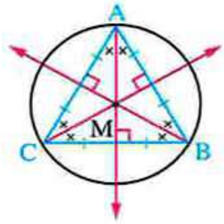
Remark

The position of the centre of the circumcircle of the triangle as M differs according to the type of the triangle as shown in the following table :

The acute-angled triangle	The right-angled triangle	The obtuse-angled triangle
		
M is inside the triangle	M is the midpoint of the hypotenuse	M is outside the triangle

• **A special case :**

The centre of the circumcircle of the equilateral triangle is :



- The point of intersection of its sides axes.
- The point of intersection of its altitudes.
- The point of intersection of its medians.
- The point of intersection of the bisectors of its interior angles.

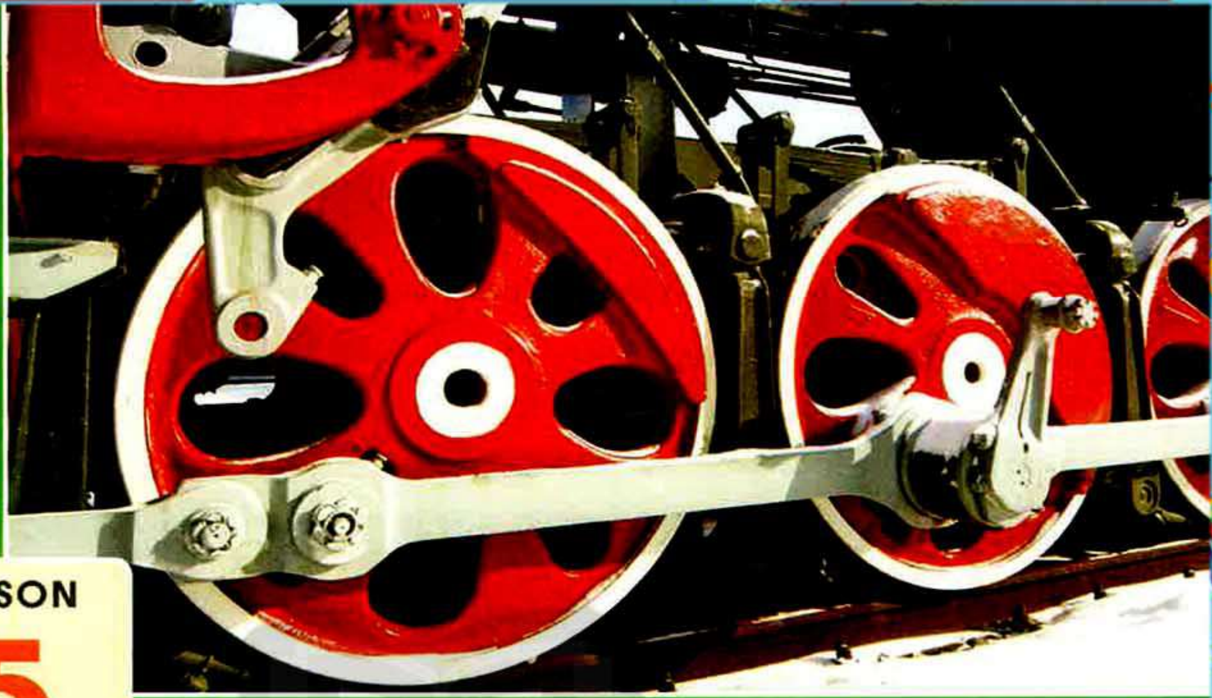
Remark

“ We can draw a circle passing through the vertices of (the rectangle , the square or the isosceles trapezium) while we cannot draw a circle passing through the vertices of (the parallelogram , the rhombus or the trapezium which is not isosceles). ”

Answers of try by yourself

1 Draw by yourself , only one solution.

2 Draw by yourself.



LESSON
5

The relation between the chords of a circle and its centre

Prelude

If M is a circle , $\overline{AB}$ and $\overline{CD}$ are two chords of it at distances MX and MY from its centre respectively , then the following figures determine three cases of these chords with respect to their distances from the centre of the circle M

Fig. (1)

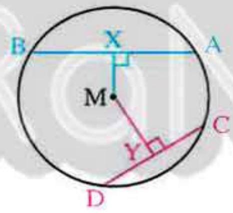


Fig. (2)

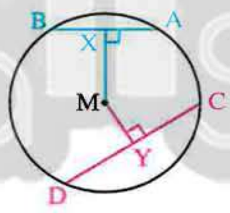
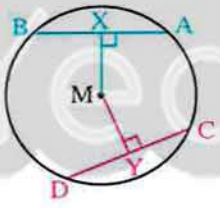


Fig. (3)



Using the ruler , you can check by yourself the truth of the following information :

Fig. (1)	Fig. (2)	Fig. (3)
AB > CD , MX < MY	AB < CD , MX > MY	AB = CD , MX = MY

From the figures (1) , (2) , we deduce that :

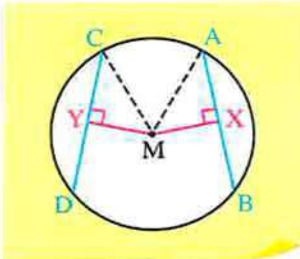
The closer the chord is from the centre of the circle , the longer its length is and vise versa.

i.e. There is a relation between the length of the chord and its distance from the centre of the circle.

The relation between the chords of a circle and its centre :

Theorem

If chords of a circle are equal in length, then they are equidistant from the centre.



Given $AB = CD$, $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{CD}$

R.T.P. $MX = MY$

Construction Draw $\overline{MA}$ and $\overline{MC}$

Proof $\because \overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$

$\therefore AX = \frac{1}{2} AB$

$\because \overline{MY} \perp \overline{CD}$ $\therefore$ Y is the midpoint of $\overline{CD}$

$\therefore CY = \frac{1}{2} CD$

$\because AB = CD$ (given) $\therefore AX = CY$

$\because \Delta\Delta$ AXM and CYM , both have $\begin{cases} AX = CY \text{ (by proof)} \\ MA = MC = r \\ m(\angle AXM) = m(\angle CYM) = 90^\circ \end{cases}$

$\therefore \Delta AXM \equiv \Delta CYM$, then we get : $MX = MY$ (Q.E.D.)

Corollary

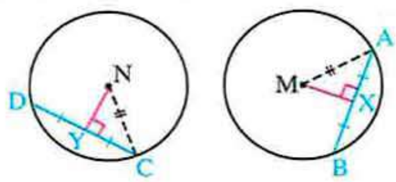
In congruent circles, chords which are equal in length are equidistant from the centres.

In the opposite figure :

If M and N are two congruent circles ,

$AB = CD$, $\overline{MX} \perp \overline{AB}$ and $\overline{NY} \perp \overline{CD}$,

then $MX = NY$



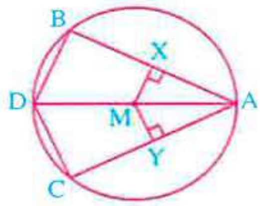
Example 1 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , $\overline{AD}$ is a diameter of it

$\overrightarrow{MX} \perp \overline{AB}$ and intersects it at X

$\overrightarrow{MY} \perp \overline{AC}$ and intersects it at Y

Prove that : $BD = DC$



Solution

Given $AB = AC$, $\overline{AD}$ is a diameter of the circle M , $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AC}$

R.T.P. $BD = DC$

Proof

$\because \overline{MX} \perp \overline{AB}$ $\quad \therefore$ X is the midpoint of $\overline{AB}$

$\because \overline{MY} \perp \overline{AC}$ $\quad \therefore$ Y is the midpoint of $\overline{AC}$

$\because AB = AC$ $\quad \therefore MX = MY$ (theorem)

In Δ ADB : $\because$ M is the midpoint of $\overline{AD}$ and X is the midpoint of $\overline{AB}$

$\therefore MX = \frac{1}{2} BD$

In Δ ADC : $\because$ M is the midpoint of $\overline{AD}$ and Y is the midpoint of $\overline{AC}$

$\therefore MY = \frac{1}{2} DC$

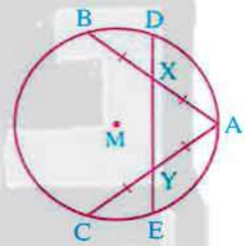
But MX = MY (by proof)

$\therefore \frac{1}{2} BD = \frac{1}{2} DC$ $\quad \therefore BD = DC$ (Q.E.D.)

Example 2 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$ and $\overleftrightarrow{XY}$ intersects the circle M at D and E

Prove that : XD = YE

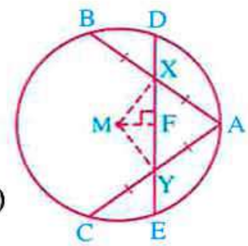


Solution

Given $AB = AC$, X is the midpoint of $\overline{AB}$ and Y is the midpoint of $\overline{AC}$

R.T.P. $XD = YE$

Construction Draw $\overline{MX}$ and $\overline{MY}$ and draw $\overline{MF} \perp \overleftrightarrow{XY}$ to intersect it at F



Proof

$\because$ X is the midpoint of $\overline{AB}$ $\quad \therefore \overline{MX} \perp \overline{AB}$

$\because$ Y is the midpoint of $\overline{AC}$ $\quad \therefore \overline{MY} \perp \overline{AC}$

$\because AB = AC$ $\quad \therefore MX = MY$ (theorem)

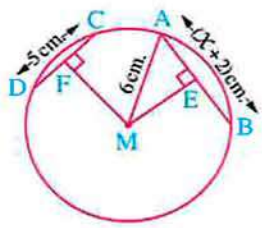
$\therefore$ In Δ MXY : MX = MY and $\overline{MF} \perp \overline{XY}$

$\therefore XF = YF$ $\quad$ (1)

$\because \overline{MF} \perp \overline{DE}$ $\quad \therefore DF = FE$ $\quad$ (2)

Subtracting (1) from (2) : $\therefore DF - XF = EF - YF$ $\quad \therefore XD = YE$ (Q.E.D.)

Example 3 **In the opposite figure :**



M is a circle , $\overline{AB}$ is a chord in it

, $\overline{ME} \perp \overline{AB}$, $\overline{MF} \perp \overline{CD}$,

AM = 6 cm. , CD = 5 cm. and AB = $(X + 2)$ cm.

If MF > ME , find the values of X which satisfy these data.

Solution

Given $\overline{ME} \perp \overline{AB}$, $\overline{MF} \perp \overline{CD}$, AM = 6 cm. , CD = 5 cm. , AB = $(X + 2)$ cm. and MF > ME

R.T.F. The values of X

Proof $\because \overline{ME} \perp \overline{AB}$, $\overline{MF} \perp \overline{CD}$ and MF > ME $\qquad \therefore$ AB > CD

$\therefore X + 2 > 5 \qquad \therefore X > 3 \qquad (1)$

$\because \overline{AB}$ is a chord not passing through the centre.

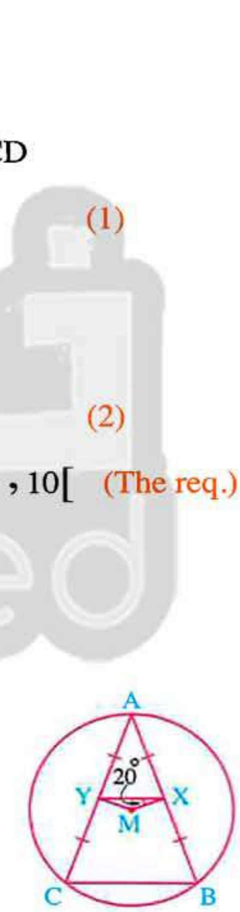
$\therefore$ AB < the length of the diameter of the circle.

$\therefore AB < 12 \qquad \therefore X + 2 < 12 \qquad \therefore X < 10 \qquad (2)$

From (1) and (2) : $\therefore 3 < X < 10 \qquad \therefore X \in]3 , 10[$ (The req.)

TRY 1 by yourself

In the opposite figure :



ABC is a triangle drawn in the circle M where AB = AC ,

X is the midpoint of $\overline{AB}$ and Y is the midpoint of $\overline{AC}$

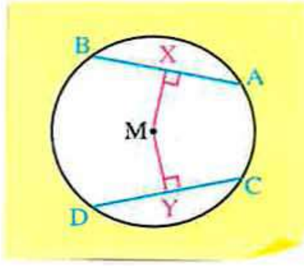
If m (∠ YXM) = 20° , **then find :** m (∠ CYX)

UNIT 4

Converse of the theorem

In the same circle (or in congruent circles) ,
chords which are equidistant from the centre (s) are equal in length.

i.e. **In the opposite figure :**

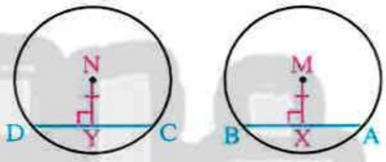


If $\overline{AB}$ and $\overline{CD}$ are two chords of the circle M ,

$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$ and MX = MY , then AB = CD

Also in the opposite figure :

If M and N are two congruent circles , $\overline{AB}$ is a chord of circle M and $\overline{CD}$ is a chord of circle N , $\overline{MX} \perp \overline{AB}$, $\overline{NY} \perp \overline{CD}$ and MX = NY , then AB = CD

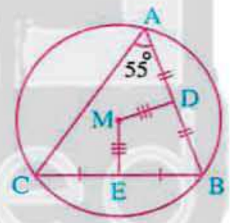


Example 4 **In the opposite figure :**

ABC is a triangle drawn inside the circle M

If m (∠ A) = 55° , D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$ and MD = ME

Find : m (∠ B)



Solution

Given m (∠ A) = 55° , D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$ and MD = ME

R.T.F. m (∠ B)

Proof

∵ D is the midpoint of $\overline{AB}$ ∴ $\overline{MD} \perp \overline{AB}$

∵ E is the midpoint of $\overline{BC}$ ∴ $\overline{ME} \perp \overline{BC}$

∵ MD = ME ∴ AB = BC

∴ m (∠ C) = m (∠ A) = 55°

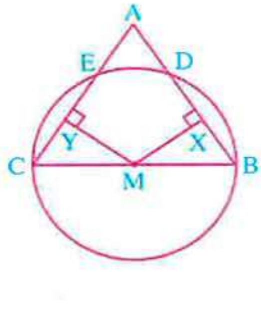
∴ m (∠ B) = 180 – (55° + 55°) = 70° (The req.)

Example 5 ABC is a triangle in which AB = AC , a circle M is drawn such that $\overline{BC}$ is a diameter of it , the circle cuts $\overline{AB}$ at D and $\overline{AC}$ at E

Draw $\overrightarrow{MX} \perp \overline{BD}$ to intersect it at X and $\overline{MY} \perp \overline{CE}$ to intersect it at Y

Prove that : BD = CE

Solution



Given AB = AC , $\overline{BC}$ is a diameter of the circle M ,

$\overline{MX} \perp \overline{BD}$ and $\overline{MY} \perp \overline{CE}$

R.T.P. BD = CE

Proof In Δ MXB and Δ MYC :

$$\begin{cases} MB = MC \text{ (two radii)} \\ m(\angle MXB) = m(\angle MYC) = 90^\circ \\ m(\angle B) = m(\angle C) \text{ (because } AB = AC) \end{cases}$$

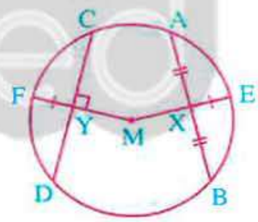
$\therefore \Delta MXB \equiv \Delta MYC$, then we deduce that : MX = MY

, $\because \overline{MX} \perp \overline{BD}$ and $\overline{MY} \perp \overline{CE}$

$\therefore$ BD = CE (Q.E.D.)

TRY 2 by yourself

In the opposite figure :



$\overline{AB}$ and $\overline{CD}$ are two chords of the circle M ,

$\overline{MF} \perp \overline{CD}$ and intersects it at Y , X is the midpoint of $\overline{AB}$

and XE = YF

Prove that : AB = CD

Answers of try by yourself

1 m ($\angle$ CYX) = 110° [Hint : Prove that $\overline{MY} \perp \overline{AC}$ and Δ XYM is an isosceles triangle]

2 Prove by yourself [Hint : Prove that MX = MY]

UNIT 5

Angles and arcs in the circle

Lessons of the unit :

1. Central angles and measuring arcs.
2. The relation between the inscribed and central angles subtended by the same arc – Well known problems.
3. Inscribed angles subtended by the same arc.
4. The cyclic quadrilateral and its properties.
5. Cases of proving the cyclic quadrilateral.
6. The relation between the tangents of a circle.
7. Angles of tangency.

Unit Objectives :

By the end of this unit, student should be able to :

- Recognize the central angle and the inscribed angle.
- Calculate the measure of an arc of a circle and calculate its length.
- Recognize the relation between the inscribed and central angles subtended by the same arc.
- Recognize the relation between the measure of the inscribed angle and the measure of its subtended arc.
- Recognize the relation among the measures of the inscribed angles subtended by the same arc.
- Recognize the inscribed angle in a semicircle.
- Recognize the cyclic quadrilateral and its properties.
- Determine when a quadrilateral be cyclic.
- Recognize the relation between two tangent-segments drawn to a circle from a point outside it.
- Recognize the angle of tangency and the relation between the angle of tangency and the inscribed angle subtended by the same arc.
- Prove that a ray drawn from one of the vertices of a triangle is a tangent to the circumcircle of this triangle.

LESSON 1

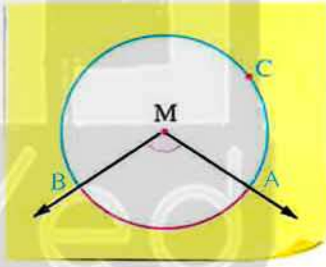
Central angles and measuring arcs

The central angle

It is the angle whose vertex is the centre of the circle and each side of its sides contains a radius of the circle.

In the opposite figure :

$\angle$ AMB is a central angle because its vertex M is the centre of the circle and each of its sides $\overrightarrow{MA}$ and $\overrightarrow{MB}$ contains a radius of the circle , they are : $\overline{MA}$ and $\overline{MB}$



Notice that :

The two sides of $\angle$ AMB divide the circle M into two arcs they are :

The minor arc AB and it is denoted by $\widehat{AB}$	The major arc AB and it is denoted by $\widehat{ACB}$ or the major arc $\widehat{AB}$

Notice that : The symbol $\widehat{AB}$ means the minor arc unless there is other stating.

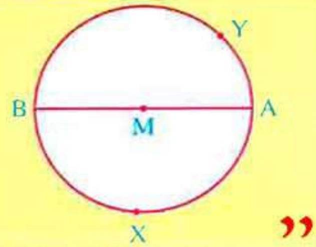
Remark

If $\overline{AB}$ is a diameter of the circle M , then :

$\angle$ AMB is a straight central angle ,

then each of $\widehat{AXB}$ and $\widehat{AYB}$

is called a semicircle.

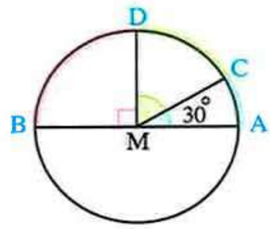


The measure of the arc

It is the measure of the central angle which subtends this arc and it is measured by the measuring units of the angle (degrees , minutes , seconds ...)

For example:

In the opposite figure :



If $\overline{AB}$ is a diameter of the circle M , C and D are two points on the circle M where m ($\angle$ AMC) = 30° , m ($\angle$ AMD) = 90° , then :

1. m ($\overset{\frown}{AC}$) = m ($\angle$ AMC) = 30°
2. m ($\overset{\frown}{CD}$) = m ($\angle$ CMD) = 90° − 30° = 60°
3. m ($\overset{\frown}{DB}$) = m ($\angle$ DMB) = 90°
4. m ($\overset{\frown}{DB}$ the major) = m ($\angle$ DMB the reflex) = 360° − 90° = 270°
5. m ($\overset{\frown}{AB}$) = m ($\angle$ AMB) = 180° (**Notice that :** $\overset{\frown}{AB}$ represents a semicircle)

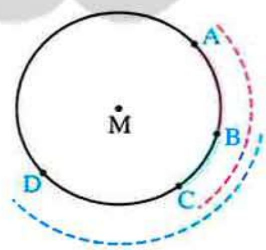
i.e.

The measure of the semicircle = 180°
and then the measure of the circle = 2 × 180° = 360°

Remark

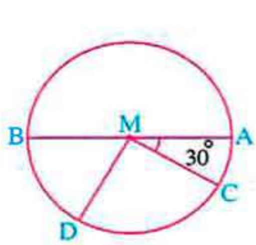
" The two adjacent arcs are two arcs in the same circle that have only one point in common. "

In the opposite figure :



- $\overset{\frown}{AB}$ and $\overset{\frown}{BC}$ are two adjacent arcs in the circle M because they have one common point only B , then it will be m ($\overset{\frown}{AB}$) + m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{ABC}$)
- $\overset{\frown}{AC}$ and $\overset{\frown}{BD}$ are not adjacent arcs because they have more than one common point (they have the common arc $\overset{\frown}{BC}$)

Example 1 **In the opposite figure :**



$\overline{AB}$ is a diameter of the circle M , m ($\angle$ AMC) = 30°

If m ($\overset{\frown}{CD}$) : m ($\overset{\frown}{DB}$) = 3 : 2

Find : m ($\overset{\frown}{AD}$)

Solution

Given $\overline{AB}$ is a diameter of the circle M ,

$m(\angle AMC) = 30°$, $m(\overset{\frown}{CD}) : m(\overset{\frown}{DB}) = 3 : 2$

R.T.F. $m(\overset{\frown}{AD})$

Proof $\because$ $\overline{AB}$ is a diameter of the circle M $\quad \therefore m(\overset{\frown}{ACB}) = 180°$

$\because m(\overset{\frown}{AC}) = m(\angle AMC) = 30° \quad \therefore m(\overset{\frown}{CDB}) = 180° - 30° = 150°$

$\because m(\overset{\frown}{CD}) : m(\overset{\frown}{DB}) = 3 : 2$

Assuming that : $m(\overset{\frown}{CD}) = 3x$, $m(\overset{\frown}{DB}) = 2x$

$\therefore 3x + 2x = 150° \quad \therefore 5x = 150° \quad \therefore x = 30°$

$\therefore m(\overset{\frown}{CD}) = 3 \times 30° = 90° \quad \therefore m(\overset{\frown}{AD}) = 30° + 90° = 120°$ (The req.)

The length of the arc

It is part of a circle's circumference proportional to its measure and it is measured by length units (centimetre , metre , ...)

To calculate the length of the arc , you can use the following rule :

$$\text{The length of the arc} = \frac{\text{the measure of the arc}}{\text{the measure of the circle}} \times \text{the circumference of the circle}$$

$$= \frac{\text{the measure of the arc}}{360°} \times 2\pi r$$

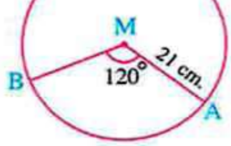
Where r is the radius length of the circle and π is the approximated ratio.

Example 2 **In the opposite figure :**

A circle of centre M , its radius length = 21 cm.

A and B are two points on the circle M

such that $m(\angle AMB) = 120°$



Find : The length of $\overset{\frown}{AB}$ $\left(\text{Consider} : \pi = \frac{22}{7}\right)$

Solution

$\because m(\angle AMB) = 120° \quad \therefore m(\overset{\frown}{AB}) = 120°$

$\therefore$ The length of $\overset{\frown}{AB} = \frac{m(\overset{\frown}{AB})}{360°} \times 2\pi r$

$= \frac{120°}{360°} \times 2 \times \frac{22}{7} \times 21 = 44$ cm.

Example 3 Find the measure of the arc which represents $\frac{1}{3}$ the measure of the circle and if the radius length of the circle is 15 cm.

Find : The length of this arc. (Consider : $\pi = 3.14$)

Solution The measure of the arc $= \frac{1}{3}$ the measure of the circle $= \frac{1}{3} \times 360° = 120°$

The length of the arc $= \frac{\text{the measure of the arc}}{360°} \times 2\pi r$

$= \frac{120°}{360°} \times 2 \times 3.14 \times 15 = 31.4$ cm.

Remark

" The length of the semicircle $= \frac{1}{2}$ the circumference of the circle $= \pi r$ length unit "

TRY 1 by yourself

1. A circle of centre M of radius length 14 cm. If A and B are two points on the circle such that m (∠ AMB) = 45° Find the length of $\overset{\frown}{AB}$ (Consider : $\pi = \frac{22}{7}$)

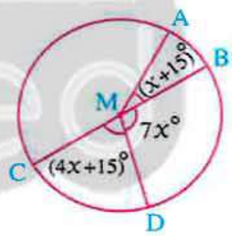
2. Find the measure of the arc which represents $\frac{2}{5}$ the measure of the circle and if the length of the diameter of the circle is 70 cm. , find the length of this arc. (Consider : $\pi = \frac{22}{7}$)

Example 4 **In the opposite figure :**

If $\overline{BC}$ is a diameter of the circle M of radius length 7 cm.

Find : 1 m ($\overset{\frown}{AD}$)

2 The length of $\overset{\frown}{AD}$ (Consider : $\pi = \frac{22}{7}$)



Solution

Given $\overline{BC}$ is a diameter in the circle M , r = 7 cm. , m (∠ AMB) = $(x + 15)°$,

m (∠ BMD) = $7x°$ and m (∠ DMC) = $(4x + 15)°$

R.T.F. 1 m ($\overset{\frown}{AD}$) 2 The length of $\overset{\frown}{AD}$

Proof ∵ $\overline{BC}$ is a diameter in the circle M ∴ m ($\overset{\frown}{BDC}$) = 180°

∴ m ($\overset{\frown}{BD}$) + m ($\overset{\frown}{DC}$) = 180°

∵ m ($\overset{\frown}{BD}$) = m (∠ BMD) = $7x°$, m ($\overset{\frown}{DC}$) = m (∠ DMC) = $(4x + 15)°$

∴ $7x° + (4x° + 15°) = 180°$ ∴ $11x° + 15° = 180°$

$\therefore 11x° = 180° - 15° = 165°$ $\quad \therefore x = \frac{165°}{11} = 15°$

$\therefore m(\overset{\frown}{BD}) = 7 \times 15° = 105°$, $m(\overset{\frown}{AB}) = 15° + 15° = 30°$

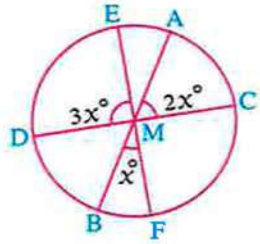
$\therefore m(\overset{\frown}{AD}) = 105° + 30° = 135°$ (First req.)

, the length of $\overset{\frown}{AD} = \frac{135°}{360°} \times 2\pi r = \frac{3}{8} \times 2 \times \frac{22}{7} \times 7 = 16.5$ cm. (Second req.)

TRY 2 *by yourself*

In the opposite figure :

$\overline{AB}$, $\overline{CD}$ and $\overline{EF}$ are three diameters in the circle M of radius length 3.5 cm.



Find : 1 $m(\overset{\frown}{AD})$ 2 The length of $\overset{\frown}{FD}$

Important corollaries

Corollary 1

In the same circle (or in congruent circles) , if the measures of arcs are equal , then the lengths of the arcs are equal , and vice versa.

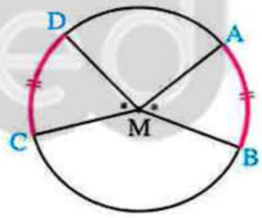
In the opposite figure :

If M is a circle in which $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$

, then the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$

and vice versa : If the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$

, then $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$



Corollary 2

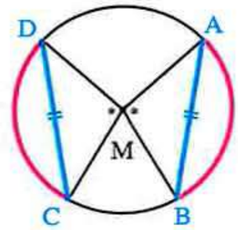
In the same circle (or in congruent circles) , if the measures of arcs are equal , then their chords are equal in length , and vice versa.

In the opposite figure :

If M is a circle in which

$m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$, then AB = CD

and vice versa : If AB = CD , then $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$



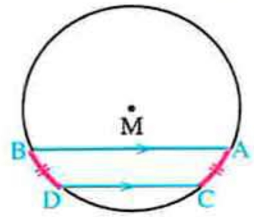
Corollary 3

If two parallel chords are drawn in a circle , then the measures of the two arcs between them are equal.

In the opposite figure :

If $\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $\overline{AB}$ // $\overline{CD}$, then m $(\overset{\frown}{AC})$ = m $(\overset{\frown}{BD})$



Corollary 4

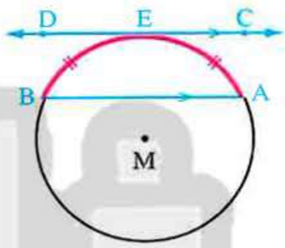
If a chord is parallel to a tangent of a circle , then the measures of the two arcs between them are equal.

In the opposite figure :

If $\overline{AB}$ is a chord in the circle M and

$\overleftrightarrow{CD}$ touches the circle M at E ,

$\overleftrightarrow{CD}$ // $\overline{AB}$, then m $(\overset{\frown}{EA})$ = m $(\overset{\frown}{EB})$



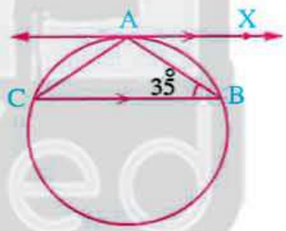
Example 5

In the opposite figure :

$\overleftrightarrow{AX}$ is a tangent to the circle at A ,

the chord $\overline{BC}$ // $\overleftrightarrow{AX}$, m (∠ B) = 35°

Find : m (∠ BAC)



Solution

Given $\overleftrightarrow{AX}$ is a tangent to the circle at A , $\overline{BC}$ // $\overleftrightarrow{AX}$, m (∠ B) = 35°

R.T.F. m (∠ BAC)

Proof ∵ $\overleftrightarrow{AX}$ // $\overline{BC}$

∴ m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{AC})$ ∴ AB = AC

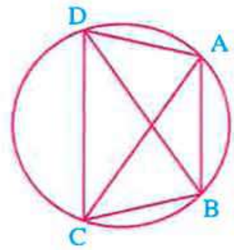
In Δ ABC :

∵ AB = AC

∴ m (∠ C) = m (∠ B) = 35°

∴ m (∠ BAC) = 180° – (35° + 35°) = 110° (The req.)

Example 6 **In the opposite figure :**



ABCD is a quadrilateral inscribed in a circle.

If AC = BD

Prove that : AD = BC

Solution

Given ABCD is a quadrilateral inscribed in a circle , AC = BD

R.T.P. AD = BC

Proof

$\because$ AC = BD (Given)

$\therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{BD})$

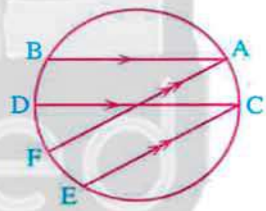
Subtracting $m(\overset{\frown}{AB})$ from both sides :

$\therefore m(\overset{\frown}{AC}) - m(\overset{\frown}{AB}) = m(\overset{\frown}{BD}) - m(\overset{\frown}{AB})$

$\therefore m(\overset{\frown}{BC}) = m(\overset{\frown}{AD})$

$\therefore$ BC = AD (Q.E.D.)

Example 7 **In the opposite figure :**



$\overline{AB}$ and $\overline{CD}$ are two parallel chords in the circle ,

$\overline{AF}$ and $\overline{CE}$ are two parallel chords in the circle.

Prove that : $m(\overset{\frown}{BD}) = m(\overset{\frown}{EF})$

Solution

Given $\overline{AB} \parallel \overline{CD}$, $\overline{AF} \parallel \overline{CE}$

R.T.P. $m(\overset{\frown}{BD}) = m(\overset{\frown}{EF})$

Proof

$\because \overline{AB} \parallel \overline{CD}$

$\therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{BD})$ (1)

$\because \overline{AF} \parallel \overline{CE}$

$\therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{EF})$ (2)

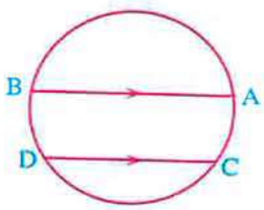
From (1) and (2) :

$\therefore m(\overset{\frown}{BD}) = m(\overset{\frown}{EF})$ (Q.E.D.)

TRY 3 *by yourself*

Complete the following :

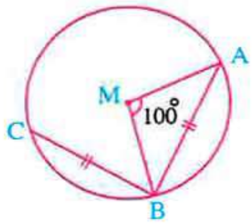
1



m $(\overset{\frown}{AB}) = 160°$, m $(\overset{\frown}{CD}) = 100°$,

then m $(\overset{\frown}{AC}) =$ ....°

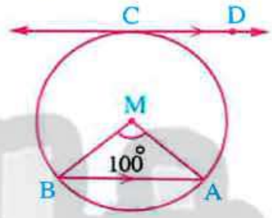
2



m $(\overset{\frown}{BC}) =$ ....°

3 $\overleftrightarrow{CD}$ is a tangent to the circle M at C ,

then m $(\overset{\frown}{AC}) =$ ....°



Answers *of try by yourself*

1 [1] 11 cm. [2] 144° , 88 cm.

2 [1] 120° [2] 5.5 cm.

3 [1] 50° [2] 100° [3] 130°

LESSON 2

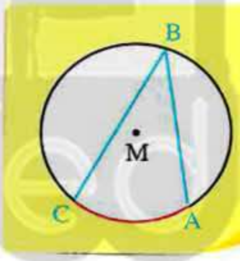
The relation between the inscribed and central angles subtended by the same arc

The inscribed angle

It is the angle whose vertex lies on the circle and its sides contain two chords of the circle.

In the opposite figure :

- ∠ ABC is an inscribed angle
  because its vertex B belongs to the circle M
  and its sides $\overrightarrow{BA}$ and $\overrightarrow{BC}$ carry the two chords
  $\overline{BA}$ and $\overline{BC}$ in the circle M



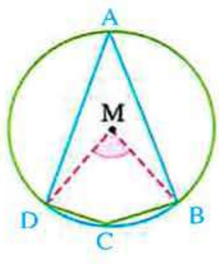
- The inscribed angle ∠ ABC is subtended by $\overset{\frown}{AC}$

Remark

"For each inscribed angle , there is one central angle subtended by the same arc."

In the opposite figure :

- The inscribed angle ∠ BAD is subtended with the central angle ∠ BMD by the arc $\overset{\frown}{BCD}$
- While the inscribed angle ∠ BCD is subtended with the reflex central angle BMD by the arc $\overset{\frown}{BAD}$



Theorem 1

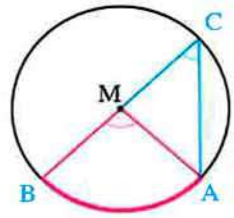
The measure of the inscribed angle is half the measure of the central angle , subtended by the same arc.

Given In the circle M : ∠ ACB is an inscribed angle , ∠ AMB is a central angle

R.T.P. m (∠ ACB) = $\frac{1}{2}$ m (∠ AMB)

Proof

The first case If M belongs to one of the sides of the inscribed angle ACB :



∵ ∠ AMB is an exterior angle of Δ AMC

∴ m (∠ AMB) = m (∠ A) + m (∠ C) (1)

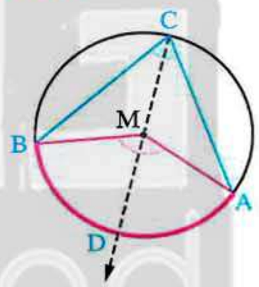
, ∵ MA = MC (two radii lengths)

∴ m (∠ A) = m (∠ C) (2)

From (1) and (2) we get : m (∠ AMB) = 2 m (∠ ACB)

∴ m (∠ ACB) = $\frac{1}{2}$ m (∠ AMB) (Q.E.D.)

The second case If M lies inside the inscribed angle ACB :



Const. Draw $\overrightarrow{CM}$ to cut the circle at D

From the first case :

m (∠ ACD) = $\frac{1}{2}$ m (∠ AMD) ,

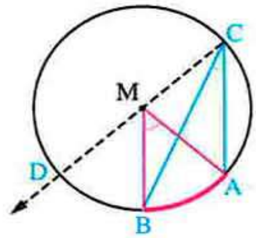
m (∠ BCD) = $\frac{1}{2}$ m (∠ BMD)

Adding :

∴ m (∠ ACD) + m (∠ BCD) = $\frac{1}{2}$ m (∠ AMD) + $\frac{1}{2}$ m (∠ BMD)

∴ m (∠ ACB) = $\frac{1}{2}$ m (∠ AMB) (Q.E.D.)

The third case If M lies outside the inscribed angle ACB:



Const. Draw $\overrightarrow{CM}$ to cut the circle at D

From the first case :

m (∠ ACD) = $\frac{1}{2}$ m (∠ AMD)

, m (∠ BCD) = $\frac{1}{2}$ m (∠ BMD)

Subtracting :

∴ m (∠ ACD) – m (∠ BCD) = $\frac{1}{2}$ m (∠ AMD) – $\frac{1}{2}$ m (∠ BMD)

∴ m (∠ ACB) = $\frac{1}{2}$ m (∠ AMB) (Q.E.D.)

Remark

The measure of the central angle **equals twice the measure of the inscribed** angle subtended by the same arc.

Example 1 **In each of the following figures :**

Find the measure of the angle denoted by the sign (?) given that M is the centre of the circle.

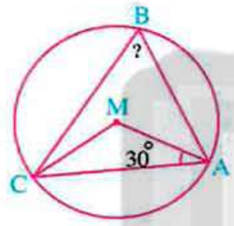


Fig. (1)

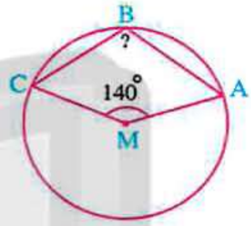


Fig. (2)

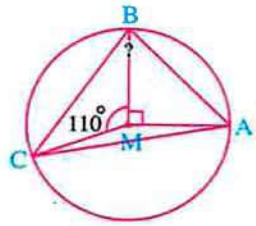


Fig.(3)

Solution

Fig. (1) In Δ AMC : $\because$ MA = MC (two radii lengths)

$\therefore$ m ($\angle$ MCA) = m ($\angle$ MAC) = 30°

$\therefore$ m ($\angle$ AMC) = 180° – (30° + 30°) = 120°

, $\because$ $\angle$ ABC is an inscribed angle and $\angle$ AMC is a central angle subtended by $\overset{\frown}{AC}$

$\therefore$ m ($\angle$ ABC) = $\frac{1}{2}$ m ($\angle$ AMC) = 60° (The req.)

Fig. (2) $\because$ m ($\angle$ AMC) = 140°

$\therefore$ m (reflex $\angle$ AMC) = 360° – 140° = 220°

, $\because$ $\angle$ ABC is an inscribed angle and reflex $\angle$ AMC is a central angle subtended by the major $\overset{\frown}{AC}$

$\therefore$ m ($\angle$ ABC) = $\frac{1}{2}$ m (reflex $\angle$ AMC) = 110° (The req.)

Fig. (3) $\because$ m ($\angle$ AMB) = 90° , m ($\angle$ BMC) = 110°

$\therefore$ m ($\angle$ AMC) = 360° – (90° + 110°) = 160°

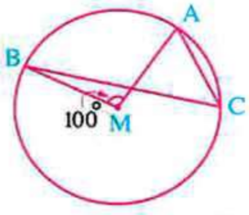
, $\because$ $\angle$ ABC is an inscribed angle subtended by the same arc $\overset{\frown}{AC}$ with the central angle AMC

$\therefore$ m ($\angle$ ABC) = $\frac{1}{2}$ m ($\angle$ AMC) = 80° (The req.)

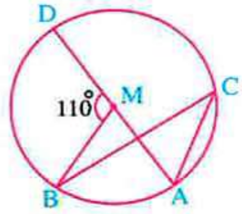
TRY 1 *by yourself*

In each of the following, find m (∠ ACB) given that M is the centre of the circle :

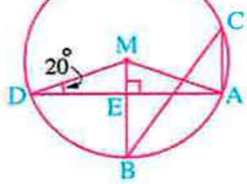
1



2



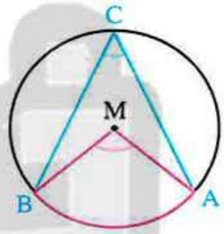
3



Corollary 1

The measure of an inscribed angle is half the measure of the subtended arc.

In the opposite figure :



m (∠ C) = $\frac{1}{2}$ m (∠ AMB)

(inscribed and central angles with common arc $\overset{\frown}{AB}$),

m (∠ AMB) = m ($\overset{\frown}{AB}$)

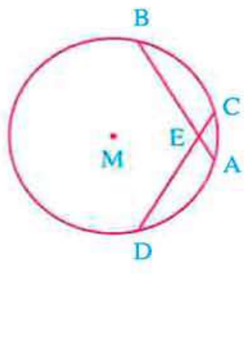
∴ m (∠ C) = $\frac{1}{2}$ m ($\overset{\frown}{AB}$)

Remark

"The measure of the arc equals twice the measure of the inscribed angle subtended by this arc."

In the previous figure : m ($\overset{\frown}{AB}$) = 2 m (∠ C)

Example 2 **In the opposite figure :**



$\overline{AB} \cap \overline{CD} = \{E\}$, AB = CD

Prove that : EA = EC

Solution

Given $\overline{AB} \cap \overline{CD} = \{E\}$, AB = CD

R.T.P. EA = EC

Construction Draw $\overline{BD}$

Proof

$\because AB = CD$ (1)

$\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$

Subtracting $m(\overset{\frown}{AC})$ from both sides :

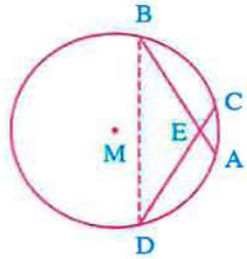
$\therefore m(\overset{\frown}{BC}) = m(\overset{\frown}{AD})$

But $m(\angle D) = \frac{1}{2} m(\overset{\frown}{BC})$, $m(\angle B) = \frac{1}{2} m(\overset{\frown}{AD})$

$\therefore m(\angle D) = m(\angle B)$

$\therefore EB = ED$ (2)

Subtracting (2) from (1) : $\therefore EA = EC$ (Q.E.D.)

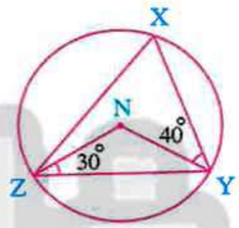


TRY 2 by yourself

In the opposite figure :

XYZ is a triangle inscribed in the circle N ,

$m(\angle XYN) = 40°$, $m(\angle NZY) = 30°$



Complete the following :

1 $m(\angle N) = ...°$

2 $m(\angle X) = ...°$

3 $m(\overset{\frown}{XZ}) = ...°$

4 $m(\overset{\frown}{XY}) = ...°$

Corollary 2

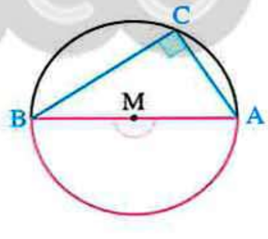
The inscribed angle in a semicircle is a right angle.

In the opposite figure :

$\because m(\angle C) = \frac{1}{2} m(\overset{\frown}{AB})$ (corollary 1)

, $\because m(\overset{\frown}{AB}) = 180°$

$\therefore m(\angle C) = 90°$



Remarks

1. The inscribed angle which is right angle is drawn in a semicircle.
2. The inscribed angle which is subtended by an arc of measure less than the measure of a semicircle is an acute angle.
3. The inscribed angle which is subtended by an arc of measure greater than the measure of a semicircle is an obtuse angle.

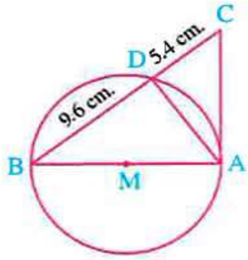
Example 3 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M , $\overline{AC}$ touches it at A ,

$\overline{BC}$ intersects it at D

If BD = 9.6 cm. , DC = 5.4 cm.

Find : The length of the radius of the circle M



Solution

Given $\overline{AB}$ is a diameter in the circle M ,

$\overline{AC}$ touches it at A ,

BD = 9.6 , DC = 5.4 cm.

R.T.F. The radius length of the circle M

Proof $\because$ $\overline{AC}$ touches the circle at A , $\overline{AB}$ is a diameter in it

$\therefore$ m ($\angle$ BAC) = 90°

$\because$ $\overline{AB}$ is a diameter in the circle M

$\therefore$ m ($\angle$ ADB) = 90°

$\therefore$ Δ ABC is right-angled at A , $\overline{AD} \perp \overline{BC}$

$\therefore$ $(AB)^2 = BD \times BC = 9.6 \times (9.6 + 5.4) = 144$

$\therefore$ AB = 12 cm.

$\therefore$ The radius length of the circle M equals 6 cm. (The req.)

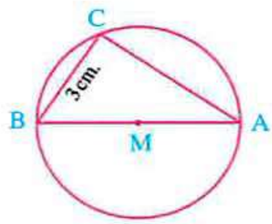
TRY 3 by yourself

In the opposite figure :

$\overline{AB}$ is a diameter in a circle M ,

the radius length = 2.5 cm.

If BC = 3 cm. ,



Find :

1 The perimeter of Δ ABC

2 The area of Δ ABC

UNIT 5

Well known problems on theorem (1) and its corollaries

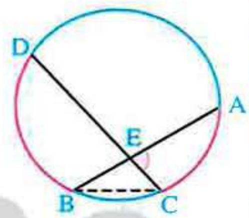
Well known problem 1

If two chords intersect at a point inside a circle , then the measure of the included angle equals half of the sum of the two measures of the two opposite arcs.

Given $\overline{AB}$, $\overline{CD}$ are two chords in a circle intersecting at the point E

R.T.P.
1. $m(\angle AEC) = \frac{1}{2}\left[m(\widehat{AC}) + m(\widehat{BD})\right]$
2. $m(\angle CEB) = \frac{1}{2}\left[m(\widehat{BC}) + m(\widehat{AD})\right]$

Construction Draw $\overline{BC}$ (or $\overline{AD}$)



Proof

$\because \angle AEC$ is an exterior angle of ΔEBC

$\therefore m(\angle AEC) = m(\angle B) + m(\angle C)$

$\because m(\angle B) = \frac{1}{2} m(\widehat{AC})$, $m(\angle C) = \frac{1}{2} m(\widehat{BD})$

$\therefore m(\angle AEC) = \frac{1}{2} m(\widehat{AC}) + \frac{1}{2} m(\widehat{BD})$

$= \frac{1}{2}\left[m(\widehat{AC}) + m(\widehat{BD})\right]$ (Q.E.D. 1)

Similarly , if we draw $\overline{AC}$ (or $\overline{BD}$) , we can prove that :

$m(\angle CEB) = \frac{1}{2}\left[m(\widehat{BC}) + m(\widehat{AD})\right]$ (Q.E.D. 2)

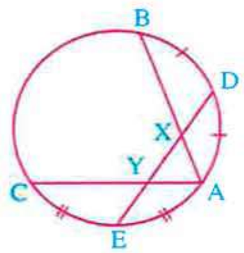
Example 4 $\overline{AB}$ and $\overline{AC}$ are two chords of a circle , D is the midpoint of $\widehat{AB}$, E is the midpoint of $\widehat{AC}$, $\overline{DE}$ is drawn to cut $\overline{AB}$ at X and $\overline{AC}$ at Y

Prove that : ΔAXY is isosceles.

Solution

Given D is the midpoint of $\widehat{AB}$, E is the midpoint of $\widehat{AC}$

R.T.P. ΔAXY is isosceles



Proof

$\because m(\angle AXE) = \frac{1}{2}\left[m(\widehat{AE}) + m(\widehat{BD})\right]$

$\therefore m(\angle AYD) = \frac{1}{2}\left[m(\widehat{CE}) + m(\widehat{AD})\right]$

$\because$ D is the midpoint of $\widehat{AB}$, E is the midpoint of $\widehat{AC}$

$\therefore m(\widehat{AE}) = m(\widehat{CE})$, $m(\widehat{BD}) = m(\widehat{AD})$

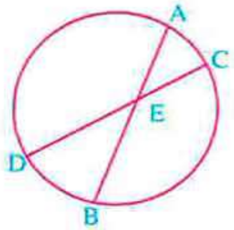
$\therefore m(\angle AXE) = m(\angle AYD)$ $\therefore AX = AY$

$\therefore \Delta AXY$ is isosceles. (Q.E.D.)

TRY 4 by yourself

Complete the following :

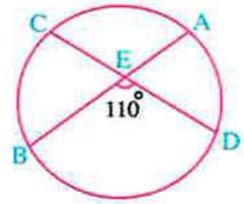
1



If m $(\overset{\frown}{AC})$ + m $(\overset{\frown}{BD}) = 80°$,

then m (∠ AEC) = ...°

2

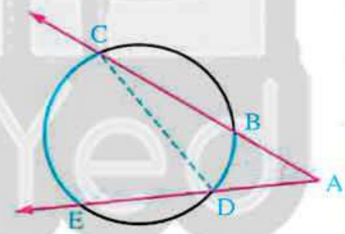


If m (∠ DEB) = 110° ,

m $(\overset{\frown}{BC}) = 70°$, then m $(\overset{\frown}{AD})$ = ...°

Well known problem 2

If two rays carrying two chords in a circle are intersecting outside it, then the measure of their intersecting angle equals half of the measure of the major arc subtracted from it half of the measure of the minor arc in which both are included by the two sides of this angle.



Given $\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$

R.T.P. m (∠ A) = $\frac{1}{2}$ [m $(\overset{\frown}{CE})$ – m $(\overset{\frown}{BD})$]

Construction Draw $\overline{CD}$ (or $\overline{BE}$)

Proof ∵ ∠ CDE is an exterior of Δ ADC

∴ m (∠ CDE) = m (∠ A) + m (∠ C)

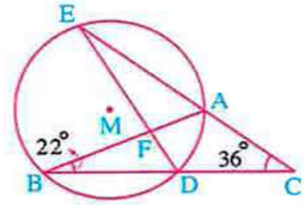
∴ m (∠ A) = m (∠ CDE) – m (∠ C)

∵ m (∠ CDE) = $\frac{1}{2}$ m $(\overset{\frown}{CE})$, m (∠ C) = $\frac{1}{2}$ m $(\overset{\frown}{BD})$

∴ m (∠ A) = $\frac{1}{2}$ m $(\overset{\frown}{CE})$ – $\frac{1}{2}$ m $(\overset{\frown}{BD})$

= $\frac{1}{2}$ [m $(\overset{\frown}{CE})$ – m $(\overset{\frown}{BD})$] (Q.E.D.)

Example 5 In the opposite figure :



$\overrightarrow{EA} \cap \overrightarrow{BD} = \{C\}$, m ($\angle$ C) = 36° ,

m ($\angle$ ABD) = 22° **Find :** m ($\widehat{BE}$)

Solution

Given $\because$ $\overrightarrow{EA} \cap \overrightarrow{BD} = \{C\}$, m ($\angle$ C) = 36° , m ($\angle$ ABD) = 22°

R.T.F. m ($\widehat{BE}$)

Proof $\because$ m ($\angle$ ABD) = 22° $\therefore$ m ($\widehat{AD}$) = 2 m ($\angle$ ABD) = 44°

$\because$ $\overrightarrow{EA} \cap \overrightarrow{BD} = \{C\}$ $\therefore$ m ($\angle$ C) = $\frac{1}{2}$ [m ($\widehat{BE}$) − m ($\widehat{AD}$)]

$\therefore$ 36° = $\frac{1}{2}$ [m ($\widehat{BE}$) − 44°] $\therefore$ 72° = m ($\widehat{BE}$) − 44°

$\therefore$ m ($\widehat{BE}$) = 116° (The req.)

Another proof $\because$ $\angle$ EAB is an exterior angle of Δ ACB

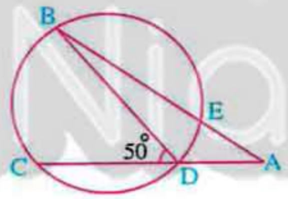
$\therefore$ m ($\angle$ EAB) = m ($\angle$ C) + m ($\angle$ B) = 36° + 22° = 58°

$\therefore$ m ($\widehat{EB}$) = 2 m ($\angle$ EAB) = 2 × 58° = 116° (The req.)

TRY 5 by yourself

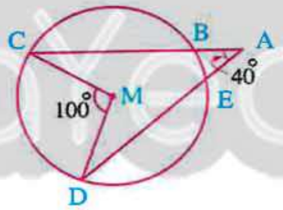
Complete the following :

1



If m ($\angle$ BDC) = 50° , m ($\widehat{DE}$) = 30° , then m ($\angle$ A) = ...°

2



If m ($\angle$ A) = 40° , m ($\angle$ CMD) = 100° , then m ($\widehat{BE}$) = ...°

Answers of try by yourself

1 (1) 50° (2) 35° (3) 35°

2 (1) 120° (2) 60° (3) 140° (4) 100°

3 (1) 12 cm. (2) 6 cm.²

4 (1) 40° (2) 70°

5 (1) 35° (2) 20°

LESSON 3

Inscribed angles subtended by the same arc

Theorem 2

In the same circle , the measures of all inscribed angles subtended by the same arc are equal.

Given: $\angle C$, $\angle D$ and $\angle E$ are inscribed angles subtended by $\overset{\frown}{AB}$

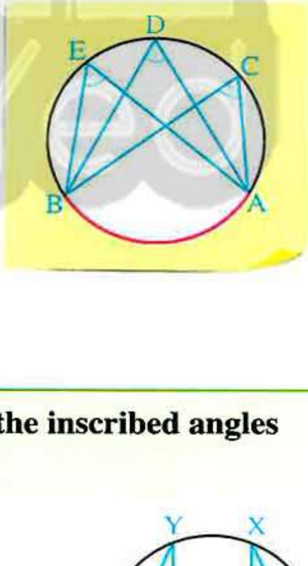
R.T.P. $m(\angle C) = m(\angle D) = m(\angle E)$

Proof: $\because m(\angle C) = \frac{1}{2} m(\overset{\frown}{AB})$

$, m(\angle D) = \frac{1}{2} m(\overset{\frown}{AB})$

$, m(\angle E) = \frac{1}{2} m(\overset{\frown}{AB})$

$\therefore m(\angle C) = m(\angle D) = m(\angle E)$ (Q.E.D.)



Corollary

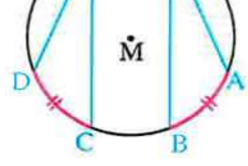
In the same circle (or in any number of circles) the measures of the inscribed angles subtended by arcs of equal measures are equal.

i.e. In the circle M

If $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$,

then $m(\angle X) = m(\angle Y)$

In this case, the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$

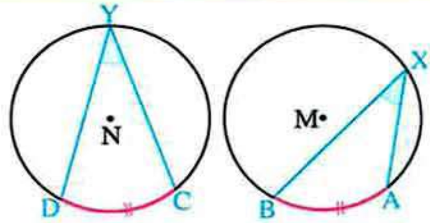


Also : If M and N are two congruent circles

and m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{CD}$) ,

then m (∠ X) = m (∠ Y)

In this case , the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$

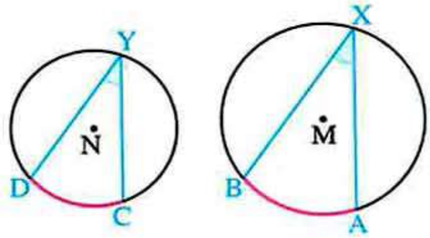


Similarly : In any two circles M and N

If m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{CD}$) ,

then m (∠ X) = m (∠ Y)

In this case , the length of $\overset{\frown}{AB}$ ≠ the length of $\overset{\frown}{CD}$



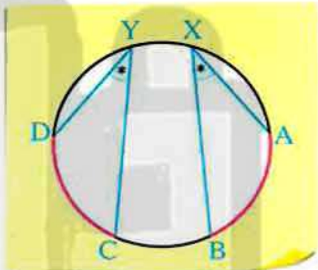
The converse of the previous corollary is true also

i.e. **In the same circle (or in any number of circles) the inscribed angles of equal measures subtend arcs of equal measures.**

In the opposite figure :

If m (∠ X) = m (∠ Y) ,

then m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{CD}$)

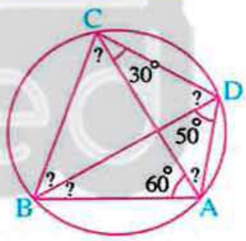


Example 1 **In the opposite figure :**

m (∠ BAC) = 60° , m (∠ ADB) = 50°

, m (∠ ACD) = 30°

Find : The measures of the angles denoted by the sign (?)



Solution

Given m (∠ BAC) = 60° , m (∠ ADB) = 50° , m (∠ ACD) = 30°

R.T.F. The measures of the angles denoted by the sign (?)

Proof m (∠ ACB) = m (∠ ADB) = 50° (inscribed angles subtended by $\overset{\frown}{AB}$)

, m (∠ ABD) = m (∠ ACD) = 30° (inscribed angles subtended by $\overset{\frown}{AD}$)

, m (∠ BDC) = m (∠ BAC) = 60° (inscribed angles subtended by $\overset{\frown}{BC}$)

, ∵ The sum of measures of the interior angles of Δ ACD = 180°

∴ m (∠ DAC) = 180° – (30° + 50° + 60°) = 40°

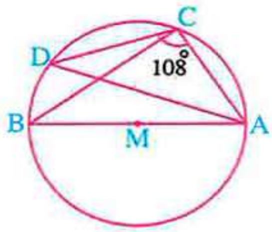
∴ m (∠ DBC) = m (∠ DAC) = 40°(inscribed angles subtended by $\overset{\frown}{DC}$)

(The req.)

Example 2 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M , m (∠ ACD) = 108°

Find : m (∠ BAD)



Solution

Given $\overline{AB}$ is a diameter in the circle M , m (∠ ACD) = 108°

R.T.F. m (∠ BAD)

Proof ∵ $\overline{AB}$ is a diameter in the circle M ∴ m (∠ ACB) = 90°

∴ m (∠ BCD) = m (∠ ACD) – m (∠ ACB) = 108° – 90° = 18°

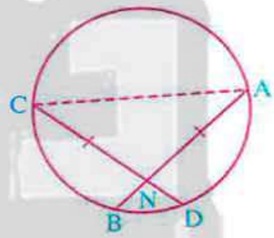
∴ m (∠ BAD) = m (∠ BCD) = 18° (inscribed angles subtended by $\overset{\frown}{BD}$)

(The req.)

Example 3 $\overline{AB}$ and $\overline{CD}$ are two chords of a circle intersecting at the point N

If AN = NC **Prove that :** AB = CD

Solution



Given AN = NC

R.T.P. AB = CD

Construction Draw $\overline{AC}$

Proof In Δ NAC : ∵ NA = NC (given)

∴ m (∠ C) = m (∠ A) ∴ m $(\overset{\frown}{AD})$ = m $(\overset{\frown}{BC})$

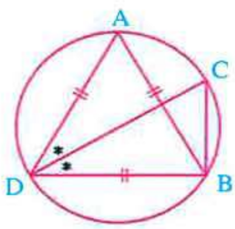
Adding m $(\overset{\frown}{DB})$ to both sides :

∴ m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{CD})$ ∴ AB = CD (Q.E.D.)

TRY 1 by yourself

Complete , given that M is the centre of the circle :

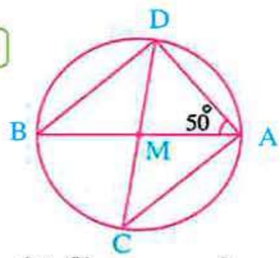
1



• m (∠ C) = ...°

• m (∠ CBA) = ...°

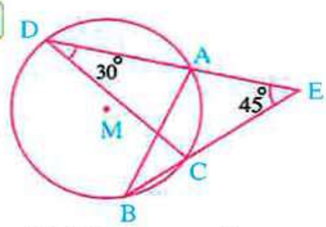
2



• m (∠ C) = ...°

• m (∠ BDC) = ...°

3



• m (∠ B) = ...°

• m (∠ DAB) = ...°

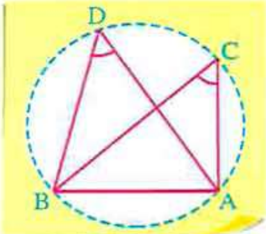
UNIT 5

The converse of theorem 2

If two angles subtended by the same base and on the same side of it have the same measure , then their vertices are on an arc of a circle and the base is a chord of it.

In the opposite figure :

If $\angle$ C and $\angle$ D are drawn on the base $\overline{AB}$ and on the same side of it , m ($\angle$ C) = m ($\angle$ D) , then the points A , B , D and C lie on a unique circle , then $\overline{AB}$ is a chord of it.

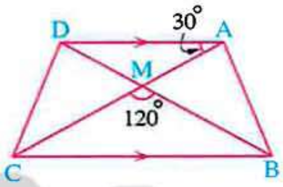


Example 4 **In the opposite figure :**

ABCD is a quadrilateral in which : $\overline{AD}$ // $\overline{BC}$

m ($\angle$ DAC) = 30°, m ($\angle$ BMC) = 120°

Prove that : The points A , B , C and D pass through them one circle.



Solution

Given ABCD is a quadrilateral , $\overline{AD}$ // $\overline{BC}$, m ($\angle$ DAC) = 30° , m ($\angle$ BMC) = 120°

R.T.P. The points A , B , C and D pass through them one circle.

Proof $\because$ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore$ m ($\angle$ ACB) = m ($\angle$ DAC) = 30° (alternate angles)

In Δ BMC : m ($\angle$ MBC) = 180° – (120° + 30°) = 30°

$\therefore$ m ($\angle$ DAC) = m ($\angle$ DBC)

and they are drawn on $\overline{DC}$ and on one side of it.

$\therefore$ The points A , B , C and D pass through them one circle. (Q.E.D.)

TRY 2 by yourself

In the opposite figure :

ABCD is a quadrilateral whose diagonals intersected at E , m ($\angle$ BDC) = 60° , AB = BC = AC

Prove that : the points A , B , C and D pass through them one circle.

Answers *of try by yourself*

1 [1] m ($\angle$ C) = 60° , m ($\angle$ CBA) = 30° [2] m ($\angle$ C) = 40° , m ($\angle$ BDC) = 40°

[3] m ($\angle$ B) = 30° , m ($\angle$ DAB) = 75°

2 Prove by yourself [Hint : Prove that m ($\angle$ BAC) = m ($\angle$ BDC)]

LESSON

4

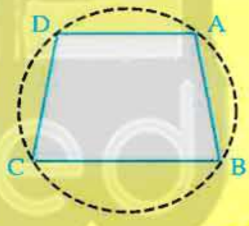
The cyclic quadrilateral and its properties

The cyclic quadrilateral

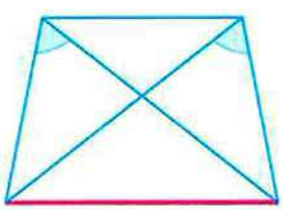
The cyclic quadrilateral is a quadrilateral whose four vertices belong to one circle.
i.e. we can draw a unique circle passes through its four vertices.

In the opposite figure :

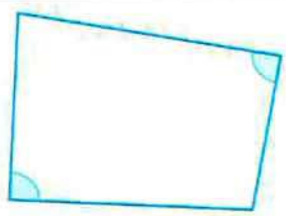
ABCD is a cyclic quadrilateral because its four vertices lie on one circle.



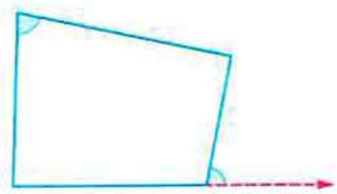
Properties of the cyclic quadrilateral



Each two angles are drawn on a side of its sides as a base and on one side of it are equal in measure



Each two opposite angles are supplementary



The measure of the exterior angle at any vertex equals the measure of the interior angle opposite to its adjacent

and next we will study each property in details with clarification by the examples.

UNIT 5

In the cyclic quadrilateral , each two angles drawn on a side of its sides as a base and on one side of it are equal in measure.

"Because they are two inscribed angles subtended by the same arc".

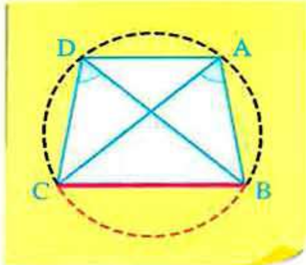
For example:

In the opposite figure :

If ABCD is a cyclic quadrilateral

, then m (∠ BAC) = m (∠ BDC)

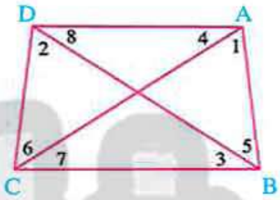
"drawn on $\overline{BC}$ and on one side of it"



Generally

If the figure ABCD is a cyclic quadrilateral , then we can deduce the following :

- m (∠ 1) = m (∠ 2) , m (∠ 3) = m (∠ 4) ,
  m (∠ 5) = m (∠ 6) , m (∠ 7) = m (∠ 8)
- The line segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$, $\overline{AC}$ and $\overline{BD}$ are chords in this circle.

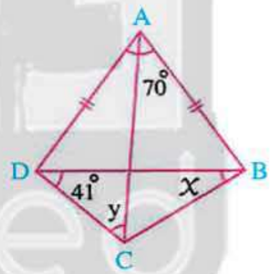


Example 1 **In the opposite figure :**

ABCD is a cyclic quadrilateral in which AB = AD ,

m (∠ BAD) = 70° and m (∠ BDC) = 41°

Find : The value of each of x and y



Solution

Given ABCD is a cyclic quadrilateral , AB = AD ,

m (∠ BAD) = 70° , m (∠ BDC) = 41°

R.T.F. The value of each of x and y

Proof ∵ ABCD is a cyclic quadrilateral

∴ m (∠ BAC) = m (∠ BDC) = 41° (drawn on the same base $\overline{BC}$)

∵ m (∠ BAD) = 70° ∴ m (∠ CAD) = 70° – 41° = 29°

∵ m (∠ CBD) = m (∠ CAD) (drawn on the same base $\overline{CD}$)

∴ x = 29°

∵ Δ ABD in which AB = AD

∴ m (∠ ABD) = m (∠ ADB) = $\frac{180° - 70°}{2}$ = 55°

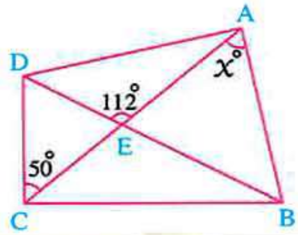
∵ m (∠ ACD) = m (∠ ABD) (drawn on the same base $\overline{AD}$)

∴ y = 55° (The req.)

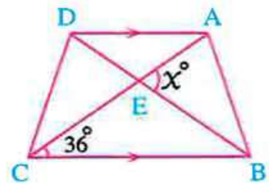
TRY 1 by yourself

In each of the two following figures , if ABCD is a cyclic quadrilateral , find the value of x :

1

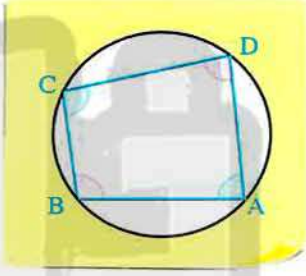


2



Theorem 3

In a cyclic quadrilateral , each two opposite angles are supplementary.



Given ABCD is a cyclic quadrilateral

R.T.P.

1 $m(\angle A) + m(\angle C) = 180°$

2 $m(\angle B) + m(\angle D) = 180°$

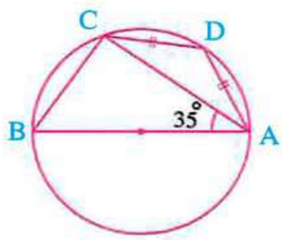
Proof $\because m(\angle A) = \frac{1}{2}\, m(\widehat{BCD})$ and $m(\angle C) = \frac{1}{2}\, m(\widehat{BAD})$

$\therefore m(\angle A) + m(\angle C) = \frac{1}{2}\left[m(\widehat{BCD}) + m(\widehat{BAD})\right]$

$= \frac{1}{2}$ the measure of the circle $= \frac{1}{2} \times 360° = 180°$

Similarly : $m(\angle B) + m(\angle D) = 180°$ (Q.E.D.)

Example 2 **In the opposite figure :**



ABCD is a quadrilateral inscribed in a circle ,

$\overline{AB}$ is a diameter in it and AD = DC

If $m(\angle BAC) = 35°$

Find : 1 $m(\angle D)$ 2 $m(\angle BCD)$

Solution

Given ABCD is a cyclic quadrilateral , $\overline{AB}$ is a diameter in the circle , AD = DC and $m(\angle BAC) = 35°$

R.T.F. 1 m (∠ D) 2 m (∠ BCD)

Proof ∵ $\overline{AB}$ is a diameter in the circle

∴ m (∠ ACB) = 90° ∴ m (∠ ABC) = 90° – 35° = 55°

∵ The figure ABCD is a cyclic quadrilateral (given)

∴ m (∠ B) + m (∠ D) = 180°

∴ m (∠ D) = 180° – 55° = 125° (First req.)

In Δ ADC : ∵ m (∠ D) = 125°

∴ m (∠ DAC) + m (∠ DCA) = 180° – 125° = 55°

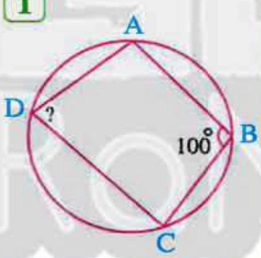
∵ DA = DC (given) ∴ m (∠ DAC) = m (∠ DCA) = $\frac{55°}{2}$ = 27.5°

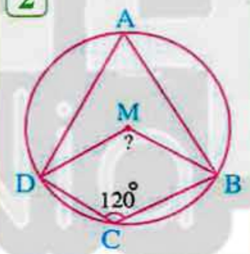
∴ m (∠ BCD) = m (∠ BCA) + m (∠ ACD)

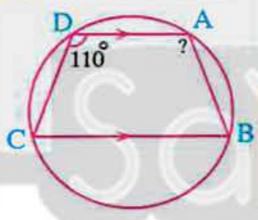
= 90° + 27.5° = 117.5° (Second req.)

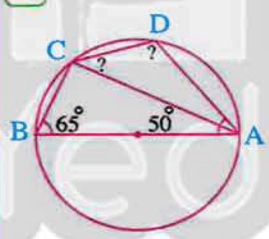
TRY 2 *by yourself*

In each of the following figures , find the measure of each angle denoted by (?) :

1


2


3


4


Corollary

The measure of the exterior angle at a vertex of a cyclic quadrilateral is equal to the measure of the interior angle at the opposite vertex.

In the opposite figure :

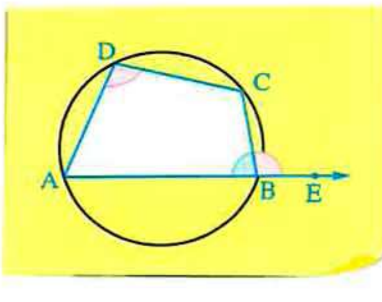
If ABCD is a cyclic quadrilateral

, ∠ CBE is an exterior angle of it ,

then m (∠ ABC) + m (∠ D) = 180°

but m (∠ ABC) + m (∠ CBE) = 180°

∴ m (∠ CBE) = m (∠ D)

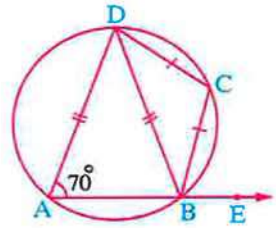


Example 3 **In the opposite figure :**

ABCD is a quadrilateral inscribed in a circle in which : m (∠ A) = 70° and E ∈ $\overrightarrow{AB}$

If CD = CB and DB = DA

Find : m (∠ EBC)



Solution

Given CD = CB , DB = DA and m (∠ A) = 70°

R.T.F. m (∠ EBC)

Proof In Δ DBA :

∵ DB = DA

∴ m (∠ DBA) = m (∠ A) = 70°

∴ m (∠ BDA) = 180° − (70° + 70°) = 40°

∵ ABCD is a cyclic quadrilateral

∴ m (∠ C) + m (∠ A) = 180°

∴ m (∠ C) = 180° − 70° = 110°

In Δ CBD :

∵ CB = CD , m (∠ C) = 110°

∴ m (∠ CDB) = m (∠ CBD) = $\frac{180° - 110°}{2}$ = 35°

∴ m (∠ CDA) = m (∠ CDB) + m (∠ BDA) = 35° + 40° = 75°

∵ ∠ EBC is an exterior angle of the cyclic quadrilateral ABCD

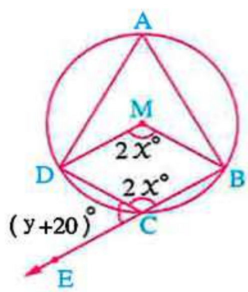
∴ m (∠ EBC) = m (∠ CDA) = 75° (The req.)

Example 4 **In the opposite figure :**

E ∈ $\overrightarrow{BC}$, m (∠ BMD) = m (∠ BCD) = 2 x°

and m (∠ DCE) = (y + 20)°

Find : The value of each of x and y



Solution

Given m (∠ BMD) = m (∠ BCD) = 2 x° and m (∠ DCE) = (y + 20)°

R.T.F. The value of each of x and y

Proof ∵ ∠ A is an inscribed angle and ∠ BMD is a central angle subtended by $\widehat{BD}$

∴ m (∠ A) = $\frac{1}{2}$ m (∠ BMD) = x

∵ The figure ABCD is a cyclic quadrilateral

∴ m (∠ A) + m (∠ BCD) = 180° ∴ x + 2 x = 180°

∴ 3 x = 180° ∴ x = 60°

∵ ∠ DCE is an exterior angle of the cyclic quadrilateral ABCD

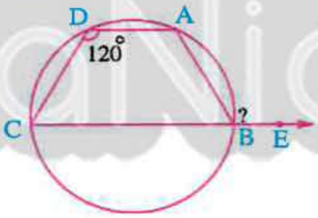
∴ m (∠ DCE) = m (∠ A) ∴ y + 20° = 60°

∴ y = 40° (The req.)

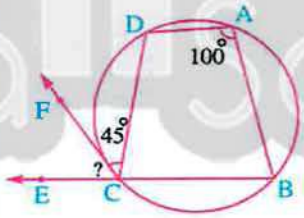
TRY 3 by yourself

In each of the following figures , find the measure of each angle denoted by (?) :

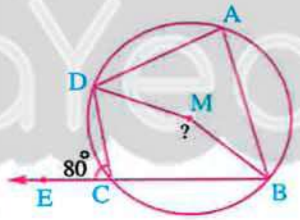
1



2



3



Answers *of try by yourself*

1 (1) 62° (2) 72°

2 (1) 80° (2) 120° (3) 110° (4) m (∠ D) = 115° , m (∠ ACD) = 40°

3 (1) 120° (2) 55° (3) 160°

LESSON
5

Cases of proving the cyclic quadrilateral

In this lesson we will answer the next question :

When a quadrilateral is a cyclic quadrilateral ?

First

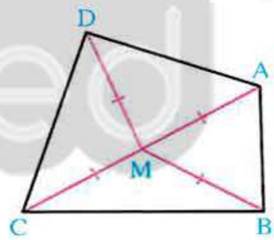
The quadrilateral is a cyclic quadrilateral if there is a point in its plane at equal distances from its vertices.

For example:

If M is a point where :

$MA = MB = MC = MD$

, then ABCD is a cyclic quadrilateral.



Second

The quadrilateral is a cyclic quadrilateral if there are two angles equal in measure drawn on one of its bases and on one side of this base.

According to this , each of the following four figures is a cyclic quadrilateral.

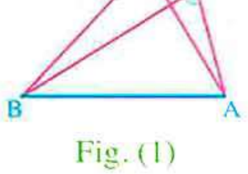


Fig. (1)

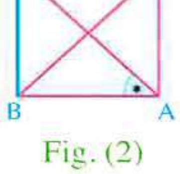


Fig. (2)

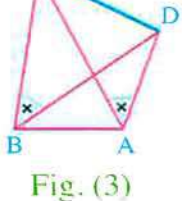


Fig. (3)

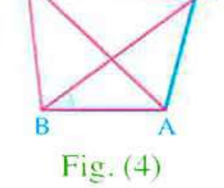


Fig. (4)

UNIT 5

Remarks

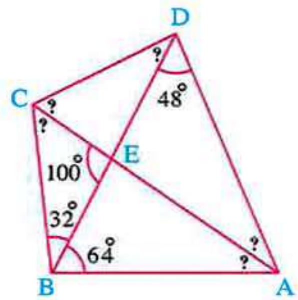
1. If there are two angles drawn on one of the bases of a quadrilateral, and on the same side of it and they are not equal in measure, then the quadrilateral is not cyclic.
2. The rectangle, the square and the isosceles trapezium are cyclic quadrilaterals while the parallelogram, the rhombus and the trapezium that is not isosceles are not cyclic quadrilaterals.

Example 1 **In the opposite figure :**

ABCD is a quadrilateral, its diagonals intersect at E

If m ($\angle$ ADB) = 48°, m ($\angle$ DBC) = 32°,

m ($\angle$ BEC) = 100° and m ($\angle$ ABD) = 64°



1 Prove that :

The figure ABCD is a cyclic quadrilateral.

2 Find :

The measures of the angles denoted by (?)

Solution

Given: m ($\angle$ ADB) = 48°, m ($\angle$ DBC) = 32°, m ($\angle$ BEC) = 100°, m ($\angle$ ABD) = 64°

R.T.P. The figure ABCD is a cyclic quadrilateral.

R.T.F. The measures of the angles denoted by (?)

Proof: In Δ BEC : $\because$ m ($\angle$ BCE) = 180° – (100° + 32°) = 48°

$\therefore$ m ($\angle$ ADB) = m ($\angle$ ACB)

and they are drawn on $\overline{AB}$ and on one side of it.

$\therefore$ The figure ABCD is a cyclic quadrilateral. (First req.)

$\therefore$ m ($\angle$ DAC) = m ($\angle$ DBC) = 32°

(drawn on $\overline{DC}$ and on the same side of it)

, m ($\angle$ ACD) = m ($\angle$ ABD) = 64°

(drawn on $\overline{AD}$ and on the same side of it)

$\because$ $\angle$ BEC is an exterior angle of Δ ABE

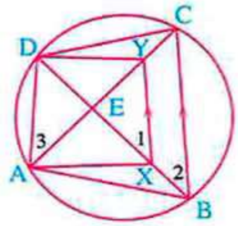
$\therefore$ m ($\angle$ BEC) = m ($\angle$ EAB) + m ($\angle$ EBA)

$\therefore$ m ($\angle$ EAB) = 100° – 64° = 36°

$\therefore$ m ($\angle$ BDC) = m ($\angle$ BAC) = 36°

(drawn on $\overline{BC}$ and on the same side of it) (Second req.)

Example 2 **In the opposite figure :**



ABCD is a quadrilateral inscribed in a circle its diagonals intersect at E , $X \in \overline{BE}$, $Y \in \overline{CE}$ such that $\overline{XY} \parallel \overline{BC}$

Prove that :

1 AXYD is a cyclic quadrilateral.

2 m (∠ BAX) = m (∠ CDY)

Solution

Given ABCD is a cyclic quadrilateral , $\overline{XY} \parallel \overline{BC}$

R.T.P. 1 AXYD is a cyclic quadrilateral. 2 m (∠ BAX) = m (∠ CDY)

Proof ∵ $\overline{XY} \parallel \overline{BC}$

∴ m (∠ 1) = m (∠ 2) (corresponding angles)

but m (∠ 2) = m (∠ 3) (two inscribed angles subtended by $\overset{\frown}{CD}$)

∴ m (∠ 1) = m (∠ 3)

and they are drawn on the base $\overline{YD}$ and on one side of it.

∴ AXYD is a cyclic quadrilateral. (Q.E.D. 1)

∵ ABCD is a cyclic quadrilateral.

∴ m (∠ BAC) = m (∠ BDC) (1)

∵ AXYD is a cyclic quadrilateral.

∴ m (∠ XAY) = m (∠ XDY) (2)

Subtracting (2) from (1) we deduce that :

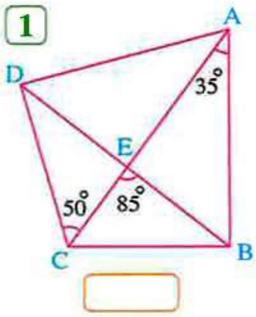
m (∠ BAC) – m (∠ XAY) = m (∠ BDC) – m (∠ XDY)

∴ m (∠ BAX) = m (∠ CDY) (Q.E.D. 2)

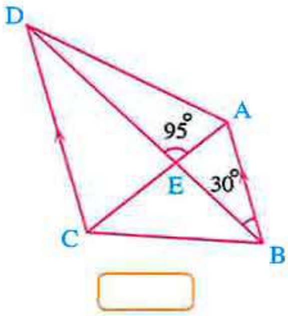
TRY 1 by yourself

Put (✓) for the cyclic quadrilateral in the following (Given that : $\overline{AC} \cap \overline{BD} = \{E\}$) :

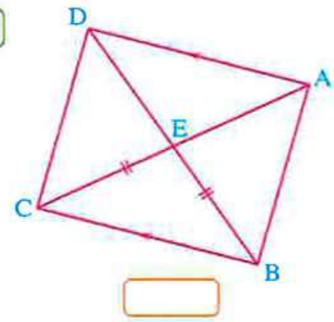
1



2



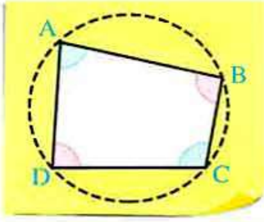
3



Third The converse of theorem 3

If two opposite angles of a quadrilateral are supplementary , then the quadrilateral is cyclic.

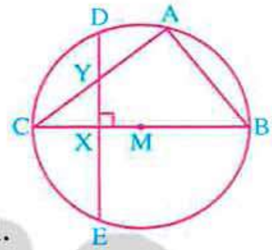
In the opposite figure :



If m (∠ B) + m (∠ D) = 180° or m (∠ A) + m (∠ C) = 180°

, then the figure ABCD is a cyclic quadrilateral.

Example 3 **In the opposite figure :**



$\overline{BC}$ is a diameter in the circle M , A $\in \widehat{BC}$,

the chord $\overline{DE} \perp \overline{BC}$ where

$\overline{DE} \cap \overline{BC} = \{X\}$ and $\overline{DE} \cap \overline{AC} = \{Y\}$

Prove that : The figure ABXY is a cyclic quadrilateral.

Solution

Given $\overline{BC}$ is a diameter in the circle M , $\overline{DE} \perp \overline{BC}$

R.T.P. The figure ABXY is a cyclic quadrilateral.

Proof $\because$ $\overline{BC}$ is a diameter in the circle M

$\therefore$ m (∠ BAC) = 90°

, $\because$ $\overline{DE} \perp \overline{BC}$ $\therefore$ m (∠ DXB) = 90°

$\therefore$ In the figure ABXY :

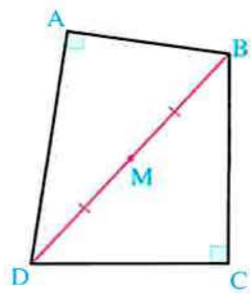
m (∠ BAY) + m (∠ YXB) = 90° + 90° = 180°

$\therefore$ The figure ABXY is a cyclic quadrilateral. (Q.E.D.)

Remark

" If one of a cyclic quadrilateral's angles is right , then the diagonal opposite to this angle is a diameter of the circumcircle of this cyclic quadrilateral and the midpoint of this diagonal is the centre of this circle. "

In the opposite figure :



If ABCD is a cyclic quadrilateral , m (∠ A) = m (∠ C) = 90°

, then $\overline{BD}$ is a diameter of the circumcircle of the figure ABCD and the point M (the midpoint of $\overline{BD}$) is the centre of this circle whose radius length = MB = MD = MA = MC

Fourth Corollary

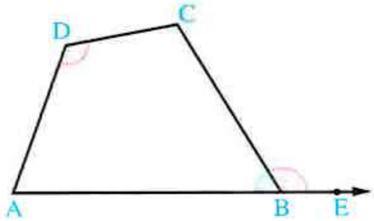
If the measure of the exterior angle at a vertex of a quadrilateral figure is equal to the measure of the interior angle at the opposite vertex , then the figure is a cyclic quadrilateral.

In the opposite figure :

If ABCD is a quadrilateral

and m (∠ CBE) (the exterior angle) = m (∠ D) ,

then the figure ABCD is a cyclic quadrilateral.



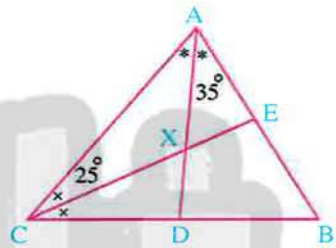
Example 4 **In the opposite figure :**

$\overrightarrow{AD}$ bisects ∠ BAC ,

$\overrightarrow{CX}$ bisects ∠ ACB , m (∠ BAD) = 35° ,

m (∠ ACE) = 25° and $\overline{AD} \cap \overline{EC} = \{X\}$

Prove that : The figure BEXD is a cyclic quadrilateral.



Solution

Given $\overrightarrow{AD}$ bisects ∠ BAC , $\overrightarrow{CX}$ bisects ∠ ACB ,

m (∠ BAD) = 35° , m (∠ ACE) = 25°

R.T.P. The figure BEXD is a cyclic quadrilateral.

Proof In Δ ABC : ∵ $\overrightarrow{AD}$ bisects ∠ BAC

∴ m (∠ BAC) = 35° × 2 = 70°

, ∵ $\overrightarrow{CE}$ bisects ∠ ACB ∴ m (∠ ACB) = 25° × 2 = 50°

∴ m (∠ B) = 180° – (70° + 50°) = 60° (1)

, ∵ ∠ AXE is an exterior angle of Δ AXC

∴ m (∠ AXE) = m (∠ XAC) + m (∠ ACX) = 35° + 25° = 60° (2)

From (1) and (2) and in the figure BEXD :

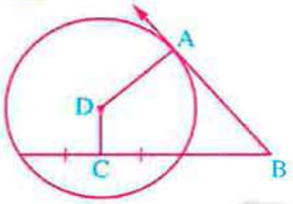
∴ m (∠ AXE) (the exterior angle) = m (∠ B) (the opposite to the vertex X)

∴ The figure BEXD is a cyclic quadrilateral. (Q.E.D.)

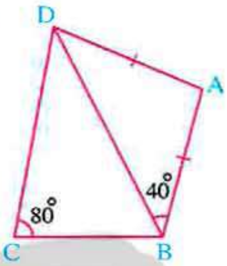
TRY 2 by yourself

In each of the following , is the figure ABCD a cyclic quadrilateral or not ?

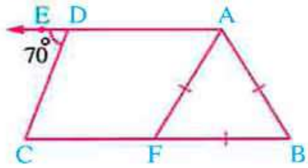
1



2



3



Answers *of try by yourself*

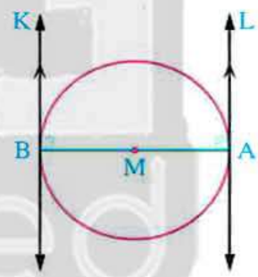
1 The cyclic quadrilaterals are (1) and (3)

2 The cyclic quadrilaterals are (1) and (2)

LESSON
6
The relation between the tangents of a circle

First — The two tangents drawn at the two ends of a diameter in a circle are parallel

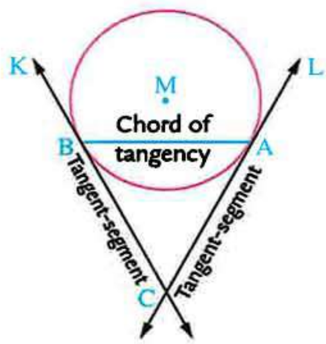
i.e. **In the opposite figure :**



If $\overline{AB}$ is a diameter in the circle M and the two straight lines L and K are two tangents to the circle at A and B respectively,
then the straight line L // the straight line K
(because the straight line L $\perp$ $\overline{AB}$ and the straight line K $\perp$ $\overline{AB}$)

Second — The two tangents drawn at the two ends of a chord of a circle are intersecting

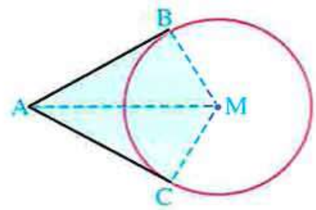
i.e. **In the opposite figure :**



If $\overline{AB}$ is a chord in the circle M and the two straight lines L and K are two tangents to the circle at A and B respectively, then the two straight lines L and K are intersecting at a point outside the circle M (Say C) and $\overline{AC}$, $\overline{BC}$ are called tangent - segments and $\overline{AB}$ is called a chord of tangency.

Theorem 4

The two tangent-segments drawn to a circle from a point outside it are equal in length.



Given A is a point outside the circle M ,
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C respectively.

R.T.P. AB = AC

Construction Draw $\overline{MB}$, $\overline{MC}$, $\overline{MA}$

Proof $\because \overleftrightarrow{AB}$ is a tangent to the circle M

$\therefore m(\angle ABM) = 90°$

$\because \overleftrightarrow{AC}$ is a tangent to the circle M

$\therefore m(\angle ACM) = 90°$

In $\Delta\Delta$ ABM , ACM :

$$\begin{cases} MB = MC \text{ (the lengths of two radii)} \\ \overline{AM} \text{ is a common side.} \\ m(\angle ABM) = m(\angle ACM) = 90° \text{ (proved)} \end{cases}$$

$\therefore \Delta ABM \equiv \Delta ACM$, and we deduce that : AB = AC (Q.E.D.)

Corollaries of theorem (4)

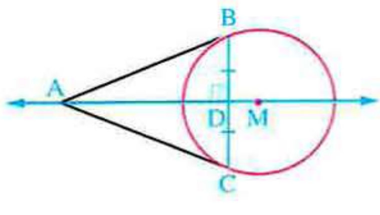
Corollary 1

The straight line passing through the centre of the circle and the intersection point of the two tangents is an axis of symmetry to the chord of tangency of those two tangents.

In the opposite figure :

If $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two tangents to the circle M at B and C respectively, then $\overleftrightarrow{AM}$ is the axis of symmetry to $\overline{BC}$

i.e. $\overleftrightarrow{AM} \perp \overline{BC}$, BD = CD

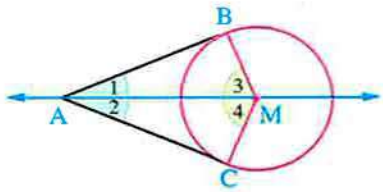


Corollary 2

The straight line passing through the centre of the circle and the intersection point of its two tangents bisects the angle between these two tangents. It also bisects the angle between the two radii passing through the two points of tangency.

In the opposite figure :

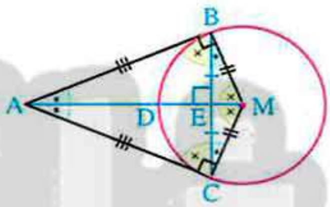
If $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two tangents to the circle M at B and C respectively , then :



- $\overrightarrow{AM}$ bisects $\angle$ BAC $\quad \therefore$ m ($\angle$ 1) = m ($\angle$ 2)
- $\overrightarrow{MA}$ bisects $\angle$ BMC $\quad \therefore$ m ($\angle$ 3) = m ($\angle$ 4)

Remarks on theorem (4) and its corollaries

In the opposite figure :



1. AB = AC
2. MB = MC = r
3. BE = CE , $\overleftrightarrow{AM} \perp \overline{BC}$
4. m ($\angle$ ABM) = m ($\angle$ ACM) = 90°

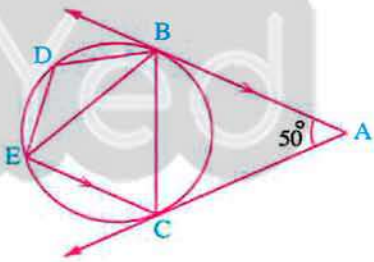
   i.e. The figure ABMC is a cyclic quadrilateral.
5. m ($\angle$ BAM) = m ($\angle$ BCM) = m ($\angle$ CAM) = m ($\angle$ CBM)
6. m ($\angle$ AMB) = m ($\angle$ ACB) = m ($\angle$ AMC) = m ($\angle$ ABC)

Example 1 **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ touch the circle at B and C ,

$\overrightarrow{AB}$ // $\overline{CE}$, m ($\angle$ A) = 50°

Find by proof : m ($\angle$ BDE)



Solution

Given $\overrightarrow{AB}$ and $\overrightarrow{AC}$ touch the circle at B and C, $\overrightarrow{AB}$ // $\overline{CE}$, m ($\angle$ A) = 50°

R.T.F. m ($\angle$ BDE)

Proof $\because$ $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle $\quad \therefore$ AB = AC

$\therefore$ In Δ ABC : m ($\angle$ ABC) = m ($\angle$ ACB) = $\frac{180° - 50°}{2}$ = 65°

$\because$ $\overrightarrow{AB}$ // $\overline{CE}$ and $\overleftrightarrow{BC}$ is a transversal to them.

$\therefore$ m ($\angle$ ABC) = m ($\angle$ BCE) = 65° (alternate angles)

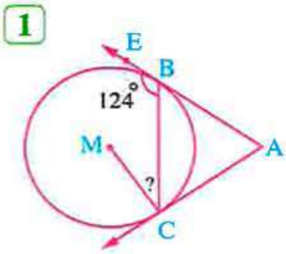
$\because$ The figure DBCE is a cyclic quadrilateral.

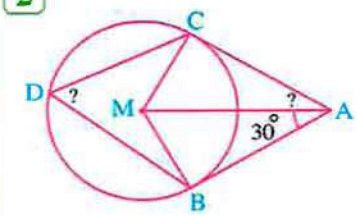
$\therefore$ m ($\angle$ BDE) = 180° – 65° = 115° (The req.)

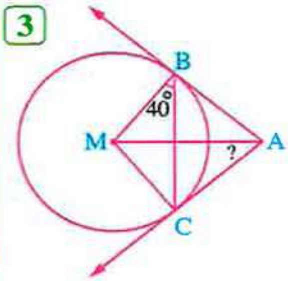
UNIT 5

TRY 1 *by yourself*

In each of the following , find the measure of the angle denoted by (?)

1


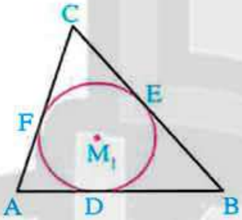
2


3


Definition

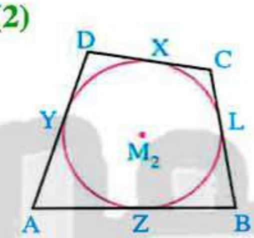
The inscribed circle of a polygon is the circle which touches all of its sides internally.

Fig. (1)



M_1 is the inscribed circle of the triangle ABC where : the side $\overline{AB}$ touches the circle at D , the side $\overline{BC}$ touches the circle at E and the side $\overline{CA}$ touches the circle at F

Fig. (2)



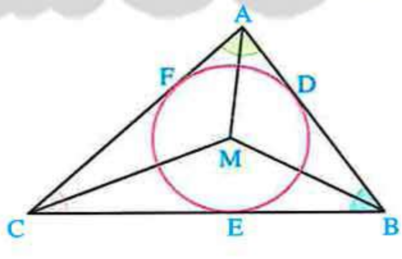
M_2 is the inscribed circle of the quadrilateral ABCD

Remark

The centre of the inscribed circle of any triangle is the point of intersection of the bisectors of its interior angles.

i.e. In the opposite figure :

If the circle M is the inscribed circle of the triangle ABC , then M is the intersection point of the bisectors of the interior angles of Δ ABC



We can prove that as follows :

$\because$ $\overline{AD}$ and $\overline{AF}$ are two tangent-segments to the circle M

$\therefore$ $\overrightarrow{AM}$ bisects $\angle$ BAC (1) , similarly

, $\overrightarrow{BM}$ bisects $\angle$ ABC (2)

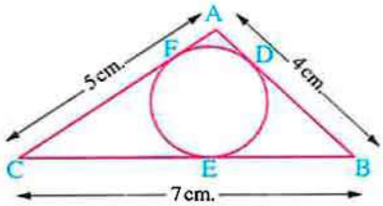
, $\overrightarrow{CM}$ bisects $\angle$ ACB (3)

From (1) , (2) and (3) , we deduce that :

M is the intersection point of the bisectors of the interior angles of the triangle **ABC**

Example 2 ABC is a triangle where AB = 4 cm. , BC = 7 cm. , AC = 5 cm.
If the inscribed circle of it touches its sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ at D , E and F respectively,
prove that : AC + BD = BC + AD , then deduce the lengths of the parts into which the sides of the triangle are divided by the points of tangency.

Solution



Given AB = 4 cm. , BC = 7 cm. , AC = 5 cm. ,
D , E and F are the points of tangency of the inscribed circle of the triangle ABC

R.T.P. AC + BD = BC + AD

R.T.F. AD , DB , BE , CE , CF , FA

Proof $\because$ $\overline{CF}$ and $\overline{CE}$ are two tangent – segments to the circle at F and E

$\therefore$ CF = CE , similarly AF = AD , BD = BE

$\therefore$ (CF + AF) + BD = (CE + BE) + AD

$\therefore$ AC + BD = BC + AD (1) (First req.)

$\because$ AC = 5 cm. , BC = 7 cm. , AD = (4 – BD) cm.

Substituting in (1) :

$\therefore$ 5 + BD = 7 + (4 – BD) $\therefore$ 5 + BD = 11 – BD

$\therefore$ 2 BD = 6 $\therefore$ BD = 3 cm. , DA = 4 – 3 = 1 cm.

$\because$ BD = BE

$\therefore$ BE = 3 cm., CE = 7 – 3 = 4 cm.

$\because$ AD = AF

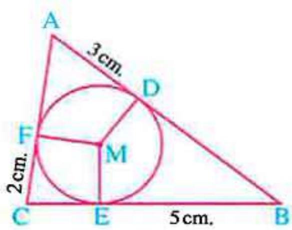
$\therefore$ AF = 1 cm., CF = CE = 4 cm. (Second req.)

TRY 2 by yourself

In the opposite figure :

If M is the centre of the inscribed circle of Δ ABC

, complete the following :



1 M is the intersection point of .... of the triangle ABC

2 The perimeter of Δ ABC = ...

3 The figure ADMF is a cyclic quadrilateral because ....

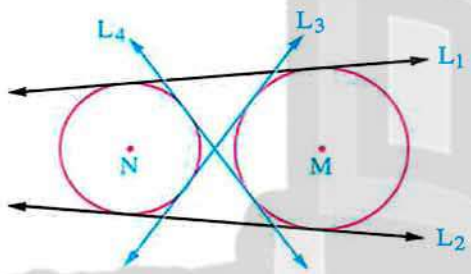
UNIT 5

The common tangents to two circles

- It is said that the tangent $\overleftrightarrow{AB}$ is an internal common tangent to the two circles M and N if the two circles M and N are on two different sides of the tangent.
- It is said that the tangent $\overleftrightarrow{AB}$ is an external common tangent to the two circles M and N if the two circles M and N are on the same side of the tangent.

The following table shows the number of the common tangents to two circles in their different situations (locations) :

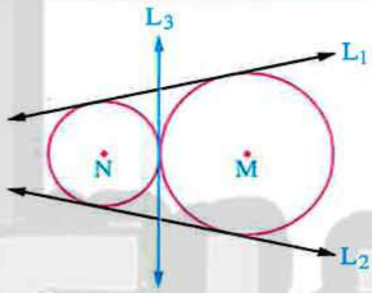
Two distant circles



4 common tangents

- L_1 and L_2 (external)
- L_3 and L_4 (internal)

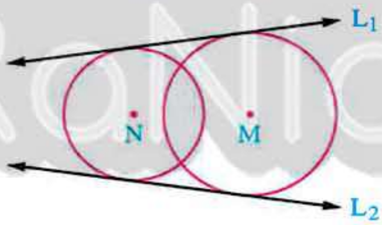
Two circles touching externally



3 common tangents

- L_1 and L_2 (external)
- L_3 (internal)

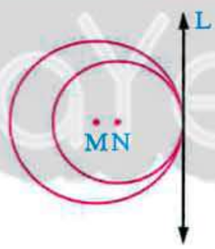
Two intersecting circles



2 common tangents

- L_1 and L_2 (external)
- There are no internal tangents

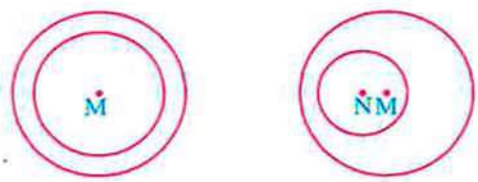
Two circles touching internally



One common tangent

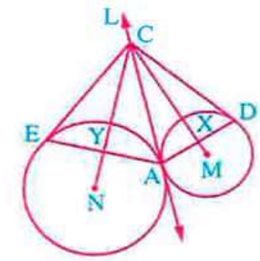
- L is the common tangent (external)
- There are no internal tangents

One circle inside the other



There are no common tangents

Example 3 **In the opposite figure :**



M and N are two circles touching externally at A

The straight line L is a common tangent to them at A , $C \in L$, from C draw more two tangents to the circles M and N to touch them at D and E respectively.

Prove that :

1 C is the centre of the circle which passes through the points D , A and E

2 $\angle$ DAE supplements $\angle$ MCN

Solution

Given L is the common tangent to the two circles M and N at A ,

$\overline{CD}$ is a tangent-segment to the circle M ,

$\overline{CE}$ is a tangent-segment to the circle N

R.T.P.

1 C is the centre of the circle which passes through the points D , A and E

2 $\angle$ DAE supplements $\angle$ MCN

Proof

$\because$ $\overline{CD}$ and $\overline{CA}$ are two tangent-segments to the circle M $\therefore$ CD = CA (1)

$\because$ $\overline{CE}$ and $\overline{CA}$ are two tangent-segments to the circle N $\therefore$ CE = CA (2)

From (1) and (2) we deduce that CD = CA = CE

$\therefore$ C is the centre of the circle which passes through the points D , A and E (Q.E.D. 1)

$\because$ $\overleftrightarrow{MC}$ is the axis of symmetry of the chord of tangency $\overline{AD}$ in the circle M

$\therefore$ m ($\angle$ AXC) = 90°

$\because$ $\overleftrightarrow{NC}$ is the axis of symmetry of the chord of tangency $\overline{AE}$ in the circle N

$\therefore$ m ($\angle$ AYC) = 90° $\therefore$ m ($\angle$ AXC) + m ($\angle$ AYC) = 90° + 90° = 180°

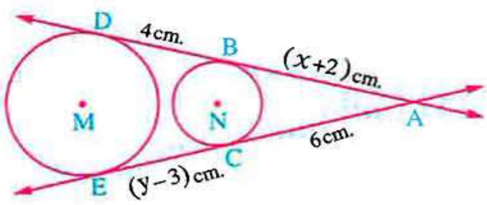
$\therefore$ The figure AXCY is a cyclic quadrilateral

$\therefore$ m ($\angle$ DAE) + m ($\angle$ MCN) = 180° (Q.E.D. 2)

TRY 3 by yourself

In the opposite figure :

Find the value of each of x and y in centimetres.



Answers of try by yourself

1 [1] 34° [2] m ($\angle$ MAC) = 30° , m ($\angle$ D) = 60° [3] 40°

2 [1] The bisectors of interior angles [2] 20 cm. [3] m ($\angle$ ADM) + m ($\angle$ AFM) = 180°

3 $x = 4$, y = 7

LESSON 7

Angles of tangency

The angle of tangency

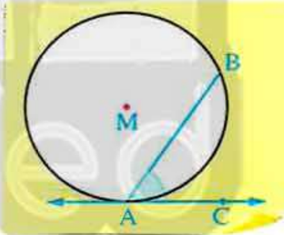
The angle of tangency is the angle which is composed of the union of two rays , one of them is a tangent to the circle and the other contains a chord of the circle passing through the point of tangency.

In the opposite figure :

If $\overleftrightarrow{AC}$ is a tangent to the circle at A and $\overrightarrow{AB}$ contains the chord $\overline{AB}$

, then $\angle$ BAC is an angle of tangency in the circle M , its chord is $\overline{AB}$

$\overline{AB}$ is called the chord of tangency of the angle of tangency BAC

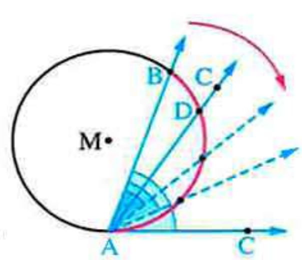


The measure of the angle of tangency

The angle of tangency is a special case of the inscribed angle because if we imagine that the side $\overrightarrow{AC}$ of an inscribed angle BAC moves around A in the direction of the arrow as shown in the opposite figure ,

then the point D gets closer to the point A till it coincides the point A ,

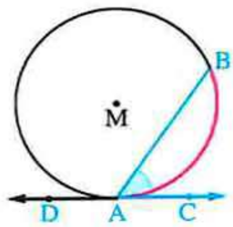
then $\overrightarrow{AC}$ becomes a tangent to the circle.



Since the inscribed angle is measured as a half the measure of the arc intercepted by its two sides , then the measure of the angle of tangency is the same.

i.e. **The measure of the angle of tangency = $\frac{1}{2}$ the measure of the arc intercepted by its sides.**

In the opposite figure :



- $\angle$ BAC is an angle of tangency that intercepts $\widehat{AB}$ between its sides.

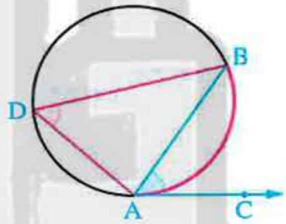
  $\therefore m(\angle BAC) = \frac{1}{2} m(\widehat{AB})$

- $\angle$ BAD is an angle of tangency that intercepts the major $\widehat{AB}$ between its sides.

  $\therefore m(\angle BAD) = \frac{1}{2} m(\widehat{AB}$ the major$)$

Theorem 5

The measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc.



Given $\angle$ BAC is an angle of tangency and $\angle$ D is an inscribed angle.

R.T.P. $m(\angle BAC) = m(\angle D)$

Proof $\because \angle$ BAC is an angle of tangency.

$\therefore m(\angle BAC) = \frac{1}{2} m(\widehat{AB})$ (1)

, $\because \angle$ D is an inscribed angle

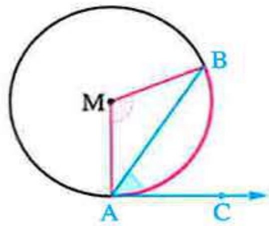
$\therefore m(\angle D) = \frac{1}{2} m(\widehat{AB})$ (2)

From (1) and (2) , we deduce that : $m(\angle BAC) = m(\angle D)$ (Q.E.D.)

Corollary

The measure of the angle of tangency is half the measure of the central angle subtended by the same arc.

In the opposite figure :



$m(\angle BAC)$ (tangency angle) $= \frac{1}{2} m(\widehat{AB})$

, $\because m(\angle AMB)$ (central angle) $= m(\widehat{AB})$

$\therefore m(\angle BAC)$ (tangency angle) $= \frac{1}{2} m(\angle AMB)$ (central angle)

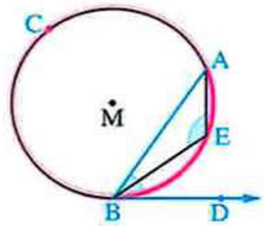
Remark

"The angle of tangency is supplementary to the drawn inscribed angle on the chord of the angle of tangency and in one side of it."

In the opposite figure :

If $\overrightarrow{BD}$ is a tangent to the circle M ,

E $\in$ the circle M



, then $m(\angle ABD) = \frac{1}{2} m(\widehat{AEB})$ (1)

and $m(\angle AEB) = \frac{1}{2} m(\widehat{ACB})$ (2)

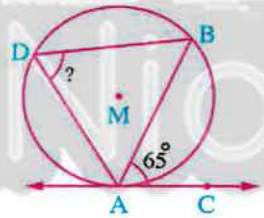
Adding (1) and (2) , we deduce that :

$$m(\angle ABD) + m(\angle AEB) = \frac{1}{2} m(\widehat{AEB}) + \frac{1}{2} m(\widehat{ACB})$$

$$= \frac{1}{2}\left[m(\widehat{AEB}) + m(\widehat{ACB})\right] = \frac{1}{2} \times 360° = 180°$$

Example 1 In each of the following , $\overleftrightarrow{AC}$ is a tangent to the circle M Find the measure of each angle denoted by the symbol (?) :

1



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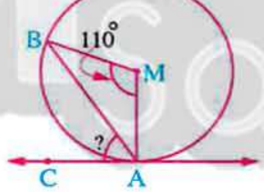
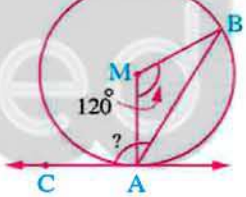


3



Solution

1 $m(\angle ADB) = m(\angle BAC) = 65°$

(inscribed and tangency angles subtended by $\widehat{AB}$)

2 $m(\angle BAC) = \frac{1}{2} m(\angle AMB) = 55°$

(tangency and central angles subtended by $\widehat{AB}$)

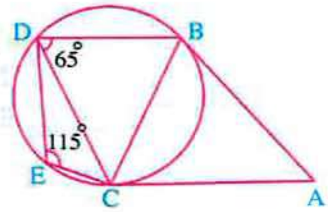
3 $\because m(\angle AMB \text{ the reflex}) = 360° - 120° = 240°$

$\therefore m(\angle BAC) = \frac{1}{2} m(\angle AMB \text{ the reflex}) = 120°$

(tangency and central angles subtended by $\widehat{BA}$ the major)

Example 2 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle ,



m (∠ BDC) = 65° and m (∠ E) = 115°

1 **Find :** m (∠ A)

2 **Prove that :** $\overrightarrow{BC}$ bisects ∠ ABD

Solution

Given $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle ,

m (∠ BDC) = 65° and m (∠ E) = 115°

R.T.F. m (∠ A)

R.T.P. $\overrightarrow{BC}$ bisects ∠ ABD

Proof ∵ m (∠ ABC) = m (∠ BDC)

(tangency and inscribed angles subtended by $\overset{\frown}{BC}$)

∴ m (∠ ABC) = 65° (1)

, ∵ $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle.

∴ AB = AC

∴ In Δ ABC : m (∠ ABC) = m (∠ ACB) = 65°

∴ m (∠ A) = 180° – (65° + 65°) = 50° (First req.)

, ∵ the figure BCED is a cyclic quadrilateral

and m (∠ E) = 115°

∴ m (∠ DBC) = 180° – 115° = 65° (2)

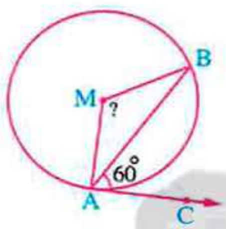
From (1) and (2) :

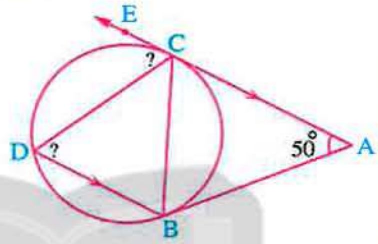
∴ m (∠ ABC) = m (∠ DBC) = 65°

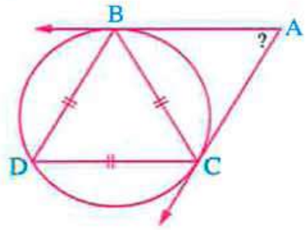
∴ $\overrightarrow{BC}$ bisects ∠ ABD (Second req.)

TRY 1 by yourself

In each of the following , find the measure of each angle denoted by the symbol (?) :

1 

2 

3 

The converse of theorem 5

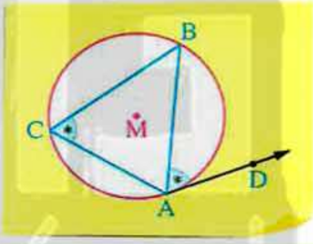
If a ray is drawn from one end of a chord of a circle so that the angle between this ray and the chord is equal in measure to the inscribed angle subtended by the chord in the alternate side , then this ray is a tangent to the circle.

Thus in the opposite figure :

If $\overline{AB}$ is a chord in the circle M ,

$\overrightarrow{AD}$ is drawn such that m (∠ BAD) = m (∠ C) ,

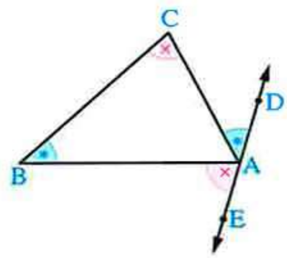
then $\overrightarrow{AD}$ is a tangent to the circle M



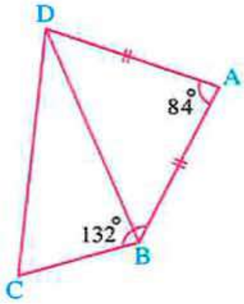
What is the benefit that we have from the converse of theorem (5) ?

The converse of theorem (5) is used to prove that the ray drawn from one of the vertices of a triangle is a tangent to the circumcircle of this triangle.

- If we want to prove that :
  $\overrightarrow{AD}$ is a tangent to the circumcircle of Δ ABC ,
  then we should prove that m (∠ CAD) = m (∠ B) (1)
- And if we want to prove that $\overrightarrow{AE}$ is a tangent to the circumcircle of Δ ABC , then we should prove that
  m (∠ BAE) = m (∠ C) (2)
- And proving one of the previous steps (1) or (2)
  that means : $\overleftrightarrow{DE}$ is a tangent to the circumcircle of Δ ABC



Example 3 **In the opposite figure :**



$AB = AD$, $m(\angle A) = 84°$, $m(\angle ABC) = 132°$

Prove that :

$\overline{BC}$ is a tangent-segment to the circle passing through A , B and D

Solution

Given $AB = AD$, $m(\angle A) = 84°$, $m(\angle ABC) = 132°$

R.T.P. $\overline{BC}$ is a tangent-segment to the circle passing through the points A , B and D

Proof In ΔABD : $\because AB = AD$, $m(\angle A) = 84°$

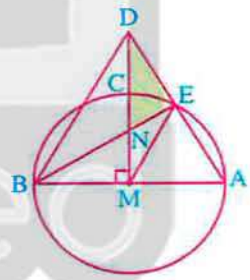
$\therefore m(\angle ABD) = m(\angle ADB) = \frac{180° - 84°}{2} = 48°$

$\because m(\angle ABC) = 132°$ $\therefore m(\angle DBC) = 132° - 48° = 84° = m(\angle A)$

$\therefore \overline{BC}$ is a tangent-segment to the circle

which passes through the points A , B and D (Q.E.D)

Example 4 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle whose centre is M ,

$\overline{MC}$ is a radius in it perpendicular to $\overline{AB}$, $D \in \overrightarrow{MC}$,

$\overline{AD} \cap$ the circle $= \{E\}$ and $\overline{BE} \cap \overrightarrow{MC} = \{N\}$

Prove that :

1 The figure DEMB is a cyclic quadrilateral.

2 $\overleftrightarrow{EM}$ is a tangent to the circumcircle of ΔNDE

Solution

Given $\overline{AB}$ is a diameter in the circle M , $\overrightarrow{DM} \perp \overline{AB}$

R.T.P.

1 The figure DEMB is a cyclic quadrilateral.

2 $\overleftrightarrow{EM}$ is a tangent to the circumcircle of ΔNDE\

Proof $\because$ $\overline{AB}$ is a diameter in the circle M $\therefore$ m $(\angle AEB) = 90°$

$\therefore$ m $(\angle BED) = 90°$,

$\because$ m $(\angle BED) =$ m $(\angle BMD) = 90°$

and they are drawn on $\overline{BD}$ and on one side of it.

$\therefore$ The figure DEMB is a cyclic quadrilateral. (Q.E.D.1)

$\because$ m $(\angle EDM) =$ m $(\angle EBM)$ (drawn on $\overline{ME}$ and on one side of it)

but , m $(\angle EBM) =$ m $(\angle MEB)$ because MB = ME = r

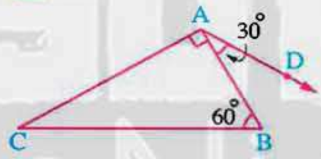
$\therefore$ m $(\angle MEB) =$ m $(\angle EDM)$

$\therefore$ $\overleftrightarrow{EM}$ is a tangent to the circumcircle of Δ NDE (Q.E.D.2)

TRY 2 by yourself

In each of the following figures , show if $\overrightarrow{AD}$ is a tangent to the circle passing through the points A , B and C or not

1



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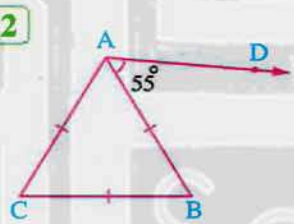

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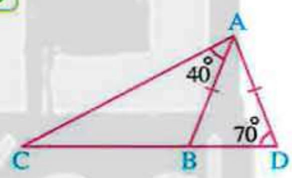

Answers of try by yourself

1 [1] 120° [2] m $(\angle D) = 65°$, m $(\angle DCE) = 65°$

[3] 60°

2 [1] $\overrightarrow{AD}$ is a tangent to the circle passing through the points A , B and C

[2] $\overrightarrow{AD}$ is not a tangent to the circle passing through the points A , B and C

[3] $\overrightarrow{AD}$ is not a tangent to the circle passing through the points A , B and C

EL-MOASSER

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Exercises

For 3rd Prep.
Second Term

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A group of supervisors

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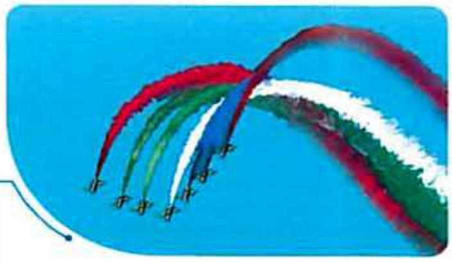
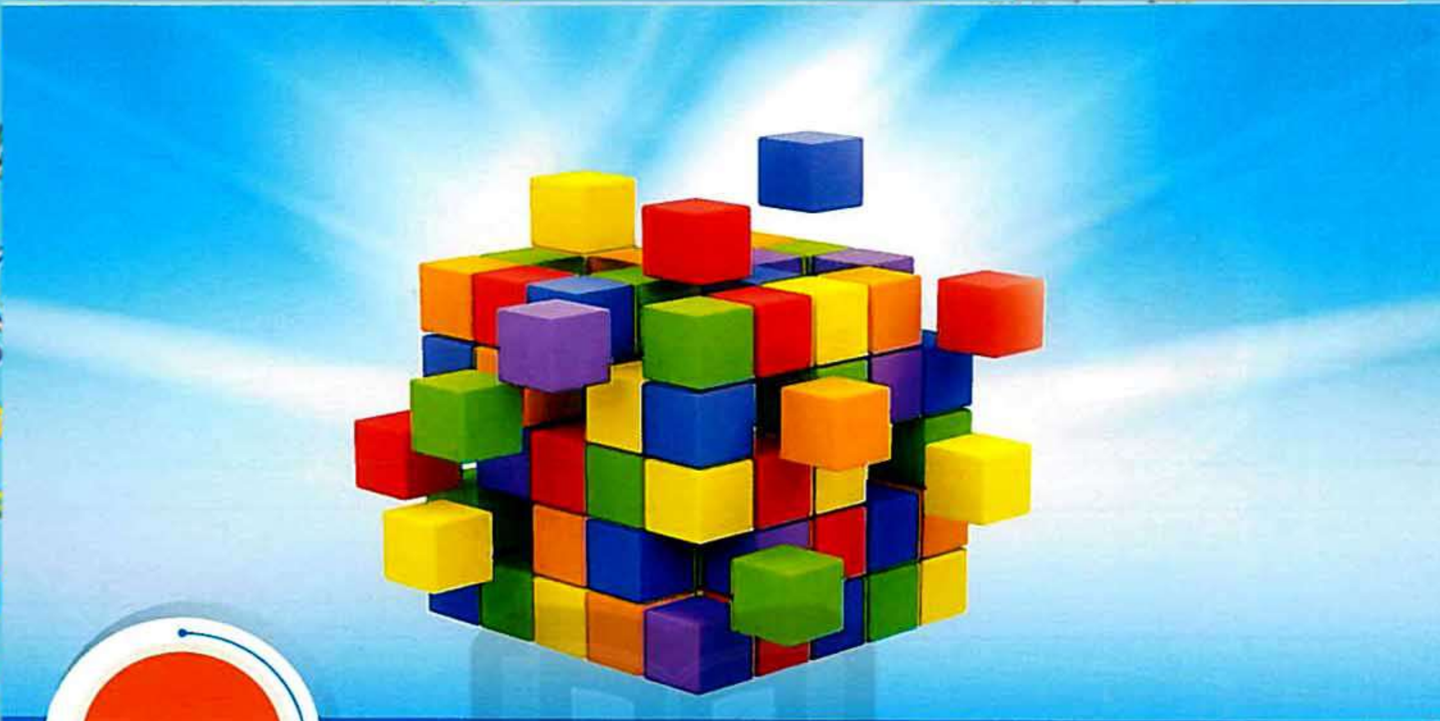


Second Geometry

First Algebra and Probability



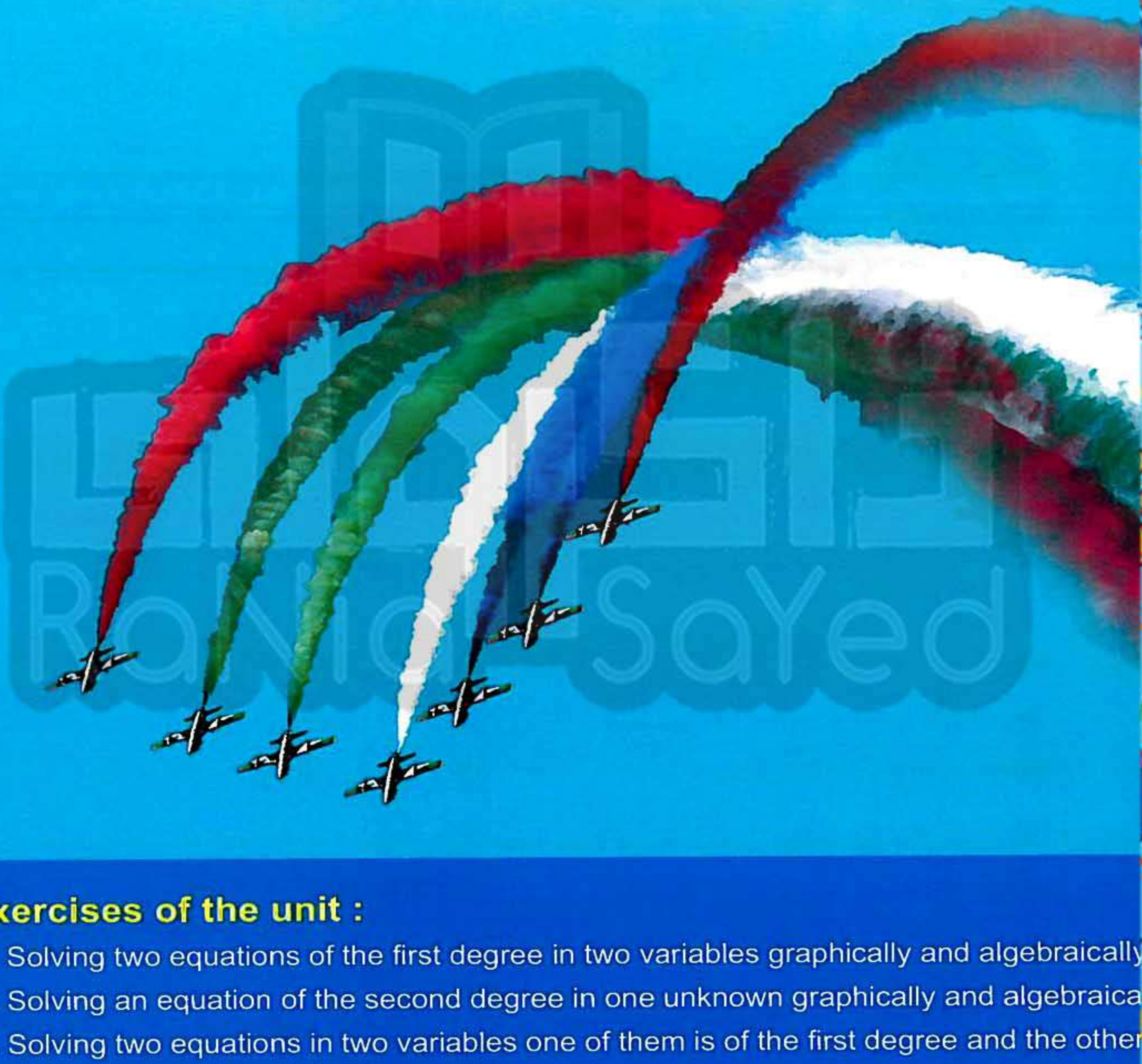
Revision

Exercise on factorizing an algebraic expressions

Factorize each of the following perfectly :

1. $25x^2 - 9y^2$
2. $2x^5 + 54x^2$
3. $2y^2 + 5y + 3$
4. $2x^4 - 18$
5. $2x^2 - 20x + 48$
6. $x^2 + 8x + 16$
7. $8x^3 + 27$
8. $y^2 - 50y - 51$
9. $25x^2 - 30x + 9$
10. $x^2 - 81$
11. $y^5 - y$
12. $3x^2 + 7x - 6$
13. $x^2 - 8x + 12$
14. $3x^3 + 2x^2 + 12x + 8$
15. $x^3 - 125$
16. $4x^2 - 12x + 9$
17. $a^3 + 3a^2 - 9a - 27$
18. $-2x^2 - 15x - 7$
19. $x^2 - 7x + 10$
20. $9x^4 - 16y^4$
21. $x^4 - 9x^2 + 20$
22. $1 - 4x^2$
23. $5x^2 - 3x - 2$
24. $3x^4 - 15x^3 + 12x^2$
25. $3x^2 - 19x + 6$
26. $4x^2 + 28xy + 49y^2$
27. $x^6 - 64y^6$
28. $2y^4 - 4y^3 + 7y - 14$
29. $x^4 - 5x^2 - 24$
30. $9x^4 - 13x^2y^2 + 4y^4$

UNIT 1 Equations



Exercises of the unit :

1. Solving two equations of the first degree in two variables graphically and algebraically
2. Solving an equation of the second degree in one unknown graphically and algebraica
3. Solving two equations in two variables one of them is of the first degree and the other is of the second degree.

- Summary of unit one.
- Unit exams.

Exercise 1 Solving two equations of the first degree in two variables graphically and algebraically

📖 From the school book

1 **Find graphically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :**

1. $y + x = 7$, $y = 2x + 1$ *(Alexandria 15)* « {(2 , 5)} »
2. $x + y = 5$, $x - y = 1$ *(South Sinai 13)* « {(3 , 2)} »
3. 📖 $3x + y = 5$, $y + 3x = 8$ « ∅ »
4. 📖 $2x + y = 4$, $8 - 2y = 4x$ « an infinite number »
5. 📖 $2x + y = 0$, $x + 2y = 3$ « {(– 1 , 2)} »
6. $y = -3$, $x - y = 5$ « {(2 , – 3)} »

2 **Find algebraically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :**

1. $x = y$, $x + 3y = 8$ « {(2 , 2)} »
2. $x - y = 2$, $x + y = 4$ *(Red Sea 18)* « {(3 , 1)} »
3. $y = x - 1$, $y + 2x = 5$ « {(2 , 1)} »
4. $x + 5y = 4$, $2x - 5y = 11$ *(Matrouh 18)* « {(5 , $\frac{-1}{5}$)} »
5. $x = y + 4$, $3x + 4y = 5$ *(El-Dakahlia 18)* « {(3 , – 1)} »
6. 📖 $2x - y = 3$, $x + 2y = 4$ *(El-Sharkia 19 , Alex. 18)* « {(2 , 1)} »
7. $3x + 2y = 4$, $x - 3y = 5$ *(Kafr El-Sheikh 19)* « {(2 , – 1)} »
8. 📖 $3x + 4y = 24$, $x - 2y = -2$ *(El-Gharbia 18 , Giza 12)* « {(4 , 3)} »
9. $3x - y = -4$, $y - 2x = 3$ *(Aswan 19)* « {(– 1 , 1)} »
10. $x + 2y = 5$, $3x = y + 8$ *(El-Sharkia 18)* « {(3 , 1)} »

11 $5y + x = 2$, $2x - 3y + 9 = 0$ « {(– 3 , 1)} »

12 $2y - 3x = 7$, $3y + 2x = 4$ « {(– 1 , 2)} »

13 $x + 2y = 1$, $2x + 4y = -5$ « ∅ »

14 $\frac{x}{6} + \frac{y}{3} = \frac{1}{3}$, $\frac{x}{2} + \frac{2y}{3} = 1$ « {(2 , 0)} »

3 **Find the solution set for each pair of the following two equations algebraically and graphically :**

1 $y = x + 4$, $x + y = 4$ *(El-Gharbia 19 , Souhag 16)* « {(0 , 4)} »

2 $3x - y + 4 = 0$, $y = 2x + 3$ « {(– 1 , 1)} »

3 $2x + y = 1$, $x + 2y = 5$ « {(– 1 , 3)} »

4 $x + 2y = 8$, $3x + y = 9$ « {(2 , 3)} »

5 $x - y = 4$, $3x + 2y = 7$ *(Damietta 13)* « {(3 , – 1)} »

6 $3x + 4y = 11$, $2x + y - 4 = 0$ « {(1 , 2)} »

4 **What is the number of solutions of each pair of the following equations :**

1 $7x + 4y = 6$, $5x - 2y = 14$

2 $4x + 2y = 10$, $y = -2x - 5$

3 $9x + 6y = 24$, $3x + 2y = 8$

5 **Find the values of a and b knowing that (3 , – 1) is the solution of the two equations :**

$a x + b y - 5 = 0$, $3 a x + b y = 17$

(Luxor 18 , Damietta 17 , El-Gharbia 16) « 2 , 1 »

6 **If (a , 2 b) is a solution for the two equations :**

$3x - y = 5$ and $x + y = -1$

, then find the values of a and b *(El-Dakahlia 17)* « 1 , – 1 »

7 If $f(x) = a x^2 + b$, $f(1) = 5$, $f(2) = 11$, then find the value of a and b

(El-Fayoum 09) « 2 , 3 »

8 **Complete the following :**

1 The two straight lines which represent the two equations : $x = 3$, $y = 1$ are intersecting at the point ....

2 The point of intersection of the two straight lines : $x + 3 = 0$, $y - 5 = 0$ lies in the .... quadrant.

3 The solution set of the two equations : $x + y = 0$, $y - 5 = 0$ in $\mathbb{R} \times \mathbb{R}$ is ....

(Alex. 11)

4 The S.S. of the two equations : $x + 3y = 4$, $3y + x = 1$ in $\mathbb{R} \times \mathbb{R}$ is ....

5 The S.S. of the two equations : $4x + y = 6$, $8x + 2y = 12$ in $\mathbb{R} \times \mathbb{R}$ is ....

6 The unique solution of the two equations : $y = 2$, $2x = y$ in $\mathbb{R} \times \mathbb{R}$ is ....

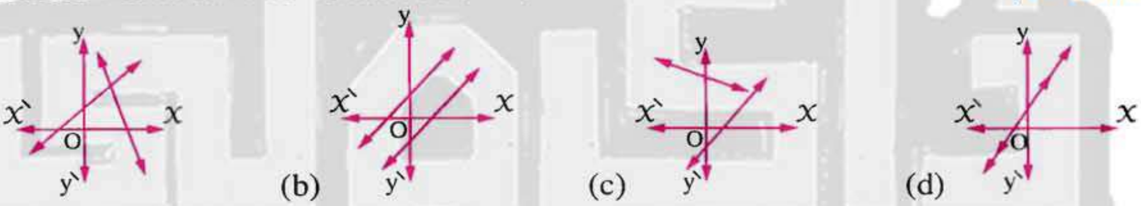
7 The S.S. of the two equations : $\frac{x}{2} + 1 = 0$, $y + 5 = 0$ in $\mathbb{R} \times \mathbb{R}$ is .... *(North Sinai 12)*

8 If the two straight lines which represent the two equations : $x + 3y = 4$, $x + ay = 7$ are parallel , then $a =$ ....

9 If there is only one solution for the two equations : $x + 2y = 1$ and $2x + ky = 2$, then k cannot equal ....

9 Choose the correct answer from those given :

1 Which of the following graphs represents two equations of the first degree in two variables which have no common solution ? *(Port Said 19)*



(a) (b) (c) (d)

2 The point of intersection of the two straight lines : $x + 2 = 0$, $y = x$ is ....

(El-Dakahlia 17)

(a) (2 , 2) (b) (2 , 0) (c) (– 2 , – 2) (d) (0 , 0)

3 The two straight lines : $3x = 7$, $2y = 9$ are .... *(Matrouh 16 , Luxor 16)*

(a) parallel. (b) coincident.

(c) intersecting and non perpendicular. (d) perpendicular.

4 The two straight lines representing the two equations : $x + 5y = 1$, $x + 5y - 8 = 0$ are .... *(El-Beheira 17 , Giza 16)*

(a) parallel. (b) coincident.

(c) perpendicular. (d) intersecting and not perpendlicular.

5 The S.S. of the two equations : $x - 2y = 1$, $3x + y = 10$ in $\mathbb{R} \times \mathbb{R}$ is ....

(Souhag 18 , Port Said 13 , El-Fayoum 11)

(a) $\{(5 , 2)\}$ (b) $\{(2 , 4)\}$ (c) $\{(1 , 3)\}$ (d) $\{(3 , 1)\}$

Unit 1

6 The two straight lines : $3x + 5y = 0$, $5x - 3y = 0$ are intersecting at ....
(Alexandria 14 , El-Beheira 11)

(a) the origin point. (b) the first quadrant.
(c) the second quadrant. (d) the fourth quadrant.

7 If the point of intersection of two straight lines : $x - 1 = 0$, $y = 2k$ lies on the fourth quadrant , then k may be equal .... *(Kafr El-Sheikh 16)*

(a) -5 (b) 0 (c) 1 (d) 5

8 The number of solutions of the two equations : $x - \frac{1}{2}y = 4$, $2x - y = 2$ in $\mathbb{R}^2$ is .... *(El-Kalyoubia 16 , El-Monofia 16)*

(a) a unique solution. (b) two solutions.
(c) an infinite number of solutions. (d) zero.

9 If there are infinite number of solutions in $\mathbb{R} \times \mathbb{R}$ of the two equations : $x + 4y = 7$, $3x + ky = 21$, then k = ... *(Souhag 19 , El-Beheira 18 , Qena 17 , Alexandria 16)*

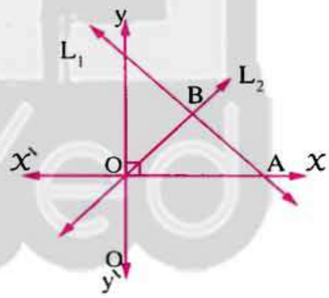
(a) 4 (b) 7 (c) 12 (d) 21

10 If $(x + y , -3) = (5 , x - y)$, then $(y , x) =$ ....

(a) $(-3 , 5)$ (b) $(5 , -3)$ (c) $(1 , 4)$ (d) $(4 , 1)$

10 **In the opposite figure :**

If the equation of straight line $L_1 : x + y = 6$
and the equation of the straight line $L_2 : y - 2x = 0$
where $L_1 \cap L_2 = \{B\}$, O is the origin point, $A \in \overleftrightarrow{xx'}$
Find : The surface area of the triangle OAB



(El-Sharkia 15) « 12 square units »

Applications on solving two equations of the first degree in two variables :

1 The sum of two natural numbers is 63 and their difference is 11
Find the two numbers. *(El-Beheira 16)* « 37 , 26 »

2 The sum of two integers is 54 , twice the first number equals the second number.
Find the two numbers. « 18 , 36 »

3 If three times a number is added to twice a second number the sum is 13 , and if the first number is added to three times the second number the sum is 16 ,
find the two numbers. *(Port Said 17)* « 1 , 5 »

Exercise 1

4 A rectangle is with a length more than its width by 4 cm. If the perimeter of the rectangle is 28 cm. **Find the area of the rectangle.** *(El-Kalyoubia 19 , Cairo 17 , Alex. 12)* « 45 cm^2 »

5 If the number of the teams participating in the African Nations Cup is 16 teams , and the number of non-Arab teams is 4 more than three times the Arab teams , **find the number of the participating Arab teams in the championship.** « 3 teams »

6 The sum of ages of a man and his son is 55 years. If the man's age is more than four times his son's age by 5 years. **Find the age of each of them.** « 45 years , 10 years »

7 If twice the number of girls in a school is more than the number of boys by 50 , and three times the number of girls is less than twice the number of boys by 50 , **find the number of boys and girls.** « 250 , 150 »

8 Two supplementary angles , the twice of the measure of their bigger equals seven times the measure of the smaller. **Find the measure of each angle.** « 140° , 40° »

9 Two acute angles in a right-angled triangle , the difference between their measures is 50° **Find the measure of each angle.** *(El-Beheira 19 , El-Kalyoubia 18 , Damietta 17)* « 70° , 20° »

10 If the price of 4 pens and two books is L.E. 22 and if the number of pens increases by one and the number of books decreases by one , then the price will become L.E. 20 **Find the price of each of the pen and the book.** « L.E. 3 , L.E. 5 »

11 A two-digit number , the sum of its two digits is three times of its units digit and its tens digit exceeds its units digit by 4 **Find this number.** « 84 »

12 A two-digit number , the sum of its digits is 11 If the two digits are reversed , then the resulted number is 27 more than the original number , what is the original number ? *(Kafr El-Sheikh 16)* « 47 »

13 A two-digit number equals 5 times the sum of its digits. If the two digits are reversed then the resulted number will be more than the origin number by 9 **Find the origin number.** « 45 »

14 A rational number , if the number 1 is subtracted from its two terms, it will be $\frac{1}{2}$ and if the number 5 is added to its denominator , it will become $\frac{1}{3}$ **Find the rational number.** « $\frac{4}{7}$ »

Unit 1

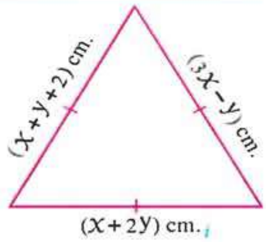
15 If the sum of the ages of Ahmed and Osama now is 43 years , and after 5 years the difference between both ages will be 3 years.
Find the age of each of them after 7 years. « 30 years , 27 years »

16 Five years ago , Magdi's age was five times the age of his daughter Dina and after four years from now , Magdi's age will become three times the age of Dina.
Find the age of each of them now. « 50 years , 14 years »

17 **In the opposite figure :**

An equilateral triangle

Find the length of its side.



« 7 cm. »

18 The two measures of the base angles of an isosceles triangle are $(5x - 5y)^\circ$, $(3x + 5y)^\circ$ and the measure of the vertex angle is $(2x)^\circ$
Find the value of each of x and y « 18 , 3.6 »

For excellent pupils

1 If $(-d , 2c)$, $(3d - 2 , 3 - c)$ are two solutions for equation : $x + y = 4$
Find the value of each of c and d « 3 , 2 »

2 If $\frac{1}{\ell} + \frac{1}{m} = 3$, $\frac{2}{\ell} + \frac{3}{m} = 10$ **Find the value of each of ℓ and m** « -1 , $\frac{1}{4}$ »

3 A rectangle of perimeter 24 cm. If its length decreased by 4 cm. and its width increased by 2 cm. became a square. **Find the area of the square.** *(Ismailia 13)* « 25 cm^2 »

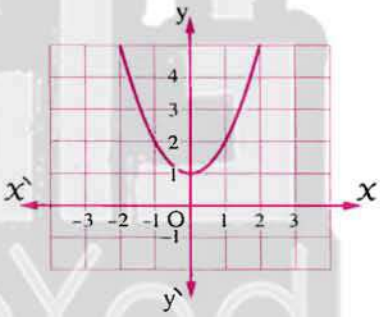
4 A sum equals L.E. 8 , it is wanted to change it to 21 banknotes , some of them are of 25 piastres and the others are of 50 piastres. **Find the number of banknotes of each type.**
« 10 of 25 piastres , 11 of 50 piastres »

Exercise 2 Solving an equation of the second degree in one unknown graphically and algebraically

From the school book

1 Choose the correct answer from those given :

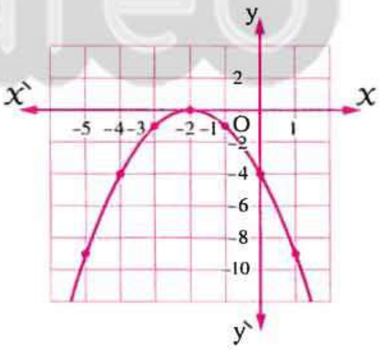
1 The opposite figure represents the curve of a quadratic function f , then the solution set of the equation $f(X) = 0$ in $\mathbb{R}$ is .... *(Cairo 16)*



(a) $\varnothing$ (b) $\{1\}$

(c) $\{0\}$ (d) $\{(0 , 1)\}$

2 **In the opposite figure :**

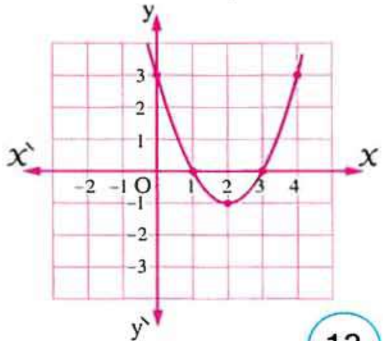


The S.S. of the equation $f(X) = 0$ in $\mathbb{R}$ is ....

(a) $\{-2\}$ (b) $\{-2 , 4\}$

(c) $\{4\}$ (d) $\varnothing$

3 **In the opposite figure :**



The S.S. of the equation $f(X) = 0$ in $\mathbb{R}$ is .... *(Cairo 15)*

(a) $(2 , -1)$ (b) $\{(3 , 1)\}$

(c) $\{3 , 1\}$ (d) $(3 , 0)$

4 If the curve of the quadratic function does not intersect the X-axis at any point , then the number of solutions of the equation $f(X) = 0$ in $\mathbb{R}$ is .... *(El-Monofia 17 , Qena 04)*

(a) a unique solution. (b) two solutions.

(c) an infinite number. (d) zero.

5 If the curve of the quadratic function f passes through the points $(-1 , 0)$, $(0 , -4)$, $(4 , 0)$ and $(0 , -6)$, then the solution set of the equation $f(X) = 0$ in $\mathbb{R}$ is ....

(El-Gharbia 19)

(a) $\{-1 , 0\}$ (b) $\{-4 , 0\}$ (c) $\{-1 , 4\}$ (d) $\{4 , -4\}$

6 The curve of the function $f : f(X) = X^2 - 5X$ intersects the X-axis at the two points ....

(a) $(2 , 0)$, $(0 , 5)$ (b) $(0 , 0)$, $(5 , 0)$

(c) $(2 , 0)$, $(-5 , 0)$ (d) $(0 , 0)$, $(-5 , 0)$

7 If the S.S. of the equation : $4X^2 + 4X + k = 0$ is $\{-\frac{1}{2}\}$, then k = ...

(a) 2 (b) 1 (c) -1 (d) -8

8 If $X = 3$ is one of the solutions of the equation : $X^2 - aX - 6 = 0$, then a = ...

(Suez 17)

(a) 3 (b) 2 (c) 1 (d) -1

9 In the equation : $aX^2 + bX + c = 0$, if $b^2 - 4ac > 0$, then this equation has .... roots in $\mathbb{R}$ *(El-Fayoum 19 , Damietta 16)*

(a) 1 (b) 2 (c) zero (d) an infinite number

10 In the equation : $aX^2 + bX + c = 0$, if $b^2 - 4ac = 0$, then the number of real solutions of the equation equals ....

(a) 1 (b) 2 (c) zero (d) an infinite number

11 In the equation : $aX^2 + bX + c = 0$, if $b^2 - 4ac < 0$, then the number of roots of the equation in $\mathbb{R}$ equals ....

(a) 1 (b) 2 (c) zero (d) an infinite number

12 If $X \in \mathbb{R}$, then the equation : $X^2 + X + 1 = 0$ has ....

(a) two roots. (b) one root.

(c) no roots. (d) an infinite number of roots.

Exercise 2

2 **Find the S.S. of the following equation in $\mathbb{R}$: $x^2 + 2x - 3 = 0$:**

1. graphically on the interval $[-4 , 2]$
2. using factorization.
3. using the general formula.
4. using the calculator.

3 Represent graphically the function $f : f(x) = x^2 - 2x$ in the interval $[-1 , 3]$, from the graph find the S.S. of the equation : $x^2 - 2x = 0$ *(Suez 12)*

4 **Graph the function $f : f(x) = x^2 + 2x + 1$ in the interval $[-4 , 2]$ and from the graph , find the solution set of the equation : $x^2 + 2x + 1 = 0$**

5 **Graph the function $f : f(x) = x^2 - 4x + 3$ on the interval $[-1 , 5]$ and from the graph , find :**

1. The minimum value of the function.
2. The equation of the axis of symmetry.
3. The S.S. of the equation $f(x) = 0$ *(El-Monofia 12)*

6 **Graph the function $f : f(x) = -x^2 + 6x - 11$ in the interval $[0 , 6]$, from the graph find the S.S. of the equation : $x^2 - 6x + 11 = 0$**

7 **Draw a graphical representation of the function f where $f(x) = 6x - x^2 - 9$ in the interval $[0 , 5]$ and from the drawing find :**

1. The maximum value or the minimum value of the function.
2. The solution set of the equation : $6x - x^2 - 9 = 0$ *(Port Said 12)*

8 **Graph the curve of the function $f : f(x) = 4x^2 - 12x + 9$ on the interval $[0 , 3]$ and from the graph find :** The S.S. of the equation $f(x) = 0$

9 **Draw the graphical representation of the function f in the given interval , then find the solution set of the equation $f(x) = 0$:**

1. $f(x) = x^2 - 2x - 4$ in the interval $[-2 , 4]$
2. $f(x) = 2x^2 + 5x$ in the interval $[-4 , 2]$ *(Souhag 13)*
3. $f(x) = 3x - x^2 + 2$ in the interval $[-1 , 4]$
4. $f(x) = x(x - 5) + 3$ in the interval $[0 , 5]$ *(El-Monofia 11)*

Unit 1

5 $f(x) = 2x^2 - 3(2 - x)$ in the interval $[-3, 2]$

6 $f(x) = 2x(x - 1) - 3(x + 2) + 5$ in the interval $[-1, 3]$

7 $f(x) = (x - 3)^2 - (x - 3) - 4$ in the interval $[1, 7]$

10 Find in $\mathbb{R}$ the S.S. of each of the following equations using the general formula :

1 $x^2 + 7x + 2 = 0$ approximating the result to the nearest tenth. *(El-Kalyoubia 16)*

2 $x^2 - 4x + 1 = 0$ approximating the result to the nearest two decimal digits. *(Giza 17 , Aswan 14 , Alexandria 13)*

3 $2x^2 - 4x + 1 = 0$ rounding the result to three decimal digits. *(El-Dakahlia 19 , Qena 12)*

4 $3x^2 - 6x + 1 = 0$ rounding the result to the nearest three decimals. *(South Sinai 18)*

5 $2x^2 + 5x = 0$ *(Alexandria 19)*

6 $x^2 + 3x + 5 = 0$ *(El-Fayoum 19)*

7 $x^2 + 8x + 9 = 0$, where $\sqrt{7} \approx 2.65$ *(Ismailia 09)*

8 $2x^2 - x - 2 = 0$, where $\sqrt{17} \approx 4.12$ *(Luxor 19)*

11 Find in $\mathbb{R}$ the solution set of each of the following equations using the general formula approximating the result to three decimal digits :

1 $x^2 = 6x - 7$

2 $2x^2 - 10x = 1$ *(Damietta 13)*

3 $x(x - 1) = 4$ *(Souhag 19)*

4 $2x^2 = 3(2 - x)$

5 $x^2 - 2x + 4 = x + 3$

6 $(x - 3)^2 - 5x = 0$

7 $x + \frac{4}{x} = 6$ *(Damietta 19)*

8 $\frac{8}{x^2} + \frac{1}{x} = 1$ *(El-Fayoum 12)*

9 $\frac{x}{3} = \frac{1}{5 - x}$

10 $\frac{x^2}{9} - \frac{4}{3}x = -2$

Applications on solving an equation of the second degree in one unknown

1 When a dolphin jumps over water surface , its pathway follows the relation $y = -0.2x^2 + 2x$ where y is the height of the dolphin above water surface and x is the horizontal distance in feet.

Find the horizontal distance that the dolphin covers when it jumps from water till it returns again to water. « 10 feet »

Exercise 2

2 A bullet is shot from a mortar cannon in a pathway follows the relation $y = 0.38 + 0.78\, x - 0.3\, x^2$, where x represents the horizontal distance in kilometre , y is the height of the bullet above the floor surface in kilometre.
Find the horizontal distance far from the cannon which the bullet reaches till it strikes the land surface.

« 3 km. »

3 A man waters his garden with a hose where the water is pumped through in a pathway identified by the relation $y = -0.06\, x^2 + 0.39\, x + 0.79$ where x is the horizontal distance that the water can reach in metre and y is the height of the water from the floor surface in metre.
Find to the nearest centimetre the maximum horizontal distance that the water can reach.

« 812 cm. »

4 A diver starts jumping from a platform of height 10 metres above water surface. If the height of the diver follows the relation $y = -4.9\, t^2 + 3.5\, t + 10$ where t is the time in seconds.
After how many seconds the diver will reach the water surface ?

« 1.83 seconds »

5 A player beats the golf ball to reach a certain place and the following relation expresses the height to which the ball will reach in feet $y = -16\, t^2 + 80\, t + 20$ where t is the time in seconds.

1. **After how many seconds will the ball reach the floor surface ?**
2. **Will the ball reach a height of 130 feet ?**

« 5.24 seconds »

6 A snake saw a hawk at a height of 160 metres and hawk was flying at a speed of 24 metre / minute to pounce on it. If the hawk is launching vertically downwards according to the relation $d = Vt + 4.9\, t^2$ where d is the distance by metre , V is the launching speed in metre / minute and t is the time in minutes.
Find the time the snake takes to escape before the hawk reaches it. « less than 3.77 seconds »

Unit 1

For excellent pupils

Complete the following :

1 If the curve of a quadratic function f intersects X-axis at the two points $(1 , 0)$, $(3 , 0)$, then the equation of the axis of symmetry of the function f is the straight line passing through the vertex of the curve at $X =$ ....

2 If the point $(-3 , 0)$ is the vertex point of the curve of the function f , then the S.S. of the equation $f(X) = 0$ is ....

3 If the point $(a - 2 , 0)$ is the vertex point of the curve of the quadratic function f , the S.S. of the equation $f(X) = 0$ is $\{5\}$, then a = ...

4 If the point $(-3 , 4)$ is the vertex point of the curve of a quadratic function f and -5 is a root of the equation $f(X) = 0$, then the other root is ....

Exercise 3 — Solving two equations in two variables one of them is of the first degree and the other is of the second degree

📖 From the school book

1 **Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :**

1. $x = y$, $x^2 + y^2 = 2$ *(Souhag 09)* « $\{(1, 1), (-1, -1)\}$ »
2. $x - 3 = 0$, $x^2 + y^2 = 25$ *(Cairo 19)* « $\{(3, 4), (3, -4)\}$ »
3. $x - 2y = 0$, $x^2 - y^2 = 3$ *(Port Said 17)* « $\{(2, 1), (-2, -1)\}$ »
4. $x - y = 0$, $x^2 + xy + y^2 = 27$ *(Alex. 19)* « $\{(3, 3), (-3, -3)\}$ »
5. $y - 2x = 0$, $xy = 18$ *(El-Sharkia 14)* « $\{(3, 6), (-3, -6)\}$ »
6. $y + 2x = 0$, $6x^2 - y^2 = 72$ « $\{(6, -12), (-6, 12)\}$ »
7. $x + y = 0$, $y^2 = x$ *(6^{th} October 11)* « $\{(0, 0), (1, -1)\}$ »
8. $x - y = 0$, $x = \frac{4}{y}$ *(El-Dakahlia 19 , Ismailia 18)* « $\{(2, 2), (-2, -2)\}$ »

2 **Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :**

1. $y = x - 1$, $y^2 + x = 7$ *(Qena 09)* « $\{(3, 2), (-2, -3)\}$ »
2. $x = 5 - y$, $x^2 - y^2 = 55$ *(Matrouh 08)* « $\{(8, -3)\}$ »
3. $x - y = 1$, $x^2 + y^2 = 25$ *(Aswan 19 , Port said 18)* « $\{(-3, -4), (4, 3)\}$ »
4. $x + y = 7$, $y^2 - x^2 = 7$ *(Kafr El-Sheikh 15)* « $\{(3, 4)\}$ »
5. $x - y - 2 = 0$, $x^2 - y^2 = 0$ *(El-Kalyoubia 09)* « $\{(1, -1)\}$ »
6. $2x + y = 10$, $x^2 + y^2 = 25$ *(El-Kalyoubia 05)* « $\{(3, 4), (5, 0)\}$ »
7. $y - x = 3$, $x^2 - 2x + 3y = 15$ *(Alex. 11)* « $\{(-3, 0), (2, 5)\}$ »
8. 📖 $y + 2x = 7$, $2x^2 + x + 3y = 19$ « $\{(\frac{1}{2}, 6), (2, 3)\}$ »

Unit 1

3 **Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :**

1. $x + y = 7$, $x y = 12$ *(Qena 17)* « $\{(4, 3), (3, 4)\}$ »
2. $x + y = 5$, $\frac{x y}{6} = 1$ *(Monofia 08)* « $\{(2, 3), (3, 2)\}$ »
3. $y - x = 2$, $x^2 + x y - 4 = 0$ *(El-Beheira 19)* « $\{(-2, 0), (1, 3)\}$ »
4. $x - 2 y - 1 = 0$, $x^2 - x y = 0$ *(Kafer El-Sheikh 19)* « $\{(0, -\frac{1}{2}), (-1, -1)\}$ »
5. $x + y = 1$, $x^2 + x y + y^2 = 3$ *(South Sinai 18)* « $\{(2, -1), (-1, 2)\}$ »
6. $x + 2 y = 4$, $x^2 + x y + y^2 = 7$ « $\{(2, 1), (-2, 3)\}$ »
7. $y - x = 3$, $x^2 + y^2 - x y = 13$ *(El-Kalyoubia 17)* « $\{(1, 4), (-4, -1)\}$ »
8. $x - y = 10$, $x^2 - 4 x y + y^2 = 52$ « $\{(-2, -12), (12, 2)\}$ »

4 **Find in $\mathbb{R} \times \mathbb{R}$ the solution set of each pair of the following equations :**

1. $x = 0$, $x^2 + y^2 + 4 x + 3 y - 10 = 0$ *(Ismailia 03)* « $\{(0, 2), (0, -5)\}$ »
2. $x - 2 y = 8$, $y^2 = x$ *(Damietta 09)* « $\{(4, -2), (16, 4)\}$ »
3. $x + 2 y = 2$, $x^2 + 2 x y = 2$ *(El-Sharkia 19)* « $\{(1, \frac{1}{2})\}$ »
4. $x + y = 2$, $x^2 + y^2 + 2 x y + y = 6$ *(New Valley 13)* « $\{(0, 2)\}$ »
5. $x + y = 2$, $\frac{1}{x} + \frac{1}{y} = 2$, where $x \neq 0$, $y \neq 0$ *(El-Menia 19)* « $\{(1, 1)\}$ »

5 **Choose the correct answer from those given :**

1. The S.S. of the two equations : $x y = 5$, $x + x y = 6$ in $\mathbb{R} \times \mathbb{R}$ is ....

   (a) $\{(1, 5)\}$ (b) $\{(5, 6)\}$
   (c) $\{(5, 2)\}$ (d) $\{(1, 5), (5, 1)\}$

2. The S.S. of the two equations : $x - y = 0$, $x y = 9$ in $\mathbb{R} \times \mathbb{R}$ is .... *(Qena 18 , El-Gharbia 11)*

   (a) $\{(0, 0)\}$ (b) $\{(-3, 3)\}$
   (c) $\{(3, 3)\}$ (d) $\{(-3, -3), (3, 3)\}$

3. The S.S. of the two equations : $x = 1$, $x^2 - y^2 = 10$ in $\mathbb{R} \times \mathbb{R}$ is ....

   (a) $\{(1, 3)\}$ (b) $\{(1, -3)\}$ (c) $\{(1, 3), (1, -3)\}$ (d) $\varnothing$

4. The S.S. of the two equations : $x + y = 0$, $x^2 + y^2 = 2$ in $\mathbb{R} \times \mathbb{R}$ is .... *(Assiut 13)*

   (a) $\{(0, 0)\}$ (b) $\{(1, -1)\}$
   (c) $\{(-1, 1)\}$ (d) $\{(1, -1), (-1, 1)\}$

5 The ordered pair which satisfies each of the two equations : $x\,y = 2$, $x - y = 1$ is .... *(El-Sharkia 12)*

(a) (1 , 1) (b) (2 , 1) (c) (1 , 2) (d) $(\frac{1}{2}, 1)$

6 One of the solutions for the two equations : $x - y = 2$, $x^2 + y^2 = 20$ is .... *(El - Kalyoubia 19 , Qena 17 , Port Said 14)*

(a) (– 4 , 2) (b) (2 , – 4) (c) (3 , 1) (d) (4 , 2)

7 If $y = 1 - x$, $(x + y)^2 + y = 5$, then y = ... *(El-Fayoum 12)*

(a) 5 (b) 3 (c) – 4 (d) 4

8 If $x^2 + x\,y = 15$, $x + y = 5$, then x = ... *(Cairo 06)*

(a) 3 (b) 4 (c) 5 (d) 6

Applications on solving two equations in two variables one of them of the first degree and the other of the second degree :

1 The sum of two real positive numbers is 17 and their product is 72

Find the two numbers. *(Alex. 09)* « 8 , 9 »

2 The sum of two real numbers is 9 and the difference between their squares equals 45

Find the two numbers. *(El-Fayoum 19 , Kafr El-Sheikh 13)* « 7 , 2 »

3 Two positive numbers , one of them exceeds three times the other by 1 and the sum of their squares is 17

What are the two numbers ? *(El-Sharkia 04)* « 1 , 4 »

4 The perimeter of a rectangle is 18 and its area is 18 cm^2.

Find its two dimensions. *(New Valley 16)* « 6 cm. , 3 cm. »

5 A length of a rectangle is 3 cm. more than its width and its area is 28 cm^2.

Find its perimeter. *(El-Fayoum 12)* « 22 cm. »

6 For a rhombus , the difference between the lengths of its diagonals equals 4 cm. and its perimeter is 40 cm.

Find the lengths of the diagonals. « 16 cm. , 12 cm. »

Unit 1

7 Two pieces of land , each of them is on the shape of a square the difference between their perimeters is 8 metres and the difference between their areas is 20 square metres.
Find the side length of each of them. « 6 metres , 4 metres »

8 A right-angled triangle of hypotenuse length 13 cm. and its perimeter is 30 cm.
Find the lengths of the other two sides. *(El-Monofia 15)* « 5 cm. , 12 cm. »

9 A right-angled triangle in which the length of one of the sides of right-angle is 5 cm. and its perimeter is 30 cm. find the area of the triangle. (Indicating the steps of the solution)
(El-Monofia 17) « 30 cm^2 »

10 A right-angled triangle , in which the length of one of the sides of the right angle is 3 cm. less than the length of the other side and the hypotenuse length is 15 cm.
Find its perimeter. « 36 cm. »

11 The length of a rectangle is x cm. and its width is y cm. and its area = 77 cm^2
If its length decreases by 2 cm. and its width increases 2 cm.
, then it will become a square.
Find the area of the square. *(North Sinai 05)* « 81 cm^2 »

12 If Ayman's age is more than three times the age of his son Bassem by one year and the sum of the squares of their ages is more than three times the product of the two ages by 181
What is the age of each of them ? « 37 years , 12 years »

13 Consider a digit in units place is twice the digit in the tens place of a two-digit number.
If the product of the two digits equals the half of the original number.
What is this number ? « 36 »

14 A two-digit number , its tens digit is more than its units digit by 1 and the product of the original number by the number resulting from reversing its two digits equals 252
Find the original number. « 21 »

15 A point moves on the straight line $5x - 2y = 1$ where its y-coordinate is twice of the square of its x-coordinate.
Find the coordinates of this point. « $(\frac{1}{4}, \frac{1}{8})$ or $(1, 2)$ »

16 A driver of a car moved a distance x kilometres towards the west , then he moved a distance y kilometres towards the south. If the sum of the two distances equals 28 kilometres and the distance between the starting point and the end point is 20 km.

Find the distance which the driver moved in each of the west direction and the south direction. « 12 km. , 16 km. »

For excellent pupils

1 **Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :**

1. $x = y$, $x^2 - 2y - 2 = 0$ « $\{(1+\sqrt{3}, 1+\sqrt{3}), (1-\sqrt{3}, 1-\sqrt{3})\}$ »
2. $x + y = 7$, $\sqrt{x} + y = 5$ « $\{(4, 3)\}$ »

2 If $(-2, 4)$ is one of the solutions of the two equations : $a x + b y = 2$, $a b x y + 2 x^2 = 0$ in $\mathbb{R} \times \mathbb{R}$ where a and b are two integers.

Find : (a, b) « $(1, 1)$ »

Summary of Unit 1

Solving two equations of the first degree in two variables graphically :

To solve two equations of the first degree in two variables graphically draw the two straight lines representing the two equations in the Cartesian plane , then the S.S. is the points of intersection of the two straight lines and we have three cases :

1. The two straight lines intersect at one point as (k , m) so , there is a unique solution which is (k , m) , the S.S. = $\{(k , m)\}$
2. The two straight lines are coincident so , there is an infinite number of solutions.
3. The two straight lines are parallel so , there is no solution , the S.S. = $\varnothing$

Solving two equations of the first degree in two variables algebraically :

This method depends on removing one of the two variables to get an equation of the first degree in one variable , then we get the value of this variable by solving this equation.

Then we substitute by this value in any of the two equations to get the value of the other variable which we have removed before and we follow one of the two methods :

1. Substituting method.
2. Omitting method.

Solving an equation of the second degree in one unknown graphically :

To solve an equation of the second degree in one unknown graphically , we do the following steps :

1. Put the equation in the form : $a X^2 + b X + c = 0$
2. Assume that : $f(X) = a X^2 + b X + c$, draw the curve of the function f
3. Determine the points of intersection of the function curve and X-axis , then the X-coordinates of these points of intersection are the solutions of the equation : $a X^2 + b X + c = 0$

and we have three cases :

(1) The curve intersects the X-axis at two points as $(\ell , 0)$, $(m , 0)$ so, there are two solutions of the equation , the S.S. = $\{\ell , m\}$

(2) The curve touches the X-axis at one point as $(\ell , 0)$ so there is a unique solution of the equation , the S.S. = $\{\ell\}$

(3) The curve does not intersect the X-axis so, there is no solution , the S.S. = $\varnothing$

Solving an equation of the second degree in one unknown using the general rule (general formula) :

If a $x^2 + b x + c = 0$ where a , b and c are real numbers , $a \neq 0$

, then $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

i.e. The solution set of the equation $= \left\{ \frac{-b + \sqrt{b^2 - 4ac}}{2a} , \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right\}$

Solving two equations in two variables one of them is of the first dergee and the other is of the second degree :

The method of solving two equations in two variables , one of them is of first degree and the other is of second degree , depends on the substituting method whose steps are as follows :

1. From the equation of the first degree we express one of the two variables in terms of the second variable.
2. Substituting in the equation of the second degree we get an equation of the second degree in one variable.
3. Solving the result equation by factorization or by general formula we get the value of one of the two variables.
4. Substituting in the equation of the first degree we get the value of the other variable.

Exams on Unit One

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 The solution set of the two equations : $x + 2y = -4$, $x - 2y = 4$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(-4, 4)\}$ (b) $\{(4, 0)\}$ (c) $\{(0, 2)\}$ (d) $\{(0, -2)\}$

2 The curve of the function $f : f(x) = x^2 - 3x + 2$ intersects the x-axis at the two points ....

(a) $(2, 0)$, $(3, 0)$ (b) $(2, 0)$, $(1, 0)$

(c) $(-2, 0)$, $(-1, 0)$ (d) $(2, 0)$, $(-1, 0)$

3 If $x^2 - y^2 = 15$, $x - y = 3$, then $x + y =$ ....

(a) -5 (b) -3 (c) 3 (d) 5

4 If $x = 1$, $x^2 + y^2 = 10$, then $y =$ ....

(a) -3 (b) ± 3 (c) 3 (d) 9

5 The point of intersection of the two straight lines : $y = 2$, $x + y = 6$ is ....

(a) $(2, 6)$ (b) $(2, 4)$ (c) $(4, 2)$ (d) $(6, 2)$

6 If the two straight lines which represent the two equations :

$x + 5y = 4$, $2x - ky = 10$ are parallel , then $k =$ ....

(a) 2 (b) 10 (c) -10 (d) -5

2 [a] Find in $\mathbb{R}$ the S.S. of the following equation using the general formula :

$x^2 = 2(x + 3)$, where $\sqrt{7} \approx 2.65$

[b] Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of the two equations :

$y - x = -5$, $x^2 - 2xy = 16$

3 **[a] Find graphically in $\mathbb{R} \times \mathbb{R}$ the S.S. of the two equations :**

$2x + y = 5$, $x = 7 - 2y$

[b] The length of the diagonal of a rectangle is 5 cm. and its perimeter is 14 cm.

Find its two dimensions.

4 **[a] Find algebraically in $\mathbb{R} \times \mathbb{R}$ the S.S. of the two equations :**

$3x + 4y = 7$, $2x - y = 1$

[b] Graph the function f where $f(x) = x^2 - 2x - 3$, $x \in [-2, 4]$, then from the graph find :

1. The vertex point of the curve.
2. The maximum of minimum value of the function.
3. The S.S. of the equation : $x^2 - 2x - 3 = 0$

5 **[a]** If $(1, -1)$ is a solution for the two equations :

$a x + b y = 7$, $a x - b y = 3$

Find the value of each of : a and b

[b] The sum of two rational numbers is 12 , three times of the smaller number exceeds twice of the greater number by 1

Find the two numbers.

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1. If the two equations : $x + 2y = 1$, $2x + ky = 2$ has a unique solution in $\mathbb{R} \times \mathbb{R}$, then $k \neq$ ....

   (a) 1 (b) 2 (c) 4 (d) -4

2. If $a^2 - b^2 = 6$, $a - b = \sqrt{3}$, then $(a + b)^2 =$ ....

   (a) $2\sqrt{3}$ (b) $3\sqrt{3}$ (c) $\sqrt{3}$ (d) 12

Unit 1

3 If the S.S. of the equation : $x^2 - a\,x + 4 = 0$ in $\mathbb{R}$ is $\{-2\}$, then a = ...

(a) -2 (b) -4 (c) 2 (d) 4

4 The S.S. of the two equations : $\frac{1}{2}x = 1$, $x + y = 1$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(2 , 1)\}$ (b) $\{(2 , -1)\}$ (c) $\{2 , -1\}$ (d) $(2 , -1)$

5 If $x = y + 1$, $(x - y)^2 + y = 3$, then y = ...

(a) 0 (b) 1 (c) 2 (d) 3

6 If the curve of the quadratic function f passes through the points $(3 , 0)$, $(1 , -4)$, $(-1 , 0)$, then the S.S. of the equation : $f(x) = 0$ in $\mathbb{R}$ is ....

(a) $\{3 , -4\}$ (b) $\{3 , -1\}$ (c) $\{3 , 1 , -4\}$ (d) $\{3 , -1 , 1\}$

2 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of the two equations :**

$x + 3y = 7$, $5x - y = 3$

[b] Find in $\mathbb{R}$ the S.S. of the following equation by using the general formula :

$5x^2 - 3x = 1$ (rounding the result to two decimal places)

3 **[a]** If the S.S. of the equation : $a\,x^2 + x + b$ in $\mathbb{R}$ is $\{0 , 1\}$,

Then find the value of each of : a and b

[b] Find in $\mathbb{R}^2$ the S.S. of the two equations :

$y - x = 3$, $x^2 + y^2 - x\,y = 13$

4 **[a]** Two numbers , if three times the first is added to twice the second the result is 13 and if the first is added to three times the second the result is 16 what are the two numbers ?

[b] two real numbers their sum is 10 and the difference betwen their squars is 40
Find the two numbers.

5 **[a]** Graph the function $f : f(x) = x^2 - 2x$ in the interval $[-2 , 4]$ and from the graph find the S.S. of the equation : $x^2 - 2x = 0$

[b] Find graphically in $\mathbb{R} \times \mathbb{R}$ the S.S. of the two equations :

$x + 2y = 1$, $2x + 4y = -5$

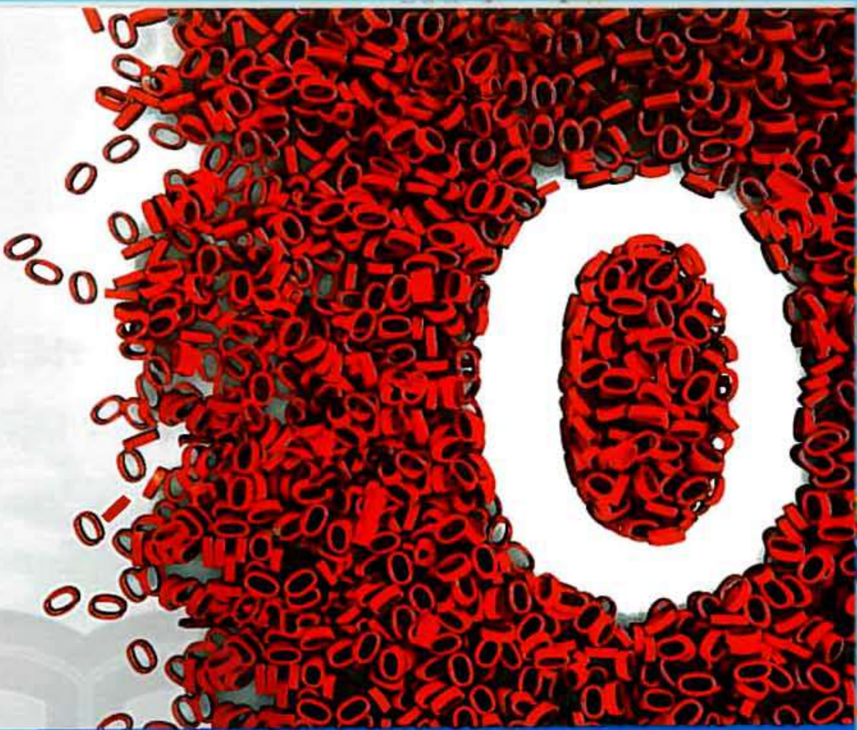
UNIT 2

Algebraic fractional functions and the operations on them

Exercises of the unit :

4. Set of zeroes of a polynomial function.
5. Algebraic fractional function.
6. Equality of two algebraic fractions.
7. Operations on algebraic fractions (Adding and subtracting algebraic fractions).
8. Operations on algebraic fractions (follow) (Multiplying and dividing algebraic fractions).

- Summary of unit two.
- Unit exams.



Exercise 4

Set of zeroes of a polynomial function

From the school book

1 **Determine the set of zeroes of the polynomial functions which are defined by the following rules in $\mathbb{R}$:**

1. $f(x) = 5x + 10$
2. $f(x) = x^2 - 2x$
3. $f(x) = x^2 - 16$
4. $f(x) = x^2 + 9$
5. $f(x) = 25 - 9x^2$
6. $f(x) = 5x^3 - 20x$
7. $f(x) = x^3 - 125$
8. $f(x) = 2x^4 + 54x$
9. $f(x) = x^2 - 3x - 4$
10. $f(x) = 2x^2 - 5x - 12$
11. $f(x) = x^3 - x^2 - 2x$
12. $f(x) = 6x^2 - 2x^3 - 4x$
13. $f(x) = 2x^4 + x^3 - 6x^2$
14. $f(x) = x(x - 5) - 14$
15. $f(x) = (x - 2)(x + 3) + 4$ *(El-Monofia 15)*
16. $f(x) = x^2 + 2x - 6$
17. $f(x) = 2x^2 - x + 5$
18. $f(x) = x^3 - 3x^2 - 4x + 12$
19. $f(x) = x^3 + x^2 - 2x - 8$
20. $f(x) = x^4 - 10x^2 + 9$

2 **Choose the correct answer from those given :**

1. The set of zeroes of the function $f : f(x) = -3x$ is .... *(Seuz 18 , Giza 17)*
   (a) $\{0\}$ (b) $\{-3\}$ (c) $\{-3, 0\}$ (d) $\mathbb{R}$
2. The set of zeroes of the function $f : f(x) = 9x^2$ is ....
   (a) $\varnothing$ (b) $\{0\}$ (c) $\{\frac{1}{3}\}$ (d) $\{-3\}$

Exercise 4

3 The set of zeroes of the function $f : f(x) = 4$ is .... *(Aswan 17)*

(a) $\{-4\}$ (b) $\{0\}$ (c) $\varnothing$ (d) $\{2\}$

4 The set of zeroes of the function $f : f(x) =$ zero is .... *(Cairo 19 , Qena 09)*

(a) $\varnothing$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R}$ (d) zero

5 The set of zeroes of the function $f : f(x) = x^2 - 25$ is .... *(Assiut 16 , South Sinai 14)*

(a) $\{5\}$ (b) $\{-5\}$ (c) $\{5 , -5\}$ (d) $\varnothing$

6 The set of zeroes of the function $f : f(x) = x^6 - 32\,x$ is .... *(Beni Suef 11)*

(a) $\{0 , 2\}$ (b) $\{2 , 16\}$ (c) $\{6 , 16\}$ (d) $\{0 , 5\}$

7 The set of zeroes of the function $f : f(x) = x(x^2 - 1)$ is ....

(a) $\{0\}$ (b) $\{0 , -1\}$ (c) $\{0 , 1 , -1\}$ (d) $\{0 , 1\}$

8 If $f(x) = x^2 + x + 1$, then the set of zeroes of the function f is .... *(El-Fayoum 06)*

(a) $\{0\}$ (b) $\{1\}$ (c) $\varnothing$ (d) $\{2\}$

9 The set of zeroes of the function $f : f(x) = x(x^2 - 2\,x + 1)$ is .... *(Alex. 13)*

(a) $\{0 , 1\}$ (b) $\{0 , -1\}$ (c) $\{0\}$ (d) $\{1\}$

10 If $z(f) = \{2\}$, $f(x) = x^3 - m$, then $m =$ .... *(Qena 15 , El-Sharkia 14)*

(a) $\sqrt[3]{2}$ (b) 2 (c) 4 (d) 8

11 If $z(f) = \{1 , -2\}$, $f(x) = x^2 + x + a$, then $a =$ ....

(a) 28 (b) 1 (c) -1 (d) -2

12 If $z(f) = \{5\}$, $f(x) = x^3 - 3\,x^2 + a$, then $a =$ .... *(Port Said 14 , Assiut 11)*

(a) -50 (b) -5 (c) 5 (d) 50

13 If $\{2\}$ is the set of zeroes of the function $f : f(x) = x^2 - 2\,a\,x + a^2$,
then $a =$ .... *(New Valley 14)*

(a) 2 (b) -2 (c) 4 (d) -4

14 If the set of zeroes of the function $f : f(x) = x^2 + a$ is $\varnothing$, then a may equal ....

(a) 25 (b) -25 (c) zero (d) -1

15 If the set of zeroes of $f : f(x) = x^2 + k\,x + 1$ is $\varnothing$, then k may equal .... *(El-Sharkia 15)*

(a) 3 (b) 2 (c) 1 (d) -2

Unit 2

3 **Complete the following :**

1. The set of zeroes of the function $f : f(X) = X - 5$ is .... *(Damietta 11)*

2. The set of zeroes of the function $f : f(X) = X^2 + 1$ is .... *(Dakahlia 09)*

3. If $f : f(X) = 4 - 2X$, then the set of zeroes of the function f is .... *(Fayoum 04)*

4. The set of zeroes of the function $f : f(X) = \frac{1}{5}(X - 3)$ is .... *(Kafr El-Sheikh 05)*

5. The set of zeroes of the function $f : f(X) = (X - 1)(X + 2)$ is ....

6. The set of zeroes of the function $f : f(X) = (X - 1)^2 (X + 2)$ is ....

7. The set of zeroes of the function $f : f(X) = X^2 - 3X$ is .... *(Suez 12)*

8. The set of zeroes of the function $f : f(X) = X(X^2 - 9) - 3(X^2 - 9)$ is ....

9. If the curve of the quadratic function f does not intersect X-axis , then $z(f) =$ ....

10. If $\{-3, 3\}$ is the set of zeroes of the function f where $f(X) = X^2 + a$, then a = ... *(Qena 18)*

4 If the function $f : f(X) = X^3 - 2X^2 - 75$

Prove that : The number 5 is the one of the zeroes of the function f

(South Sinai 18 , Beni Suef 15)

5 If the set of zeroes of the function : $f(X) = aX^2 + X + b$ is $\{0, 1\}$

Find the value of each two constants a and b *(Alex. 17)* « -1 , 0 »

6 If the set of zeroes of the function f where $f(X) = aX^2 + bX + 15$ is $\{3, 5\}$

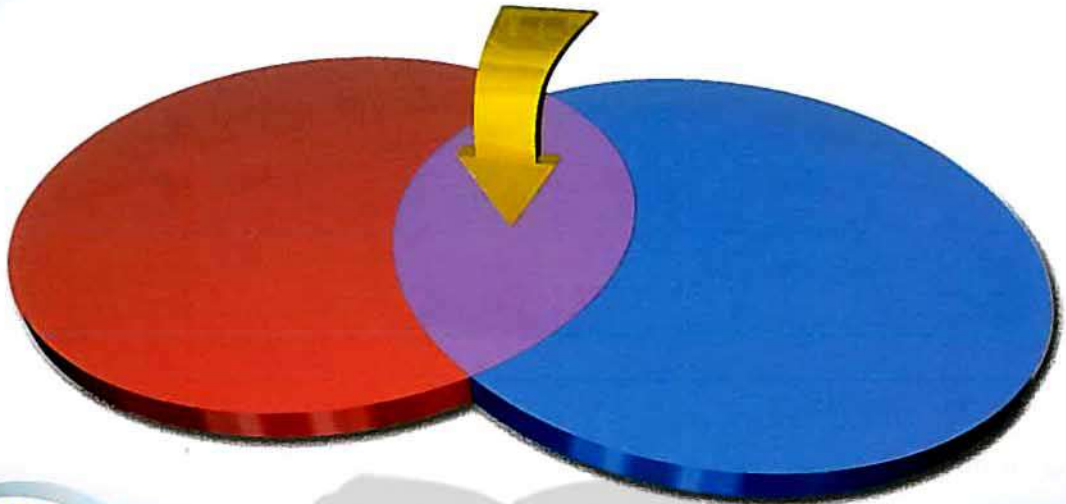
Find the values of a and b *(El-Fayoum 19)* « 1 , -8 »

For excellent pupils

1 If $z(h) = \{-2, 0\}$ where $h(X) = aX^2 + bX + c$, $h(3) = 15$ **Find :** $h(2)$ « 8 »

2 If $g(X) = aX - 3$, $f(X) = a^2X^2 - 12X + 9$ and $z(g) = z(f)$

Find the value of a , then find : $z(f)$ « 2 , $\{\frac{3}{2}\}$ »



Exercise 5 Algebraic fractional function

From the school book

1 **Determine the domain of each of the algebraic fractional functions which are defined by the following rules :**

1. $n(x) = \frac{x+1}{x-2}$
2. $n(x) = \frac{1}{x+2}$
3. $n(x) = \frac{x+3}{4}$
4. $n(x) = \frac{x-6}{x}$
5. $n(x) = \frac{x-2}{2x}$
6. $n(x) = \frac{x^2+1}{x^2-x}$
7. $n(x) = \frac{x^2+9}{x^2-16}$
8. $n(x) = \frac{x^2-1}{x^2+1}$
9. $n(x) = \frac{x^2+25}{x^3+25x}$
10. $n(x) = \frac{x^2-4}{x^2-x-6}$
11. $n(x) = \frac{x^2-4x+3}{8x^3+8}$
12. $n(x) = \frac{x^2-5x+6}{x^4-81}$
13. $n(x) = \frac{x+1}{4x^2-4x-x^3}$
14. $n(x) = \frac{x^2-3}{x^2-3x+5}$

2 **Find the common domain of the following algebraic fractions :**

1. $\frac{x}{3}, \frac{3}{x}$
2. $\frac{x+2}{x+5}, \frac{x-4}{x-7}$
3. $\frac{3x}{x-2}, \frac{x+3}{x^2-9}$ *(North Sinai 09)*
4. $\frac{x^2+x+1}{2x}, \frac{x^2-1}{x^2-x}$ *(Port Said 03)*
5. $\frac{x}{x^2-4}, \frac{3}{2-x}$
6. $\frac{x^2+3x}{x^3-9x}, \frac{x^2+3x+9}{x^3-27}$

Unit 2

7 $\frac{x-4}{x^2-5x+6}$, $\frac{2x}{x^3-9x}$ *(Luxor 19)*

8 $\frac{x^2+4}{x^2-4}$, $\frac{7}{x^2+4x+4}$

9 $\frac{x-1}{x+2}$, $\frac{x+2}{5}$, $\frac{x}{x-3}$ *(South Sinai 09)*

10 $\frac{x-2}{x+4}$, $\frac{7}{x-3}$, $\frac{x}{x^2+4}$

11 $\frac{x+3}{2}$, $\frac{3}{x^2-9}$, $\frac{3x}{x^2-3x}$

12 $\frac{x^2-4}{x^2-5x+6}$, $\frac{7}{x^2-9}$, $\frac{x^2-3x-4}{x^2+x-2}$

3 Complete the following :

1 The domain of the function $f : f(x) = \frac{1}{x^3}$ is ....

2 The domain of the function $f : f(x) = \frac{x^3-4x}{2x+4}$ is ....

3 The domain of the function $f : f(x) = \frac{x^2}{x^7-32x^2}$ is ....

4 If $n(x) = \frac{x^2-25}{x^2-7x+6}$, then $n(x)$ is meaningless if $x \in$ ....

5 The set of zeroes of the function $f : f(x) = \frac{x^2-4}{x^2-x-2}$ is .... *(Suez 05)*

6 The common domain of the two functions n_1 and n_2 , where $n_1(x) = \frac{6}{x}$, $n_2(x) = x + 6$ is .... *(Ismailia 06)*

7 If $f(x) = \frac{x+a}{x^2+1}$ and $f(2) = 1$, then a = ...

8 If $n(x) = \frac{7}{x+a}$ and the domain of the function n is $\mathbb{R} - \{-2\}$, then a = ... *(El-Monofia 11)*

9 If the function $f : f(x) = \frac{x-5}{x^2-a}$ has the domain $\mathbb{R} - \{-5 , 5\}$, then a = ... *(El-Monofia 04)*

10 If $n_1(x) = \frac{-7}{x+2}$, $n_2(x) = \frac{x}{x-k}$ and the common domain of the two functions n_1 and n_2 is $\mathbb{R} - \{-2 , 7\}$, then k = ... *(North Sinai 12)*

11 If $f : f(x) = \frac{x^2-\ell}{x^3-27}$ and the set of zeroes of this function is $\{2 , -2\}$, then ℓ = ...

4 Choose the correct answer from those given :

1 If n_1 and n_2 are two algebraic fractions , the domain of $n_1 = \mathbb{R} - X_1$ where X_1 is the set of zeroes of the denominator of n_1 , the domain of $n_2 = \mathbb{R} - X_2$ where X_2 is the set of zeroes of the denominator of n_2 , then the common domain of n_1 and $n_2 = \mathbb{R} -$ .... *(Port said 18)*

(a) $X_1 - X_2$ (b) $X \cap x_2$ (c) $X_1 \cup X_2$ (d) $\varnothing$

2 The domain of the function n : n $(x) = \frac{x-2}{x^2+1}$ is .... *(Qena 19 , Assiut 17)*

(a) $\mathbb{R} - \{-1\}$ (b) $\mathbb{R} - \{1 , -1\}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R}$

3 The domain of the function $f : f(x) = \frac{2x-4}{x^3-4x}$ is ....

(a) $\mathbb{R}$ (b) $\{-2 , 2\}$ (c) $\mathbb{R} - \{-2 , 2\}$ (d) $\mathbb{R} - \{-2 , 0 , 2\}$

4 The domain of the algebraic fraction $\frac{x-5}{3}$ equals the domain of the algebraic fraction .... *(El-Kalyoubia 16)*

(a) $\frac{x}{x^2+1}$ (b) $\frac{x}{x-3}$ (c) $\frac{3}{x-5}$ (d) $\frac{x-5}{x-3}$

5 If $f(x) = \frac{x}{x-2}$, then $f(2) =$ .... *(Qena 06)*

(a) 2 (b) 1 (c) zero (d) undefined.

6 If the domain of the algebraic fraction n is $\mathbb{R} - \{2 , 3 , 4\}$, then n (3) = ... *(El-Sharkia 19)*

(a) 3 (b) 2 (c) 4 (d) undefined

7 The set of zeroes of the function $f : f(x) = \frac{2-x}{7}$ is .... *(Cairo 16)*

(a) $\{7\}$ (b) $\{2 , 7\}$ (c) $\{2\}$ (d) $\varnothing$

8 The set of zeroes of the function $f : f(x) = \frac{(x+1)(x-3)}{x^2-4}$ is .... *(El-Menia 18)*

(a) $\{3 , -3\}$ (b) $\{-3 , -1\}$ (c) $\{3 , -1\}$ (d) $\{2 , -2\}$

9 The set of zeroes of the function $f : f(x) = \frac{x^2-x-2}{x^2+4}$ is .... *(El-Gharbia 17)*

(a) $\{2 , -2\}$ (b) $\{-2 , -1\}$ (c) $\{2 , -1\}$ (d) $\{1 , -1\}$

10 The set of zeroes of the function $f : f(x) = \frac{x^2-9}{x-2}$ is .... *(Matrouh 17)*

(a) $\{2\}$ (b) $\mathbb{R} - \{2\}$ (c) $\{3 , -3\}$ (d) $\{3 , -3 , 2\}$

11 The common domain of the two fractions $\frac{2}{x^2-1}$, $\frac{5x}{x^2-x}$ is .... *(El-Fayoum 18)*

(a) $\mathbb{R} - \{1\}$ (b) $\mathbb{R} - \{0 , 1\}$ (c) $\mathbb{R} - \{0 , 1 , -1\}$ (d) $\mathbb{R} - \{1 , -1\}$

12 The common domain of the two functions $n_1 : n_1(x) = 3x - 15$, $n_2 : n_2(x) = x^2 - 4$ is ....

(a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{2 , -2\}$ (c) $\mathbb{R} - \{5 , 2 , -2\}$ (d) $\mathbb{R}$

13 If the domain of the function n : n $(x) = \frac{x-2}{x^2+a}$ is $\mathbb{R}$, then a .... 0 *(El-Dakahlia 16)*

(a) = (b) > (c) ≤ (d) <

Unit 2

14 If the domain of the function n : $n(x) = \frac{x+2}{4x^2 + kx + 9}$ is $\mathbb{R} - \{\frac{-3}{2}\}$

, then k = ... *(Kafr El-Sheikh 19)*

(a) 15 (b) – 15 (c) 12 (d) – 12

15 If $x = 3$ is one of the zeroes of the function $f : f(x) = \frac{x^2 - 2x - k}{x^2 - 25}$, then k = ... *(Kafr El-Sheikh 18)*

(a) 3 (b) 6 (c) – 3 (d) – 6

16 If $f(x) = \frac{7+x}{7-x}$, $x \in \mathbb{R} - \{7, -7\}$, then $f(-2) =$ .... *(El-Dakahlia 16)*

(a) $\frac{-1}{f(-2)}$ (b) $\frac{-1}{f(2)}$ (c) $\frac{1}{f(2)}$ (d) $\frac{1}{f(-2)}$

5 Determine the domain of the function n : $n(x) = \frac{2x+1}{x^2 - 5x + 6}$

, then find n (0) , n (2) *(New Valley 08)*

6 If the domain of the function n : $n(x) = \frac{x-1}{x^2 - ax + 9}$ is $\mathbb{R} - \{3\}$

, then find the value of a *(Ismailia 19 , Souhag 18 , Beni Suef 17)* « 6 »

7 If n is an algebraic fraction where $n(x) = \frac{11}{4x^2 - 12x + 9}$ and n (a) is undefined

, then find the value of a « $\frac{3}{2}$ »

8 If the domain of the function f where $f(x) = \frac{x}{x^2 - 5x + m}$ is $\mathbb{R} - \{2, c\}$

, then find the value of each m and c *(El-Sharkia 16)* « 6 , 3 »

9 If the domain of the function f where $f(x) = \frac{x+b}{x+a}$ is $\mathbb{R} - \{-2\}$ and $f(0) = 3$

, then find the value of each a and b *(El-Fayoum 16)* « 2 , 6 »

10 If the set of zeroes of the function f where $f(x) = \frac{ax^2 - 6x + 8}{bx - 4}$ is $\{4\}$

and its domain is $\mathbb{R} - \{2\}$ **, then find a , b** *(El-Sharkia 17)* « 1 , 2 »

For excellent pupils

1 If the domain of the function n is $\mathbb{R} - \{1, 3\}$, where $n(x) = \frac{x+1}{x^2 + ex + a}$

Find the value of each of e and a « – 4 , 3 »

2 If $n_1(x) = \frac{x}{x^2 + 9}$, $n_2(x) = \frac{5}{x^2 - 6x - a}$ and the common domain of the two functions n_1 and n_2 is $\mathbb{R} - \{3\}$

Find the value of a « – 9 »

Exercise 6 Equality of two algebraic fractions

From the school book

1 **Reduce each of the following algebraic fractions to the simplest form showing the domain of each of them :**

1. $n(x) = \dfrac{2x+8}{x+4}$
2. $n(x) = \dfrac{x^2-2x}{x^2+3x}$
3. $n(x) = \dfrac{x^2-4x}{x^2-16}$
4. $n(x) = \dfrac{x^2-4}{x^3-8}$
5. $n(x) = \dfrac{12x^2-8x}{6x^2-4x}$
6. $n(x) = \dfrac{x^2-4}{x^2-5x+6}$
7. $n(x) = \dfrac{x^2-6x+9}{2x^3-18x}$
8. $n(x) = \dfrac{x^2+x-6}{x^2-2x-15}$
9. $n(x) = \dfrac{2x^2+7x+6}{4x^2+4x-3}$
10. $n(x) = \dfrac{x^3+1}{x^3-x^2+x}$
11. $n(x) = \dfrac{6+x-x^2}{x^2-5x+6}$
12. $n(x) = \dfrac{x^6-64}{x^4+4x^2+16}$
13. $n(x) = \dfrac{(x-2)^2-1}{x(x-3)}$
14. $n(x) = \dfrac{x+\frac{1}{x}}{4x+\frac{4}{x}}$ *(Damietta 17)*
15. $n(x) = \dfrac{x^2-x-6}{x^3+2x^2-9x-18}$
16. $n(x) = \dfrac{x^3+x^2-2}{x-1}$

Unit 2

2 **In each of the following , prove that : $n_1(x)$ and $n_2(x)$ are equal for all values of x which belong to the common domain and find this domain. (In another meaning , find the common domain in which the two functions n_1 and n_2 are equal) :**

1. $n_1(x) = \frac{4x^2-9}{6x-9}$, $n_2(x) = \frac{2x^2+3x}{3x}$ *(Port Said 2015)*
2. $n_1(x) = \frac{x^2-3x+9}{x^3+27}$, $n_2(x) = \frac{2}{2x+6}$ *(El-Sharkia 17)*
3. $n_1(x) = \frac{x^2-4}{x^2+x-6}$, $n_2(x) = \frac{x^3-x^2-6x}{x^3-9x}$ *(Kafr El-Sheikh 18)*
4. $n_1(x) = \frac{x^2+x-12}{x^2+5x+4}$, $n_2(x) = \frac{x^2-2x-3}{x^2+2x+1}$ *(Alex. 19 , Damietta 17)*

3 **In each of the following , prove that $n_1 = n_2$:**

1. $n_1(x) = \frac{3x}{3x-6}$, $n_2(x) = \frac{2x}{2x-4}$ *(Souhag 06)*
2. $n_1(x) = \frac{x}{x^2-1}$, $n_2(x) = \frac{5x}{5x^2-5}$ *(Loxur 19)*
3. $n_1(x) = \frac{2x}{2x+4}$, $n_2(x) = \frac{x^2+2x}{x^2+4x+4}$ *(El-Beheira 19 , El-Menia 17)*
4. $n_1(x) = \frac{x^3-1}{x^3+x^2+x}$, $n_2(x) = \frac{(x-1)(x^2+1)}{x^3+x}$ *(Matrouh 18)*
5. $n_1(x) = \frac{x^2-x}{x^3-2x^2}$, $n_2(x) = \frac{x^2-3x+2}{x^3-4x^2+4x}$ *(El-Dakahlia 19)*
6. $n_1(x) = \frac{x^2}{x^3-x^2}$, $n_2(x) = \frac{x^3+x^2+x}{x^4-x}$ *(Souhag 19)*
7. $n_1(x) = \frac{x^3+x}{x^3+x^2+x+1}$, $n_2(x) = \frac{x}{x+1}$

4 **In each of the following , show whether $n_1 = n_2$ or not (give reason) :**

1. $n_1(x) = \frac{x-1}{x}$, $n_2(x) = \frac{(x-1)(x^2+1)}{x(x^2+1)}$
2. $n_1(x) = \frac{2x^3+6x}{(x-1)(x^2+3)}$, $n_2(x) = \frac{2x}{x-1}$

3 $n_1(x) = \frac{x+5}{x^2-25}$, $n_2(x) = \frac{3}{3x-15}$ *(Assiut 18)*

4 $n_1(x) = \frac{x^2-9}{x^2+4x+3}$, $n_2(x) = \frac{x-3}{x+1}$ *(Giza 16)*

5 $n_1(x) = \frac{x^2-4}{x^2+x-6}$, $n_2(x) = \frac{x^2-x-6}{x^2-9}$ *(El-Gharbia 19 , Qena 18)*

6 $n_1(x) = \frac{x^3+1}{x^3-x^2+x}$, $n_2(x) = \frac{x^3+x^2+x+1}{x^3+x}$

7 $n_1(x) = 1 - \frac{1}{x}$, $n_2(x) = \frac{1-x}{x}$ *(El-Sharkia 19)*

5 Complete the following :

1 If $x \neq 2$, then the simplest form of the fraction n where $n(x) = \frac{2-x}{x-2}$ is .... *(Alex. 12)*

2 The simplest form of the function n where $n(x) = \frac{4x^2-2x}{2x}$, $x \neq 0$ is .... *(Alex. 11)*

3 If $n_1(x) = \frac{x+1}{x-2}$, $n_2(x) = \frac{x^2+x}{x^2-2x}$, then the common domain in which $n_1 = n_2$ is .... *(Kafr El-Sheikh 11)*

4 If $n_1(x) = \frac{x}{x^2+x}$, $n_2(x) = \frac{1}{x+1}$, then $n_1 = n_2$ when $x \in$ .... *(New Valley 09)*

5 If $n_1(x) = \frac{1+a}{x-2}$, $n_2(x) = \frac{4}{x-2}$ and $n_1(x) = n_2(x)$, then a = ...

6 If the simplest form of the algebraic fraction $n(x) = \frac{x(x-2)}{x+a}$, $x \neq 2$ is $n(x) = x$, then a = ...

7 If the simplest form of the algebraic fraction $n(x) = \frac{x^2-4x+4}{x^2-a}$ is $n(x) = \frac{x-2}{x+2}$, then a = ...

6 Choose the correct answer from those given :

1 If the domain of $n_1 : n_1(x) = \frac{5}{x-8}$ equals the domain of $n_2 : n_2(x) = \frac{x-3}{x+k}$, then k = ...

(a) 8 (b) – 8 (c) – 3 (d) 24

2 If $n_1(x) = \frac{x^2-4}{x-2}$, $n_2(x) = x+2$, then $n_1 = n_2$ when they have the same domain which is .... *(Fayoum 03)*

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\mathbb{R} - \{1\}$

3 If $n_1(x) = \frac{1}{x-3}$, $n_2(x) = \frac{1}{3-x}$, then $n_1 \neq n_2$ because .... *(Souhag 04)*

(a) $n_1(x) = n_2(x)$

(b) the domain of n_1 = the domain of n_2

(c) $n_1(x) \neq n_2(x)$

(d) the domain of n_1 ≠ the domain of n_2

4 If $p(x) = \frac{x^2 - 2x}{(x+2)(x-2)}$, $q(x) = \frac{x}{x+2}$, then p = q when .... *(El-Sharkia 03)*

(a) $p(x) = q(x)$ for each $x \in \mathbb{R} - \{-2\}$

(b) $p(x) = q(x)$ in the simplest form

(c) $p(x) = q(x)$ for each $x \in \mathbb{R} - \{2, -2\}$

(d) $p(x) = q(x)$ for each $x \in \mathbb{R}$

For excellent pupils

1 Reduce the algebraic fraction $n(x) = \frac{(5x+3)^2 - (3x-1)^2}{32x+8}$ to the simplest form showing the domain , then find n (0) and find the values of x which makes $(n(x))^2 = 4$

« 1 , 2 or – 6 »

2 If $n_1(x) = \frac{x}{x+a}$, $n_2(x) = \frac{x^3 + bx}{x^3 + ax^2 + x + 5}$ and $n_1 = n_2$, then find the value of each of a and b

« 5 , 1 »

Exercise 7 Adding and subtracting algebraic fractions

From the school book

1 Choose the correct answer from those given :

1. If $n(x) = \frac{3}{x} + \frac{x}{3}$, then the domain of n is .... *(El-Sharkia 18)*
   (a) $\mathbb{R} - \{3, 0\}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R} - \{3\}$ (d) $\mathbb{R}$
2. The domain of n : $n(x) = \frac{3x+4}{x^2+25} + \frac{x-2}{x^2+7}$ is ....
   (a) $\mathbb{R}$ (b) $\mathbb{R} - \{5\}$
   (c) $\mathbb{R} - \{-5, 5\}$ (d) $\mathbb{R} - \{-5, 5, -7\}$
3. The simplest form of $\frac{x^2+1}{x^2+4} + \frac{3}{x^2+4}$ is .... *(El-Fayoum 15)*
   (a) 3 (b) 4 (c) 1 (d) $\frac{1}{x^2+1}$
4. If $x \in \mathbb{R} - \{2\}$, then $\frac{x}{x-2} + \frac{2}{2-x} =$ .... *(Aswan 13)*
   (a) 1 (b) 2 (c) x (d) -1
5. The additive inverse of the fraction : $\frac{x+7}{x-5}$ is .... *(El-Fayoum 12)*
   (a) $\frac{7-x}{x+5}$ (b) $\frac{x+7}{5-x}$ (c) $\frac{-(x+7)}{5-x}$ (d) $\frac{x-7}{5-x}$
6. The function f where $f(x) = \frac{x-2}{x-5}$ has an additive inverse if the domain is .... *(Kafr El-Sheikh 16)*
   (a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{5, -2\}$ (d) $\mathbb{R} - \{2, 5\}$

Unit 2

7 If $n(x) = \frac{x}{x-3} - \frac{1}{x-3}$, then the set of zeroes of the function n is .... *(Helwan 11)*

(a) $\{3\}$ (b) $\{1\}$ (c) $\{-1\}$ (d) $\{-3\}$

2 In each of the following , find n (x) in the simplest form , showing the domain of n :

1 $n(x) = \frac{2x}{x+2} + \frac{4}{x+2}$

2 $n(x) = \frac{3x}{x-3} - \frac{9}{x-3}$

3 $n(x) = \frac{2x^2}{2x+5} + \frac{2x^2-25}{5+2x}$

4 $n(x) = \frac{x^2}{x^2-1} - \frac{x}{x^2-1}$

3 In each of the following , find n (x) in the simplest form , showing the domain of n :

1 $n(x) = \frac{x}{x^2+2x} + \frac{x+1}{x+2}$

2 $n(x) = \frac{x}{x-4} - \frac{x+4}{x^2-16}$ *(El-Kalyoubia 18 , North Sinai 17 , Aswan 16)*

3 $n(x) = \frac{x^2+x-6}{x+3} + \frac{x^2-4}{x+2}$ *(El-Kalyoubia 16)*

4 $n(x) = \frac{x^2+3x}{x^2+2x-3} - \frac{x-2}{x^2-3x+2}$ *(Suez 18 , El-Dakahlia 17)*

5 $n(x) = \frac{x^2-2x+4}{x^3+8} + \frac{x^2-1}{x^2+x-2}$ *(Damietta 19 , Assiut 08)*

6 $n(x) = \frac{2x+6}{x^2+x-6} - \frac{x^2-6x}{x^2-8x+12}$ *(El-Monofia 13)*

7 $n(x) = \frac{x-6}{2x^2-15x+18} + \frac{x-5}{15-13x+2x^2}$ *(El-Dakahlia 11)*

8 $n(x) = \frac{x^2+x-2}{x^2-1} - \frac{x+5}{x^2+6x+5}$ *(Damietta 14)*

9 $n(x) = \frac{3x+15}{x^2+7x+10} + \frac{2x^2-3x-2}{x^2-4}$ *(El-Dakahlia 15)*

10 $n(x) = \frac{3x-6}{x^2-4} - \frac{x^2-3x}{x^3-x^2-6x}$ *(Qena 12)*

4 In each of the following , find n (x) in the simplest form , showing the domain of n :

1 $n(x) = \frac{x-2}{x} + \frac{3+x}{2x}$

2 $n(x) = \frac{x}{x-2} - \frac{x}{x+2}$ *(El-Gharbia 19)*

3 $n(x) = \frac{2}{x+3} + \frac{x+3}{x^2+3x}$ *(North Sinai 14)*

4 $n(x) = \frac{x+3}{2x} - \frac{x}{2x-1}$

5 $n(x) = \frac{x}{x^2 + 2x} + \frac{x + 2}{x^2 - 4}$ *(El-Sharkia 14 , Souhag 15)*

6 $n(x) = \frac{2x - 1}{x^2 - x - 2} - \frac{1}{x - 2}$ *(Damietta 06)*

7 $n(x) = \frac{3x - 4}{x^2 - 5x + 6} + \frac{2x + 6}{x^2 + x - 6}$ *(Qena 17 , El-Beheira 14 , Cairo 11)*

8 $n(x) = \frac{x^2 + 2x + 4}{x^3 - 8} + \frac{x^2 - x - 12}{x^2 - 9}$ *(6th October 09)*

9 $n(x) = \frac{3x - 2}{3x^2 + x - 2} - \frac{3x - 4}{2x^2 - 3x - 5}$

10 $n(x) = (x + 3) - \frac{x^2}{x - 3}$

5 **In each of the following , find n (x) in the simplest form , showing the domain of n :**

1 $n(x) = \frac{x^2}{x - 1} + \frac{x}{1 - x}$ *(Giza 19 , Luxor 18)*

2 $n(x) = \frac{3x^2 + 6x}{x^2 - 4} + \frac{6}{2 - x}$ *(El-Kalyoubia 05)*

3 $n(x) = \frac{x^2 + 2x + 4}{x^3 - 8} - \frac{9 - x^2}{x^2 + x - 6}$ *(El-Monofia 18 , Alex. 17 , El-Beheira 15)*

4 $n(x) = \frac{x^2 + x}{x^2 - 1} - \frac{5 - x}{x^2 - 6x + 5}$ *(El-Dakahlia 19 , El-Menia 18 , Luxor 17)*

5 $n(x) = \frac{2x^2 - 8x}{2x^2 - 11x + 12} + \frac{3(2x + 3)}{9 - 4x^2}$ *(El-Sharkia 03)*

6 $n(x) = \frac{x + 3}{x^2 - 9} + \frac{2x + 2}{3 + 2x - x^2}$ *(Kafr El-Sheikh 02)*

7 $n(x) = \frac{3x - 6}{x^2 - 4} - \frac{9}{2 - x - x^2}$ *(El-Dakahlia 18 , El-Fayoum 12)*

8 $n(x) = \frac{x - 5}{2x^2 - 13x + 15} + \frac{x + 3}{15x - 18 - 2x^2}$ *(Aswan 08)*

9 $n(x) = \frac{x^2 - 4}{x^2 + x - 2} + \frac{5 - 10x}{3x - 1 - 2x^2}$

10 $n(x) = \frac{x - 3}{x^2 - 7x + 12} - \frac{x - 3}{3 - x}$ *(Assiut 19 , Luxor 19)*

6 If $n(x) = \frac{x^2 - 5x}{x^2 - 8x + 15} - \frac{x^2 + 3x + 9}{x^3 - 27}$

, then find n (x) in the simplest form and calculate the value of each of n (1) , n (5) if it is possible. *(El-Sharkia 17)*

Unit 2

7 **Find n ($\mathcal{X}$) in the simplest form , showing the domain of n where :**

$n(\mathcal{X}) = \frac{\mathcal{X}+3}{\mathcal{X}^2+6\mathcal{X}+9} + \frac{\mathcal{X}+2}{\mathcal{X}+3}$, then find n (– 3) and n (2016) if it is possible. *(El-Sharkia 16)*

8 **Find n ($\mathcal{X}$) in the simplest form , showing the domain of n where :**

$n(\mathcal{X}) = \frac{12}{12\mathcal{X}^2-3} + \frac{2}{2\mathcal{X}-4\mathcal{X}^2}$, then find n (0) , n (– 1) if it is possible.

9 If $n(\mathcal{X}) = \frac{\mathcal{X}^2-2\mathcal{X}}{\mathcal{X}^4-3\mathcal{X}^3+2\mathcal{X}^2} - \frac{4-\mathcal{X}^2}{\mathcal{X}^2+\mathcal{X}-2}$

, find n ($\mathcal{X}$) in the simplest form , showing the domain of n , then find the S.S. of the equation : n ($\mathcal{X}$) = 0 *(New Valley 13)* « ∅ »

10 **Find n ($\mathcal{X}$) in the simplest form , showing the domain where :**

$n(\mathcal{X}) = \frac{\mathcal{X}^2+\mathcal{X}+1}{\mathcal{X}^4-\mathcal{X}} + \frac{\mathcal{X}+3}{3-2\mathcal{X}-\mathcal{X}^2}$, and if n (a) = – 2 , find the value of a *(El-Monofia 17)* « $\frac{1}{2}$ »

11 If $f_1(\mathcal{X}) = \frac{\mathcal{X}-a}{\mathcal{X}+b}$, and the set of zeroes of f_1 is $\{5\}$, and the domain of f_1 is $\mathbb{R} - \{3\}$, then find the values of a and b

If $f_2(\mathcal{X}) = \frac{\mathcal{X}-1}{\mathcal{X}-3}$, then find $f_1(\mathcal{X}) + f_2(\mathcal{X})$ in the simplest form. *(El-Dakahlia 17)* « 5 , – 3 »

12 If the domain of the function n where $n(\mathcal{X}) = \frac{b}{\mathcal{X}} + \frac{9}{\mathcal{X}+a}$ is $\mathbb{R} - \{0 , 4\}$, n (5) = 2

, find the values of a and b *(Kafr El-Sheikh 16 , El-Beheira 15 , El-Menia 14)* « – 4 , – 35 »

For excellent pupils

1 If $n(\mathcal{X}) = \frac{5\mathcal{X}+10}{\mathcal{X}^2-\mathcal{X}-6}$, k ($\mathcal{X}$) is the additive inverse of n ($\mathcal{X}$)

, find k (2) , k (3) « 5 , undefined »

2 **Find the value of $\mathcal{X}$ if :**

1. $\frac{4\mathcal{X}}{\mathcal{X}-1} = \frac{3\mathcal{X}}{\mathcal{X}+1} + 1$ « $\frac{-1}{7}$ »
2. $\frac{3}{\sqrt{\mathcal{X}}-\sqrt{7}} = \frac{3}{\sqrt{\mathcal{X}}+\sqrt{7}} + \frac{1}{2\sqrt{7}}$ « 91 »
3. $\frac{1}{\mathcal{X}^2-4\mathcal{X}-5} + \frac{4\mathcal{X}+5}{5\mathcal{X}^2+4\mathcal{X}^3-\mathcal{X}^4} = 1$ « 1 »

Exercise 8

Multiplying and dividing algebraic fractions

From the school book

1 **Choose the correct answer from those given :**

1 If $n(x) = \frac{x-2}{x+5}$, then the domain of n^{-1} is .... *(Port Said 19 , Souhag 18)*

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{-5\}$ (d) $\mathbb{R} - \{2 , -5\}$

2 If $n(x) = \frac{1}{(x-2)^2}$, then the domain of n^{-1} is .... *(Cairo 18)*

(a) $\mathbb{R} - \{1 , 2\}$ (b) $\mathbb{R}$ (c) $\mathbb{R} - \{2\}$ (d) $\{2\}$

3 If $n(x) = \frac{x}{x^2+9}$, then the domain of n^{-1} is .... *(El-Sharkia 16)*

(a) $\varnothing$ (b) $\mathbb{R} - \{-3 , 3\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{0\}$

4 If $n(x) = \frac{x-2}{x^2-x-6}$, then the domain of n^{-1} is .... *(El-Beheira 17)*

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{-2 , 3\}$

(c) $\mathbb{R} - \{-2 , 2\}$ (d) $\mathbb{R} - \{-2 , 2 , 3\}$

5 If $n(x) = \frac{3}{x-4}$, then $n^{-1}(4)$ is ....

(a) equal to zero (b) equal to 4 (c) equal to 8 (d) undefined

6 If $n(x) = \frac{x-1}{x+2}$, then $n^{-1}(1)$ is .... *(Beni Suef 17)*

(a) equal to -1 (b) equal to zero (c) equal to 3 (d) undefined

7 If $n(x) = \frac{x}{x-5} + \frac{3}{x-5}$, then the domain in which the function n has a multiplicative inverse is $\mathbb{R}$ – ....

(a) $\{0, 5\}$ (b) $\{0, 3, 5\}$ (c) $\{5\}$ (d) $\{5, -3\}$

8 If $n(x) = \frac{x^2 - x}{x^2 - 1}$, $n^{-1}(\ell) = 3$, then ℓ = ...

(a) $-\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $1\frac{1}{3}$

2 Complete the following :

1 If the function $n : n(x) = \frac{2-x}{x-2}$ has a multiplicative inverse, then the domain of n is .... *(El-Kalyoubia 06)*

2 If $x \notin \{2, -2\}$, then the multiplicative inverse of the function $f : f(x) = \frac{x+2}{x^2-4}$ is the function $k : k(x)$ = ...

3 If $n(x) = \frac{x}{x-2} \div \frac{2}{x-2}$, then the domain of n is .... *(Damietta 05)*

4 The domain of the algebraic fraction $n : n(x) = \frac{x-2}{x-5} \div (x-2)$ is ....

5 The simplest form of the rule of the function $f : f(x) = \frac{5}{x+3} \div \frac{x}{x+3}$ is $f(x)$ = ... and its domain is .... *(Cairo 05)*

6 If $x \notin \{2, 0\}$, then $\frac{x}{x-2} \div \frac{x}{2-x}$ = ...

7 If the algebraic fraction $\frac{x-a}{x-3}$ has a multiplicative inverse which is $\frac{x-3}{x+2}$, then a = ... *(El-Beheira 11)*

3 In each of the following, find n (x) in the simplest form, showing the domain of n :

1 $n(x) = \frac{3x-15}{x+3} \times \frac{4x+12}{5x-25}$ *(Luxor 13)*

2 $n(x) = \frac{x+2}{x^2-4} \times \frac{2x-4}{x-3}$ *(Luxor 05)*

3 $n(x) = \frac{x^2+2x+1}{2x-8} \times \frac{x-4}{x+1}$ *(Suez 17, Cairo 16, Ismailia 15)*

4 $n(x) = \frac{x^3-1}{x^2-2x+1} \times \frac{2x-2}{x^2+x+1}$ *(El-Dakahlia 19, El-Kalyoubia 18, El-Monofia 18)*

5 $n(x) = \frac{2x-10}{x^2-25} \times \frac{x^2+5x}{x-3}$ *(Qena 09)*

6 $n(x) = \frac{x^2-3x-4}{x^2-1} \times \frac{x^2-x}{x^2+3x}$ *(El-Kalyoubia 16, El-Gharbia 04)*

7 $n(x) = \frac{6x^2+3x}{x+2} \times \frac{x^2+4x+4}{6x+3}$ *(Assiut 15)*

8 $n(x) = \frac{x^3-1}{x^2-x} \times \frac{x+3}{x^2+x+1}$ *(Alexandria 19)*

9 $n(x) = \frac{5x+5}{x+6} \times \frac{x^2+3x-18}{x^2-2x-3}$, then find n (2) if it is possible. *(Ismailia 09)*

10 $n(x) = \frac{x^2+2x}{x^3-27} \times \frac{x^2+3x+9}{x+2}$, then find n (6) , n (– 2) if it is possible. *(South Sinai 17)*

11 $n(x) = \frac{x^3-8}{x^2+3x-10} \times \frac{2x+6}{x^2+2x+4}$, then find $n^{-1}(x)$ when $x = 1$ *(Port Said 04)*

12 $n(x) = \frac{2x^3-16}{x^2-7x+10} \times \frac{3x^2-10x-25}{x^2+2x+4}$ *(Ismailia 09)*

13 $n(x) = \frac{x^2-2x-3}{5x^3-135} \times \frac{5x^2+15x+45}{x+1}$

14 $n(x) = \frac{x-2}{2x^2-3x} \times \frac{9-4x^2}{6+x-2x^2}$

15 $n(x) = \frac{x^2-12x+36}{x^2-6x} \times \frac{4x+24}{36-x^2}$

4 In each of the following , find n (x) in the simplest form , showing the domain of n :

1 $n(x) = \frac{3x-15}{x+3} \div \frac{5x-25}{4x+12}$ *(Luxor 18 , Beni Suef 14)*

2 $n(x) = \frac{x-1}{x^2-1} \div \frac{x^2-5x}{x^2-4x-5}$ *(Matrouh 19 , El-Menia 16 , El-Beheira 15 , Aswan 14)*

3 $n(x) = \frac{x^2+2x-3}{x+3} \div \frac{x^2-1}{x+1}$ *(Port Said 18 , Alexandria 13)*

4 $n(x) = \frac{x^2-2x-15}{x^2-9} \div \frac{2x-10}{x^2-6x+9}$ *(El-Gharbia 18 , El-Beheira 18 , Alexandria 16)*

5 $n(x) = \frac{x^3-8}{x^2+x-6} \div \frac{x^2+2x+4}{2x+6}$ *(Alexandria 09)*

6 $n(x) = \frac{x^2-2x+1}{x^3-1} \div \frac{x-1}{x^2+x+1}$ *(Suez 19 , El-Dakahlia 18 , El-Gharbia 17)*

7 $n(x) = \frac{x^3-27}{x^2-9} \div \frac{x^3+3x^2+9x}{2x}$ *(El-Fayoum 09)*

8 $n(x) = \frac{x^2-3x}{2x^2-x-6} \div \frac{2x^2-3x}{4x^2-9}$ *(Luxor 19)*

9 $n(x) = \frac{x^2-x-2}{x^2+x-6} \div \frac{x^2-4x-5}{x^2-2x-15}$

10 $n(x) = \frac{x^2-9}{2x^2+3x} \div \frac{3x^2+6x-45}{4x^2-9}$ *(Aswan 08)*

11 $n(x) = \frac{x^2-4}{3x^2+x-10} \div \frac{6x^2-5x-14}{3x^2-5x}$

12 $n(x) = \frac{x^2-3x+2}{1-x^2} \div \frac{3x-15}{x^2-6x+5}$

13 $n(x) = \frac{3x-9}{x^2-5x+6} \div \frac{2x+6}{6-x-x^2}$

14 $n(x) = \frac{x-2}{2x^2-3x} \div \frac{6+x-2x^2}{9-4x^2}$

15 $n(x) = \frac{x^2+x-6}{x^2+5x+6} \div \frac{x^3-2+x-2x^2}{x^3+2x^2+x+2}$

5 If $n(x) = \frac{x^2-2x}{(x-2)(x^2+2)}$

First : Find $n^{-1}(x)$ and identify the domain.

Second : If $n^{-1}(x) = 3$, what is the value of x ?

(Alex. 19 , El-Kalyoubia 18 , El-Gharbia 17 , Aswan 16) « 1 »

6 If $n(x) = \frac{x^3+3x^2+2x}{x^2+2x}$, find $n^{-1}(x)$ in the simplest form showing the domain of n^{-1} , then find $n^{-1}(-2)$ if it is possible. *(Ismailia 08)* « undefined »

7 If $n(x) = x + \frac{x}{x-2}$, find $n^{-1}(x)$ in the simplest form showing the domain of n^{-1}

(El-Gharbia 19)

8 If $f(x) = \frac{x^2-49}{x^3-8} \div \frac{x+7}{x-2}$, then find n (x) in the simplest form and identify its domain and find $f(1)$ *(Assiut 19 , El-Beheira 17 , El-Gharbia 12)* « $-\frac{6}{7}$ »

9 If $n(X) = \frac{X^3 - X^2 - 2X}{X^2 - 5X + 6} \div \frac{X^3 + 6X^2 + 5X}{X^2 + 2X - 15}$, then find n (X) in the simplest form showing the domain of n , then find n (7) , n (3) « 1 , undefined »

10 If $f(X) = \frac{X^2 - 2X - 15}{X^2 - 9} \div \frac{X^2 - 25}{X^2 - 3X}$ find $f(X)$ in the simplest form showing the domain of f and if $f(a) = \frac{1}{3}$ find the value of a *(Assiut 08)* « $\frac{5}{2}$ »

11 Find $n_1(X)$, $n_2(X)$, n (X) in the simplest form showing the domain of each of n_1 , n_2 and n where :

$n_1(X) = \frac{2X + 7}{4X^2 - 1} \div \frac{4X^2 + 12X - 7}{8X^3 - 1}$

$n_2(X) = \frac{12X^2 - 3}{12X^2 + 6X + 3}$, $n(X) = n_1(X) \times n_2(X)$

12 Find in the simplest form :

$n(X) = \left(\frac{3X + 15}{X^2 + 7X + 10} + \frac{2X + 1}{X + 2} \right) \times \frac{X^3 - 27}{X^2 + 3X + 9}$

Showing the domain of n and if n (X) = 2 , find the value of X *(Suez 05)* « 4 »

For excellent pupils

1 If $n_1(X) = \frac{X^2 - aX + 12}{X^2 - 3X - 4}$, $n_1^{-1}(X) = \frac{X + 1}{X - 3}$, find the value of a « 7 »

2 If $n(X) = \dfrac{X + \frac{1}{X - 2}}{4X + \frac{4}{X - 2}}$, find n (X) in the simplest form showing the domain of n

, then find n (1) , n (8) if it is possible. « undefined , $\frac{1}{4}$ »

Summary of Unit 2

- If f is a polynomial function in X , then the set of values of X which makes $f(X) = 0$ is called the set of zeroes of the function f and is denoted by $z(f)$

  i.e. $z(f)$ is the solution set of the equation $f(X) = 0$ in $\mathbb{R}$

- The domain of the algebraic fraction function is all the real numbers **except** the numbers which make the fraction undefined (*i.e.* the set of zeroes of the denominator)

  i.e. The domain of the algebraic fraction function = $\mathbb{R}$ – the set of zeroes of the denominator.

- The common domain of two algebraic fractions = $\mathbb{R}$ – the set of zeroes of the two denominators of the two fractions.

- **To reduce the algebraic fraction , do as follows :**

  1. Factorize each of the numerator and denominator perfectly.
  2. Identify the domain of the algebraic fraction before removing the common factors between the numerator and denominator.
  3. Remove the common factors between the numerator and the denominator to get the simplest form of the algebraic fraction.

- **It is said that the two functions n_1 and n_2 are equal (*i.e.* $n_1 = n_2$) if the two following conditions are satisfied together :**

  1. The domain of n_1 = the domain of n_2
  2. $n_1(X) = n_2(X)$ for each $X \in$ the common domain.

- **Adding and subtracting algebraic fractions :**

  The steps of adding or subtracting two algebraic fractions :

  1. Arrange the terms of each of the numerator and the denominator of each fraction descendingly or ascendingly according to the powers of any variable in it.
  2. Factorize the numerator and the denominator of each fraction if possible.
  3. Find the common domain which will be the domain of the result.
  4. Reduce each fraction separately to make the operation of addition or subtraction easier.
  5. Unify the denominators.
  6. Perform the operation of addition or subtraction of the terms of the numerators.
  7. Put the final result in the simplest form.

Multiplying algebraic fractions :

The steps of multiplying the algebraic fractions :

1. Arrange the terms of each of the numerator and the denominator of each fraction descendingly or ascendingly according to the powers of any variable in it.
2. Factorize the numerator and the denominator of each fraction if possible.
3. Find the common domain.
4. Remove the common factors between the numerator and the denominator of each fraction and between the numerator of a fraction and the denominator of another fraction.
5. Perform the operation of multiplication and put the result in the simplest form.

The multiplicative inverse of the algebraic fraction :

If n is an algebraic fraction where $n(X) = \frac{p(X)}{k(X)} \neq 0$, then n has a multiplicative inverse which is the algebraic fraction n^{-1} where $n^{-1}(X) = \frac{k(X)}{p(X)}$ and the domain of n^{-1} is $\mathbb{R}$ – the set of zeroes of each of the numerator and the denominator of any of the two fractions.

Dividing an algebraic fraction by another :

If n_1 and n_2 are two algebraic fractions where :

$n_1(X) = \frac{f(X)}{r(X)}$, $n_2(X) = \frac{p(X)}{k(X)}$, then $n_1(X) \div n_2(X) = n_1(X) \times n_2^{-1}(X) = \frac{f(X)}{r(X)} \times \frac{k(X)}{p(X)}$

where the domain of $n_1 \div n_2$ = the common domain of each of n_1 and n_2^{-1}

$= \mathbb{R}$ – the set of zeroes of denominator of n_1 or denominator of n_2 or numerator of n_2

$= \mathbb{R} - \{z(r) \cup z(p) \cup z(k)\}$

Exams on Unit Two

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If $n(x) = \frac{x-1}{x-2}$, then the domain of n^{-1} is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{1 , 2\}$

2 The set of zeroes of the function $f : f(x) = x^2 - 16$ is ....

(a) $\{16\}$ (b) $\{4\}$ (c) $\{4 , -4\}$ (d) $\varnothing$

3 The common domain of the two functions $n_1 : n_1(x) = \frac{x+2}{7}$, $n_2 : n_2(x) = \frac{4}{x-2}$ is ....

(a) $\{-2 , 2\}$ (b) $\mathbb{R} - \{2 , -2\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{4 , 2 , -2 , 7\}$

4 The simplest form of $n(x) = \frac{x}{x-3} \div \frac{3x}{x^2-9}$ is ....

(a) $\frac{3}{x-3}$ (b) $\frac{3}{x+3}$ (c) $\frac{x+3}{3}$ (d) $\frac{x-3}{3}$

5 The domain of the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x^2 - 4$ is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{2 , -2\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{-2\}$

6 The domain of the additive inverse of the function f where $f(x) = \frac{x-2}{x+7}$ is ....

(a) $\mathbb{R} - \{-7\}$ (b) $\mathbb{R} - \{2 , -7\}$ (c) $\mathbb{R} - \{2\}$ (d) $\{-7 , 2\}$

2 **[a]** If $n_1(x) = \frac{x^2}{x^3 - 2x^2}$, $n_2(x) = \frac{x^3 + 2x^2 + 4x}{x^4 - 8x}$

, prove that : $n_1 = n_2$

[b] Find $n(x)$ in the simplest form , showing the domain of n where :

$$n(x) = \frac{x^2 + 2x}{x^2 - 4} - \frac{2x - 6}{x^2 - 5x + 6}$$

3 **[a]** If the set of zeroes of the function f where $f(x) = a x^2 + b x + 6$ is $\{2 , 3\}$, find the values of a and b

[b] Find $n(x)$ in the simplest form , showing the domain of n where

$$n(x) = \frac{3x^2 - 6x}{x^2 - 4} \times \frac{x^2 + 3x + 2}{x^2 + x}$$

4 **[a]** If n $(x) = \frac{x^2 - 3x}{x^2 - 5x + 6}$

1 Find $n^{-1}(x)$ in the simplest form and identify the domain of n^{-1}

2 If $n^{-1}(x) = 2$, what is the value of x ?

[b] If n $(x) = \frac{x^3 + x^2 - 6x}{x^4 - 13x^2 + 36}$

1 Find n (x) in the simplest form showing the domain of n

2 Find n (x) at $x = -1$

5 **[a]** If n $(x) = \frac{x}{x+1} + \frac{2x^2}{x^3 - x}$

, then find n (x) in the simplest form showing the domain of n and find n (2) , n (1) if it is possible.

[b] If n $(x) = \frac{x^3 - 8}{x^2 - 3x + 2} \div \frac{x^3 + 2x^2 + 4x}{2x^2 + x - 3}$

, put n (x) in the simplest form showing the domain of n

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The set of zeroes of the function $f : f(x) = \frac{-3}{x-2}$ is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{3\}$ (c) $\{2\}$ (d) $\varnothing$

2 The domain of the function $f : f(x) = \frac{x-3}{4}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-4\}$ (c) $\mathbb{R} - \{-4 , 3\}$ (d) $\varnothing$

3 If n $(x) = \frac{x+1}{x-2}$, then the domain in which n has a multiplicative inverse is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{-1 , 2\}$ (c) $\mathbb{R} - \{-1\}$ (d) $\{-1 , 2\}$

4 The simplest form of the algebraic fraction n : n $(x) = \frac{4x^2 - 2x}{2x}$, $x \neq 0$ is ....

(a) $4x^2$ (b) $2x - 1$ (c) $2x$ (d) 2

5 If $n_1(x) = \frac{r_1(x)}{f_1(x)}$, $n_2 = (x) = \frac{r_2(x)}{f_2(x)}$, then the common domain of the two functions n_1 , n_2 is ....

(a) $\mathbb{R} - \left(z(f_1) \cup z(f_2)\right)$ (b) $\mathbb{R} - \left(z(f_1) \cap z(f_2)\right)$

(c) $z(f_1) \cup z(f_2)$ (d) $z(f_1) \cap z(f_2)$

Unit 2

6 If $n_1(x) = \frac{3+a}{x-5}$, $n_2(x) = \frac{4}{x-5}$ and $n_1(x) = n_2(x)$, then a = ...

(a) 1 (b) 2 (c) 3 (d) 4

2 **[a]** If $n_1(x) = \frac{x^2-4}{x^2-4x+4}$, $n_2(x) = \frac{x+2}{x-2}$, **prove that :** $n_1 = n_2$

[b] If $n(x) = \frac{x^2-2x}{x^2-4} + \frac{2x-6}{x^2-x-6}$

Find n (x) in the simplest form, showing the domain of n

3 **[a]** If $n(x) = \frac{x^2-4}{x^3-8} \div \frac{x^2-x-6}{x^2+2x+4}$

Find n (x) in the simplest form, showing the domain of n

[b] If the set of zeroes of the function f where $f(x) = \frac{x^2-ax+9}{bx+4}$ is $\{3\}$ and its domain $= \mathbb{R} - \{2\}$ **Find :** a and b

4 **[a]** Find n (x) in the simplest form , showing the domain of n where :

$n(x) = \frac{x^2+x+1}{x^4-x} + \frac{x+3}{3-2x-x^2}$ and if n (a) = – 2 , find the value of a

[b] If $n(x) = \frac{x^3-8}{x^2+x-6} \times \frac{x+3}{x^2+2x+4}$

Find n (x) in the simplest form , showing the domain of n

5 **[a]** Find the common domain in which the two functions n_1 , n_2 are equal where :

$n_1(x) = \frac{x^2+3x+2}{x^2-4}$, $n_2(x) = \frac{x^2-1}{x^2-3x+2}$

[b] If $n(x) = \frac{x+\frac{1}{x}}{4x+\frac{4}{x}}$

Find n (x) in the simplest form , showing the domain of n.

UNIT 3

Probability

Exercises of the unit :

9. Operations on events : Intersection and union of two events.
10. Operations on events (follow) :
    Complementary event and the difference between two events.

✪ Summary of unit three.

✪ Unit exam.

Exercise 9 Intersection and union of two events

From the school book

1 Choose the correct answer from those given :

1 The probability of the impossible event equals .... *(Kafr El-Sheikh 17 , Cairo 15)*

(a) ∅ (b) zero (c) $\frac{1}{2}$ (d) 1

2 The probability of the certain event = ... *(Qena 15)*

(a) zero (b) ∅ (c) 1 (d) – 1

3 If A and B are two mutually exclusive events , then P (A ∩ B) equals ....

(a) ∅ (b) P (A) (c) P (B) (d) zero *(El-Gharbia 15)*

4 If A and B are two mutually exclusive events , then P (A ∪ B) = ... *(El-Menia 16)*

(a) P (A) (b) P (B) (c) P (A ∩ B) (d) P (A) + P (B)

5 If A and B are two events in a sample space for a random experiment , A ⊂ B

, then P (A ∩ B) = ... *(Cairo 16)*

(a) P (B) (b) P (A) (c) zero (d) ∅

6 If A ⊂ B , then P (A ∪ B) equals .... *(El-Beheira 19 , El-Kalyoubia 18 , Qena 17)*

(a) zero (b) P (A) (c) P (B) (d) P (A ∩ B)

7 If a regular coin is tossed once, then the probability of getting head or tail is ....

(Alexandria 14 , El-Dakahlia 13)

(a) 100 % (b) 50 % (c) 25 % (d) zero %

8 A card is drawn randomly from 20 identical cards numbered from 1 to 20, then the probability that the number of the drawn card multiple of 7 is .... *(El-Beheira 17)*

(a) 10 %   (b) 15 %   (c) 20 %   (d) 25 %

9 If a regular die is rolled once, then the probability of getting an odd number and even number together equals .... *(Alexandria 16, El-Beheira 14, El-Fayoum 12)*

(a) zero   (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1

10 A regular die is rolled once, if the event A is "appearing a prime number" and the event B is "appearing an odd number", then $P(A \cap B) =$ .... *(El-Sharkia 11)*

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

11 A fair die is rolled once, the event A is appearing an odd number and the event B is appearing a number less than 5, then the probability of occurring one of them at least is ....

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{5}{6}$

2 **If A and B are two events in the sample space of a random experiment. Complete :**

1	2	3
$P(A) = 0.3$	$P(A) = 0.55$	$P(A) =$ ....
$P(B) = 0.6$	$P(B) = \frac{3}{10}$	$P(B) = \frac{1}{4}$
$P(A \cap B) = 0.2$	$P(A \cap B) =$	$P(A \cap B) =$ zero
$P(A \cup B) =$	$P(A \cup B) = \frac{13}{20}$	$P(A \cup B) = 0.9$

3 **If A and B are two events in the sample space of a random experiment.**

Answer the following :

1 $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{3}$, then find $P(A \cup B)$ *(Port Said 13)* «$\frac{5}{6}$»

2 $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$, $P(A \cup B) = \frac{5}{8}$, then find $P(A \cap B)$ *(Damietta 11)* «$\frac{1}{4}$»

3 $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, then find $P(A \cup B)$ in the following cases :

(i) $P(A \cap B) = \frac{1}{8}$

(ii) A and B are mutually exclusive events. *(El-Gharbia 18, Qena 18, Aswan 17)* «$\frac{17}{24}$, $\frac{5}{6}$»

4 **If A and B are two events from a sample space of a random experiment , $P(B) = \frac{1}{12}$ and $P(A \cup B) = \frac{1}{3}$, then find P (A) if :**

1 A and B are two mutually exclusive events.

2 $B \subset A$ *(Port Said 18 , Luxor 17 , North Sinai 14)* « $\frac{1}{4}$, $\frac{1}{3}$ »

5 **If A and B are two events of the sample space of a random experiment , $P(A) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{18}$ and $P(A \cup B) = \frac{4}{9}$, then find : P (B)** « $\frac{1}{3}$ »

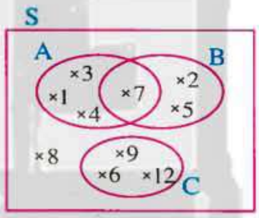
6 **If A and B are two events of the sample space of a random experiment , $A \subset B$, $P(A \cap B) = \frac{2}{5}$ and $P(A \cup B) = \frac{4}{5}$, find :**

1 P (A) 2 P (B) « $\frac{2}{5}$, $\frac{4}{5}$ »

7 **If A and B are two events from the sample space of a random experiment , if P (A) = 0.5 , $P(A \cup B) = 0.8$ and $P(B) = 2x$, then calculate the value of x if :**

1 $A \subset B$ 2 $P(A \cap B) = 0.1$ *(Kafr El-Sheikh 16)* « 0.4 , 0.2 »

8 **Use the opposite Venn diagram to find :**

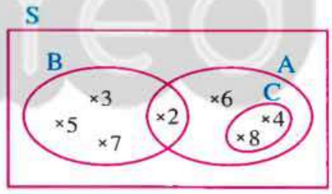


1 $P(A \cap B)$, $P(A \cup B)$

2 $P(A \cap C)$, $P(A \cup C)$

3 $P(B \cap C)$, $P(B \cup C)$ *(Assiut 11)*

9 **Use the opposite Venn diagram to find :**



1 $P(A \cap B)$ 2 $P(A \cup B)$ 3 $P(A \cap C)$

4 $P(A \cup C)$ 5 $P(B \cap C)$ 6 $P(B \cup C)$

7 $P(A) + P(B) - P(A \cap B)$

10 S is the sample space of a random experiment where its outcomes are equal , A and B are two events from S

If the number of outcomes that leads to the occurrence of the event A equals 13 and the number of all possible outcomes of the random experiment is 24 , $P(A \cup B) = \frac{5}{6}$ and $P(B) = \frac{5}{12}$ **Find :**

1 The probability of occurrence of the event A

2 The probability of occurrence the event A and B together. *(El-Menia 17 , El-Gharbia 16)* « $\frac{13}{24}$, $\frac{1}{8}$ »

11 A box contains 12 balls , 5 of them are blue , 4 are red and the left are white. A ball is randomly drawn from the box. **Find the probability that the drawn ball is :**

1 blue. 2 not red. 3 blue or red.

(Souhag 18 , Luxor 18 , Alexandria 13) « $\frac{5}{12}$, $\frac{2}{3}$, $\frac{3}{4}$ »

12 A bag contains 25 balls , all of them are identical , 4 balls are yellow , 7 balls are red and the rest of balls are black. A ball is drawn randomly.

Find the probability that the drawn ball is :

1 black. 2 yellow or black.

3 not yellow. 4 green.

« $\frac{14}{25}$, $\frac{18}{25}$, $\frac{21}{25}$, 0 »

13 In the experiment of rolling a fair die once , if A is the event of getting an even number , B is the event of getting an odd number and C is the event of getting an even prime number. **Find :**

1 The probability of occurring the two events A and B together.

2 The probability of occurring the events A or C

« 0 , $\frac{1}{2}$ »

14 A group of identical cards numbered from 1 to 8 without replacing. They are mixed well. If a card is drawn randomly.

First : Write :

1 The sample space.

2 A is the event that the drawn card carrying an even number.

3 B is the event that the drawn card carrying a prime number.

4 C is the event that the drawn card carrying a number divisible by 4

Second : Use Venn diagram to calculate the probability of :

1 The event of occuring the two events A and B together.

2 The event of occuring one of the events B or C at least.

« $\frac{1}{8}$, $\frac{3}{4}$ »

15 A card is randomly drawn from 20 identical cards numbered from 1 to 20

Calculate the probability that the number on the card is :

1 Divisible by 3 2 Divisible by 5

3 Divisible by 3 and divisible by 5

4 Divisible by 3 or divisible by 5

(Aswan 11) « $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{20}$, $\frac{9}{20}$ »

16 A box contains 30 identical cards numbered from 1 to 30 , a card is drawn randomly.

Find the probability that the written number on the card :

1 Odd and divisible by 5 2 Prime or divisible by 7

« $\frac{1}{10}$, $\frac{13}{30}$ »

Unit 3

17 A set of cards numbered from 1 to 30 and well mixed. If a card is randomly drawn.
Find the probability that the card is carrying :

1 A number multiple of 6
2 A number multiple of 8
3 A number multiple of 6 and 8 together.
4 A number multiple of 6 or 8 «$\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{30}$, $\frac{7}{30}$»

18 A box has 15 balls , 6 of them are red numbered from 1 to 6 and 9 are green numbered from 7 to 15 one ball was drawn randomly from the box.
Find the probability of each of the following events :

1 The drawn ball is red or carrying an odd number.
2 The drawn ball is green and carrying an even number. «$\frac{11}{15}$, $\frac{4}{15}$»

19 For an irregular dice the probability of the appearance of the numbers 1 , 2 , 3 , 4 and 5 are equal and the probability of the appearance of the number 6 is 3 times the probability of the appearance of the number 1 If the cube is rolled once.
Calculate the probability of :

1 The appearance of the number 6
2 The appearance of a prime odd number. «$\frac{3}{8}$, $\frac{1}{4}$»

20 If A and B are two mutually exclusive events from the sample space of a random experiment such that the probability of occurrence of event B is three times the probability of occurrence of event A , the probability of occurrence of one at least of the two events is 0.64
Find the probability of occurrence of each of the two events A and B

«0.16 , 0.48»

For excellent pupils

1 Three players A , B and C join in the competition of weight lifting. If the probability that the first player wins is equal to twice the probability of the second player to win and the probability that the player B wins is equal to the probability that the player C wins.
Find the probability that the player B or C wins , taking into consideration that one player will win. *(Matrouh 18)* «$\frac{1}{2}$»

2 If A and B are two events in the sample space of a random experiment , $P(A \cup B) = \frac{4}{5}$, $P(A) = \frac{2}{5}$ and $7\,P(A \cap B) = 2 - P(B)$
Find : P (B) «$\frac{3}{5}$»

Exercise **10**

Complementary event and the difference between two events

From the school book

1 **Complete the following :**

1. If the probability of occurrence of an event A is 65 % , then the probability that event does not occur equals .... *(Assiut 11)*

2. If $P(A) = P(\grave{A})$, then $P(A) =$ ....

3. If A and B are two mutually exclusive events in a random experiment , then $P(A - B) = P($..........$)$, $P(B - A) = P($..........$)$

4. If A and B are two events of a random experiment and $A \subset B$, then $P(A - B) = P($..........$) =$ ....

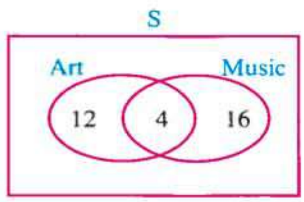
5. If A is an event of a random experiment , then :
   (1) $A \cup \grave{A} =$ .... **(2)** $A \cap \grave{A} =$ ....
   (3) $P(A \cup \grave{A}) =$ .... **(4)** $P(A \cap \grave{A}) =$ ....

6. If A and B are two events from the sample space of the random experiment , $P(A) = 0.7$ and $P(A - B) = 0.5$, then $P(A \cap B) =$ ....

7. If the English alphabet letters is a sample space which contains 26 letters , $X = \{a , b , c , d , e , f , h\}$ and $Y = \{k , u , v\}$, then $P\grave{(X - Y)} =$ ....

2 **Choose the correct answer from those given :**

1. A class of 32 students , two sets of the students are from the lovers of art and music , their number is as in the figure. If a student is chosen randomly , then the probability that the student does not love music is ....



(a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{5}{8}$ (d) 1

Unit 3

2 If A and B are two events in a sample space the event of occurrence of A only is ....

(a) $\acute{A}$ (b) $A - B$ (c) $A \cap B$ (d) $A \cup B$

(El-Menia 15)

3 If A is an event from the sample space of the random experiment , then $P(\acute{A}) =$ ....

(El-Dakahlia 17)

(a) 1 (b) -1 (c) $1 - P(A)$ (d) $P(A) - 1$

4 If S is the sample space of a random experiment , then $P(\acute{S}) =$ ....

(a) 1 (b) zero (c) $\frac{1}{2}$ (d) -1

5 If $P(A) = 4\,P(\acute{A})$, then $P(A) =$ .... *(El-Kalyoubia 18 , El-Kalyoubia 17)*

(a) 0.8 (b) 0.6 (c) 0.4 (d) 0.2

6 If A and B are two mutually exclusive events in a random experiment and $P(\acute{A}) = 0.6$, $P(A \cup B) = 0.9$, then $P(B) =$ .... *(Kafr El-Sheikh 13)*

(a) 0.5 (b) 0.4 (c) 0.6 (d) 0.3

7 If A and B are two events of the sample space of a random experiment , $P(A) = 0.6$ and $P(A \cap B) = 0.4$, then $P(A - B) =$ .... *(New Valley 14)*

(a) 0.6 (b) 0.4 (c) 0.2 (d) 0.1

8 If A and B are two events of a sample space of a random experiment , $A \subset B$, $P(A) = 0.2$ and $P(B) = 0.6$, then $P(B - A) =$ .... *(Luxor 19)*

(a) 0.6 (b) 0.2 (c) 0.8 (d) 0.4

9 For any two events C and D of a random experiment, there is : $(C - D) \cup (C \cap D) =$ .... *(El-Dakahlia 14)*

(a) 1 (b) S (c) D (d) C

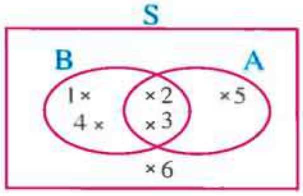
3 If S is the sample space of a random experiment , $A \subset S$, $\acute{A}$ is the complementary event to the event A and $S = \{1 , 2 , 3 , 4 , 5 , 6\}$

Complete the following table and record your observation :

event A	event $\acute{A}$	P (A)	P ($\acute{A}$)	P (A) + P ($\acute{A}$)
$\{2 , 4 , 6\}$				
	$\{3 , 6\}$			
$\{5\}$				
$\{1 , 2 , 3 , 4 , 5 , 6\}$				

4 **In the opposite figure :**

If A and B are two events of a sample space S of a random experiment then , **find :**



1 P (A ∩ B)

2 P (A – B)

3 The probability of non-occurrence of the event A

(Cairo 17) « $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{2}$ »

5 If A and B are two events of the sample space of a random experiment ,

$P(A) = \frac{1}{5}$, $P(B) = \frac{3}{5}$ and $P(A \cap B) = \frac{1}{10}$ **Find :**

1 P (Á) 2 P (B́) 3 P (A ∪ B)

4 P (A – B) 5 P (B – A)

« $\frac{4}{5}$, $\frac{2}{5}$, $\frac{7}{10}$, $\frac{1}{10}$, $\frac{1}{2}$ »

6 If X and Y are two events of a sample space S , P (X) = 0.35 , P (Y) = 0.48 and P (X ∪ Y) = 0.6 **Find :**

1 P (X̀) , P (Ỳ) 2 P (X ∩ Y)

3 P (X – Y) 4 P (X ∩ Ỳ)

« 0.65 , 0.52 , 0.23 , 0.12 , 0.77 »

7 If A and B are two events of a sample space of a random experiment , $P(B) = \frac{1}{3}$ and $P(A - B) = \frac{1}{4}$ **Find : P (A) if :**

1 $P(A \cap B) = \frac{1}{12}$ 2 A and B are mutually exclusive.

3 B ⊂ A

« $\frac{1}{3}$, $\frac{1}{4}$, $\frac{7}{12}$ »

8 If X and Y are two events in a sample space of a random experiment where :

$P(Y) = \frac{2}{5}$, P (X) = P (X̀) , $P(X \cap Y) = \frac{1}{5}$ **Find :**

1 P (X) 2 P (X ∪ Y)

(Kafr El-Sheikh 18 , El-Kalyoubia 16 , El-Dakahlia 14) « $\frac{1}{2}$, $\frac{7}{10}$ »

9 If A and B are two events of a sample space of a random experiment , P (A) = P (Á) , $P(A \cap B) = \frac{1}{16}$ and $P(B) = \frac{5}{8} P(A)$ **Find :**

1 P (B) 2 P (A ∪ B)

3 P (A – B)

(El-Fayoum 19) « $\frac{5}{16}$, $\frac{3}{4}$, $\frac{7}{16}$ »

Unit 3

10 A bag contains 12 balls numbered from 1 to 12 , if a ball is drawn randomly , and the event A is «getting an odd number» and the event B is «getting a prime number» , then find : P (A) , P (B) , P (Á) , (A ∪ B) , P (A − B) «$\frac{1}{2}$, $\frac{5}{12}$, $\frac{1}{2}$, $\frac{7}{12}$, $\frac{1}{6}$»

11 A box contains 20 balls which have the same shape , size and weight are well mixed , 8 of them are red , 7 are white and the rest of the balls are green. A ball is drawn randomly. **Find the probability that the drawn ball is :**

1 Red. 2 White or green. 3 Not white. «$\frac{2}{5}$, $\frac{3}{5}$, $\frac{13}{20}$»

12 If A and B are two events from the sample space of a random experiment , P (A) = 0.8 , P (B) = 0.7 and P (A ∩ B) = 0.6 **Find :**

1 The probability of non occurrence of the events A and B together.

2 The probability of occurrence of at least one of the two events. « 0.4 , 0.9 »

13 If A and B are two events of a sample space of a random experiment , P (A) = 0.5 , P (B) = 0.6 and P (A ∩ B) = 0.4 **Find :**

1 The probability of occurrence at least one of the two events.

2 The probability of occurrence of B and non occurrence of A

3 The probability of non occurrence of A

4 The probability of non occurrence of any of them.

5 The probability of occurrence of one of the events but not the other.

6 The probability of occurrence of the event A only. « 0.7 , 0.2 , 0.5 , 0.3 , 0.3 , 0.1 »

14 If A and B are two events of the sample space of a random experiment , the probability of non occurrence of the event A is $\frac{1}{4}$, the probability of non occurrence of the event B is $\frac{1}{2}$ and the probability of occurring one of them at most is $\frac{3}{5}$

Find the probability of each of the following :

1 The occurrence of the event A

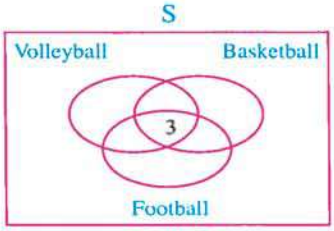
2 The occurrence of the two events together.

3 The occurrence of any of the two events.

4 The occurrence of the event A only.

5 The occurrence of one of the two events but not the other. «$\frac{3}{4}$, $\frac{2}{5}$, $\frac{17}{20}$, $\frac{7}{20}$, $\frac{9}{20}$»

15 A class of 32 students , 22 of them play football , 10 of them play basketball and 12 play volleyball.
If 3 of them are playing the three games , 5 play football and basketball , 8 play football and volleyball and 6 play basketball and volleyball.



1 How many students play football and basketball only ?

2 How many students don't play any of these games ?

3 If a student is chosen randomly , then what is the probability that the student be a football player only ?

« 2 , 4 , $\frac{3}{8}$ »

16 A classroom contains 40 students. 18 of them read Al-Akhbar Newspaper , 15 read Al-Ahram Newspaper and 8 read both Newspapers. If a student is selected randomly.
Calculate the probability that the student :

1 Reads Al-Akhbar Newspaper.

2 Does not read Al-Akhbar Newspaper.

3 Reads Al-Ahram Newspaper.

4 Reads both Newspapers.

5 Reads Al-Akhbar Newspaper only.

6 Reads Al-Ahram Newspaper only.

7 Reads Al-Akhbar only or Al-Ahram only.

« $\frac{9}{20}$, $\frac{11}{20}$, $\frac{3}{8}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{7}{40}$, $\frac{17}{40}$ »

17 45 students participated in some sports activities. 27 of them are members in the school football team. 15 in basketball team and 9 in both, football and basketball team. A student is randomly selected. **Represent this situation using a Venn diagram , then find the probability that the selected student is :**

1 A member in the football team.

2 A member in the basketball team.

3 A member in the basketball team and football team.

4 Not a member in any of the two previous teams.

« $\frac{3}{5}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{4}{15}$ »

18 60 students participated in one of the school's sports activities , 36 students participated in football , 27 students participated in basketball and 12 students participated in football and basketball. A student was chosen randomly.
Represent this using the Venn diagram , then find the probability that the chosen student :

1 A participant in the football team and not a participant in the basketball team.

2 A participant of at least one team from the two teams.

3 Not a participant of any of the two teams.

« $\frac{2}{5}$, $\frac{17}{20}$, $\frac{3}{20}$ »

Unit 3

19 The number of all outcomes of a random experiment of equal choices of all outcomes is 30 If A and B are two events of the sample space of this experiment and the number of outcomes of occurring the event B equals 12 outcomes and $P(A \cup B) = \frac{13}{15}$, $P(A) = 0.6$ **Find :**

1. The probability of occurrence of the two events together.
2. The probability of occurring one of the two events but not the other. « $\frac{2}{15}$, $\frac{11}{15}$ »

20 In a survey of 6000 birth cases in a province selected randomly. Researchers paid much attention to find a relation between mother's age when she gives birth and the place where she lives. The following table shows the number of births in urban and rural villages :

Mother's age	Place of living	
	Urban	Rural villages
Less than 20 years	120	240
From 20 years to less than 22 years	240	360
From 22 years to less than 30 years	1740	1440
From 30 years and more	1500	360

1. If the event A expresses the mother who gave birth and lives in the urban area and the event B expresses the mother who gave birth whose age is not more than 22 years.

   Find :

   (1) $P(A)$ **(2)** $P(B)$ « $\frac{3}{5}$, $\frac{4}{25}$ »

2. Represent the sets A and B using the Venn diagram , then find :

   (1) $P(A \cap B)$ **(2)** $P(A \cup B)$ **(3)** $P(A - B)$ **(4)** $P(A \cup B)'$

   « $\frac{3}{50}$, $\frac{7}{10}$, $\frac{27}{50}$, $\frac{3}{10}$ »

3. Predict the number of births if the mother lives in the urban area and aged 30 years or more, take into consideration that the number of births is 9000 in the province.

   « 2250 cases »

For excellent pupils

1 A farm contains cows of the two colours white and brown. If the probability that the cow is white = $\frac{5}{7}$ and the probability that the cow is brown = $\frac{11}{28}$

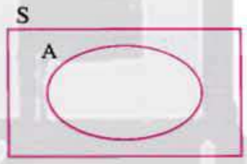
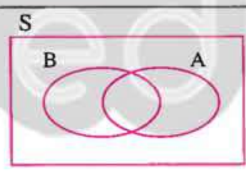
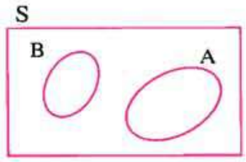
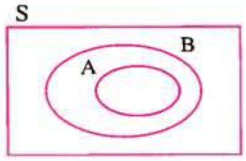
Find the probability of each of the following :

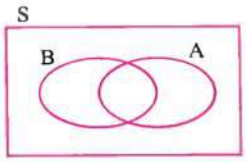
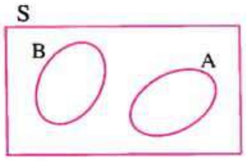
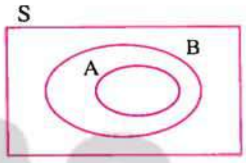
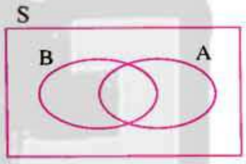
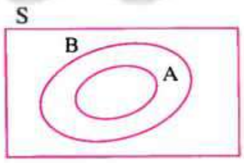
1. The cow is mixed colour of the two colours.
2. The cow is only white. «$\frac{3}{28}$, $\frac{17}{28}$»

2 A coin is tossed twice. **What is the probability of :**

1. Non occurrence of a head in the second toss ?
2. Non occurrence of a head in the two tosses together ? «$\frac{1}{2}$, $\frac{3}{4}$»

Summary of Unit 3

Words representation of the event	Probability of the event	Representing event by Venn diagram
Probability of occuring the certain event = 1	$P(S) = 1$	
Probability of occuring the impossible event = zero	$P(\varnothing) = \text{zero}$	
Probability of occuring the event A	$P(A) = \frac{n(A)}{n(S)}$	
The complementary event : Probability of occuring the complementary event of the event A or probability of non occuring event A	$P(\grave{A}) = \frac{n(\grave{A})}{n(S)} = 1 - P(A)$	
Intersecting of two events ($A \cap B$): Probability of occuring A and B together	$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$	
	• If A and B are mutually exclusive events , then $P(A \cap B) = \text{zero}$	
	• If $A \subset B$, then $P(A \cap B) = P(A)$	

Union of two events ($A \cup B$) : • Probability of occuring the events A or B or both of them. • Probability of occuring one of the two events at least. • Probability of occuring any of the two events.	$P(A \cup B) = \frac{n(A \cup B)}{n(S)}$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
	• If A and B are two mutually exclusive events , then $P(A \cup B) = P(A) + P(B)$	
	• If $A \subset B$, then $P(A \cup B) = P(B)$	
The difference between events ($A - B$): • Probability of occuring the event A and non occuring of event B • Probability of occuring the event A only.	$P(A - B) = \frac{n(A - B)}{n(S)}$, $P(A - B) = P(A) - P(A \cap B)$	
	• If A and B are two mutually exclusive events , then $P(A - B) = P(A)$	
	• If $A \subset B$, then $P(A - B) = P(\varnothing) = \text{zero}$	

Exam on Unit Three

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If A and B are two mutually exclusive events , then $P(A \cap B) = \dots\dots$

(a) $\varnothing$ (b) $P(A)$ (c) $P(B)$ (d) zero

2 If $A \subset S$ of a random expriment , $P(A) = 0.35$, then $P(\grave{A}) = \dots\dots$

(a) -0.65 (b) 0.65 (c) 0.35 (d) 0.4

3 If $A \subset B$, then $P(A \cup B) = \dots\dots$

(a) $P(A)$ (b) $P(B)$ (c) zero (d) $P(A \cap B)$

4 If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{8}$, then $P(A \cup B) = \dots\dots$

(a) $\frac{5}{8}$ (b) $\frac{17}{24}$ (c) $\frac{1}{6}$ (d) $\frac{13}{24}$

5 If $\grave{A}$ is the complement event of A , then $A \cup \grave{A} = \dots\dots$

(a) S (b) $\frac{1}{2}$ (c) 1 (d) $\varnothing$

6 If A and B are two events from the sample space of a random experiment , $P(\grave{A}) = 0.3$ and $P(A - B) = 0.5$, then $P(A \cap B) = \dots\dots$

(a) 0.6 (b) 0.4 (c) 0.3 (d) 0.2

2 **[a]** If A and B are two events of the sample space of a random experiment , $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cap B) = \frac{1}{4}$

(1) Find : $P(A \cup B)$ **(2)** Find : $P(A - B)$ **(3)** Prove that : $P(\grave{A}) = P(B)$

[b] If A and B are two events of the sample space of a random experiment where $2\,P(A) = 3\,P(\grave{A})$, $P(B) = \frac{2}{5}$

(1) Find : $P(A)$

(2) If the two events are mutually exclusive events , find the probability of the occurrence of at least one of the two events.

3 **A card is drawn randomly from 20 identical cards numbered from 1 to 20 , calculate the probability that the number on the drawn card is :**

1. A number divisible by 5
2. A number divisible by 4
3. A number divisible by 5 and divisible by 4
4. A number divisible by 5 or divisible by 4

4 If A and B are two events of the sample space of a random experiment ,

$P(A) = 0.6$, $P(B) = 0.7$ and $P(A \cap B) = 0.4$

Find :

1. The probability of non occurrence of the events A and B together.
2. The probability of occurrence of at least one of the two events.
3. The probability of non occurrence of A
4. The probability of occurrence of the event A only.

5 **[a]** If a bag contains 21 identical balls , 8 white balls , 6 red balls and the rest are black if one ball is drawn randomly , **find the probability that this ball is :**

(1) White **(2)** Not black **(3)** Red or black

[b] If A and B are two mutually exclusive events of a random experiment ,

$P(A) = \frac{1}{3}$ and $P(A \cup B) = \frac{7}{12}$, find : $P(\acute{B})$

TIMSS Problems

Accumulative basic skills

Choose the correct answer from the given ones :

1 $(-1)^{37} - (-1)^{36} = \dots\dots$ *(El-Menia 17)*

(a) -2 (b) zero (c) 1 (d) 2

2 $\sqrt[3]{a} + \sqrt[3]{-a} = \dots\dots$ *(Ismailia 16)*

(a) $\sqrt[3]{-a^2}$ (b) $2\sqrt[3]{a}$ (c) -1 (d) zero

3 $2^7 \times 3^7 = 6^k$, then $k = \dots\dots$ *(El-Fayoum 19 , El-Kalyoubia 17 , El-Sharkia 16)*

(a) 14 (b) 7 (c) 6 (d) 5

4 If $2^5 \times 3^5 = m \times 6^4$, then $m = \dots\dots$ *(El-Sharkia 17)*

(a) 1 (b) 2 (c) 3 (d) 6

5 The solution set of the inequality : $x \leq 1$ in $\mathbb{N}$ is $\dots\dots$ *(Matrouh 17)*

(a) $\{1\}$ (b) $\{0\}$ (c) $\{0 , 1\}$ (d) $\{1 , 0 , -1 , ...\}$

6 $[2 , 5]$ is the solution set of the inequality $\dots\dots$ *(Alexandria 16)*

(a) $1 \leq x - 1 \leq 4$ (b) $1 < x - 1 < 4$ (c) $1 \leq x - 1 < 4$ (d) $1 < x - 1 \leq 4$

7 If $\frac{1}{5} x = \frac{1}{10}$, then $2 x = \dots\dots$ *(New Valley 17)*

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 50

8 If x is a negative number , then the greatest number from the following numbers is $\dots\dots$

(El-Monofia 18 , El-Kalyoubia 17 , Giza 17 , Cairo 16)

(a) $5 - x$ (b) $5 + x$ (c) $\frac{5}{x}$ (d) $5 x$

9 $\sqrt{64+36} = 8 + x$, then $x =$ .... *(Aswan 17)*

(a) 9 (b) 6 (c) 2 (d) 10

10 If $(5 , x - 4) = (y + 2 , 3)$, then $x + y =$ .... *(Luxor 17)*

(a) 6 (b) 8 (c) 10 (d) 12

11 If x is the additive identity , y is the multiplicative identity , then $2^x + 3^y =$ .... *(Ismailia 19)*

(a) 2 (b) 3 (c) 4 (d) 5

12 The multiplicative inverse of the number $\frac{\sqrt{2}}{3}$ is .... *(North Sinai 17 , El-Menia 16)*

(a) $\frac{-\sqrt{2}}{3}$ (b) $\frac{3\sqrt{2}}{2}$ (c) $\frac{2\sqrt{3}}{3}$ (d) $\frac{\sqrt{2}}{3}$

13 If $(x - 5)^{\text{zero}} = 1$ for every $x \in$ .... *(Souhag 16)*

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{-5\}$ (d) $\mathbb{R} - \{1\}$

14 If $a b = 3$, $a b^2 = 12$, then $b =$ .... *(Damietta 19 , Kafr El-Sheikh 17)*

(a) 4 (b) 2 (c) -2 (d) ± 2

15 If $2 x y = 6$, $x^2 y + x y^2 = 6$, then $x + y =$ .... *(Souhag 16)*

(a) 1 (b) 2 (c) 6 (d) $\frac{1}{2}$

16 If $y^{-3} = 8$, then $y =$ .... *(Alexandria 16)*

(a) $\frac{1}{512}$ (b) $\frac{1}{8}$ (c) 2 (d) $\frac{1}{2}$

17 The curve $y = a x^2 + bx + c$ intersects the y-axis at the point .... *(El-Menia 18)*

(a) (0 , b) (b) (b , 0) (c) (c , 0) (d) (0 , c)

18 If $x + y = 5$, $x - y = 3$, then $x^2 - y^2 =$ ....

(a) 2 (b) 8 (c) 16 (d) 15

19 The additive inverse of the number $(1 - \sqrt{2})$ is .... *(Ismailia 16)*

(a) $1 + \sqrt{2}$ (b) $-1 - \sqrt{2}$ (c) $\sqrt{2} - 1$ (d) $\sqrt{2}$

20 The arithmetic mean of the values : 2 , 3 , 4 , 7 and 9 is .... *(El-Fayoum 16)*

(a) 4 (b) 5 (c) 6 (d) 8

Basic skills

21 If the function f is a function from the set X to the set Y , then the domain of the function is .... *(Beni Suef 16)*

(a) X (b) Y (c) $X \times Y$ (d) $Y \times X$

22 If $a^2 - b^2 = a + b$, then $b - a =$ ....
where $a + b \neq 0$ *(El-Gharbia 18)*

(a) 2 (b) -2 (c) 1 (d) -1

23 If $x = 2$, $y = 3$, then $(y - 2x)^{10} =$ .... *(Damietta 19 , Alex. 17 , Luxor 16)*

(a) 1 (b) -1 (c) 5 (d) 10

24 If $x + 3y = 7$, then $x + 3(y + 5) =$ .... *(El-Beheira 18)*

(a) 22 (b) 21 (c) 7 (d) 3

25 $4^{15} + 4^{15} =$ .... *(El-Monofia 19)*

(a) 4^{30} (b) 4^0 (c) 8^{15} (d) 2^{31}

26 The rule which discribes the pattern : $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, ... in terms of n
where $n \in \mathbb{Z}_+$ is .... *(El-Fayoum 19)*

(a) $\frac{2}{n+1}$ (b) $n + \frac{1}{2}$ (c) $\frac{n}{n+1}$ (d) $\frac{2n-1}{n+1}$

27 If $a < \sqrt{3} < b$, then (a , b) may be .... *(El-Monofia 17)*

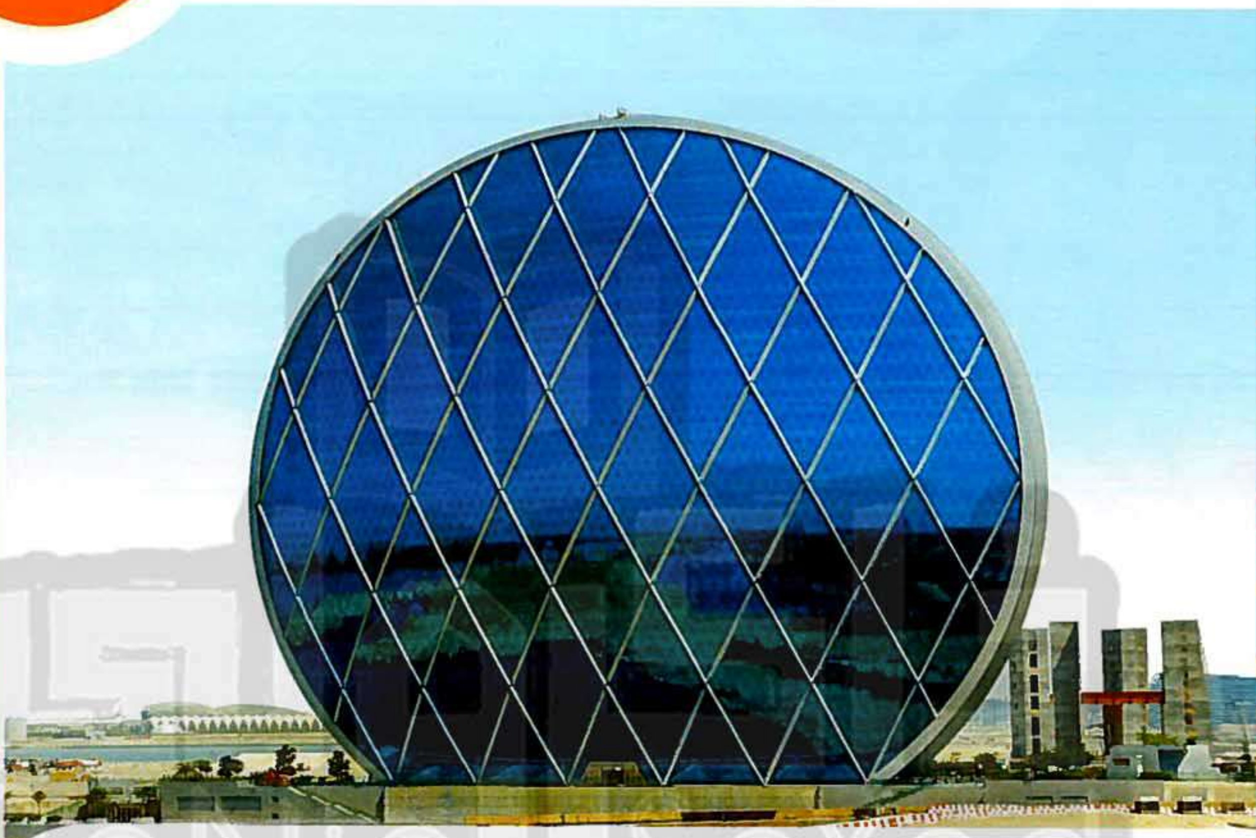
(a) (0 , 1) (b) (2.5 , 3.5) (c) (1 , 2) (d) (2 , 3)

28 If $ab = 12$, $bc = 20$, $ac = 15$ and $a , b , c \in \mathbb{R}_+$, then $abc =$ .... *(Damietta 17)*

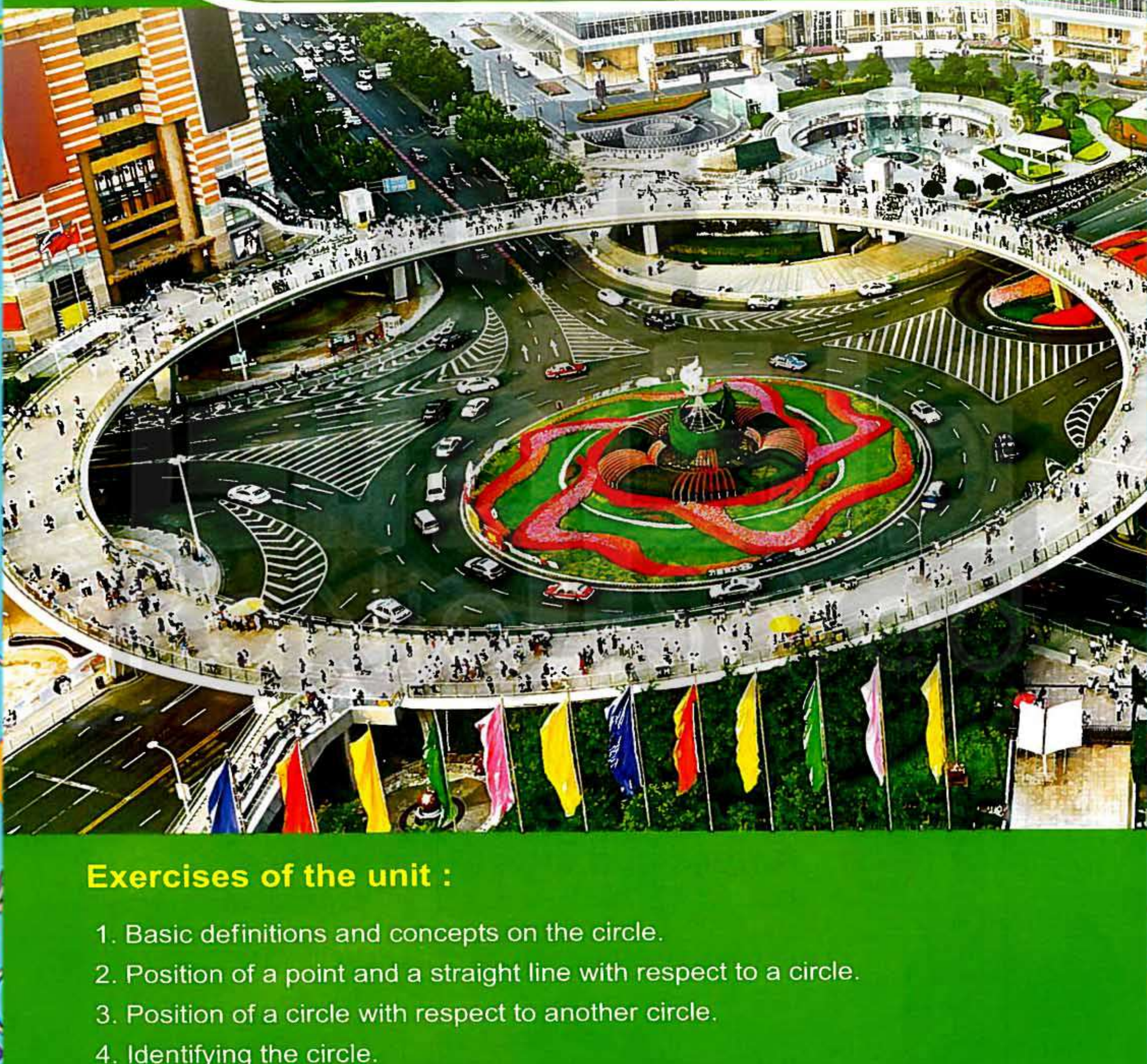
(a) 360 (b) 3600 (c) 60 (d) 36

Second

Geometry



UNIT 4 The circle



Exercises of the unit :

1. Basic definitions and concepts on the circle.
2. Position of a point and a straight line with respect to a circle.
3. Position of a circle with respect to another circle.
4. Identifying the circle.
5. The relation between the chords of a circle and its centre.

- Summary of unit four.
- Unit exams.

Exercise **1**

Basic definitions and concepts on the circle

From the school book

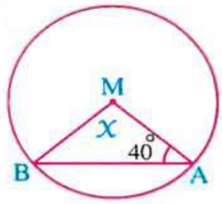
1 **Complete the following :**

1. .... is a line segment with one endpoint at the centre of the circle and the other endpoint on the circle.
2. The line segment whose endpoints are any two points on the circle and not passing through the centre is called .... *(New Valley 06)*
3. The longest chord of the circle is called .... *(Damietta 09)*
4. Any straight line passing through the centre of the circle is called .... of it. *(Sharkia 09)*
5. The number of axes of symmetry of the circle is .... and the number of axes of symmetry of a semicircle is ....
6. The straight line passing through the centre of the circle and the midpoint of any chord of it is .... *(Kafr El-Sheikh 11)*
7. The straight line passing through the centre of a circle and perpendicular to any chord of it .... this chord. *(Souhag 12)*
8. The straight line which is perpendicular to any chord of the circle from its midpoint passes through .... *(Alex. 06)*
9. The product of the approximating ratio (π) by the length of the diameter of a circle = ... *(Luxor 04)*
10. If the area of a circle is 9π cm^2. , then its radius length = ... cm. *(Alex. 06)*
11. A chord of length 8 cm. in a circle of diameter 10 cm. , then the distance of the chord from the centre of the circle is .... cm.

Unit 4

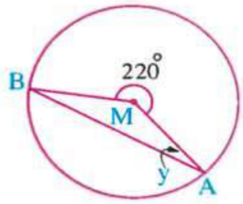
2 **In each of the following figures , find the value of the used symbol in measuring where M is the centre of the circle :**

1



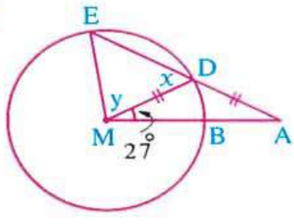
x = ...°

2



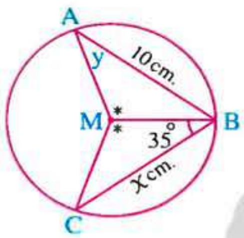
y = ...°

3



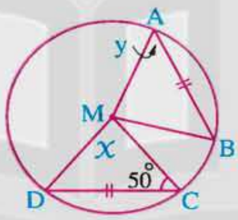
x = ...°
y = ...°

4



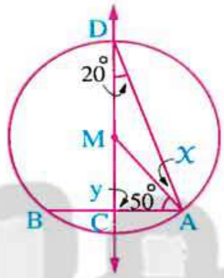
x = ... cm.
y = ...°

5



x = ...°
y = ...°

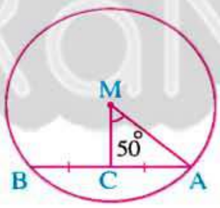
6



x = ...°
y = ...°

3 **In each of the following figures , M is a circle , complete :**

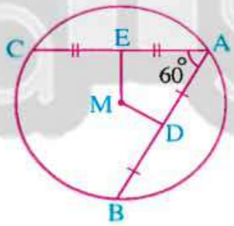
1



m (∠ MAC) = ...°

(Luxor 14 , Assiut 11)

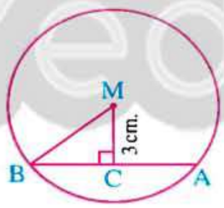
2



m (∠ DME) = ...°

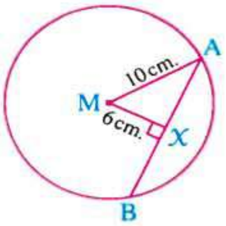
(Luxor 14 , Giza 15)

3



If AB = 8 cm.
, then MB = ... cm.

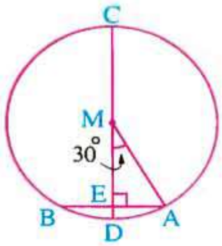
4



AB = ... cm.

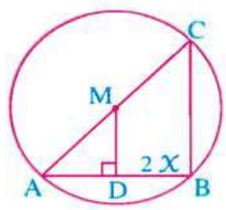
(Red Sea 12)

5



If AB = 10 cm.
, then CD = ... cm.

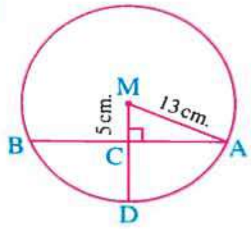
6



x = ...°

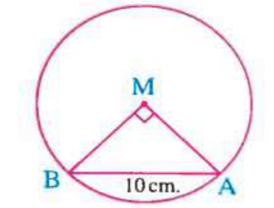
Exercise 1

7



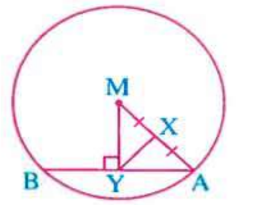
AB = ... cm.
CD = ... cm.

8



m (∠ A) = ... °
MA = ... cm.

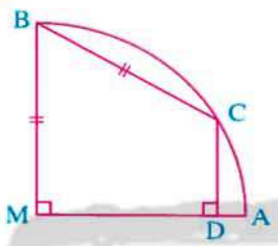
9



XY = 7 cm. , $\pi \approx \frac{22}{7}$
The area of the circle = ... cm^2.

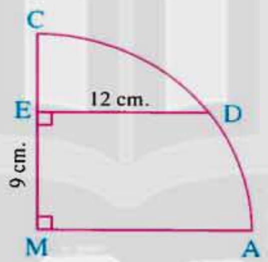
4 **Each of the following figures represents a quarter of a circle M , complete :**

1



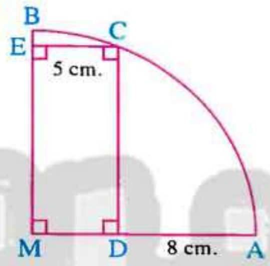
m (∠ BCD) = ... °

2



The length of $\overline{EC}$ = ... cm.

3



The area of the rectangle = ... cm^2.

5 **In the opposite figure :**

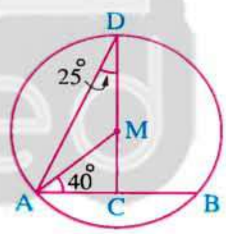
$\overline{AB}$ is a chord of the circle M ,

m (∠ D) = 25°

and m (∠ MAC) = 40°

Prove that :

C is the midpoint of $\overline{AB}$



(Kafr El-Sheikh 09)

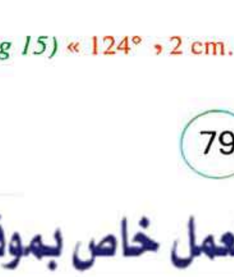
6 **In the opposite figure :**

$\overline{AB}$ and $\overline{BC}$ are two chords in circle M ,

which has radius length of 5 cm.,

$\overrightarrow{MD} \perp \overline{AB}$ intersects $\overline{AB}$ at D and inersects the circle M at E ,

X is the midpoint of $\overline{BC}$, AB = 8 cm., m (∠ ABC) = 56°



Find : 1 m (∠ DMX) 2 The length of $\overline{DE}$

(El-Menia 19 , El-Gharbia 17 , Souhag 15) « 124° , 2 cm. »

Unit 4

7 **In the opposite figure :**

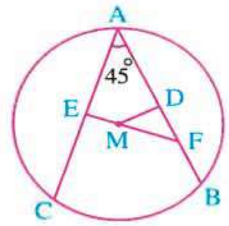
$\overline{AB}$ and $\overline{AC}$ are two chords of the circle M ,

m (∠ BAC) = 45° ,

D and E are the midpoints

of $\overline{AB}$ and $\overline{AC}$ respectively.

Prove that : Δ DFM is an isosceles triangle.



(New Valley 05)

8 **In the opposite figure :**

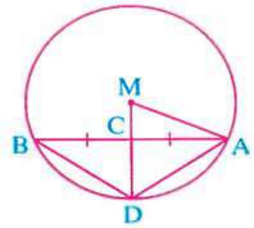
M is a circle of radius length 13 cm. ,

$\overline{AB}$ is a chord of length 24 cm. ,

C is the midpoint of $\overline{AB}$

and $\overrightarrow{MC}$ ∩ circle M = {D}

Find : The area of the triangle ADB



(El-Dakahlia 13) « 96 cm.^{2} »

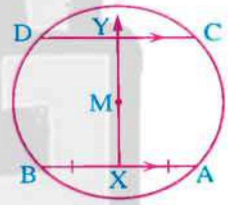
9 **In the opposite figure :**

M is a circle , $\overline{AB}$ // $\overline{CD}$,

X is the midpoint of $\overline{AB}$

and $\overrightarrow{XM}$ is drawn to cut $\overline{CD}$ at Y

Prove that : Y is the midpoint of $\overline{CD}$



(El-Menia 18 , Assiut 18 , Aswan 15 , Alexandria 13)

10 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords in circle M

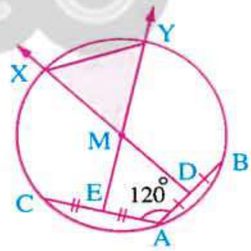
that includes an angle of measure 120° ,

D and E are the two midpoints of $\overline{AB}$ and $\overline{AC}$

respectively , $\overrightarrow{DM}$ and $\overrightarrow{EM}$ are drawn to intersect

the circle at X and Y respectively.

Prove that : The triangle XYM is an equilateral triangle.



(Aswan 16 , Beni Suef 15)

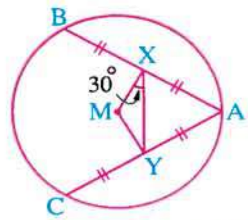
11 **In the opposite figure :**

AC = AB , X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{AC}$,

m (∠ MXY) = 30°

Prove that : The triangle AXY is equilateral.



(Assiut 14)

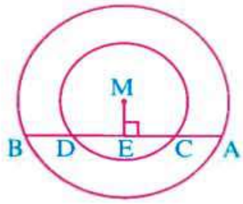
12 **In the opposite figure :**

Two concentric circles with centre M ,

$\overline{AB}$ is a chord of the greater circle

and intersects the smaller circle at C , D

and $\overline{ME} \perp \overline{AB}$

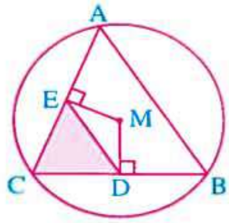


Prove that : AC = BD

(El-Gharbia 18 , Qena 18 , Qena 17 , Red Sea 12)

13 **In the opposite figure :**

ABC is a triangle drawn inside a circle with centre M (inscribed triangle) , $\overline{MD} \perp \overline{BC}$ and $\overline{ME} \perp \overline{AC}$



Prove that :

1 $\overline{ED}$ // $\overline{AB}$

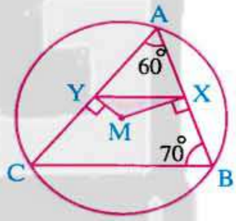
2 The perimeter of Δ CDE = $\frac{1}{2}$ the perimeter of Δ ABC

(Kafr El-Sheikh 16 , El-Beheira 13)

14 **In the opposite figure :**

In circle M , $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, m ($\angle$ A) = 60° and m ($\angle$ B) = 70°



Find : The measures of the interior angles of the triangle MXY

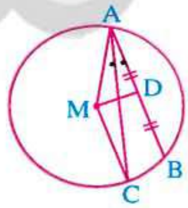
« 120° , 20° , 40°»

15 **In the opposite figure :**

$\overline{AB}$ is a chord of circle M ,

$\overrightarrow{AC}$ bisects $\angle$ BAM and intersects circle M at C

If D is the midpoint of $\overline{AB}$



Prove that : $\overline{DM} \perp \overline{CM}$

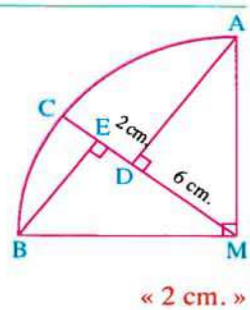
(El-Beheira 19 , El-Gharbia 17 , Souhag 14)

16 **In the opposite figure :**

A quarter of the circle M , $\overline{AM} \perp \overline{MB}$

, $\overline{AD} \perp \overline{MC}$, $\overline{BE} \perp \overline{MC}$

If MD = 6 cm. , DE = 2 cm.



Find : The length of $\overline{EC}$

« 2 cm. »

Unit 4

17 **In the opposite figure :**

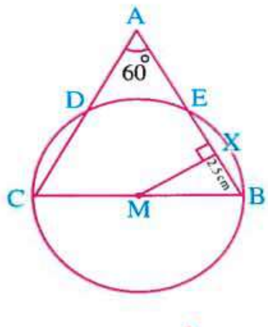
$\overline{BC}$ is a diameter of the circle M ,

AB = AC ,

m (∠ BAC) = 60° ,

BX = 2.5 cm. and $\overline{MX} \perp \overline{AB}$

Find : The length of $\overline{AE}$



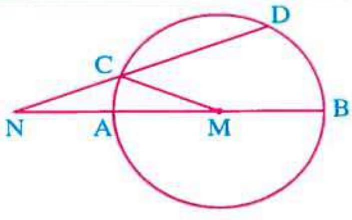
« 5 cm. »

18 **In the opposite figure :**

$\overline{AB}$ is a diameter in circle M

, $\overrightarrow{BA} \cap \overrightarrow{DC} = \{N\}$

Prove that : NC > NA



« El-Beheira 18 »

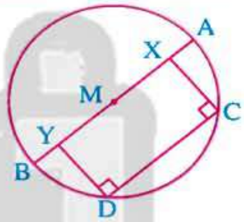
19 **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M ,

$\overline{CD}$ is a chord of it , $\overline{XC} \perp \overline{CD}$

and $\overline{YD} \perp \overline{CD}$

Prove that : AX = BY



(Sharkia 09)

20 $\overline{AB}$ and $\overline{CD}$ are two parallel chords in circle M , AB = 12 cm. , CD = 16 cm.
Find the distance between those two chords if the radius length of circle M equals 10 cm.
Are there any other answers ? Explain your answer. « 14 cm. or 2 cm. »

Connecting with analytical geometry

21 In a cartesian coordinates plane , if $\overline{AB}$ is a diameter of the circle M where A (3 , 4) and B (3 , – 3) , find the coordinates of M , then calculate the circumference of the circle.

« $\left(3 , \frac{1}{2}\right)$, 22 length units »

22 In a cartesian coordinates plane , if M (– 1 , 2) , A (2 , 6) and B (2 , – 2)
Prove that M is the centre of a circle passing through the two points A and B , then calculate the perpendicular distance between the chord $\overline{AB}$ and the centre of the circle.

« 3 length units »

23 In a cartesian coordinates plane , $\overline{AB}$ is a chord of the circle M , D is the midpoint of $\overline{AB}$
If A (4 , 1) and B (– 4 , 5) **Find :** The equation of $\overleftrightarrow{MD}$ « $y = 2X + 3$ »

For excellent pupils

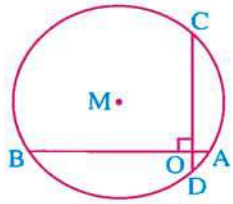
1 **In the opposite figure :**

A circle M has a radius length of 7 cm. ,

$\overline{AB}$ and $\overline{CD}$ are two perpendicular and intersecting chords at O

If AB = 12 cm. and CD = 10 cm.

Find : The length of $\overline{MO}$



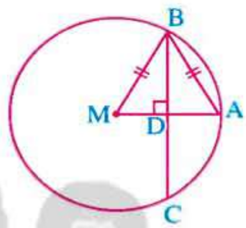
« $\sqrt{37}$ cm. »

2 **In the opposite figure :**

$\overline{AB}$ and $\overline{BC}$ are two chords of the circle M , AB = BM ,

$\overline{MA} \perp \overline{BC}$ and BC = $14\sqrt{3}$ cm.

Find : The radius length of the circle M



« 14 cm. »

Exercise 2
Position of a point and a straight line with respect to a circle

From the school book

1 **A circle M is of radius length 8 cm. , A is a point in the plane of the circle M Complete :**

1. If MA = 10 cm. , then A lies .... the circle.
2. If MA = 8 cm. , then A lies .... the circle.
3. If MA = $5\sqrt{2}$ cm. , then A lies .... the circle.
4. If MA = zero , then A lies .... the circle and it is represented by ....

2 **Complete the following :**

1. If the straight line L $\cap$ the circle M = $\varnothing$, then the straight line L lies ....
2. If the straight line L intersects the circle M at the two points A and B , then the straight line L $\cap$ the surface of the circle M = ...
3. The tangent to the circle is perpendicular to .... drawn from the tangency point.
4. The straight line which is drawn perpendicular to a diameter of a circle at one of its endpoints is .... *(Port Said 12)*
5. The two tangents to a circle at the two endpoints of a diameter of it are .... *(Kafr El-Sheikh 08)*
6. If M is a circle with circumference 8 π cm. , A is a point on the circle , then MA = ... cm. *(Kafr El-Sheikh 11)*

3 **If M is a circle with radius length 7 cm. , $\overline{MA} \perp$ L where A $\in$ L**
Complete the following :

1. If MA = $4\sqrt{3}$ cm. , then the straight line L ....

2 If $MA = 3\sqrt{7}$ cm. , then the straight line L ....

3 If $2\ MA - 5 = 9$ cm. , then the straight line L ....

4 If the straight line L intersects the circle M and $MA = 3\ x - 5$, then $x \in$ ....

5 If the straight line L is a tangent to the circle M and $MA = x^2 - 2$, then $x \in$ ....

4 **Choose the correct answer from those given :**

1 If M is a circle , its diameter length = 6 cm. and A is a point on the circle , then ....

(a) $MA > 6$ cm. (b) $MA = 6$ cm.

(c) $MA = 3$ cm. (d) $MA < 3$ cm.

2 If a straight line L is a tangent to the circle M whose diameter length is 8 cm. , then L is at a distance of .... cm. from its centre. *(Souhag 19 , El-Kalyoubia 18)*

(a) 3 (b) 4 (c) 6 (d) 8

3 A circle M is of radius length 5 cm. , A is a point outside the circle , then MA equals .... cm. *(Gharbia 03)*

(a) 3 (b) 5 (c) 8 (d) 4

4 If the diameter length of a circle is 8 cm. and the straight line L is at distance of 3 cm. from its centre , then the straight line L is .... *(Damietta 16 , El-Menia 14)*

(a) a tangent to the circle. (b) a secant to the circle.

(c) outside the circle. (d) an axis of symmetry of the circle.

5 If M is a circle its diameter length = 14 cm. , $MA = (2\ x + 3)$ cm. where A is a point on the circle , then $x =$ .... *(El-Kalyoubia 17 , El-Sharkia 15)*

(a) 5 (b) 3 (c) 2 (d) 1

6 If M is a circle , its radius length is 7 cm. , A is a point in the plane of the circle , $MA = (2\ x - 3)$ cm. , where A is outside the circle , then ....

(a) $x = 5$ (b) $x \in [5 , \infty[$

(c) $x \in]5 , \infty[$ (d) $x \in]-\infty , 5[$

7 $\overline{AB}$ is a diameter in a circle M , $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are two tangents to the circle , then $\overleftrightarrow{AC}$ .... $\overleftrightarrow{BD}$ *(Alexandria 13)*

(a) intersects (b) is perpendicular to

(c) is parallel to (d) is coincident to

8 A circle is of a circumference 6π cm. , and the straight line L is distant from its centre by 3 cm. , then the straight line L is .... *(Red Sea 19 , Red Sea 17 , El-Monofia 15)*

(a) a tangent to the circle. (b) a secant.

(c) outside the circle. (d) a diameter of the circle.

9 If the area of the circle M is 16π cm^2. , A is a point in its plane where MA = 8 cm. , then A lies .... the circle M *(Qena 17 , El-Sharkia 09)*

(a) inside (b) outside (c) on (d) at the centre of

10 M is a circle with diameter of length 8 cm. If the straight line L is outside the circle , then the distance between the centre of the circle and the straight line L $\in$ ....

(a) $]4 , \infty[$ (b) $[0 , 4[$ (c) $]0 , 4[$ (d) $[0 , 8]$ *(Kafr El-Sheikh 14)*

11 A circle with diameter length $(2x + 5)$ cm. , the straight line L is at a distance $(x + 2)$ cm. from its centre , then the straight line L is .... *(Port Said 17)*

(a) a secant to the circle at the two points.

(b) outside the circle.

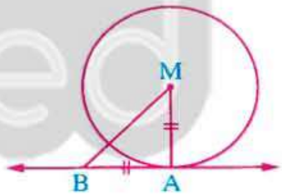
(c) a tangent to the circle.

(d) an axis of symmetry of the circle.

5 Using each of the following figures , choose the correct answer from those given :

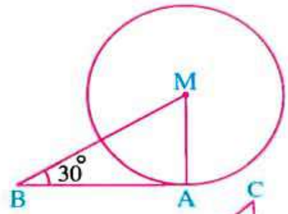
1 If $\overleftrightarrow{AB}$ is a tangent to the circle M at A , AB = AM , then m ($\angle$ M) = ...

(a) 30° (b) 45° (c) 60° (d) 90°



2 $\overrightarrow{AB}$ is a tangent to the circle M , m ($\angle$ B) = 30° , AM = 6 cm. , then MB = ... cm. *(Red Sea 18)*

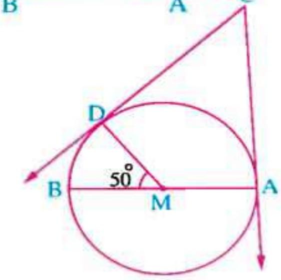
(a) 3 (b) 6 (c) 9 (d) 12



3 $\overline{AB}$ is a diameter of the circle M , $\overrightarrow{CA}$ and $\overrightarrow{CD}$ touch the circle at A and D , if m ($\angle$ DMB) = 50° , then m ($\angle$ C) = ...

(a) 50° (b) 130°

(c) 90° (d) 40°



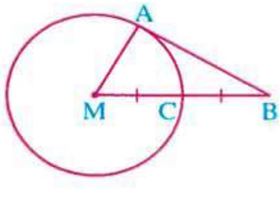
Exercise 2

4 If $\overline{AB}$ touches the circle M at A ,

$\overline{MB} \cap$ the circle M $= \{C\}$

where MC = BC , then m ($\angle$ B) = ...

(a) 30° (b) 45° (c) 60° (d) 90°



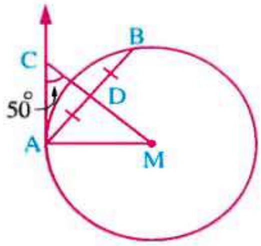
5 If $\overrightarrow{AC}$ touches the circle M at A ,

D is the midpoint of the chord $\overline{AB}$

, m ($\angle$ ACM) = 50° , then m ($\angle$ BAM) = ...

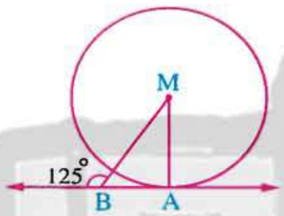
(a) 40° (b) 45°

(c) 50° (d) 90°



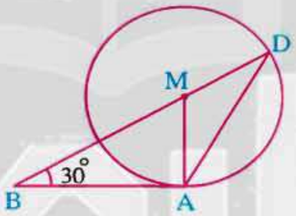
6 **M is a circle in each of the following figures and $\overleftrightarrow{AB}$ is a tangent. Complete :**

1



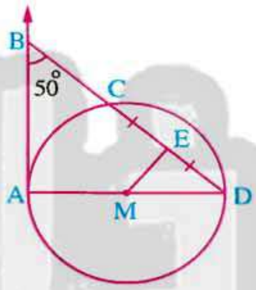
m ($\angle$AMB) = ...°

2



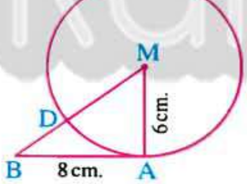
m ($\angle$ADB) = ...°

3



m ($\angle$AME) = ...°

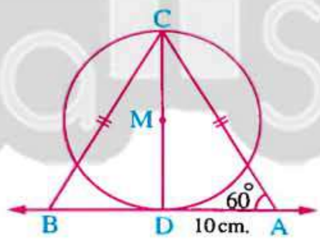
4



DB = ... cm.

(Gharbia 12)

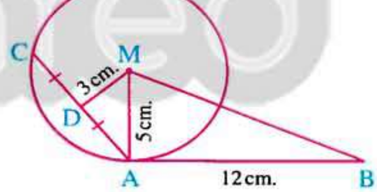
5



The perimeter of Δ ABC = ... cm.

(Alexandria 11)

6



The perimeter of the figure ABMD = ... cm.

7 **In the opposite figure :**

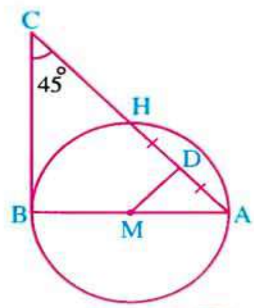
$\overrightarrow{BC}$ is a tangent at B

, m ($\angle$ C) = 45°

, D is the midpoint of $\overline{AH}$

Prove that : DA = DM

(Aswan 11)



8 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

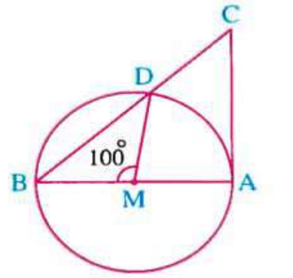
$\overrightarrow{AC}$ is a tangent to the circle at A,

m (∠ DMB) = 100°

Find by proof :

1 m (∠ ACB)

2 m (∠ CDM)



(El-Menia 11) « 50° , 140° »

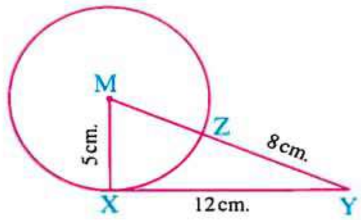
9 **In the opposite figure :**

M is a circle with radius length 5 cm. ,

XY = 12 cm. , $\overline{MY} \cap$ circle M = $\{Z\}$

and ZY = 8 cm.

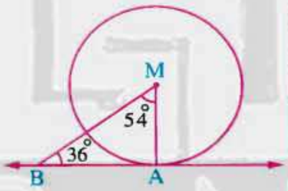
Prove that : $\overleftrightarrow{XY}$ is a tangent to the circle M at X



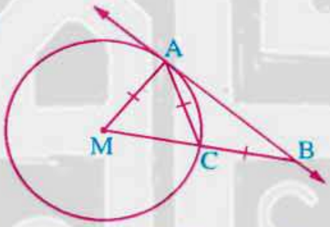
(Matrouh 17 , South Sinai 16 , Qena 15 , El-Beheira 14)

10 **In each of the following figures , explain why $\overleftrightarrow{AB}$ is a tangent to circle M :**

1

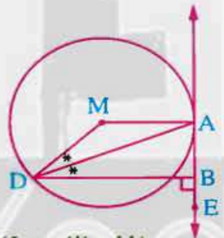


2



(Ismailia 17 , El-Gharbia 16)

3



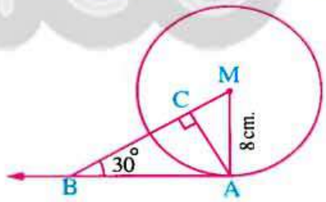
(Ismailia 11)

11 **In the opposite figure :**

$\overrightarrow{AB}$ is a tangent to the circle M at A ,

MA = 8 cm. , m (∠ ABM) = 30° and $\overline{AC} \perp \overline{MB}$

Find : The length of each of $\overline{AB}$ and $\overline{AC}$



(Giza 19 , Matrouh 18 , New Valley 18 , El-Monofia 14) « $8\sqrt{3}$ cm. , $4\sqrt{3}$ cm. »

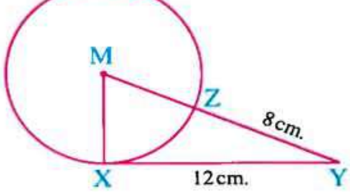
12 **In the opposite figure :**

M is a circle , $\overleftrightarrow{XY}$ is a tangent to the circle at X

, $\overline{MY} \cap$ the circle M = $\{Z\}$,

XY = 12 cm. , YZ = 8 cm.

Find : The radius length of the circle.



(El-Menia 13) « 5 cm. »

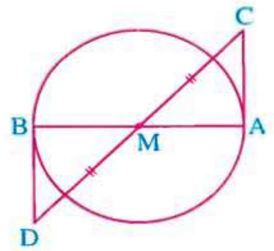
13 **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M ,

$\overrightarrow{AC}$ is a tangent to it at A , $\overrightarrow{CM}$ is drawn ,

D is a point on it such that CM = MD

Prove that : $\overrightarrow{BD}$ is a tangent to the circle M at B

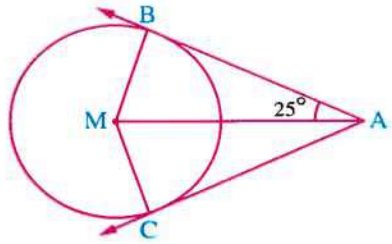


14 **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M

, touch it at B , C respectively

and m (∠ BAM) = 25°



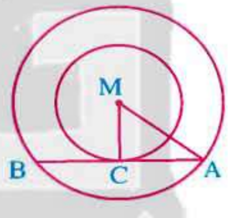
1 **Prove that :** $\overrightarrow{MA}$ bisects ∠ BMC

2 **Find :** m (∠ BMC) *(Port Said 17)* « 130° »

15 **In the opposite figure :**

$\overline{AB}$ is a chord of the great circle and touches the small circle at C , AB = 8 cm. and the radius length of the great circle = 5 cm.

Find : The radius length of the small circle. *(Souhag 09)* « 3 cm. »



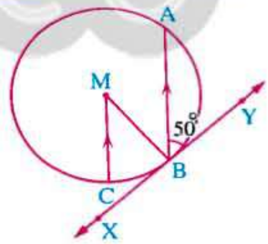
16 **In the opposite figure :**

M is a circle , the chord $\overline{BA}$ // $\overline{MC}$,

$\overleftrightarrow{YX}$ is a tangent to the circle at B

If m (∠ ABY) = 50°

Find : m (∠ CBX) « 20° »



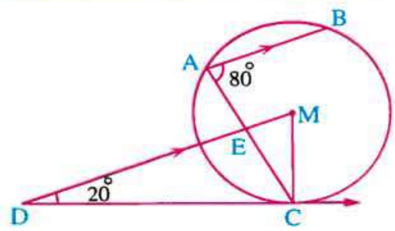
17 **In the opposite figure :**

$\overleftrightarrow{DC}$ touches the circle M at C , $\overline{AB}$ // $\overline{MD}$,

m (∠ BAC) = 80° , m (∠ MDC) = 20°

and $\overline{AC} \cap \overline{MD} = \{E\}$

Find : m (∠ ECM) *(Beni Suef 05)* « 30° »



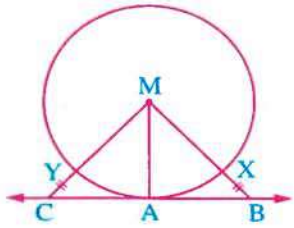
18 **In the opposite figure :**

$\overleftrightarrow{BC}$ is a tangent to the circle M at A ,

$\overline{MB} \cap$ the circle M = $\{X\}$, $\overline{MC} \cap$ the circle M = $\{Y\}$

If BX = CY

Prove that : m ($\angle$ BMA) = m ($\angle$ CMA)

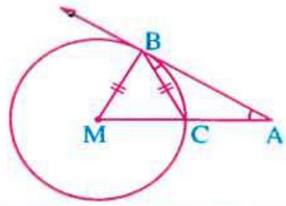


19 **In the opposite figure :**

$\overline{BC}$ is a chord of the circle M , A $\in \overleftrightarrow{MC}$,

m ($\angle$ A) = m ($\angle$ CBA) , BC = BM

Prove that : $\overrightarrow{AB}$ is a tangent to the circle M at B



20 $\overline{AB}$ is a diameter in a circle of area 36 π cm^2 , $\overleftrightarrow{BC}$ is drawn a tangent to the circle at B , if m ($\angle$ ACB) = 60° , then calculate the area of Δ ABC *(El-Dakahlia 14)* « $24\sqrt{3}$ cm^2 »

21 $\overline{AB}$ is a diameter in circle M , $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are two tangents of the circle M , $\overrightarrow{CM}$ intersects the circle M at X and Y and intersects $\overleftrightarrow{BD}$ at E

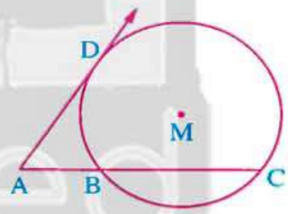
Prove that : CX = YE

22 **In the opposite figure :**

M circle with radius length of 5 cm. , A is a point outside the circle , $\overrightarrow{AD}$ is a tangent to circle M at D , $\overrightarrow{AB}$ intersects the circle at B and C respectively where AB = 4 cm. and AC = 12 cm.

1 Find the distance of the chord $\overline{BC}$ from the centre of the circle.

2 Calculate the length of $\overline{AD}$ « 3 cm. , $4\sqrt{3}$ cm. »



Connecting with analytical geometry

23 **Determine the positions of the following points with respect to the circle M whose radius length is 5 length units and its centre is the origin point.**

1 A (– 3 , 4) 2 B (2 , 3) 3 C (6 , 8)

24 **Prove that :** The points A (3 , – 1) , B (– 4 , 6) and C (2 , – 2) are located in circle whose centre is the point M (– 1 , 2) , then find the circumference of the circle.

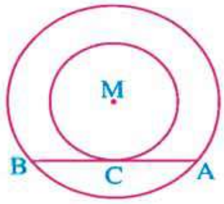
(El-Beheira 11) «10 π length units »

25 If $\overline{CD}$ is a diameter of circle M where M (1 , 1) , D (3 , –2)

Find : The equation of the tangent to M at C *(El-Dakahlia 11)* « $y = \frac{2}{3} x + 4\frac{2}{3}$ »

For excellent pupils

1 **In the opposite figure :**



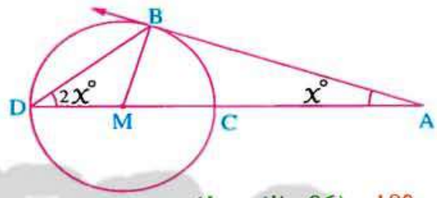
Two circles are concentric at M

, $\overline{AB}$ is a chord in the greater circle and touches

the smaller circle at C , if AB = 14 cm.

Find : The area of the part included between the two circles. *(El-Dakahlia 19)* « 49π cm^2 »

2 **In the opposite figure :**

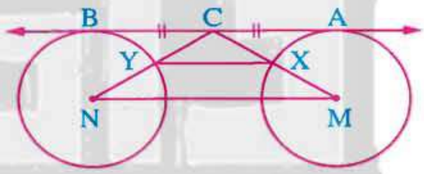


$\overrightarrow{AB}$ touches the circle M at B , $\overline{CD}$ is a diameter of it ,

m ($\angle$ BAM) = x° and m ($\angle$ MDB) = 2 x°

Find : The value of x in degrees. *(Ismailia 06)* « 18° »

3 **In the opposite figure :**



M and N are two congruent circles ,

$\overleftrightarrow{AB}$ is a common tangent to them ,

C is the midpoint of $\overline{AB}$,

the circle M $\cap$ $\overline{MC}$ = $\{X\}$, the circle N $\cap$ $\overline{NC}$ = $\{Y\}$

Prove that : 1 $\overline{AB}$ // $\overline{MN}$

2 Δ CMN is an isosceles triangle.

3 $\overline{XY}$ // $\overline{MN}$ *(El-Kalyoubia 04)*

Exercise 3

Position of a circle with respect to another circle

From the school book

1 **Complete the following :**

1. If the surface of the circle M $\cap$ the surface of the circle N = $\varnothing$, then the two circles M and N are .... *(Souhag 11)*
2. If the surface of the circle M $\cap$ the surface of the circle N = $\{A\}$, then the two circles M and N are .... *(New Valley 12)*
3. If the surface of the circle M $\cap$ the surface of the circle N = the surface of the circle N , then the two circles M and N are .... or ....
4. If the circle M $\cap$ the circle N = $\varnothing$, then the two circles M and N are .... or ....
5. If the circle M $\cap$ the circle N = $\{A\}$, then the two circles are .... or ....
6. The centres line of two intersecting circles is perpendicular to .... and .... *(Beheira 12)*
7. The centres line of two touching circles is perpendicular to .... *(El-Sharkia 06)*
8. The axis of symmetry of the two circles M and N that are intersecting at A and B is ....
9. If M and N are the two centres of two circles of radii lengths r_1 and r_2 , where MN > $r_1 + r_2$, then the two circles are ....
10. M and N are two circles of radii lengths r_1 and r_2 If $r_1 - r_2 < MN < r_1 + r_2$, then the two circles are ....
11. If the sum of lengths of the two radii of two circles equals the distance between the two centres , then the two circles are ....

2 **Choose the correct answer from those given :**

1 M and N are two circles touching internally , their radii lengths are 3 cm. and 5 cm. , then MN = ... cm. *(Beni Suef 17 , El-Gharbia 15)*

(a) 8 (b) 6 (c) 4 (d) 2

2 M and N are two circles touching externally , if their radii lengths are 4 cm. and 2 cm. , then MN = ... cm. *(Cairo 15)*

(a) zero (b) 2 (c) 6 (d) 7

3 M and N are two circles of radii lengths are 9 cm. and 4 cm. respectively , MN = 5 cm. , then the two circles are .... *(El-Dakahlia 17 , El-Gharbia 14)*

(a) touching externally. (b) touching internally.
(c) intersecting. (d) distant.

4 M and N are two circles , their radii lengths are 8 cm. and 3 cm. , if MN = 11 cm. , then the two circles M and N are .... *(El-Menia 13)*

(a) distant. (b) concentric.
(c) intersecting. (d) touching externally.

5 M and N are two circles , their radii lengths are 4 cm. and 3 cm. If MN = 9 cm. , then the two circles are .... *(Port Said 09)*

(a) distant. (b) intersecting.
(c) touching. (d) one is inside the other.

6 If the radii lengths of the two circles M and N are 6 cm. , 3 cm. , if MN = 2 cm. , then the two circles M , N are .... *(El-Dakahlia 18)*

(a) intersecting. (b) one is inside the other.
(c) touching externally. (d) distant.

7 If the radius length of the circle M = 3 cm. and the radius length of the circle N = 5 cm. , MN = 6 cm. , then the two circles M and N are .... *(El-Gharbia 08)*

(a) distant. (b) one is inside the other.
(c) intersecting. (d) touching externally.

8 M and N are two intersecting circles their radii lengths are 3 cm. and 5 cm. respectively , then MN ∈ .... *(Alexandria 16 , Cairo 16 , Suez 11)*

(a) $]0 , 2[$ (b) $]2 , 8[$ (c) $]8 , \infty[$ (d) $]2 , \infty[$

9 Two circles M and N with radii lengths 8 cm. and 5 cm. respectively , are touching when MN ∈ .... *(El-Dakahlia 16)*

(a) $]13 , 3[$ (b) $]3 , 13[$ (c) $\mathbb{R} - [3 , 13]$ (d) $\{13 , 3\}$

10 M and N are two intersecting circles at A and B , then the axis of symmetry of $\overline{AB}$ is .... *(El-Monofia 04)*

(a) $\overline{MN}$ (b) $\overrightarrow{NM}$ (c) $\overleftrightarrow{MN}$ (d) $\overrightarrow{MN}$

11 If the radius length of the circle M = the radius length of the circle N = MN , then the two circles are .... *(Alexandria 05)*

(a) one is inside the other. (b) touching externally.
(c) distant. (d) intersecting.

12 If the two circles M and N are touching internally , the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle = ... cm. *(Giza 17)*

(a) 12 (b) 11 (c) 6 (d) 5

13 M and N are two touching circles where MN = 6 cm. , the radius length of the greater circle is 10 cm. , then the radius length of the smaller circle = ... cm. *(El-Sharkia 05)*

(a) 16 (b) 12 (c) 8 (d) 4

14 M , N and L are three circles touching externally two-by-two , their radii lengths are 5 cm. , 6 cm. and 4 cm. , then the perimeter of the triangle MNL = ... cm. *(El-Monofia 11)*

(a) 15 (b) 30 (c) 4 (d) 60

15 If the two circles M and N are touching externally , the radius length of the circle M is 4 cm. , if MN = 7 cm. , then the circumference of the circle N is .... cm. *(El-Monofia 16)*

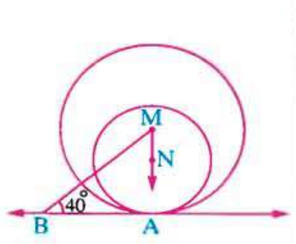
(a) 4π (b) 6π (c) 7π (d) π

16 A circle M of radius length 4 cm. touches a circle N internally , MN = 7 cm. , then the circumference of the circle M : the circumference of the circle N = ... *(El-Dakahlia 09)*

(a) 4 : 7 (b) 3 : 4 (c) 4 : 3 (d) 4 : 11

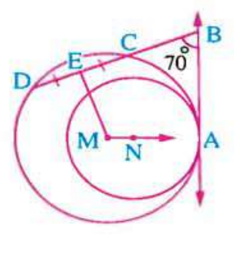
3 **In each of the following figures , the circles are touching two-by-two use information of each figure to complete :**

1



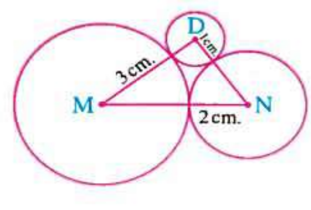
m (∠ BMN) = ...°

2



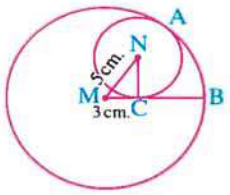
m (∠ EMN) = ...°

3



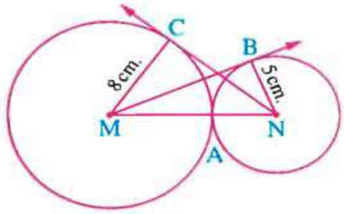
m (∠ MDN) = ...°

4



BC = ... cm.

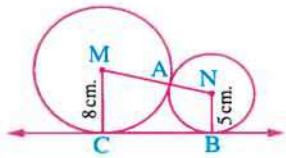
5



MB = ... cm.

NC = ... cm.

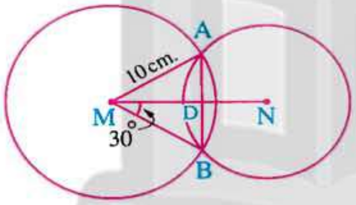
6



BC = ... cm.

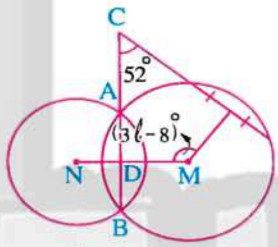
4 📖 **In each of the following figures , M and N are two intersecting circles at A and B , complete :**

1



AB = ... cm.

2



ℓ = ...

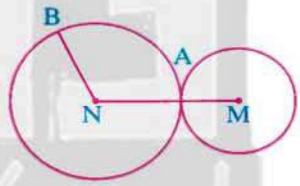
5 **In the opposite figure :**

M and N are two circles touching at A ,

the distance between their centres MN = 12 cm.

If NB = 7 cm.

Find : The length of $\overline{MA}$



(Kafr El-Sheikh 06) « 5 cm. »

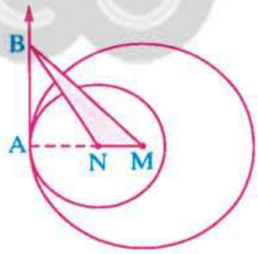
6 📖 **In the opposite figure :**

M and N are two circles with radii lengths of 10 cm. and 6 cm. respectively and they are touching internally at A ,

$\overleftrightarrow{AB}$ is a common tangent for both.

If the area of Δ BMN = 24 cm^2.

Find : The length of $\overline{AB}$



(El-Kalyoubia 18 , Luxor 16 , Port Said 14) « 12 cm. »

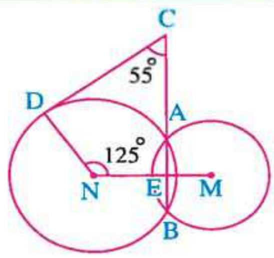
7 📖 **In the opposite figure :**

M and N are two intersecting circles at A and B ,

C $\in \overrightarrow{BA}$, D $\in$ the circle N ,

m ($\angle$ MND) = 125° and m ($\angle$ BCD) = 55°

Prove that : $\overleftrightarrow{CD}$ is a tangent to circle N at D



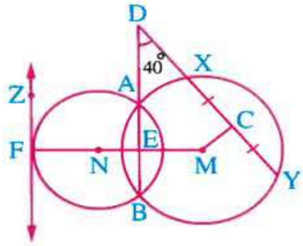
(Red Sea 19 , Kafr El-Sheikh 17 , Souhag 15)

8 **In the opposite figure :**

M and N are two intersecting circles at A and B ,

C is the midpoint of $\overline{XY}$, m (∠ D) = 40° ,

$\overleftrightarrow{FZ}$ is a tangent to the circle N at F where $\overleftrightarrow{MN} \cap \overleftrightarrow{FZ} = \{F\}$



1 **Find :** m (∠ CME) « 140° »

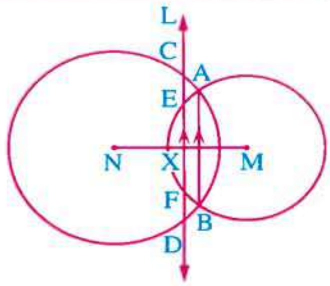
2 **Prove that :** $\overleftrightarrow{FZ}$ // $\overline{AB}$

(El-Fayoum 11)

9 **In the opposite figure :**

$\overline{AB}$ is the common chord of the intersecting circles M and N the straight line L // $\overline{AB}$ and cuts the circle M at E and F and cuts the circle N at C and D

Prove that : CE = FD

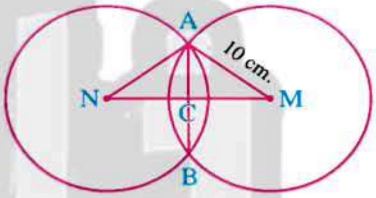


10 **In the opposite figure :**

Two congruent circles M and N are intersecting at A and B

If MA = 10 cm. , AB = 12 cm.

Find by proof : The length of $\overline{MN}$ *(El-Menia 17)* « 16 cm. »



11 M and N are two intersecting circles at A and B , MA = 12 cm , NA = 9 cm. and MN = 15 cm.

Find : The length of $\overline{AB}$ *(Port Said 11)* « 14.4 cm. »

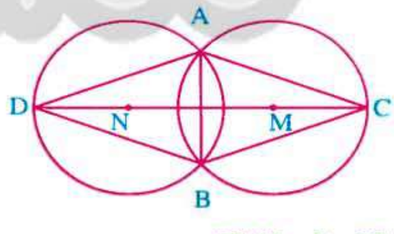
12 **In the opposite figure :**

M and N are two intersecting circles at A and B

where C is a point on the circle M ,

D is a point on the circle N , C ∈ $\overleftrightarrow{MN}$, D ∈ $\overleftrightarrow{MN}$

Prove that : m (∠ CAD) = m (∠ CBD)



(El-Sharkia 15)

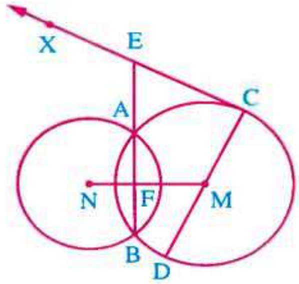
13 **In the opposite figure :**

M and N are two intersecting circles at A and B ,

$\overline{CD}$ is a diameter in circle M , $\overrightarrow{CX}$ is a tangent to the circle M at C where $\overrightarrow{CX} \cap \overrightarrow{BA} = \{E\}$

and $\overrightarrow{MN} \cap \overline{AB} = \{F\}$

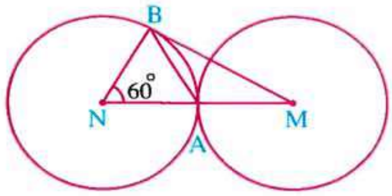
Prove that : m (∠ DMN) = m (∠ CEB)



14 **In the opposite figure :**

M and N are two congruent circles touching externally at A , in the circle N draw the radius $\overline{NB}$ such that m (∠ ANB) = 60°

Prove that : $\overline{MB}$ touches the circle N at B

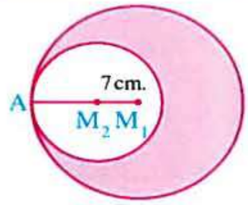


15 **In the opposite figure :**

Two circles are touching internally at A , the area of the shaded part = 550 cm^2 ,

$M_1M_2 = 7$ cm.

Find : The sum of the two lengths of their radii. ($\pi \approx \frac{22}{7}$)



« 25 cm. »

16 If AB = 3 cm. , and a circle is drawn such that its centre is the point A and passes through the point B , and another circle is drawn such that B is its centre and passes through the point A. If the two circles intersect at C and D

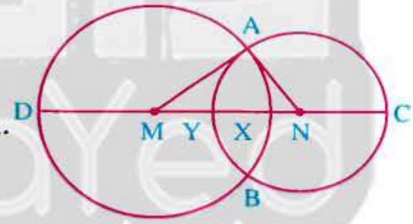
Find : 1 m (∠ ACB)

2 The length of the common chord $\overline{CD}$

« 60° , $3\sqrt{3}$ cm. »

17 **In the opposite figure :**

M and N are two intersecting circles at A and B their radii lengths are 8 cm. and 6 cm. respectively and XY = 4 cm.



Study the figure , then answer the following questions :

1 Complete : YM = ... cm. , CX = ... cm. and CD = ... cm.

2 Is the perimeter of Δ ANM = the length of $\overline{CD}$? Explain your answer.

3 What is the measure of ∠ NAM ?

4 Find the area of Δ NAM

5 What is the length of the common chord $\overline{AB}$?

Connecting with analytical geometry

18 In a cartesian coordinates plane , the two circles M and N are drawn with radii lengths 6 and 4 length units respectively. Show the position of each of them with respect to the other in each of the following cases :

1 M (– 4 , 8) , N (5 , – 4)　　2 M (2 , 1) , N (6 , – 2)

19 In a cartesian coordinates plane, if the two circles M and N are intersecting at A and B , where A (0 , 3) and B (– 4 , – 1)

Find : The equation of $\overleftrightarrow{MN}$ « $y = -x - 1$ »

20 If M (3 , 5) and N (– 3 , – 7) are the two centres of two circles whose radii lengths are $4\sqrt{5}$ length units and $2\sqrt{5}$ length units respectively , A (– 1 , – 3)

Prove that : The two circles are touching at A showing the kind of tangency. *(Helwan 09)*

For excellent pupils

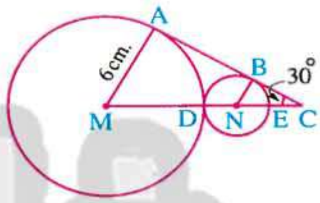
1 **In the opposite figure :**

M and N are two circles touching externally at D

If the common tangent to them is drawn to meet the centres line at C and m (∠ C) = 30°

If the radius length of the circle M = 6 cm.

Find : The radius length of the circle N « 2 cm. »

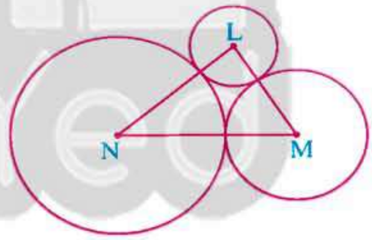


2 **In the opposite figure :**

Three circles with centres M , N and L , each touches the other two externally.

If LM = 5 cm. , MN = 8 cm. and LN = 7 cm.

Calculate : The radius length of each of them.



« 2 cm. , 3 cm. , 5 cm. »

Exercise **4**

Identifying the circle

From the school book

1 **Choose the correct answer from those given :**

1 It is possible to draw .... passing through a given point. *(New Valley 05)*

(a) one circle (b) two circles

(c) three circles (d) an infinite number of circles

2 The number of circles which passes through two given points is .... *(Giza 12)*

(a) 1 (b) 2

(c) 3 (d) an infinite number.

3 The number of circles passing through three collinear points is ....

(Souhag 18 • Giza 16 • Ismailia 15)

(a) zero (b) one (c) three (d) an infinite number.

4 The number of circles passing through three non-collinear points is .... *(El-Menia 17)*

(a) 1 (b) zero (c) 2 (d) 3

5 We can identify the circle if we are given .... *(El-Sharkia 08)*

(a) three collinear points. (b) two points.

(c) three non-collinear points. (d) one point.

6 The centres of the circles passing through the two points A and B lie on ....

(El-Dakahlia 17)

(a) the axis of symmetry of $\overline{AB}$ (b) $\overline{AB}$

(c) the perpendicular to $\overline{AB}$ (d) the midpoint of $\overline{AB}$

Unit 4

7 The centre of the circumcircle of a triangle is the point of intersection of ....

(El-Fayoum 19 , Kafr El-Sheikh 17 , Qena 17)

(a) the bisectors of its interior angles. (b) the bisectors of its exterior angles.

(c) its altitudes. (d) the symmetry axes of its sides.

8 If Δ ABC is right-angled at B , then the centre of its circumcircle is .... *(Ismailia 03)*

(a) the midpoint of $\overline{AB}$ (b) the midpoint of $\overline{AC}$

(c) the midpoint of $\overline{BC}$ (d) outside the triangle.

9 It is (impossible) to draw a circle passing through the vertices of ....

(Beni Suef 17 , El-Dakahlia 13 , El-Sharkia 12)

(a) a rectangle. (b) a triangle. (c) a square. (d) a rhombus.

10 It is possible to draw a circle passing through the vertices of ....

(El-Sharkia 19 , Souhag 18 , Giza 17 , Beni Suef 16)

(a) a rhombus. (b) a rectangle. (c) a trapezium. (d) a parallelogram.

11 If $\overline{AB}$ is a line segment of length 4 cm. , then the radius length of the smallest circle which passes through the two points A and B = ... cm. *(El-Monofia 16)*

(a) 2 (b) 3 (c) 4 (d) 5

12 If AB = 6 cm. , then the area of the smallest circle which passes through the two points A and B = ... cm^2. *(El-Sharkia 15)*

(a) 3 π (b) 6 π (c) 8 π (d) 9 π

2 **Complete the following :**

1 The circle is identified if its centre is given and the length of .... *(Ismailia 04)*

2 The circle which passes through the vertices of a triangle is called .... *(North Sinai 09)*

3 The number of circles that can pass through any three vertices of a parallelogram is ....

4 If AB = 6 cm. , then the number of circles of radius length for each is 5 cm. and pass through the two points A and B is ....

5 A and B are two points in a plane where AB = 5.4 cm. , then the number of circles which the radius length of each = 2.7 cm. , and pass through the two points A and B are ....

6 The greatest length of line segment whose two terminals lie on a circle of radius length is 7 cm. equals ....

3 L is a straight line in the plane , point A is at a distance of 2 cm. from L
Show how to draw a circle of radius length 3 cm. such that it passes through A and its centre lies on the straight line L

How many solutions can be carried out ?

Exercise 4

4 **If $A \in L$, draw the circle M passing through A and its radius length = 3 cm. if :**

1 $M \in$ the straight line L , how many circles can be drawn ?

2 $M \notin$ the straight line L , how many circles can be drawn ? *(Assiut 11)*

5 A and B are two points where AB = 6 cm. Draw a circle of radius length 5 cm. and passes through the two points A and B

Find :

1 The number of circles can be drawn.

2 The distance of the centre of the circle from $\overline{AB}$ by proof. *(Damietta 17)* « 4 cm. »

6 **Using your geometric tools , draw $\overline{AB}$ of length 4 cm. , then draw on one figure :**

1 A circle passing through the two points A and B and its diameter length is 5 cm. What are the possible solutions ?

2 A circle passing through the two points A and B and its radius length is 2 cm. What are the possible solutions ?

3 A circle passing through the two points A and B and its diameter length is 3 cm. What are the possible solutions ?

7 $\overline{AB}$ is a line segment of length 6 cm. Draw the circle that passes through the two points A and B and its radius length is the smallest length. *(Luxor 05)*

8 Using the geometric tools and draw $\overline{AB}$ with length 6 cm. , then draw $\overrightarrow{AC}$ where m ($\angle$ CAB) = 60° , draw the circle that passes through the points A , B and its centre lies on $\overrightarrow{AC}$ and calculate the length of its radius (Don't remove the arcs). *(El-Dakahlia 17)* « 6 cm. »

9 Draw a circle with radius length of 3 cm. and touches to the straight line L What is the number of possible solutions ? *(Giza 06)*

10 Draw the two parallel straight lines L_1 and L_2 given that the distance between them is 5 cm. , then draw a circle such that its centre lies on L_1 and touches L_2

11 Using the geometric tools , draw the triangle ABC in which AB = 4 cm. , BC = 5 cm. and CA = 6 cm. Draw a circle passing through the points A , B and C. What is the kind of the triangle ABC with respect to the measures of its angles ? Where is the centre of the circle located with respect to the triangle ?

12 Draw the right-angled triangle ABC at B where AB = 4 cm. and BC = 3 cm., then draw the circumcircle of this triangle. Where does the centre of the circle lie with respect to the sides of this triangle ? *(Damietta 18)*

13 **Draw the equilateral triangle ABC of side length of 4 cm. Draw the circumcircle of this triangle ABC**

1 Locate the position of the centre of the circle with respect to :
heights of the triangle – medians of the triangle – bisectors of the angles.

2 How many axes of symmetry are there in the equilateral triangle ?

14 Using geometrical instruments, draw the isosceles triangle ABC in which $m(\angle ABC) = 120°$, $BC = 4$ cm. Determine the centre of the circumcircle of it and find its radius length. *(El-Dakahlia 11)* « 4 cm. »

15 Draw Δ ABC in which : AB = 6 cm. , AC = 4 cm. , $m(\angle BAC) = 60°$, then draw a circle passes through the two points A and C where its centre lies on $\overline{AB}$

16 Draw Δ ABC in which : AB = 5 cm. , BC = 4 cm. , and CA = 3 cm. What is the type of the triangle with respect to the measures of its angles ? then draw a circle whose centre is the point A and touches $\overleftrightarrow{BC}$, another circle whose centre is B and touches $\overleftrightarrow{AC}$ and a third circle whose centre is C and touches $\overleftrightarrow{AB}$ *(Beni Suef 06)*

17 Draw the triangle ABC in which : AB = 6 cm. , $m(\angle A) = 40°$ and the radius length of the circumcircle of the triangle ABC equals 5 cm. If D is the midpoint of $\overline{AB}$, then calculate the length of $\overline{MD}$ where M is the centre of the circumcircle of the triangle ABC « 4 cm. »

Connecting with analytical geometry

18 If A (2 , 0) and B (– 2 , 3) , draw a circle M of radius length 4 length units and passes through the two points A and B
How many solutions are there for this problem ? *(North Sinai 09)*

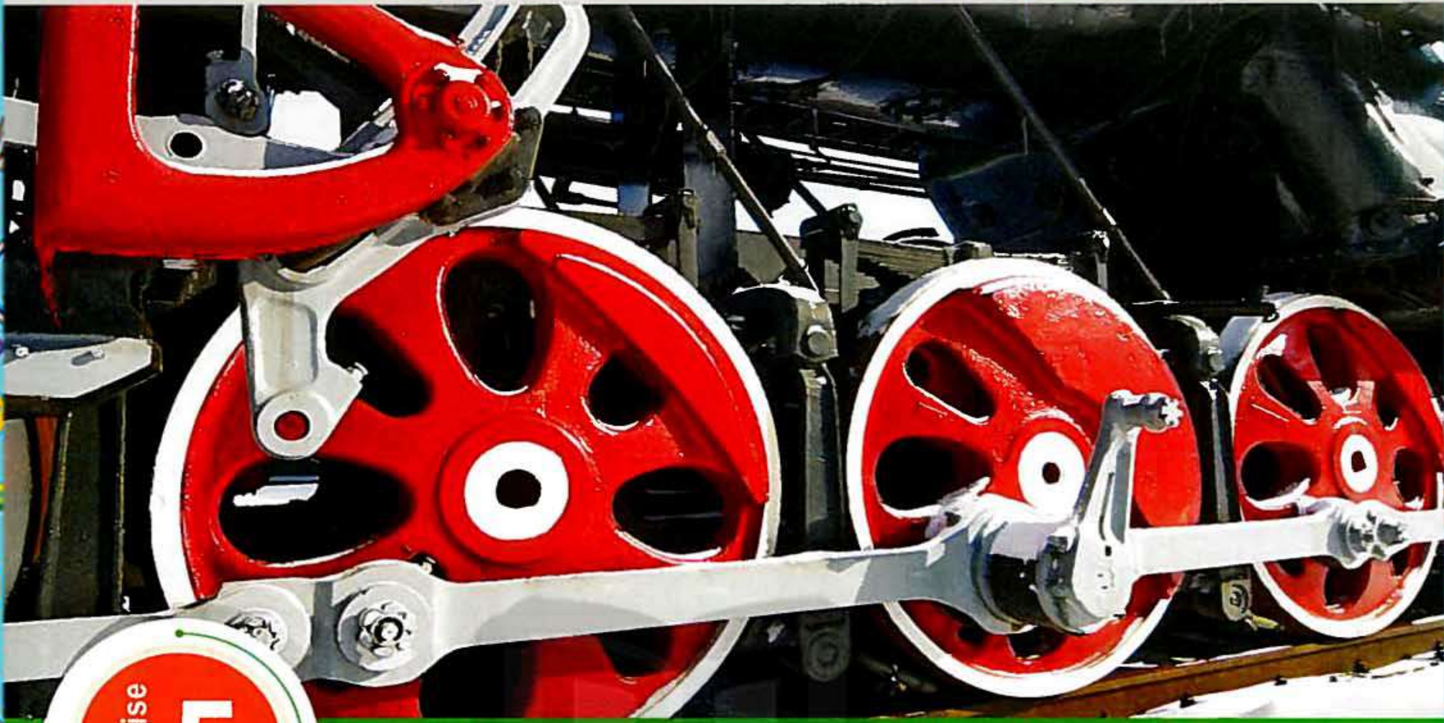
19 If A (1 , 3) , B (1 , – 1) and C (– 3 , – 1) , find the coordinates of M the centre of the circumcircle of Δ ABC « (– 1 , 1) »

For excellent pupils

Draw Δ ABC which is right-angled at B , where AB = 4 cm. and $m(\angle A) = 30°$, then draw the circle M such that $\overline{AB}$ is a chord of it , $\overline{AC}$ is a tangent to it at A

1 **Prove that :** Δ ABM is equilateral and calculate its area. « $4\sqrt{3}$ cm.^{2} »

2 **Calculate :** The area of the circle M « 16π cm.^{2} »



Exercise 5 The relation between the chords of a circle and its centre

From the school book

1 **Complete the following :**

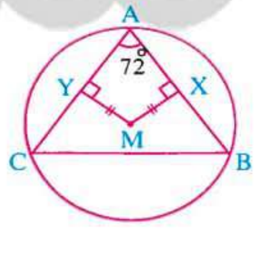
1. If the chords of a circle are equal in length , then they are .... from the .... (Cairo 16)

2. In the same circle if the chords are equidistant from the centre then they are .... (El-Gharbia 12)

3. The square which is inscribed in a circle , its sides are .... from the centre of the circle. (North Sinai 09)

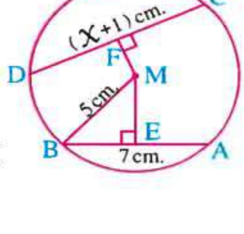
4. $\overline{AB}$ and $\overline{CD}$ are two chords in a circle AB = 5 cm. and CD = 3 cm. , then the chord which is nearer to the centre of the circle is ....

5. **In the opposite figure :**



Δ ABC is inscribed in the circle M ,
$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$, MX = MY
and m ($\angle$ A) = 72° , then m ($\angle$ B) = ...

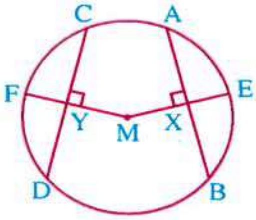
6. **In the opposite figure :**



$\overline{CD}$ is a chord which does't pass through the centre of the circle M
, $\overline{MF} \perp \overline{CD}$, $\overline{ME} \perp \overline{AB}$, MF < ME
$\because$ MF < ME $\quad \therefore$ CD > ....
$\therefore x + 1 >$ .... $\quad \therefore x >$ ....
$\because$ $\overline{CD}$ is a chord which does't pass through the centre of the circle M
$\therefore$ CD < .... $\quad \therefore x <$ .... , $\quad \therefore$ .... $< x <$ ....
i.e. $x \in$ ....

2 **Study the figure , then complete :**

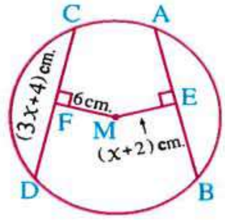
1



If AB = CD , then MX = ...

$\because$ ME = ... $\therefore$ EX = ...

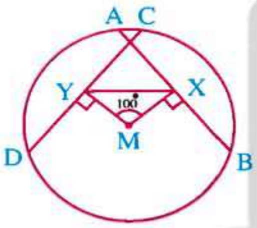
2



If AB = CD , then ME = ...

$\therefore$ X = ... cm. $\therefore$ CD = ... cm.

3



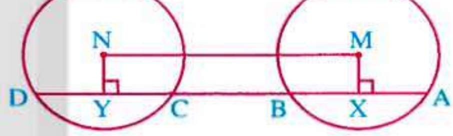
If AB = CD , then MX = ...

In Δ MXY

$\because$ m ($\angle$ XMY) = 100°

$\therefore$ m ($\angle$ MXY) = ... °

4



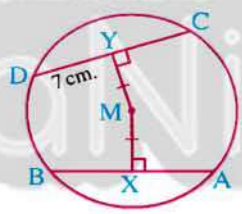
If M and N are two congruent circles ,

AB = CD , then MX = ...

and the figure MXYN is ....

3 **Study the figure and complete :**

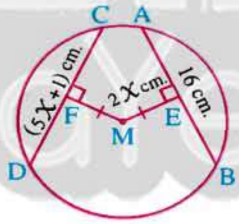
1



If MX = MY , YD = 7 cm.

, then AB = ... cm.

2

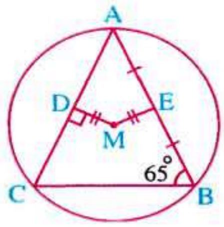


If ME = MF , then CD = ...

$\therefore$ X = ... cm. , EM = ... cm.

AM = ... cm.

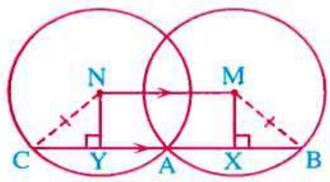
3



If MD = ME , m ($\angle$ B) = 65°

, then m ($\angle$ A) = ...°

4



$\because$ $\overleftrightarrow{MN}$ // $\overleftrightarrow{BC}$ $\therefore$ MX = ...

$\because$ The two circles M and N

are .... , A $\in$ $\overline{BC}$

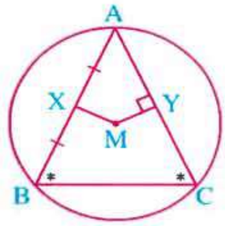
$\therefore$ AB = ...

4 **In the opposite figure :**

The triangle ABC is an inscribed triangle inside a circle M ,

m (∠ B) = m (∠ C) ,

X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$



Prove that : MX = MY *(Giza 19 , El-Beheira 19 , Matrouh 17 , Fayoum 15)*

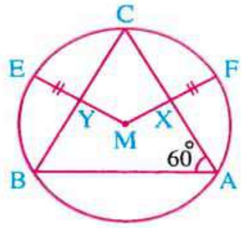
5 **In the opposite figure :**

M is a circle , m (∠ A) = 60°

, X is the midpoint of $\overline{AC}$

, Y is the midpoint of $\overline{BC}$

, FX = EY



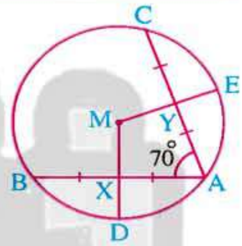
Prove that : Δ ABC is an equilateral triangle *(El-Sharkia 18)*

6 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M

, X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{AC}$ and m (∠ CAB) = 70°



1 **Calculate :** m (∠ DME) « 110°»

2 **Prove that :** XD = YE *(New Valley 19 , Port said 18 , Matrouh 18 , Cairo 17)*

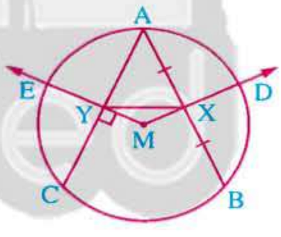
7 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M

, X is the midpoint of $\overline{AB}$,

$\overrightarrow{MX}$ intersects the circle at D , $\overrightarrow{MY} \perp \overline{AC}$

intersects it at Y and intersects the circle at E



Prove that : **1** XD = YE **2** m (∠YXB) = m (∠ XYC) *(Assiut 18 , El-Gharbia 13)*

8 $\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , m (∠ MXY) = 30°

Prove that : **1** Δ MXY is an isosceles triangle.

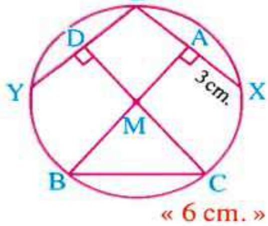
2 Δ AXY is an equilateral triangle. *(New Valley 16)*

9 **In the opposite figure :**

M is a circle , $\overline{BA} \perp \overline{XE}$, $\overline{CD} \perp \overline{YE}$,

$\overline{AB} \cap \overline{CD} = \{M\}$, AB = CD and AX = 3 cm.

Find : The length of $\overline{EY}$

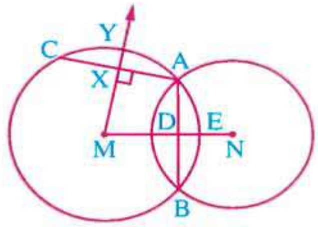


« 6 cm. »

10 **In the opposite figure :**

M and N are two circles intersecting at A and B , $\overleftrightarrow{MX} \perp \overline{AC}$ and intersects $\overline{AC}$ at X and intersects the circle M at Y , $\overline{MN}$ to intersects $\overline{AB}$ at D and intersects the circle M at E, if AC = AB

Prove that : XY = DE



(El-Kalyoubia 18)

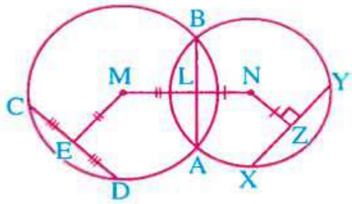
11 **In the opposite figure :**

The circle M $\cap$ the circle N = $\{A , B\}$

, E is the midpoint of $\overline{CD}$

, ME = ML , NL = NZ , $\overline{NZ} \perp \overline{YX}$

Prove that : CD = XY



12 $\overline{AB}$ and $\overline{AC}$ are two chords in the circle M , $\overline{MX} \perp \overline{AB}$, Y is the midpoint of $\overline{AC}$, m (∠ ABC) = 75° , MX = MY

1 **Find :** m (∠ BAC) « 30° »

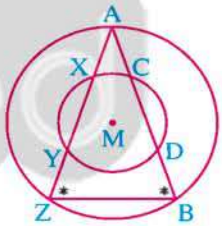
2 **Prove that :** The perimeter of Δ AXY = $\frac{1}{2}$ the perimeter of Δ ABC

(Kafr-El-Sheikh 18 , Alexandria 16)

13 **In the opposite figure :**

Two concentric circles at M , $\overline{AB}$ is a chord in the greater circle and cuts the smaller circle at C and D , $\overline{AZ}$ is a chord in the greater circle and cuts the smaller circle at X and Y If m (∠ ABZ) = m (∠ AZB)

Prove that : CD = XY



(El-Kalyoubia 17 , Souhag 13)

14 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords of the circle M ,

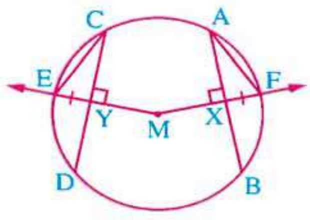
$\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle at F ,

$\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E ,

FX = EY

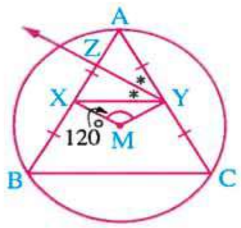
Prove that :

1 AB = CD 2 AF = CE



(El-Gharbia 16 , Kafr El-Sheikh 11)

15 **In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords of the circle M , equal in length

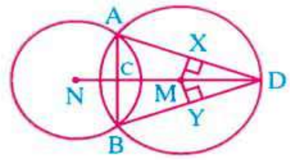
X and Y are their midpoints respectively.

If m ($\angle$ XMY) = 120° , $\overrightarrow{YZ}$ bisects $\angle$ AYX

Prove that : $\overrightarrow{YZ} // \overleftrightarrow{MX}$

(Cairo 08)

16 **In the opposite figure :**



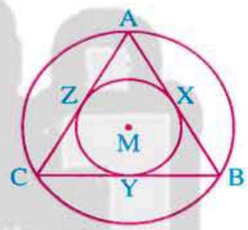
The circle M $\cap$ the circle N = $\{A , B\}$, $\overleftrightarrow{AB} \cap \overleftrightarrow{MN} = \{C\}$,

D $\in \overleftrightarrow{MN}$, $\overline{MX} \perp \overline{AD}$ and $\overline{MY} \perp \overline{BD}$

Prove that : MX = MY

(El-Kalyoubia 19 , El-Sharkia 11)

17 **In the opposite figure :**



The concentric circles of radii 4 cm. , 2 cm.

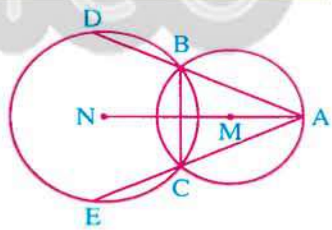
Δ ABC is drawn such that its vertices lie on the greater circle and its sides touch the smaller circle at X , Y , Z

Prove that :

Δ ABC is an equilateral triangle and find its area.

(El-Fayoum 19) « $12\sqrt{3}$ cm^2. »

18 **In the opposite figure :**

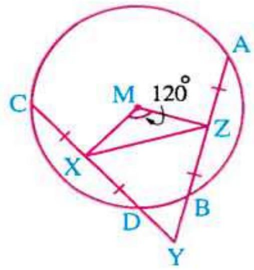


M , N are two intersecting circles at B , C , A $\in \overleftrightarrow{MN}$

Prove that : BD = CE

(El-Dakahlia 17)

19 **In the opposite figure :**



$\overline{AB}$ and $\overline{CD}$ are two chords of the circle M equal in length , $\overrightarrow{AB} \cap \overrightarrow{CD} = \{Y\}$,

Z is the midpoint of $\overline{AB}$, X is the midpoint of $\overline{CD}$ and m ($\angle$ ZMX) = 120°

Prove that : Δ ZYX is an equilateral triangle.

Unit 4

20 **In the opposite figure :**

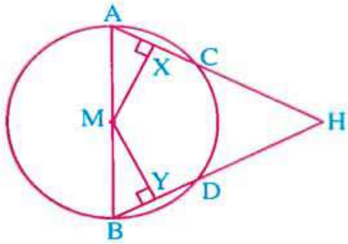
$\overline{AB}$ is a diameter of the circle M , $\overline{AC}$ and $\overline{BD}$ are two chords in it ,

$MX = MY$, $\overline{MX} \perp \overline{AC}$, $\overline{MY} \perp \overline{DB}$

Prove that :

1 Δ HAB is isosceles triangle.

2 $HC = HD$



(Beni Suef 12)

21 **In the opposite figure :**

Δ ABC is inscribed in the circle M ,

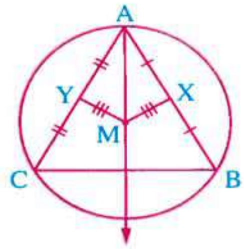
m ($\angle$ BAC) = 60° , X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{AC}$ and MX = MY

Prove that :

1 ABC is an equilateral triangle.

2 $\overrightarrow{AM} \perp \overline{BC}$

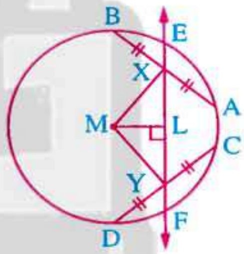


(Giza 05)

22 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords of the circle M , equal in length , X and Y are the two midpoints of $\overline{AB}$ and $\overline{CD}$ respectively. $\overleftrightarrow{XY}$ is drawn to cut the circle at E and F , $\overline{ML}$ is drawn $\perp \overleftrightarrow{XY}$

Prove that : XE = YF



(Cairo 03)

23 **In the opposite figure :**

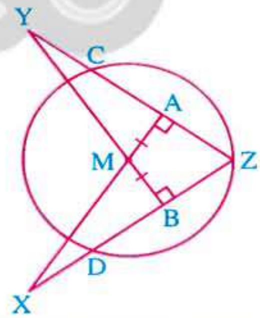
$\overline{ZC}$ and $\overline{ZD}$ are two chords of the circle M ,

$A \in \overline{ZC}$ such that : $\overrightarrow{AM} \perp \overline{ZC}$, $\overrightarrow{AM} \cap \overrightarrow{ZD} = \{X\}$,

$B \in \overline{ZD}$ such that : $\overrightarrow{BM} \perp \overline{ZD}$, $\overrightarrow{BM} \cap \overrightarrow{ZC} = \{Y\}$

If MA = MB

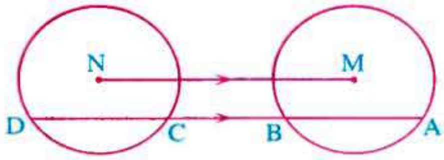
Prove that : CY = DX



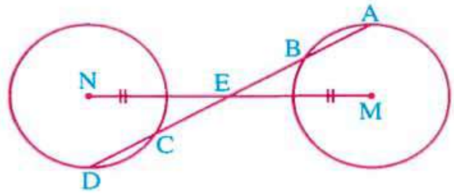
24 **In the opposite figure :**

M and N are two congruent circles , $\overleftrightarrow{AB} \parallel \overleftrightarrow{MN}$ and intersects circle M at A and B and intersects the circle N at C and D

Prove that : AC = BD



25 📖 **In the opposite figure :**

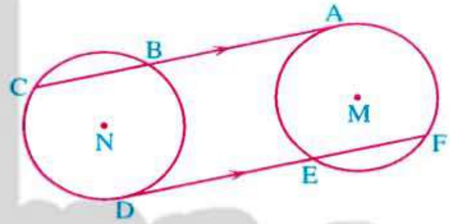


M , N are two congruent and distant circles , E is the midpoint of $\overline{MN}$, $\overleftrightarrow{AE}$ is drawn to cut the circle M at A , B and to cut the circle N at C , D

Prove that :

1 AB = CD 2 E is the midpoint of $\overline{AD}$

26 **In the opposite figure :**



M and N are two congruent circles ,

$\overline{AC}$ touches the circle M at A ,

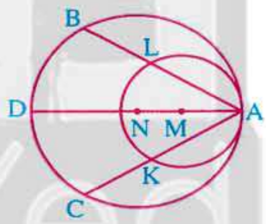
$\overline{DF}$ touches the circle N at D ,

$\overline{AC}$ // $\overline{DF}$

Prove that :

1 BC = EF 2 AB = ED

27 **In the opposite figure :**

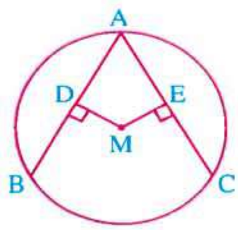


M and N are two circles touching internally at A , $\overline{AB}$ and $\overline{AC}$ are two chords drawn in the greater circle N such that they are equal in length to cut the smaller circle M at L and K respectively.

Prove that : AL = AK *(Dakahlia 09)*

Connecting with analytical geometry

28 **In the opposite figure :**



M is a circle , $\overline{MD} \perp \overline{AB}$

, $\overline{ME} \perp \overline{AC}$

, A (2 , 2) , D (1 , 0) and E (3 , 4)

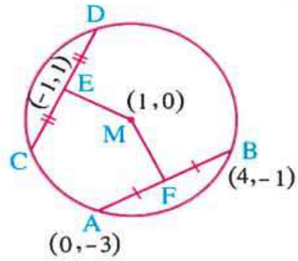
Prove that : ME = MD *(Kafr El-Sheikh 13)*

29 If $\overline{AB}$ and $\overline{AC}$ are two chords equal in length in a circle M , if M (2 , 1) , A (4 , 3) and B (0 , 3) , find the distance between the chord $\overline{AC}$ and the centre of the circle M « 2 length units »

Unit 4

30 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M , F and E are the midpoints of $\overline{AB}$ and $\overline{CD}$ respectively. If A (0 , – 3) , B (4 , – 1) , E (– 1 , 1) and M (1 , 0)

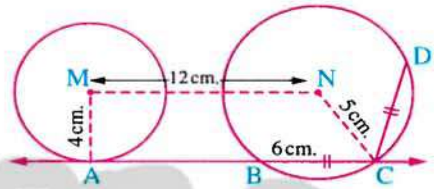


Prove that : AB = CD

For excellent pupils

In the opposite figure :

M and N are two circles of radii lengths 4 cm. and 5 cm. , $\overline{AC}$ touches the circle M at A and cuts the circle N at B and C , where BC = 6 cm. and MN = 12 cm.



1 Prove that the quadrilateral MACN is a trapezium then calculate its area. « 54 cm² »

2 If CD = CB , find the distance between N and $\overline{CD}$ *(Sharkia 06)* « 4 cm. »

Summary of Unit 4

- The circle is the set of points of the plane which are at a constant distance from a fixed point in the same plane.
- The surface of the circle is the set of points on the circle $\cup$ the set of points inside it.
- The radius of the circle is a line segment with one endpoint at the centre of the circle and the other endpoint on the circle.
- The chord of the circle is a line segment whose endpoints are any two points on the circle.
- The diameter of the circle is a chord passing through the centre of the circle.
- Symmetry in the circle : Any straight line passing through the centre of the circle is an axis of symmetry of it and the circle has an infinite number of axes of symmetry.
- The straight line passing through the centre of the circle and the midpoint of any chord of it is perpendicular to this chord.
- The straight line passing through the centre of the circle and perpendicular to any chord of it bisects this chord.
- The perpendicular bisector to any chord of a circle passes through the centre of the circle.

Position of a point with respect to a given circle

If M is a circle of radius length r and A is a point in its plane , then :

1. A is outside the circle M , if $MA > r$
2. A is on the circle M , if $MA = r$
3. A is inside the circle M , if $MA < r$

Position of a straight line with respect to a given circle

If M is a circle with radius length r and L is a straight line in its plane , and we draw $\overrightarrow{MA} \perp L$ to cut it at the point A , then there are three cases :

1. If $MA > r$, then the straight line L lies outside the circle M
2. If $MA = r$, then the straight line L is a tangent to the circle M at A and A is called the point of tangency.
3. If $MA < r$, then the straight line L is a secant to the circle M

- The tangent to a circle is perpendicular to the radius drawn from the point of tangency.
- The straight line which is perpendicular to the diameter of a circle at one of its endpoints is a tangent to the circle.
- The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.

Position of a circle with respect to another circle

If M and N are two circles , their radii lengths are r_1 and r_2 respectively , $r_1 > r_2$, then the two circles M and N takes one of the following six positions :

1. If $MN > r_1 + r_2$, then the two circles are distant.
2. If $MN = r_1 + r_2$, then the two circles are touching externally.
3. If $r_1 - r_2 < MN < r_1 + r_2$, then the two circles are intersecting.
4. If $MN = r_1 - r_2$, then the two circles are touching internally.
5. If $MN < r_1 - r_2$, then the two circles are one inside the other.
6. If $MN = 0$, then the two circles are concentric.

- The line of centres of two touching circles passes through the point of tangency and is perpendicular to the common tangent at this point.
- The line of centres of two intersecting circles is perpendicular to the common chord and bisects it.
- We can draw an infinite number of circles passing through a given point.
- There is an infinite number of circles that can be drawn to pass through the two points A and B and all their centres lie on the axis of symmetry of $\overline{AB}$
- It is impossible to draw a circle passing through three collinear points.
- For any three non-collinear points , there is a unique circle can be drawn to pass through them.
- The circle which passes through the vertices of a triangle is called the circumcircle of this triangle.
- The perpendicular bisectors of the sides of a triangle intersect at a point which is the centre of the circumcircle of the triangle.
- We can draw a circle passing through the vertices of a rectangle , a square or an isosceles trapezium while we cannot draw a circle passing through the vertices of a parallelogram , rhombus or , trapezium which is not isosceles.
- If chords of a circle are equal in length , then they are equidistant from the centre.
- In congruent circles , chords which are equal in length are equidistant from the centres.
- In the same circle (or in congruent circles) , chords which are equidistant from the centre(s) are equal in length.

Exams on Unit Four

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If M and N are two circles touching externally , their radii lengths are 2 cm. , 4 cm. , then the area of the circle whose diameter is $\overline{MN}$ is ………. cm^2.

(a) 36π (b) 9π (c) 16π (d) 4π

2 The number of circles passing through three collinear points is ……….

(a) zero. (b) one. (c) three. (d) an infinite number.

3 The axis of symmetry of the common chord $\overline{AB}$ of the two intersecting circles M and N is ……….

(a) $\overleftrightarrow{MA}$ (b) $\overleftrightarrow{MB}$ (c) $\overleftrightarrow{MN}$ (d) $\overleftrightarrow{NA}$

4 A circle is of diameter length $2X$ cm. and the straight line L is at distance of $(X + 1)$ cm. from its centre , then the straight line L is ……….

(a) a tangent to the circle. (b) a secant to the circle.

(c) outside the circle. (d) an axis of symmetry of the circle.

5 M and N are two intersecting circles their radii lengths are 5 cm. and 2 cm. respectively , then MN $\in$ ……….

(a) $]3 , 7[$ (b) $[3 , 7[$ (c) $]3 , 7]$ (d) $[3 , 7]$

6 The centre of the circumcircle of a triangle is the point of intersection of ……….

(a) the bisectors of its interior angles. (b) the bisectors of its exterior angles.

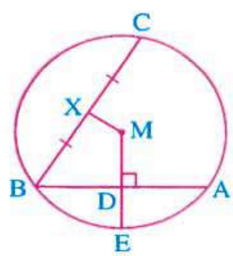
(c) its altitudes. (d) the symmetry axes of its sides.

2 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{BC}$ are two chords in circle M , which has radius length of 10 cm.

, $\overrightarrow{MD} \perp \overline{AB}$ intersecting $\overline{AB}$ at D and inersecting the circle M at E

, X is the midpoint of $\overline{BC}$, AB = 16 cm. , m ($\angle$ ABC) = 54°



Find : 1 m ($\angle$ DMX)

2 The length of $\overline{DE}$

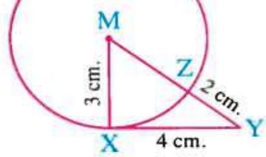
[b] In the opposite figure :

M is a circle with radius length 3 cm.

, XY = 4 cm. , $\overline{MY} \cap$ The circle M = $\{Z\}$

and ZY = 2 cm.

Prove that : $\overleftrightarrow{XY}$ is a tangent to the circle M at X



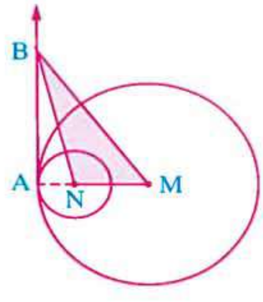
3 **[a] In the opposite figure :**

M and N are two circles with radii lengths of 6 cm. and 2 cm. respectively and they are touching internally at A

, $\overleftrightarrow{AB}$ is a common tangent for both

If the area of Δ BMN = 12 cm^2.

, **find :** The length of $\overline{AB}$



[b] Draw $\overline{AB}$ is of length 8 cm. , then draw a circle passing through the two points A and B and its radius length is 5 cm. **How many circles you can draw ?**

4 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords of the circle M

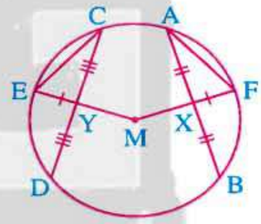
, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{CD}$, XF = YE

Prove that :

1 AB = CD

2 AF = CE



[b] In the opposite figure :

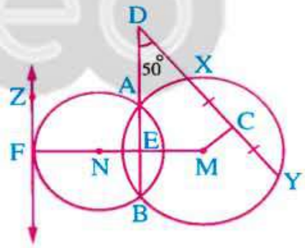
M and N are two intersecting circles at A and B

, C is the midpoint of $\overline{XY}$, m ($\angle$ D) = 50°

, $\overleftrightarrow{FZ}$ is a tangent to the circle N at F where $\overleftrightarrow{MN} \cap \overleftrightarrow{FZ} = \{F\}$

1 **Find :** m ($\angle$ CME)

2 **Prove that :** $\overleftrightarrow{FZ}$ // $\overleftrightarrow{AB}$



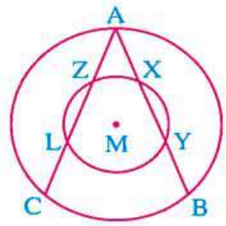
5 **[a]** Draw the right-angled triangle ABC at B , where AB = 4 cm. and BC = 3 cm. , then draw the circumcircle of this triangle. Where does the centre of the circle lie with respect to the sides of this triangle ?

[b] In the opposite figure :

Two concentric circles at M

, AB = AC

Prove that : XY = ZL



Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1. If the diameter length of a circle is 8 cm. and straight line L is at distance 4 cm. from its centre , then the straight line L is ....
   (a) a tangent to the circle. (b) a secant to the circle.
   (c) a diameter to the circle. (d) outside the circle.

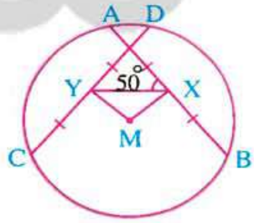
2. M and N are two distant circles and the lengths of their radii are 8 cm. , 6 cm. respectively , then MN .... 14 cm.
   (a) $<$ (b) $>$ (c) $=$ (d) $\leq$

3. We can draw a circle passing through the vertices of ....
   (a) a trapezium (b) a parallelogram (c) a rectangle (d) a rhombus

4. If the surface of the circle M $\cap$ the surface of the circle N = {A} and the radius length of one of them is 3 cm. , MN = 8 cm. , then the radius length of the other circle = ... cm.
   (a) 5 (b) 6 (c) 11 (d) 16

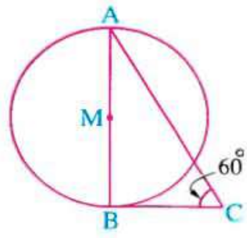
5. If $\overleftrightarrow{AB} \cap$ the circle M = $\{A , B\}$, then $\overleftrightarrow{AB} \cap$ the surface of the circle M = ...
   (a) $\{A , B\}$ (b) $\overline{AB}$ (c) $\overrightarrow{AB}$ (d) $\overrightarrow{BA}$

6. **In the opposite figure :**



   $\overline{AB}$, $\overline{CD}$ are two equal chords at circle M , X , Y are the midpoints of $\overline{AB}$, $\overline{CD}$ respectively
   if m ($\angle$ AXY) = 50° , then m ($\angle$ XMY) = ...°
   (a) 40 (b) 50 (c) 90 (d) 100

2 **[a] In the opposite figure :**



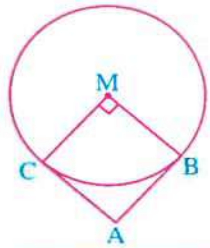
A circle M of circumference 44 cm.
, $\overline{AB}$ is a diameter of it , $\overline{BC}$ is a tangent to it at B
, m ($\angle$ C) = 60°
Find : The length of $\overline{BC}$ $\left(\pi = \frac{22}{7}\right)$

Unit 4

[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to circle M at B , C , m ($\angle$ BMC) = 90°

Prove that : ABMC is a square.



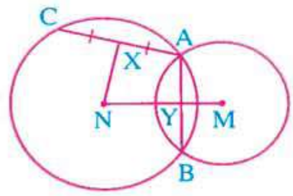
3 **[a] In the opposite figure :**

M and N are two intersecting circles

, $\overline{MN} \cap \overline{AB} = \{Y\}$

, AB = AC , X is the midpoint of $\overline{AC}$

Prove that : NX = NY

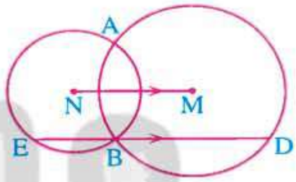


[b] In the opposite figure :

M , N are two intersecting circles at A , B

, $\overleftrightarrow{BD}$ // $\overline{MN}$, $\overleftrightarrow{BD}$ cuts the two circles at D , E

Prove that : DE = 2 MN



4 **[a]** Using the geometrical tools draw $\overline{AB}$ of length 6 cm. , then draw the circle that passes through the points A , B and its radius length is 4 cm. What is the length of the radius of the smallest circle passes through the points A , B ?

[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two chords at circle M , $\overline{MX} \perp \overline{AB}$

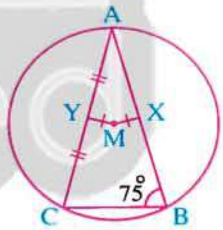
, Y is the midpoint of $\overline{AC}$

, m ($\angle$ ABC) = 75° , MX = MY

Find with proof :

1 m ($\angle$ BAC)

2 $\dfrac{\text{The perimeter of } \Delta \text{ ABC}}{\text{The perimeter of } \Delta \text{ AXY}}$

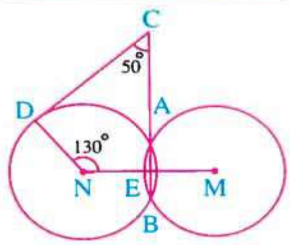


5 **[a] In the opposite figure :**

M and N are two intersecting circles at A , B , C $\in \overrightarrow{BA}$

, D $\in$ the circle N

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle N at D



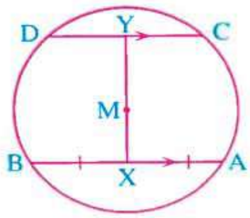
[b] In the opposite figure :

M is a circle , $\overline{AB}$ // $\overline{CD}$

, X is the midpoint of $\overline{AB}$

, $\overrightarrow{XM}$ is drawn to cut $\overline{CD}$ at Y

Prove that : Y is the midpoint of $\overline{CD}$



UNIT 5

Angles and arcs in the circle

Exercises of the unit :

6. Central angles and measuring arcs.
7. The relation between the inscribed and central angles subtended by the same arc - Well known problems.
8. Inscribed angles subtended by the same arc.
- ✪ Summary of the first part of unit five.
- ✪ Exams on the first part of unit five.
9. The cyclic quadrilateral and its properties.
10. Cases of proving the cyclic quadrilateral.
11. The relation between the tangents of a circle.
12. Angle of tangencey.
- ✪ Summary of the second part of unit five.
- ✪ Exams on the second part of unit five.

Exercise 6

Central angles and measuring arcs

From the school book

1 **Complete the following :**

1 In the same circle (or in congruent circles) , the equal arcs in measure are equal in .... *(Matrouh 11)*

2 In the same circle (or in the congruent circles) , if the measures of arcs are equal , then their chords are .... *(South Sinai 12)*

3 If two parallel chords are drawn in a circle , then the measures of the two arcs between them are .... *(Alex. 11)*

4 If a chord is parallel to a tangent of a circle , then the measures of the two arcs between them are .... *(Cairo 12)*

5 The measure of the circle = ... *(Cairo 08)*

6 The measure of the semicircle equals .... while the length of the arc of the semicircle whose radius length is r equals .... *(Port Said 06)*

7 If a square ABCD is inscribed in a circle M , then m $(\widehat{AB})$ = ... *(Beni Suef 09)*

2 Find the measure of the arc which represents $\frac{1}{3}$ the measure of the circle, then calculate the length of this arc if the length of the radius is 21 cm. $(\pi \approx \frac{22}{7})$ (Show steps)

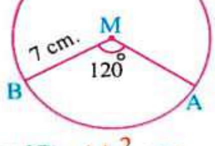
(El-Monofia 16) « 120° , 44 cm. »

3 **In the opposite figure :**

M is a circle of radius length 7 cm. ,

m (∠ AMB) = 120°

Find the length of $\widehat{AB}$ $(\pi \approx \frac{22}{7})$



(Suez 17) «$14\frac{2}{3}$ cm. »

4 **Choose the correct answer from those given :**

1 The central angle whose measure is 90° subtends an arc of length = ... the circumference of the circle. *(Assiut 11)*

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

2 The circumference of a circle = 36 cm. , then the measure of an arc of it with length = 6 cm. is .... *(Monofia 09)*

(a) 60° (b) 30° (c) 90° (d) 120°

3 The length of the arc opposite to a central angle whose measure = 120° in a circle of radius length r equals .... *(Suez 09)*

(a) $\frac{1}{3}\pi r$ (b) πr (c) $\frac{2}{3}\pi r$ (d) $3\pi r$

4 The length of the arc which represents $\frac{1}{4}$ the circumference of the circle = ... cm. *(El-Dakahlia 17 , El-Kalyoubia 16)*

(a) $2\pi r$ (b) πr (c) $\frac{1}{2}\pi r$ (d) $4\pi r$

5 The measure of the arc which represents $\frac{1}{6}$ the circumference of the circle = ... *(Cairo 15)*

(a) 60° (b) 90° (c) 120° (d) 300°

6 The length of the arc opposite to a central angle of measure 30° in a circle of circumference 36 cm. = ... cm. *(Souhag 09)*

(a) 18 (b) 9 (c) 3 (d) 4.5

7 An arc in a circle , its length = $\frac{1}{3}\pi r$, then it is opposite to a central angle of measure .... *(Beni Suef 16)*

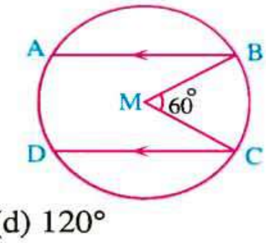
(a) 30° (b) 60° (c) 120° (d) 240°

8 If A and B are two points belonging to a circle M such that the length of $\overset{\frown}{AB} = \pi r$, then $\overline{AB}$ is .... in the circle M

(a) a radius (b) a chord not passing through the centre

(c) a diameter (d) an axis of symmetry of the circle

9 **In the opposite figure :**



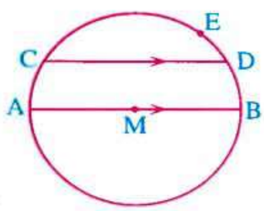
M is a circle , $\overline{AB} // \overline{CD}$

, m (∠ BMC) = 60°

, then m $(\overset{\frown}{AD})$ = ... *(Aswan 18)*

(a) 30° (b) 40° (c) 60° (d) 120°

10 **In the opposite figure :**



If $\overline{AB}$ is a diameter in the circle M

, $\overline{AB} // \overline{CD}$

, m $(\overset{\frown}{DEC})$ = 80°

, then m $(\overset{\frown}{AC})$ = ... *(El-Sharkia 18)*

(a) 40° (b) 50° (c) 80° (d) 100°

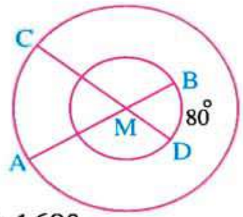
Unit 5

11 **In the opposite figure :**

Two concentric circles with centre M

, $\overline{AB} \cap \overline{CD} = \{M\}$, if m $(\widehat{BD}) = 80°$

, then m $(\widehat{AC}) = \dots\dots\dots$ *(Suez 13)*

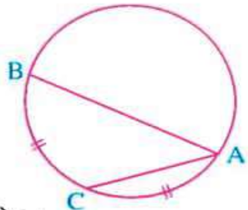


(a) 40° (b) 60° (c) 80° (d) 160°

12 **In the opposite figure :**

If C is the midpoint of $\widehat{AB}$

, then AB $\dots\dots\dots$ 2 AC *(Giza 17)*

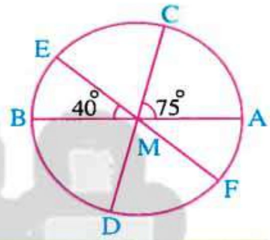


(a) < (b) > (c) ≥ (d) =

5 **In the opposite figure :**

$\overline{AB}$, $\overline{CD}$ and $\overline{EF}$ are diameters of the circle M

Complete :



1 m $(\widehat{AC}) = \dots\dots\dots°$

2 m $(\widehat{ACE}) = \dots\dots\dots°$

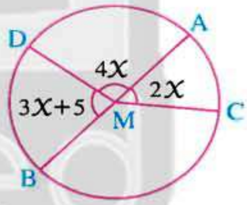
3 m $(\widehat{ACD}) = \dots\dots\dots°$

4 m $(\widehat{AFE}) = \dots\dots\dots°$

6 **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M ,

study the figure , then complete :



1 $x = \dots\dots\dots°$

2 m $(\widehat{AC}) = \dots\dots\dots°$

3 m $(\widehat{AD}) = \dots\dots\dots°$

4 m $(\widehat{BC}) = \dots\dots\dots°$

5 m $(\widehat{CAD}) = \dots\dots\dots°$

6 m $(\widehat{CBD}) = \dots\dots\dots°$

7 m $(\widehat{ACD}) = \dots\dots\dots°$

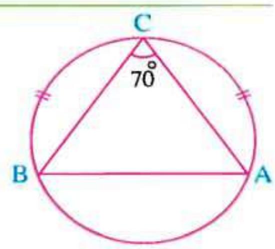
8 m $(\widehat{ADC}) = \dots\dots\dots°$

7 **In the opposite figure :**

If m $(\widehat{AC})$ = m $(\widehat{BC})$

, m $(\angle ACB) = 70°$

Find : m $(\angle ABC)$



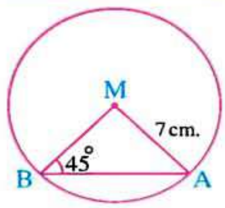
« 55° »

8 **In the opposite figure :**

A and B are two points belonging to the circle M

such that : m $(\angle MBA) = 45°$, AM = 7 cm.

Find : The length of $\widehat{AB}$ $(\pi = \frac{22}{7})$

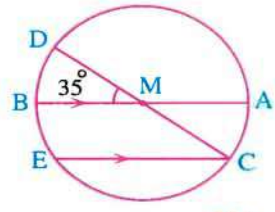


« 11 cm. »

9 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two diameters in the circle M such that : m (∠ DMB) = 35° , $\overline{CE}$ // $\overline{AB}$

Find : m $(\widehat{BE})$

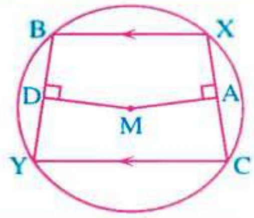


« 35° »

10 **In the opposite figure :**

$\overline{XB}$ // $\overline{CY}$, $\overline{MA} \perp \overline{XC}$, $\overline{MD} \perp \overline{BY}$

Prove that : MA = MD



(Giza 17)

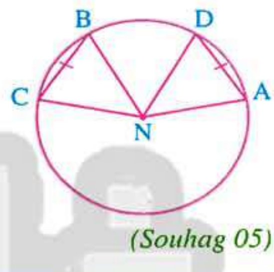
11 **In the opposite figure :**

A and B are two points belonging to the circle N

, D ∈ $\widehat{AB}$, C ∈ the major arc $\widehat{AB}$

such that AD = BC

Prove that : m (∠ ANB) = m (∠ CND)



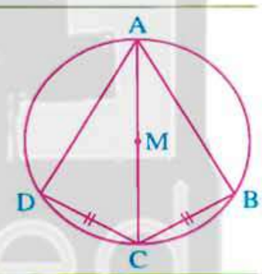
(Souhag 05)

12 **In the opposite figure :**

ABCD is a quadrilateral inscribed in a circle M

, $\overline{AC}$ is a diameter in the circle , CB = CD

Prove that : m $(\widehat{AB})$ = m $(\widehat{AD})$

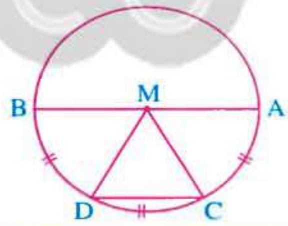


13 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m $(\widehat{AC})$ = m $(\widehat{CD})$ = m $(\widehat{DB})$

Prove that : Δ MCD is equilateral.



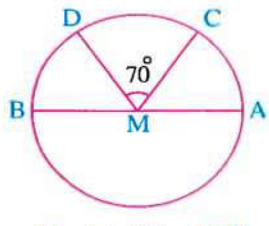
14 **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M

, m (∠ CMD) = 70°

, m $(\widehat{AC})$: m $(\widehat{DB})$ = 5 : 6

Find : m $(\widehat{ACD})$



(Assiut 12) « 120° »

15 ABCD is a quadrilateral inscribed in the circle M such that AB = CD

Prove that : AC = BD

Unit 5

16 ABCD is a quadrilateral inscribed in a circle. If $\overline{AB} \parallel \overline{DC}$, E is the midpoint of $\widehat{AB}$

Prove that : CE = DE

(Damietta 16 , Giza 05)

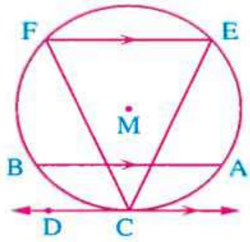
17 **In the opposite figure :**

M is a circle , $\overleftrightarrow{CD}$ is a tangent to the circle at C ,

$\overline{AB}$ and $\overline{EF}$ are two chords in the circle

, where $\overline{AB} \parallel \overline{EF} \parallel \overleftrightarrow{CD}$

Prove that : CE = CF



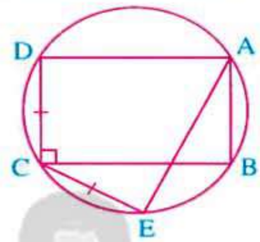
(Assiut 19 , El-Beheira 14 , Alex. 11)

18 **In the opposite figure :**

ABCD is a rectangle inscribed in a circle. Draw the chord $\overline{CE}$

, where CE = CD

Prove that : AE = BC



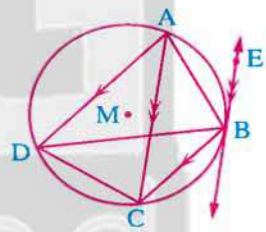
(Aswan 19 , Port Said 18 , El-Monofia 16 , Souhag 15)

19 **In the opposite figure :**

$\overleftrightarrow{BE}$ is a tangent to the circle M at B

, $\overline{BC} \parallel \overline{AD}$, $\overleftrightarrow{BE} \parallel \overline{AC}$

Prove that : Δ BCD is isosceles.

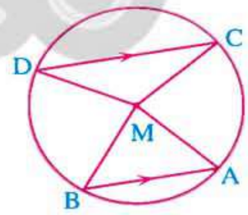


20 **In the opposite figure :**

M is a circle with radius length of 15 cm. , $\overline{AB}$ and $\overline{CD}$ are two parallel chords of the circle , m $(\widehat{AC}) = 80°$

, length of $\widehat{AC}$ = length of $\widehat{AB}$ **Find :**

1 m (∠ MAB)   2 m $(\widehat{CD})$ 3 The length of $\widehat{CD}$



(New valley 18) « 50° , 120° , 31.4 cm. »

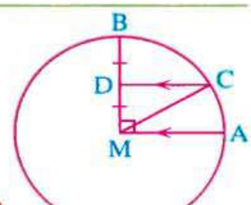
21 $\overline{AB}$ and $\overline{AC}$ are two tangent line segments to a circle of centre M at B and C

If m (∠ BAC) = 35° **Find :** m ($\widehat{BC}$ the major) « 215° »

22 **In the opposite figure :**

$\overline{AM} \parallel \overline{CD}$, MD = DB

, m (∠ AMB) = 90° **Find :** m $(\widehat{AC})$

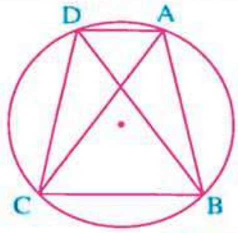


(Loxur 19) « 30° »

Exercise 6

23 If A and B are two points belonging to the circle M and m (∠ AMB) = $\frac{1}{4}$ m (∠ AMB the reflex) **Find :** m ($\widehat{AB}$) « 72° »

24 **In the opposite figure :**

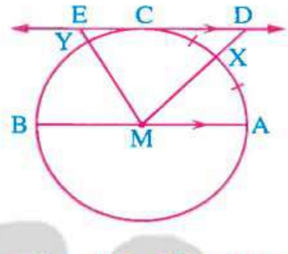


ABCD is a quadrilateral inscribed in a circle in which AC = BD , AB = (3 x – 5) cm. , CD = (x + 3) cm.

Find with proof : The length of $\overline{AB}$

« 7 cm. »

25 **In the opposite figure :**



$\overline{AB}$ is a diameter in a circle M

, $\overleftrightarrow{DE}$ is a tangent to it at C

, $\overline{AB}$ // $\overline{DE}$, X is the midpoint of $\widehat{AC}$

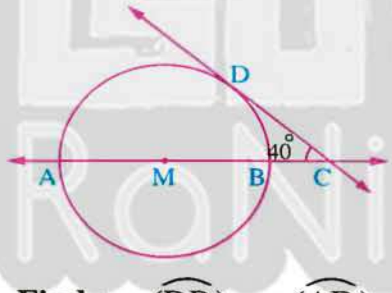
, m ($\widehat{BY}$) = 2 m ($\widehat{CY}$)

Find : The measures of the angles of Δ MDE *(Damietta 13)* « 45° , 75° , 60° »

26 **In each of the following figures :**

$\overleftrightarrow{CD}$ is a tangent to the circle M at D

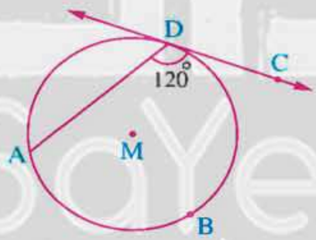
1



Find : m ($\widehat{DB}$) , m ($\widehat{AD}$)

« 50° , 130° »

2

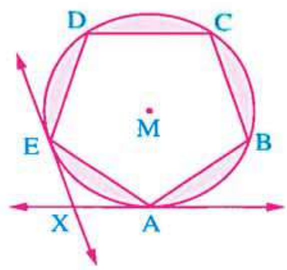


Find : m ($\widehat{ABD}$)

« 240° »

For excellent pupils

1 **In the opposite figure :**



ABCDE is a regular pentagon inscribed in the circle M

, $\overleftrightarrow{AX}$ is a tangent to the circle at A

, $\overleftrightarrow{EX}$ is a tangent to the circle at E

where $\overleftrightarrow{AX} \cap \overleftrightarrow{EX} = \{X\}$ **Find :**

1 m ($\widehat{AE}$)

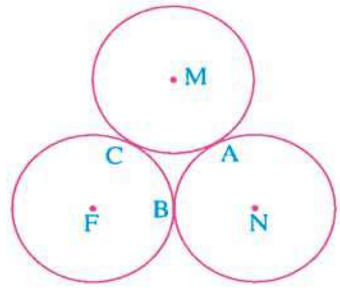
2 m (∠ AXE)

« 72° , 108° »

Unit 5

2 **In the opposite figure :**

M , N and F are three congruent circles and touching at A , B and C , the radius length of each is 10 cm.



1 **Prove that :** The length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{BC}$ = the length of $\overset{\frown}{AC}$

2 Find the perimeter of the figure ABC « 31.4 cm. »

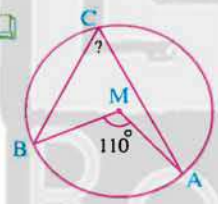
Exercise **7**

The relation between the inscribed and central angles subtended by the same arc - well Known problems

From the school book

First : problems on theorem (1) and its corollaries :

1 In each of the following , find the measure of each angle or arc denoted by (?) given that M is the centre of the circle :

1


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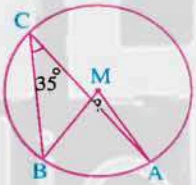

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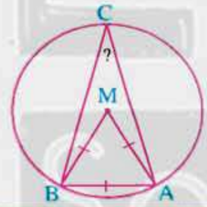

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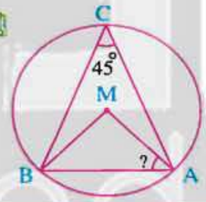

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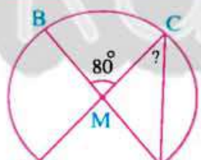

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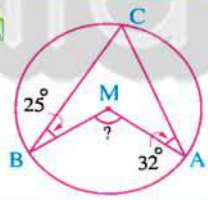

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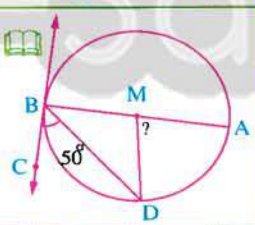

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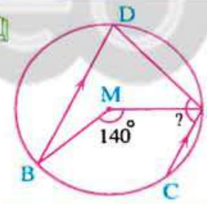

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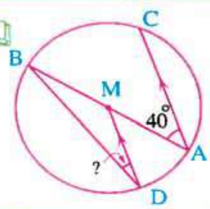

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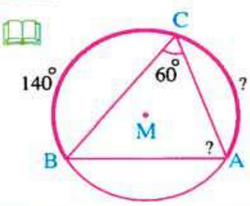
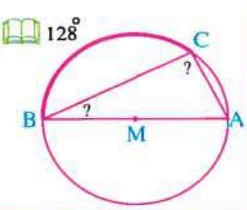
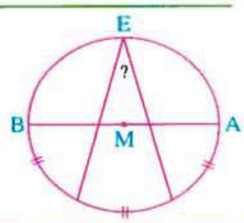


11 128°


12


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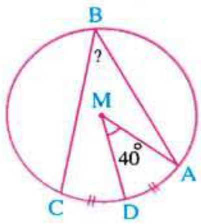
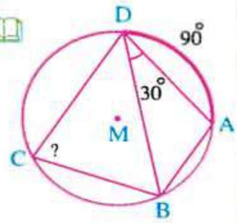


14


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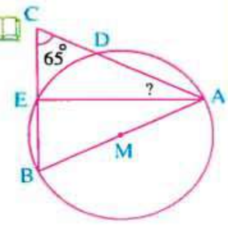

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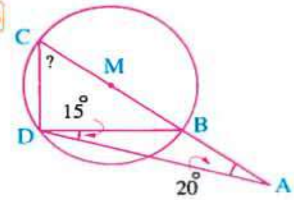
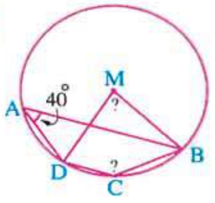
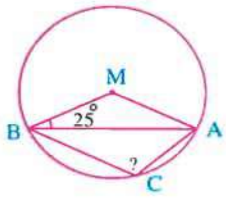
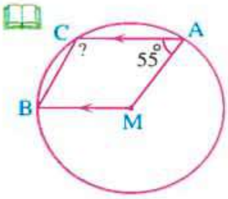
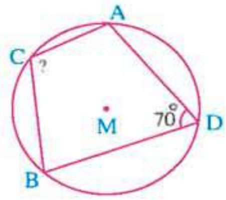


Unit 5

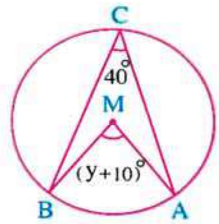
17 

18 

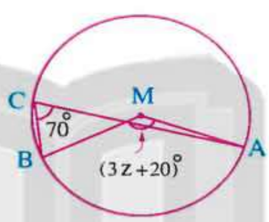
19 

20 

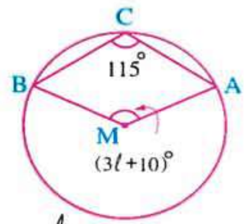
3 M is a circle , in each of the following , find the value of the symbol used in measuring :

1 

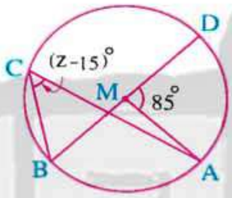
y = ...

2 

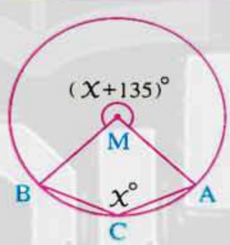
z = ...

3 

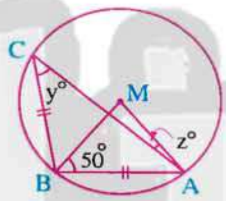
ℓ = ...

4 

z = ...

5 

x = ...

6 

y = ...

, z = ...

3 **Choose the correct answer :**

1 The measure of the inscribed angle equals .... the measure of the central angle subtended by the same arc. *(Giza 17 , Aswan 15 , El-Menia 14)*

(a) half (b) twice (c) quarter (d) third

2 The ratio between the measure of the central angle and the measure of the inscribed angle that has the same subtended arc is .... *(El-Fayoum 11)*

(a) 3 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 3

3 If the measure of a central angle is 100° , then the measure of the inscribed angle that has the same subtended arc is .... *(Giza 11)*

(a) 200° (b) 100° (c) 50° (d) 25°

4 The measure of the arc that is opposite to an inscribed angle of measure 40° is ....°

(a) 20 (b) 40 (c) 80 (d) 90

5 The inscribed angle which is drawn in a semicircle is ....

(a) obtuse. (b) acute. (c) right. (d) straight.

Exercise 7

6 The type of the inscribed angle which is opposite to an arc greater than the semicircle is .... *(New Valley 18)*

(a) acute. (b) obtuse. (c) right. (d) straight.

7 The inscribed angle which is subtended by minor arc in a circle is .... *(Alex. 17 , Qena 16)*

(a) reflex. (b) right. (c) obtuse. (d) acute.

8 ABC is an equilateral triangle inscribed in a circle , then m $(\overset{\frown}{AB})$ = ... *(El-Fayoum 18)*

(a) 30° (b) 60° (c) 90° (d) 120°

9 The length of the arc that is opposite to a right inscribed angle in a circle whose circumference is 44 cm. equals .... cm. *(Dakahlia 12)*

(a) 22 (b) 11 (c) $\frac{22}{7}$ (d) $\frac{44}{7}$

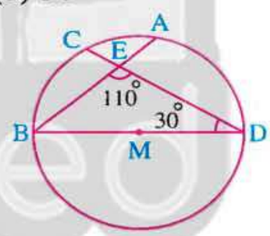
10 The measure of the inscribed angle which is drawn in $\frac{1}{3}$ a circle equals .... *(Cairo 09)*

(a) 240° (b) 120° (c) 60° (d) 30°

11 The measure of the inscribed angle which is subtended by an arc representing $\frac{1}{3}$ a circle equals ....

(a) 240° (b) 120° (c) 60° (d) 30°

12 **In the opposite figure :**

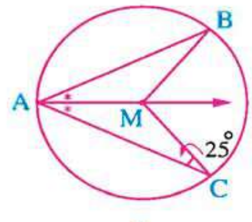


$\overline{AB} \cap \overline{CD} = \{E\}$, m (∠ D) = 30° , m (∠ DEB) = 110° , then m $(\overset{\frown}{AD})$ = ...

(a) 80° (b) 70°

(c) 40° (d) 60° *(kalyoubia 05)*

13 **In the opposite figure :**

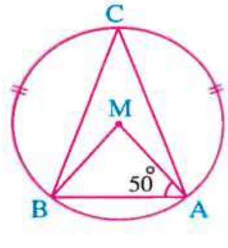


If $\overrightarrow{AM}$ bisects ∠ BAC , m (∠ ACM) = 25° , then m $(\overset{\frown}{BC})$ = ...

(a) 25° (b) 50°

(c) 100° (d) 140°

14 **In the opposite figure :**



m (∠ CAM) = ...

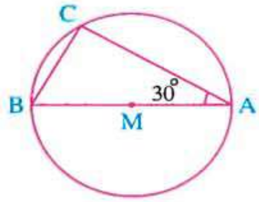
(a) 20° (b) 30°

(c) 40° (d) 50°

Unit 5

15 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M of radius length 4 cm. , m (∠ A) = 30° , then BC = ... cm.



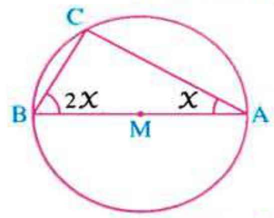
(a) 2 (b) 4

(c) 6 (d) 8

(Matrouh 11)

16 **In the opposite figure :**

If $\overline{AB}$ is a diameter in the circle M , then x = ...



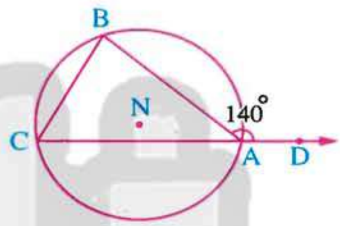
(a) 40° (b) 20°

(c) 30° (d) 60°

(Menia 12)

17 **In the opposite figure :**

A circle is of centre N , if m (∠ DAB) = 140° , m ($\overparen{AB}$) = 120° , then m (∠ B) = ...

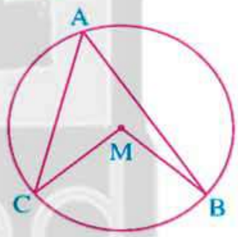


(a) 40° (b) 50°

(c) 80° (d) 120°

18 **In the opposite figure :**

M is a circle , m (∠ M) – m (∠ A) = 50° , then m (∠ A) = ... *(El-Menia 17 , Port Said 13)*



(a) 40° (b) 50°

(c) 100° (d) 130°

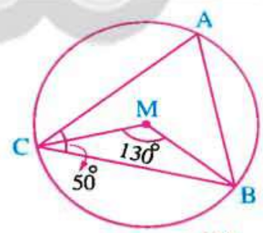
4 **In the opposite figure :**

Δ ABC is inscribed in the circle M

, m (∠ BMC) = 130°

, m (∠ ACB) = 50°

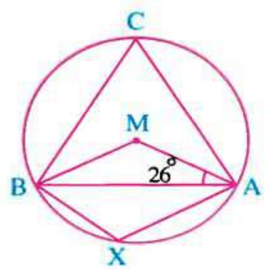
Find : m (∠ ABC)



« 65° »

5 **In the opposite figure :**

Δ ABC is inscribed in the circle M , $\overline{MA}$ and $\overline{MB}$ are two radii in it , m (∠ MAB) = 26° , X ∈ $\overparen{AB}$



Find by proof :

1 m (∠ AMB) **2** m (∠ ACB)

3 m (∠ AXB) **4** m ($\overparen{AXB}$)

« 128° , 64° , 116° , 128° »

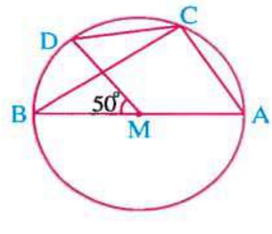
6 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

m (∠ BMD) = 50°

Find with proof :

m (∠ ACD)



(Damietta 14) « 115° »

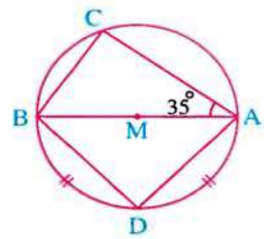
7 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

the length of $\widehat{AD}$ = the length of $\widehat{BD}$,

m (∠ CAB) = 35°

Find by proof : m (∠ CBD)



(El-Menia 11) « 100° »

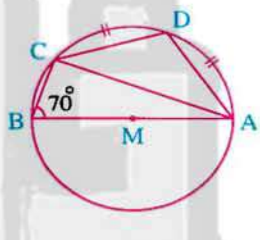
8 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

the length of $(\widehat{AD})$ = the length of $(\widehat{DC})$,

m (∠ ABC) = 70°

Find each of : m (∠ DCA) , m (∠ CAB)



(El-Ismailia 05) « 35° , 20° »

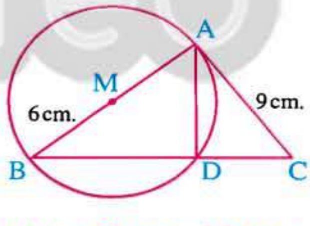
9 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overline{AC}$ touches the circle at A

If AC = 9 cm. , BM = 6 cm.

Find the length of each of : $\overline{BC}$, $\overline{AD}$

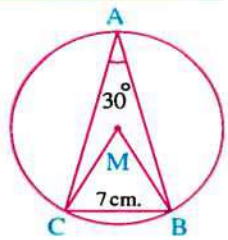


(Souhag 17 , Kafr El-Sheikh 04) « 15 cm. , 7.2 cm. »

10 **In the opposite figure :**

m (∠ A) = 30° , BC = 7 cm.

Find : The area of the circle M $\left(\pi = \frac{22}{7}\right)$



(Gharbia 09) « 154 cm^2 »

Unit 5

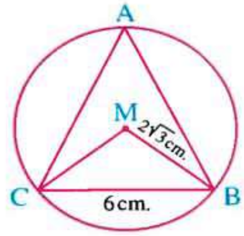
11 **In the opposite figure :**

A circle M , BC = 6 cm.

, BM = $2\sqrt{3}$ cm.

Find : m (∠ BAC)

(Hint : Draw $\overline{MD} \perp \overline{BC}$)



(New Valley 13) « 60° »

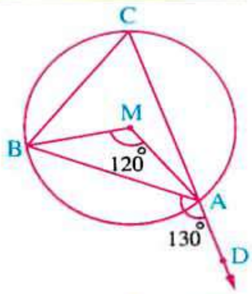
12 **In the opposite figure :**

Δ ABC is inscribed in the circle M

, D ∈ $\overrightarrow{CA}$, m (∠ BAD) = 130°

, m (∠ AMB) = 120°

Find : m (∠ MBC)



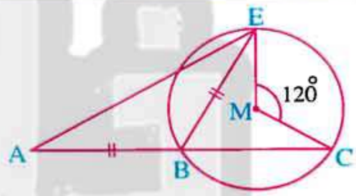
(Monofia 09) « 40° »

13 **In the opposite figure :**

M is a circle , m (∠ EMC) = 120°

and BE = AB

Find with proof : m (∠ A)

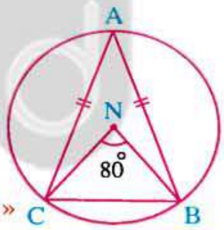


(South Sinai 16) « 30° »

14 **Using the opposite figure :**

Write the given data then find :

1 m (∠ ABC)   2 m ($\widehat{BC}$ the major)



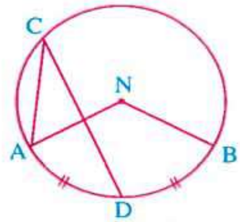
(Cairo 19 , New Valley 06) « 70° , 280° »

15 **In the opposite figure :**

D is the midpoint of $\widehat{AB}$

Prove that :

m (∠ ACD) = $\frac{1}{4}$ m (∠ ANB)



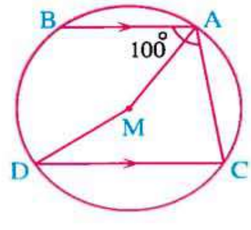
(Beni Suef 04)

16 **In the opposite figure :**

$\overline{AB}$, $\overline{CD}$ are two chords in the circle M

, m (∠ BAC) = 100° , $\overline{AB}$ // $\overline{CD}$

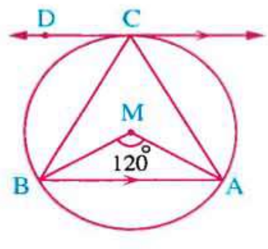
Find : m (∠ AMD)



(El-Dakahlia 18) « 160° »

Exercise 7

17 **In the opposite figure :**



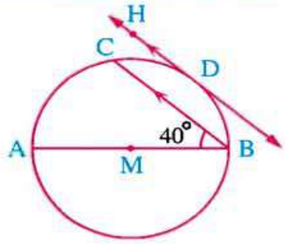
$\overleftrightarrow{CD}$ is a tangent to the circle at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$, m (∠ AMB) = 120°

Prove that : Δ CAB is equilateral.

(Giza 19 , Alex. 19 , South Sinai 18 , Alexandria 16 , Ismailia 13)

18 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M ,

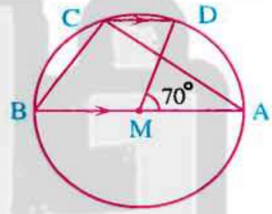
M (∠ B) = 40° , $\overleftrightarrow{DH}$ is a tangent to the circle M at D ,

$\overleftrightarrow{DH}$ // $\overline{BC}$

Find : m ($\overparen{DC}$)

(El-Monofia 17) « 50° »

19 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

$\overline{DC}$ // $\overline{AB}$, m (∠ AMD) = 70°

Find by proof : m (∠ ACD) , m (∠ ABC)

(El-Menia 17) « 35° , 55° »

20 $\overline{AB}$ is a diameter in the circle M , $\overline{AC}$ is a chord such that m (∠ BAC) = 30° , draw $\overline{BC}$ and draw $\overrightarrow{MD} \perp \overline{AC}$ and to intersect it at D

1 **Prove that :** $\overline{MD}$ // $\overline{BC}$

2 **Prove that :** length of $\overline{BC}$ = length of the radius of this circle.

(El-Monofia 17)

21 M , N are two touching externally circles at A , $\overleftrightarrow{BA}$, $\overleftrightarrow{CA}$ are two secants cut the circle M at B , C and the circle N at D , E respectively, m (∠ BMC) = 140°

Find : m ($\overparen{ED}$)

(El-Dakahlia 2016) « 140° »

22 $\overline{BC}$ is a diameter in the circle M , $\overline{BY}$ is a chord in it , E ∈ $\overrightarrow{BY}$ where Y is the midpoint of $\overline{BE}$ **Prove that :** m (∠ YMC) = 2 m (∠ BEC)

(El-Dakahlia 18)

23 A is a point outside the circle M , $\overrightarrow{AB}$ is a tangent to the circle at B, $\overrightarrow{AM}$ intersects the circle M at C and D respectively, m (∠ A) = 40°

Find by proof : m (∠ BDC)

« 25° »

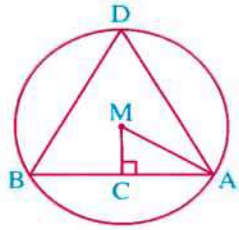
Unit 5

24 **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M ,

$\overline{MC} \perp \overline{AB}$

Prove that : m (∠ AMC) = m (∠ ADB)



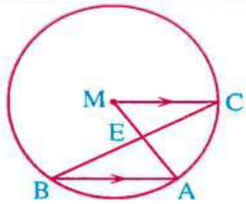
(Port Said 14 , El-Beheira 13)

25 **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M ,

$\overline{CM}$ // $\overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$

Prove that : BE > AE



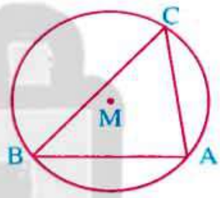
(El-Monofia 18 , El-Gharbia 18 , El-Gharbia 17 , Beni Suef 16 , Port Said 15 , El-Gharbia 14)

26 **In the opposite figure :**

ABC is an inscribed triangle in circle M

, m $(\widehat{AB})$: m $(\widehat{BC})$: m $(\widehat{AC})$ = 4 : 5 : 3

Find : m (∠ ACB)



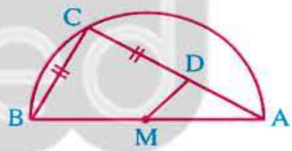
(Alexandria 16) « 60° »

27 **In the opposite figure :**

$\overline{AB}$ is a diameter in the semicircle M

, BC = CD = r

Find : m (∠ ADM)

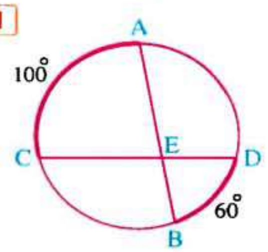


« 105° »

Second : Problems on well known problems :

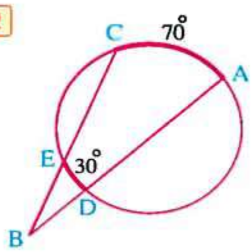
1 **Study each of the following figures , then complete :**

1



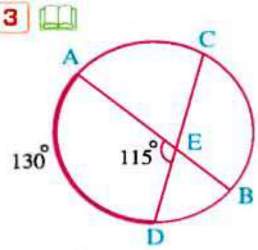
m (∠ AEC) = ...°

2



m (∠ B) = ...°

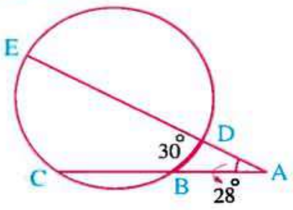
3



m $(\widehat{BC})$ = ...°

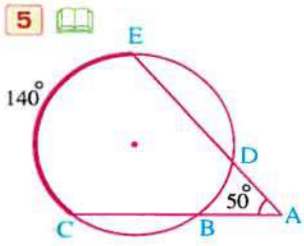
Exercise 7

4



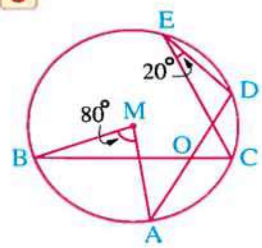
m $(\widehat{EC})$ = ...°

5



m $(\widehat{DB})$ = ...°

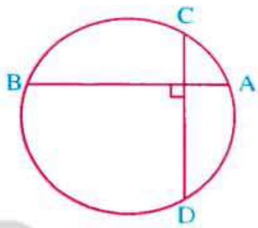
6



m (∠ AOB) = ...°

2 **Choose the correct answer from those given :**

1 **In the opposite figure :**



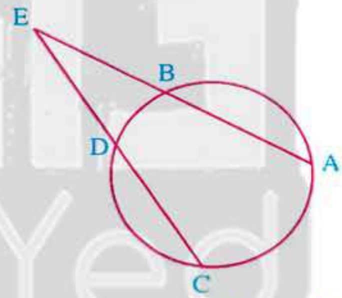
m $(\widehat{AC})$ + m $(\widehat{BD})$ = ...

(a) 45° (b) 90°

(c) 180° (d) 270°

(Cairo 16 , El-Monofia 15)

2 **In the opposite figure :**



If m $(\widehat{AC})$ − m $(\widehat{BD})$ = 70°

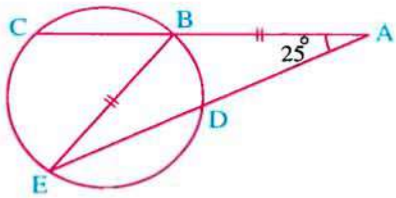
, then m (∠ E) = ...

(a) 35° (b) 70°

(c) 110° (d) 140°

(Luxor 11)

3 **In the opposite figure :**



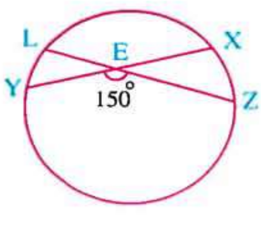
AB = BE , m (∠ EAC) = 25°

, then m $(\widehat{CE})$ = ...

(a) 25 ° (b) 50 °

(c) 100 ° (d) 180 °

4 **In the opposite figure :**



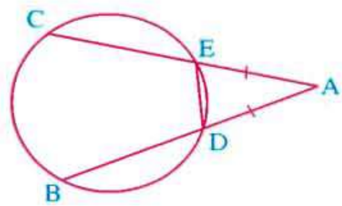
If m (∠ ZEY) = 150°

, then m $(\widehat{XZ})$ + m $(\widehat{LY})$ = ...

(a) 30 ° (b) 60 °

(c) 90 ° (d) 100 °

5 In the opposite figure :



If m $(\overset{\frown}{BC}) = 112°$, m $(\overset{\frown}{DE}) = 44°$, AD = AE ,

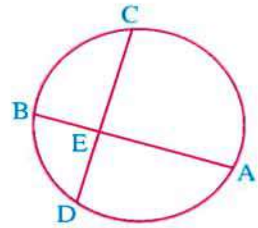
then m (∠ ADE) = ...

(a) 75° (b) 73°

(c) 70° (d) 76°

(Kafr El-Sheikh 08)

3 In the opposite figure :



$\overline{AB}$ and $\overline{CD}$ are two chords in the circle ,

$\overline{AB} \cap \overline{CD} = \{E\}$, if m $(\overset{\frown}{BD}) = 60°$

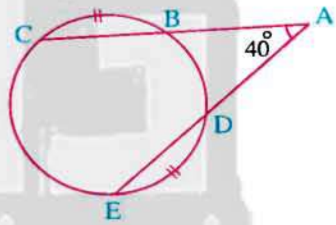
, m $(\overset{\frown}{AD}) = 100°$,

m $(\overset{\frown}{AC}) = 120°$

Calculate : 1 m $(\overset{\frown}{CB})$ 2 m (∠ CEB)

(Alex. 05) « 80° , 90° »

4 In the opposite figure :



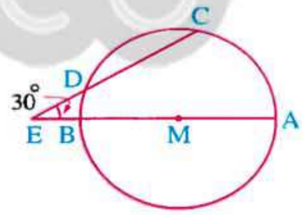
m (∠ A) = 40° , m $(\overset{\frown}{BD}) = 60°$

, m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{DE})$

Find : 1 m $(\overset{\frown}{EC})$ 2 m $(\overset{\frown}{BC})$

(Port Said 17 , North Sinai 17) « 140° , 80° »

5 In the opposite figure :



$\overline{AB}$ is a diameter in the circle M

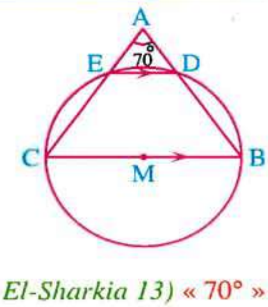
, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$

, m (∠ AEC) = 30° , m $(\overset{\frown}{AC}) = 80°$

Find : m $(\overset{\frown}{CD})$

(Alex. 18 , El-Sharkia 17 , Aswan 17) « 80° »

6 In the opposite figure :



M is a circle , $\overline{BC}$ is a diameter in it

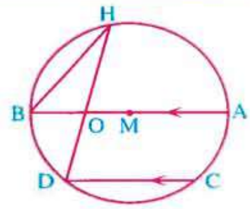
, m (∠ A) = 70° , $\overline{DE} \,//\, \overline{BC}$

Find : m $(\overset{\frown}{BD})$

(El-Dakahlia 17 , El-Sharkia 13) « 70° »

Exercise 7

7 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M ,

$\overline{AB} // \overline{DC}$, m $(\widehat{DC}) = 80°$,

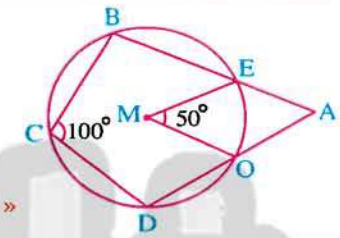
m $(\widehat{AH}) = 100°$

Find by proof : m (∠ DHB) , m (∠ AOH) *(El-Menia 17)* « 25° , 75° »

8 A is a point outside the circle M , $\overrightarrow{AC}$ is drawn to cut the circle at B and C , $\overrightarrow{AE}$ is drawn to cut the circle at D and E , if m (∠CME) = 100° , m (∠ BMD) = 40°

Find : m (∠ A) « 30° »

9 **In the opposite figure :**

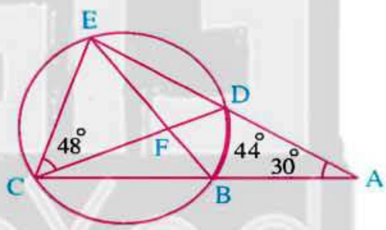


M is a circle , m (∠ M) = 50°

, m (∠ C) = 100°

Find : m (∠ A) *(Luxor 19)* « 55° »

10 **In the opposite figure :**



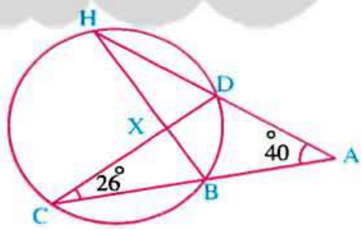
$\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$, $\overline{BE} \cap \overline{CD} = \{F\}$

If m (∠ A) = 30° , m $(\widehat{BD}) = 44°$

, m (∠ DCE) = 48°

Find : 1 m $(\widehat{CE})$ 2 m $(\widehat{BC})$ « 104° , 116° »

11 **In the opposite figure :**



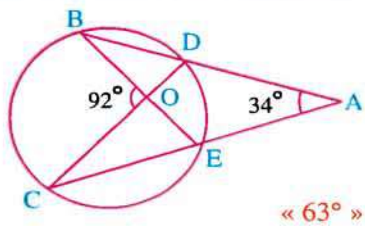
$\overrightarrow{CB} \cap \overrightarrow{HD} = \{A\}$, m (∠ A) = 40°

, $\overline{DC} \cap \overline{BH} = \{X\}$ and m (∠ C) = 26°

Find :

1 m $(\widehat{CH})$ 2 m (∠ HXC) *(Red Sea 19 , El-Gharbia 17 , Ismailia 16)* « 132° , 92° »

12 **In the opposite figure :**



m (∠ A) = 34°

, m (∠ BOC) = 92°

Find : m (∠ CDB) « 63° »

Unit 5

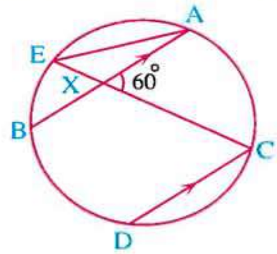
13 **In the opposite figure :**

$\overline{AB} \parallel \overline{CD}$, m (∠ AXC) = 60° , m $(\widehat{AC})$ = 80°

Find by proof :

1 m (∠ AEC) 2 m $(\widehat{BD})$

3 m $(\widehat{BE})$



« 40° , 80° , 40° »

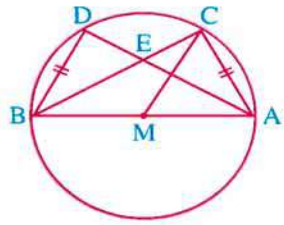
14 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

$\overline{AC}$ and $\overline{BD}$ are two chords equal in length ,

$\overline{AD} \cap \overline{BC} = \{E\}$

Prove that : m (∠ AMC) = m (∠ AEC)



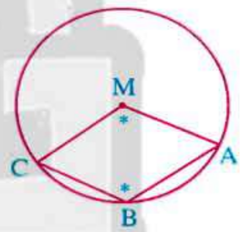
For excellent pupils

1 **In the opposite figure :**

If M is the centre of the circle

, m (∠ AMC) = m (∠ B)

Find : m (∠ B)

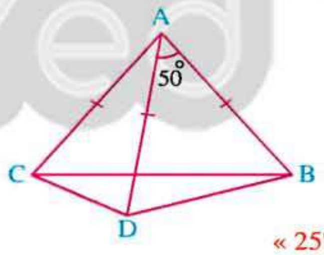


(Monofia 06) « 120° »

2 **In the opposite figure :**

AB = AD = AC , m (∠ BAD) = 50°

Find by proof : m (∠ BCD)



« 25° »

Exercise **8**

Inscribed angles subtended by the same arc

From the school book

1 **Complete the following :**

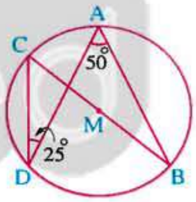
1 The inscribed angles subtended by the same arc in the same circle are ....

(Luxor 12)

2 The inscribed angles subtended by equal arcs in measure in the same circle are ....

(El-Sharkia 04)

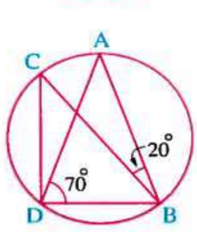
3 **In the opposite figure :**



m (∠ C) = ...°

, m (∠ B) = ... °

4 **In the opposite figure :**

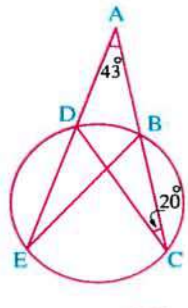


If AB = AD , then

m (∠ C) = ...°

, m (∠ BDC) = ...°

5 **In the opposite figure :**



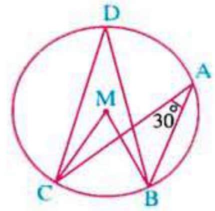
m (∠ BED) = ... °

, m (∠ ABE) = ... °

Unit 5

2 **Choose the correct answer from those given :**

1 **In the opposite figure :**



(El-Dakahlia 06)

If m (∠ BAC) = 30° , then

First : m (∠ BDC) = ...

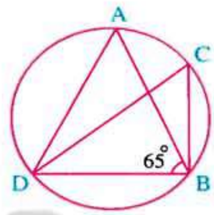
(a) 15° (b) 30°

(c) 60° (d) 150°

Second : m (∠ BMC) = ...

(a) 30° (b) 90° (c) 60° (d) 120°

2 **In the opposite figure :**



If m (∠ ABD) = 65°

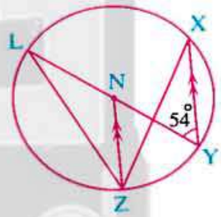
, AB = AD

, then m (∠ BCD) = ...

(Beni Suef 12)

(a) 15° (b) 25° (c) 30° (d) 50°

3 **In the opposite figure :**



A circle N , $\overline{XY}$ // $\overline{NZ}$

If m (∠ XYL) = 54° , then

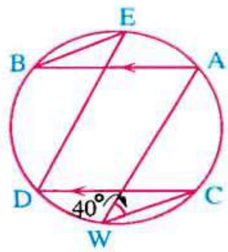
First : m (∠ XZL) = ...

(a) 27° (b) 54° (c) 100° (d) 108°

Second : m (∠ YXZ) = ...

(a) 27° (b) 54° (c) 100° (d) 108°

4 **In the opposite figure :**

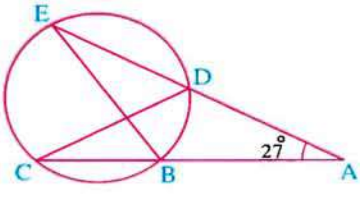


$\overline{AB}$ // $\overline{CD}$, m (∠ AWC) = 40° ,

then m (∠ DEB) = ... *(El-Sharkia 17)*

(a) 50° (b) 40° (c) 30° (d) 45°

5 **In the opposite figure :**



$\overrightarrow{AD}$ intersects the circle at D and E ,

$\overrightarrow{AB}$ intersects it at B and C

If m (∠ A) = 27° , AB = BE , then m (∠ CDE) = ...

(a) 13.5° (b) 54° (c) 27° (d) 36°

3 **In each of the following figures, find the value of the symbol used in measure, knowing that M is the centre of the circle :**

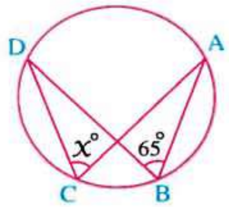


Fig. (1)

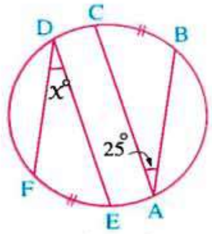


Fig. (2)

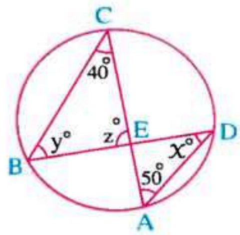


Fig. (3)

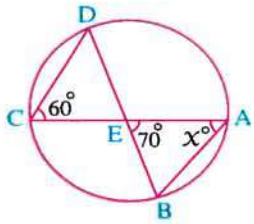


Fig. (4)

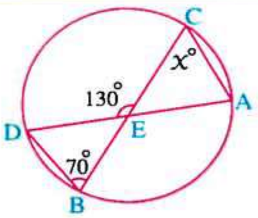


Fig. (5)

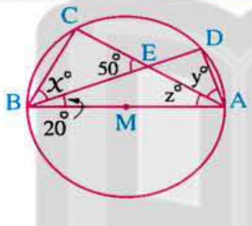


Fig. (6)

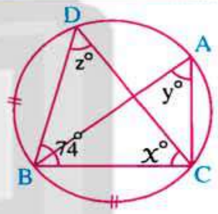


Fig. (7)

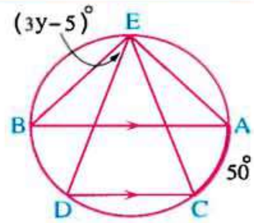


Fig. (8)

(El-Dakahlia 12)

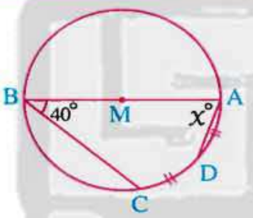


Fig. (9)

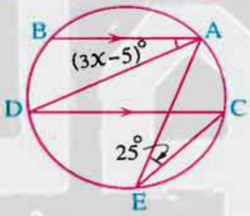


Fig. (10)

(El-Kalyoubia 18)

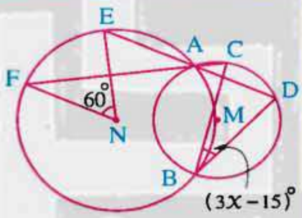


Fig. (11)

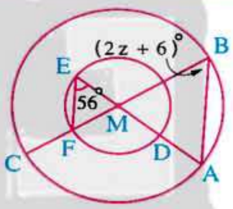


Fig. (12)

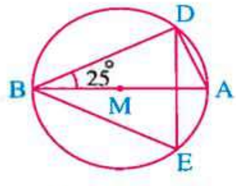
4 Prove that the inscribed angles subtended by the same arc in the circle are equal in measure. *(Matrouh 19, Kafr El-Sheikh 16)*

5 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m (∠ ABD) = 25°

Find : m (∠ DEB)



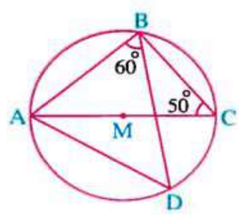
(Suez 11) « 65° »

6 **In the opposite figure :**

$\overline{AC}$ is a diameter in the circle M

, m (∠ C) = 50° , m (∠ ABD) = 60°

Find with proof : m (∠ CBD) and m (∠ BAD)



(Ismailia 19, Kafr El-Sheikh 13) « 30° , 70° »

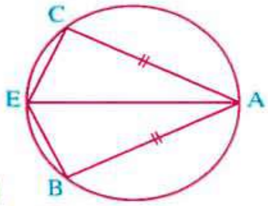
Unit 5

7 In the opposite figure :

$AB = AC$, $E \in \widehat{BC}$

Prove that :

m (∠ AEB) = m (∠ AEC) *(El-Menia 19 , Suez 18 , North Sinai 17)*

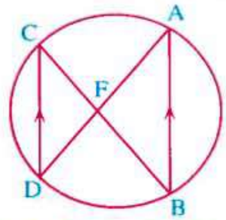


8 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two parallel chords in the circle

, $\overline{AD} \cap \overline{CB} = \{F\}$

Prove that : AF = FB



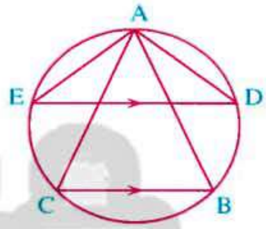
(Kafr El-Sheikh 08)

9 **In the opposite figure :**

ABC is a triangle inscribed in a circle ,

$\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)



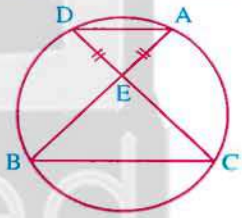
(Matrouh 19 , Ismailia 18 , El-Fayoum 17 , El-Gharbia 16)

10 **In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{E\}$

, EA = ED

Prove that : EB = EC



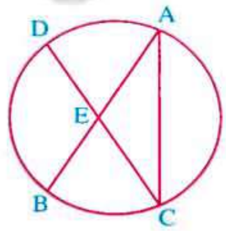
(Ismailia 19 , Kafr El-Sheikh 17 , El-Sharkia 16 , Suez 15 , El-Beheira 14 , S. Sinai 13)

11 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two equal chords in length in the circle

, $\overline{AB} \cap \overline{CD} = \{E\}$

Prove that : The triangle ACE is an isosceles triangle.



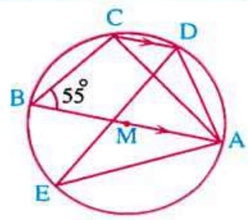
(El-Beheira 19 , El-Monofia 18 , El-Kalyoubia 11)

12 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

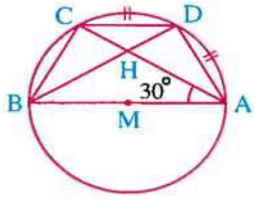
, $\overline{DC}$ // $\overline{AB}$, m (∠ ABC) = 55°

Find : m (∠ AED)



« 35° »

13 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M , C ∈ the circle M ,

m (∠ CAB) = 30° , D is the midpoint of $\overset{\frown}{AC}$,

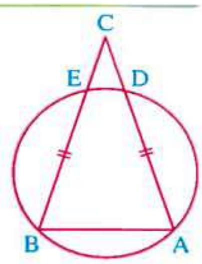
$\overline{DB} \cap \overline{AC} = \{H\}$

1 **Find :** m (∠ BDC) and m $(\overset{\frown}{AD})$

2 **Prove that :** $\overline{AB}$ // $\overline{DC}$

(Damietta 18 , Cairo 17) « 30° , 60° »

14 **In the opposite figure :**



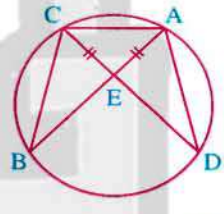
$\overline{AD}$ and $\overline{BE}$ are two equal chords in length in the circle

, $\overrightarrow{AD} \cap \overrightarrow{BE} = \{C\}$

Prove that : CD = CE

(El-Dakahlia 18 , Damietta 17 , Beni Suef 14 , El-Kalyoubia 13)

15 **In the opposite figure :**



$\overline{AB}$ and $\overline{DC}$ are two chords inside circle M

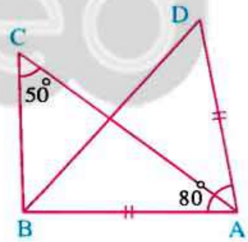
and are intersecting in E

If AE = CE ,

prove that : m (∠ ACB) = m (∠ CAD)

(Red Sea 11)

16 **In the opposite figure :**



AB = AD , m (∠ BAD) = 80°

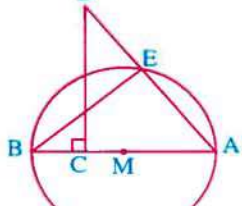
and m (∠ C) = 50°

Prove that :

The points A , B , C and D have one circle passing through them.

(Suez 16 , South Sinai 15 , Port Said 14)

17 **In the opposite figure :**



$\overline{AB}$ is a diameter in circle M in which

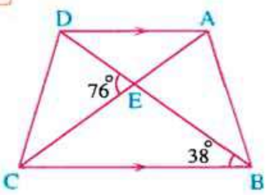
$\overline{AE}$ is a chord and $\overrightarrow{CD} \perp \overline{AB}$, $\overrightarrow{CD}$ intersects $\overrightarrow{AE}$ at D

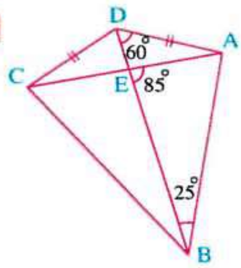
Prove that :

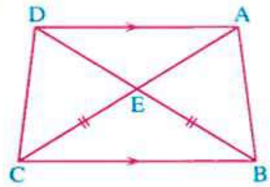
The points D , E , C and B have one circle passing through them.

Unit 5

18 **In each of the following figures prove that : a circle passing through the points A , B , C and D :**

1 

2 

3 

19 ABC is an isosceles triangle which has AB = AC , D is the midpoint of $\overline{BC}$, draw $\overrightarrow{BE} \perp \overline{AC}$, where $\overrightarrow{BE} \cap \overline{AC} = \{E\}$

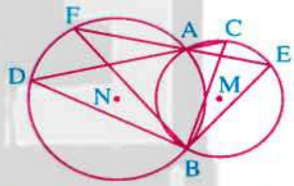
Prove that : The points A , B , D and E have one circle passing through them.

(Dakahlia 12)

20 $\overline{AB}$ is a diameter in the circle M , C $\in$ The circle where m ($\angle$ ABC) = 40° , D $\in \widehat{BC}$

Find : m ($\angle$ CDB) « 130° »

21 **In the opposite figure :**

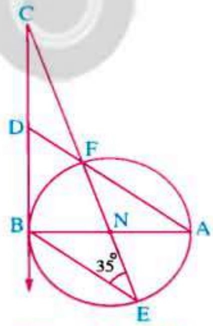


M and N are two intersecting circles at A and B , $\overleftrightarrow{AC}$ intersects the circle M at C and intersects the circle N at D , $\overleftrightarrow{AE}$ intersects the circle M at E and intersects the circle N at F

Prove that : m ($\angle$ EBC) = m ($\angle$ FBD)

(Qena 17 , El-Beheira 13)

22 **In the opposite figure :**

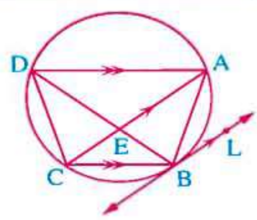


$\overline{AB}$ is a diameter in a circle of centre N , $\overrightarrow{CB}$ is a tangent to the circle at B , $\overrightarrow{CN}$ is drawn to cut the circle at F and E and $\overrightarrow{AF}$ is drawn to cut $\overrightarrow{CB}$ at D

If m ($\angle$ BEC) = 35°

Find : 1 m ($\angle$ BNC) 2 m ($\angle$ BCN) 3 m ($\angle$ BDA) « 70° , 20° , 55° »

23 **In the opposite figure :**



ABCD is a quadrilateral inscribed in a circle where $\overline{BC}$ // $\overline{AD}$, $\overline{AC} \cap \overline{BD} = \{E\}$, $\overleftrightarrow{BL}$ is a tangent to the circle at B where $\overleftrightarrow{BL}$ // $\overline{AC}$

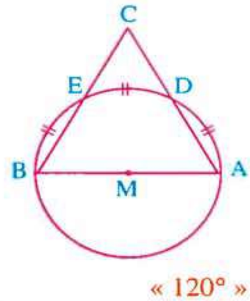
Prove that : 1 $\overrightarrow{DB}$ bisects $\angle$ ADC 2 m ($\angle$ CBD) = m ($\angle$ CDB)

24 ABC is an equilateral triangle inscribed in the circle M

Draw the diameter $\overline{CD}$

Prove that : m (∠ ABD) = m (∠ CBM) = m (∠ ACD) *(Beni Suef 08)*

25 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

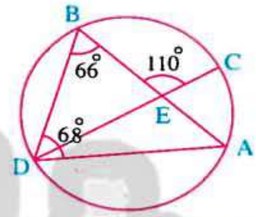
, D and E belong to $\overset{\frown}{AB}$ such that m $(\overset{\frown}{AD})$ = m $(\overset{\frown}{DE})$ = m $(\overset{\frown}{EB})$

If $\overrightarrow{AD} \cap \overrightarrow{BE} = \{C\}$

Prove that : CA = CB , then

Find : m $(\overset{\frown}{DEB})$ « 120° »

26 **In the opposite figure :**

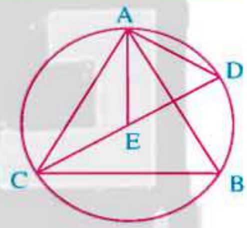


m (∠ B) = 66°

, m (∠ BEC) = 110° , m (∠ ADB) = 68°

Prove that : $\overline{CD}$ is a diameter in the circle.

27 **In the opposite figure :**



ABC is an equilateral triangle inscribed in a circle

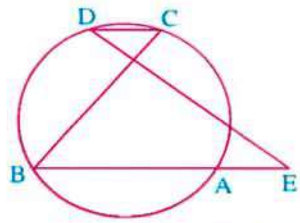
, D ∈ $\overset{\frown}{AB}$, E ∈ $\overline{DC}$, where AD = DE

Prove that : The triangle ADE is equilateral.

(Kafr El-Sheikh 18 , Matrouh 16 , Fayoum 15 , Alex. 11)

For excellent pupils

1 **In the opposite figure :**

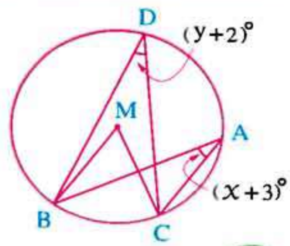


E is a point outside the circle.

Prove that : m (∠ E) < m (∠ BCD)

(Kalyoubia 12)

2 **In the opposite figure :**



M is a circle , ∠ A and ∠ D are two inscribed angles of measures $(x + 3)°$ and $(y + 2)°$ respectively. If $y^2 - x^2 = 53$

Find : m (∠ CMB) « 58° »

Summary of the first part of Unit 5
"From lesson 1 to lesson 3"

Summary
1.
2.
3.

The central angle :

It is the angle whose vertex is the centre of the circle and the two sides contain two radii in the circle.

The measure of the arc :

It is the measure of the central angle which subtends this arc.

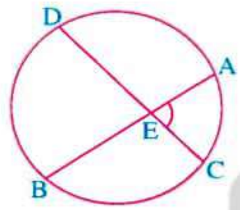
- The length of the arc $= \frac{\text{The measure of the arc}}{360°} \times 2 \pi r$
- In the same circle (or in congruent circles) , if the measures of arcs are equal , then the lengths of the arcs are equal and vice versa.
- In the same circle (or in congruent circles) , if the measures of arcs are equal , then their chords are equal in length , and vice versa.
- If two parallel chords are drawn in a circle , then the measures of the two arcs between them are equal.
- If a chord is parallel to a tangent of a circle , then the measures of the two arcs between them are equal.

The inscribed angle :

It is the angle whose vertex lies on the circle and its sides contain two chords of the circle.

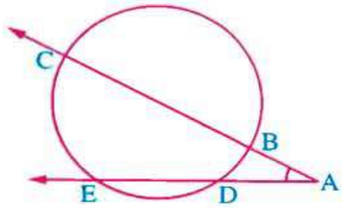
- The measure of the inscribed angle is half the measure of the central angle , subtended by the same arc.
- The measure of an inscribed angle is half the measure of the subtended arc.
- The inscribed angle in a semicircle is a right angle.
- The inscribed angle which is subtended by an arc of measure less than the measure of a semicircle is an acute angle.
- The inscribed angle which is subtended by an arc of measure greater than the measure of a semicircle is an obtuse angle.

Well known problem 1



$$m(\angle AEC) = \frac{1}{2}\left[m(\widehat{AC}) + m(\widehat{BD})\right]$$

Well known problem 2



$$m(\angle A) = \frac{1}{2}\left[m(\widehat{CE}) - m(\widehat{BD})\right]$$

- In the same circle , the measures of all inscribed angles subtended by the same arc are equal.
- In the same circle (or in any number of circles) the measures of the inscribed angles subtended by arcs of equal measures are equal.
- In the same circle (or in any number of circles) the inscribed angles of equal measures subtend arcs of equal measures.
- If two angles subtended by the same base and on the same side of it have the same measure , then their vertices are on an arc of a circle and the base is a chord of it.

Exams on the first part of unit Five
from lesson (1) to lesson (3)

Model 1

Answer the following questions :

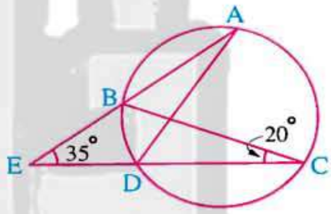
1 **Choose the correct answer from those given :**

1 The inscribed angle which is subtended by major arc in a circle is ....
(a) reflex. (b) right. (c) obtuse. (d) acute.

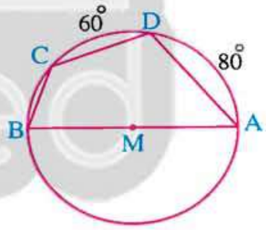
2 If the length of an arc of a circle is $\frac{1}{3}\pi$ r cm. , then its opposite central angle of measure equals ....
(a) 30° (b) 60° (c) 120° (d) 240°

3 The ratio between the measure of the inscribed angle and the measure of the central angle that has the same subtended arc equals 2 : ....
(a) 1 (b) 3 (c) 4 (d) 6

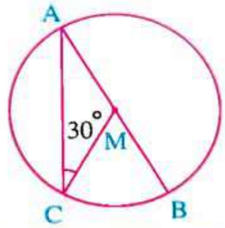
4 **In the opposite figure :**
If m (∠ E) = 35° , m (∠ C) = 20°
, then m ($\widehat{AC}$) = ...
(a) 135° (b) 110°
(c) 65° (d) 55°



5 **In the opposite figure :**
$\overline{AB}$ is a diameter in the circle M
, m ($\widehat{AD}$) = 80° , m ($\widehat{CD}$) = 60°
, then m (∠ DAB) = ...
(a) 30° (b) 40°
(c) 50° (d) 100°

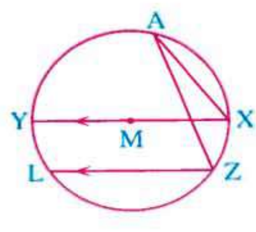


6 **In the opposite figure :**
m (∠ BMC) = ...
(a) 30° (b) 60°
(c) 90° (d) 120°



2 **[a]** A is a point outside the circle M , $\overrightarrow{AB}$ is a tangent to the circle at B , $\overrightarrow{AM}$ intersects the circle M at C and D respectively , m (∠ A) = 40°
Find by proof : m (∠ BDC)

[b] In the opposite figure :
$\overline{XY}$ is a diameter in the circle M , $\overline{XY}$ // $\overline{ZL}$
, m ($\widehat{ZL}$) = 70°
Find : m (∠ A)

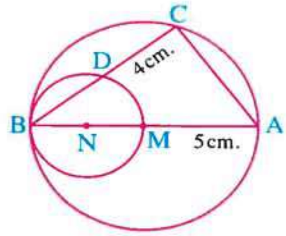


3 **[a] In the opposite figure :**

M and N are two circles touching internally at B

, AM = 5 cm. , CD = 4 cm.

Find with proof the length of $\overline{AC}$

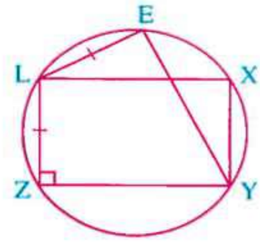


[b] In the opposite figure :

XYZL is a rectangle inscribed in a circle

, the chord $\overline{LE}$ is drawn , where LE = LZ

Prove that : YE = XL

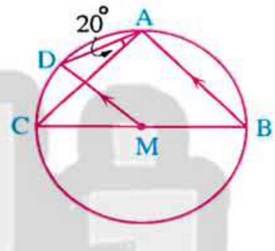


4 **[a] In the opposite figure :**

$\overline{BC}$ is a diameter in the circle M

, $\overline{MD} \,//\, \overline{BA}$, m (∠ CAD) = 20°

Find : m (∠ ACB)

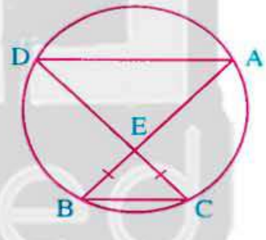


[b] In the opposite figure :

$\overline{AB} \cap \overline{DC} = \{E\}$

, EB = EC

Prove that : EA = ED



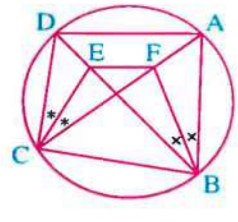
5 **[a] In the opposite figure :**

ABCD is a quadrilateral which has $\overrightarrow{CE}$ bisects ∠ ACD

and $\overrightarrow{BF}$ bisects ∠ ABD

Prove that :

The points B , F , E and C have one circle passing through them.

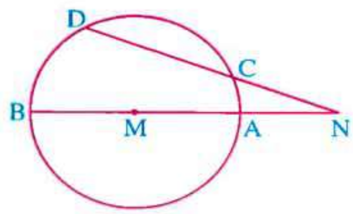


[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{BA} \cap \overrightarrow{DC} = \{N\}$

Prove that : NB > ND



Unit 5

Model 2

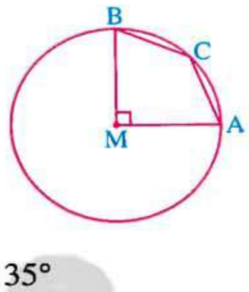
Answer the following questions :

1 Choose the correct answer from those given :

1 If the measure of a central angle is 80° , then the measure of the inscribed angle subtended by the same arc equals ....

(a) 40° (b) 80° (c) 160° (d) 100°

2 **In the opposite figure :**



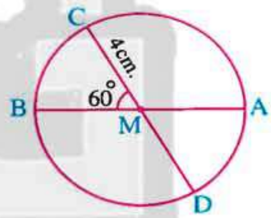
M is a circle

, $\overline{AM} \perp \overline{MB}$

, then m (∠ ACB) = ...

(a) 45° (b) 90° (c) 145° (d) 135°

3 **In the opposite figure :**



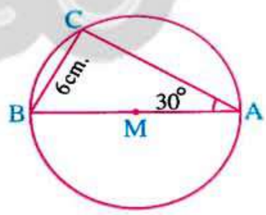
M is a circle , MC = 4 cm. , m (∠ CMB) = 60°

, the length of $(\overset{\frown}{BD})$ = ... cm.

(a) 4 π (b) 8 π

(c) $\frac{8}{3}$ π (d) 16 π

4 **In the opposite figure :**



If $\overline{AB}$ is a diameter in the circle M

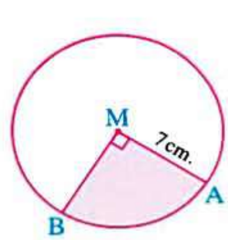
, m (∠ A) = 30° , BC = 6 cm.

, then the radius length of the circle = ...

(a) 6 cm. (b) 12 cm.

(c) 18 cm. (d) 2 cm.

5 **In the opposite figure :**



$\overline{MA}$, $\overline{MB}$ are two perpendicular radii

in the circle M whose radius length is 7 cm.

, then the perimeter of the shaded part = ... cm. $(\pi = \frac{22}{7})$

(a) 14 (b) 11

(c) 38 $\frac{1}{2}$ (d) 25

6 **In the opposite figure :**

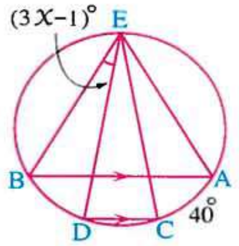
$\overline{AB} \parallel \overline{CD}$, m $(\widehat{AC}) = 40°$

, m (∠ DEB) = $(3\,x - 1)°$

, then x = ...

(a) $\left(\frac{41}{3}\right)°$ (b) 7°

(c) 21° (d) 41°



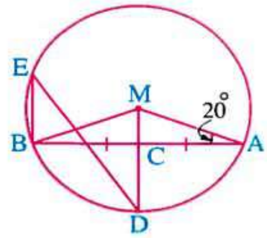
2 **[a] In the opposite figure :**

C is the midpoint of $\overline{AB}$

, $\overrightarrow{MC} \cap$ the circle M = {D}

, m (∠ MAB) = 20°

Find : m (∠ BED) , m (∠ ADB)



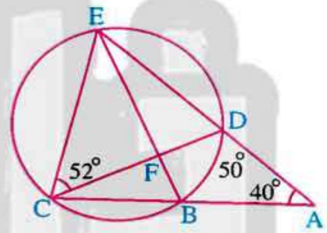
[b] In the opposite figure :

$\overrightarrow{CB} \cap \overrightarrow{ED}$ = {A} , $\overline{BE} \cap \overline{CD}$ = {F}

If m (∠ A) = 40° , m $(\widehat{BD}) = 50°$

, m (∠ DCE) = 52°

Find : 1 m $(\widehat{CE})$ 2 m $(\widehat{BC})$



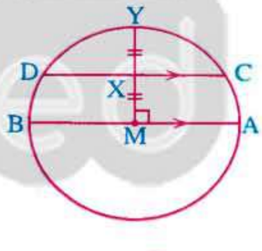
3 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overline{CD} \parallel \overline{AB}$, X is the midpoint of $\overline{MY}$

, $\overline{MY} \perp \overline{AB}$

Find : m $(\widehat{AC})$, m $(\widehat{CY})$

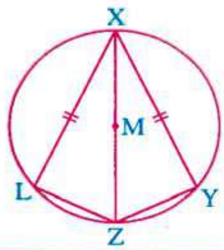


[b] In the opposite figure :

XYZL is a quadrilateral inscribed in the circle M

, $\overline{XZ}$ is a diameter in it , XY = XL

Prove that : m $(\widehat{YZ})$ = m $(\widehat{LZ})$



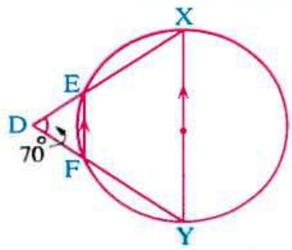
4 **[a] In the opposite figure :**

$\overline{XY}$ is a diameter in the circle

, $\overline{EF}$ is a chord in it

where $\overline{XY} \parallel \overline{EF}$, m (∠ D) = 70°

Find : m $(\widehat{EX})$



Unit 5

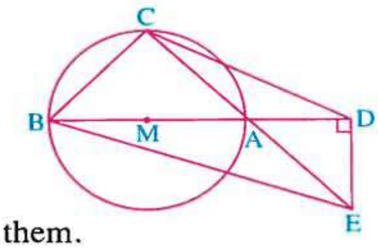
[b] In the opposite figure :

$\overline{AB}$ is a diameter of the circle M

, $\overrightarrow{DE} \perp \overrightarrow{BA}$, $\overrightarrow{CA} \cap \overrightarrow{DE} = \{E\}$

Prove that :

The points D , E , B and C have one circle passing through them.

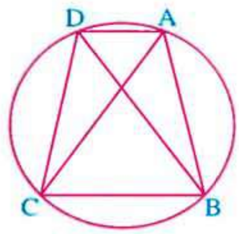


5 **[a] In the opposite figure :**

ABCD is quadrilateral inscribed in a circle where AC = BD

, AB = $(2\,\mathcal{X} - 1)$ cm. , CD = $(\mathcal{X} + 3)$ cm.

Find : The length of $\overline{AB}$



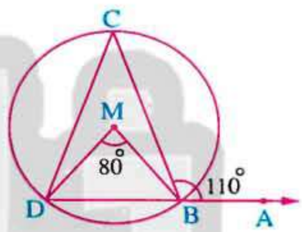
[b] In the opposite figure :

M is a circle , m (∠ BMD) = 80°

, m (∠ ABC) = 110°

1 **Find :** m (∠ CDB)

2 **Prove that :** CB = CD



Exercise 9

The cyclic quadrilateral and its properties

From the school book

1 **In each of the following figures:**
If ABCD is a cyclic quadrilateral $\overline{AC} \cap \overline{BD} = \{F\}$, find the measures of the angles denoted by (?) :

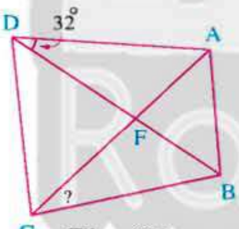


Fig. (1)

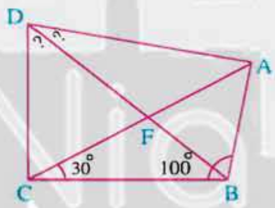


Fig. (2)

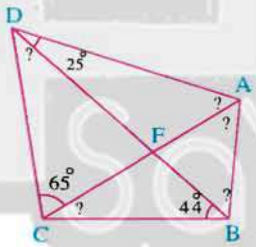


Fig. (3)

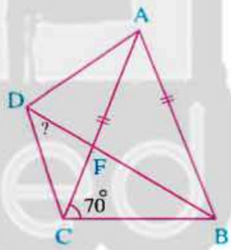


Fig. (4)

2 **In each of the following figures , find the measure of the angle denoted by the sign (?) given that M is the centre of the circle :**

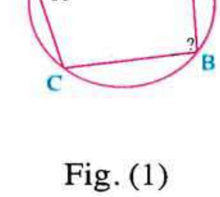


Fig. (1)

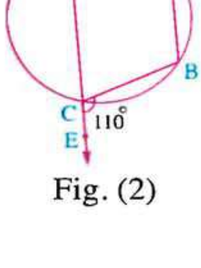


Fig. (2)

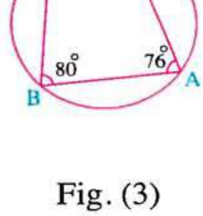


Fig. (3)

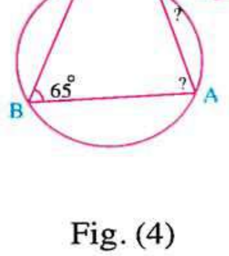


Fig. (4)

Unit 5

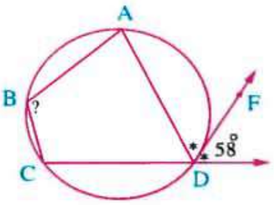


Fig. (5)

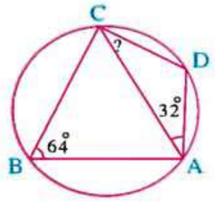


Fig. (6)

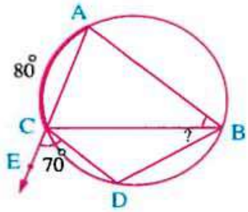


Fig. (7)

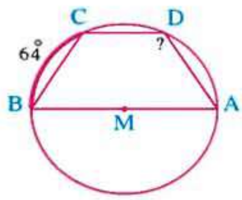


Fig. (8)

3 **In each of the following figures , find the value of the symbol used in measure :**

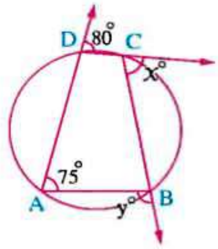


Fig. (1)

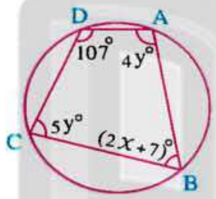


Fig. (2)

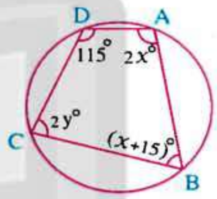


Fig. (3)

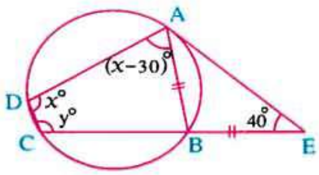


Fig. (4)

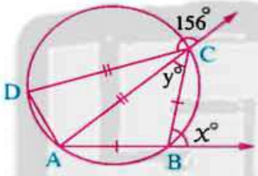


Fig. (5)

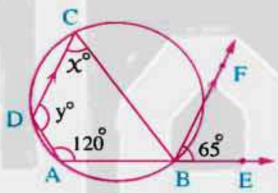


Fig. (6)

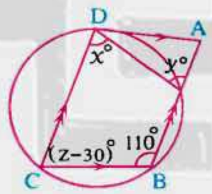


Fig. (7)

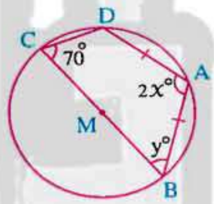


Fig. (8)

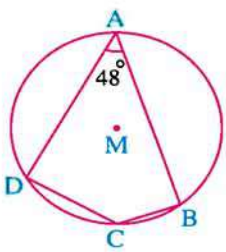
4 **Complete the following :**

1. If the quadrilateral is cyclic , then each two opposite angles in it are .... *(Cairo 17)*
2. The measure of the exterior angle at a vertex of the cyclic quadrilateral is equal to the measure of the .... angle. *(Dakahlia 12)*
3. In the cyclic quadrilateral ABCD , if m (∠ C) = 115° , then m (∠ A) = ...° *(Alex. 05)*
4. If the figure ABCD is a cyclic quadrilateral , m (∠ A) = 60° , then the measure of the exterior angle at the vertex C equals .... °
5. **In the opposite figure :**

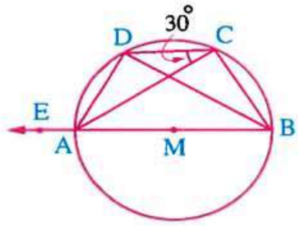
   If M is a circle , m (∠ A) = 48°

   , then m (∠ C) = ... °

   and m ($\widehat{BD}$ the major) = ... ° *(New Valley 17)*



6 **In the opposite figure :**



A circle of centre M

, if m (∠ DCA) = 30° , then :

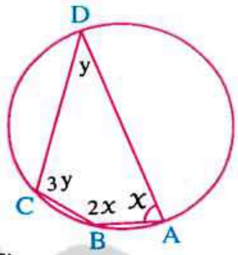
First : m (∠ DBA) = ... °

Second : m (∠ DAE) = ... °

7 If ABCD is a cyclic quadrilateral and m (∠ B) = $\frac{1}{4}$ m (∠ D) ,

then m (∠ B) = ... °

8 **In the opposite figure :**



The figure ABCD is a cyclic quadrilateral

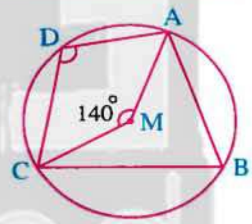
, then x = ... , y = ...

9 In the cyclic quadrilateral ABCD , if m (∠ A) = 2 m (∠ B) = 5 m (∠ C) ,

then m (∠ D) = ...° *(Alex. 06)*

5 Choose the correct answer from those given :

1 **In the opposite figure :**



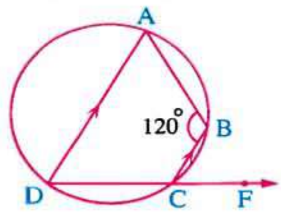
In the circle M

, if m (∠ AMC) = 140°

, then m (∠ ADC) = ... *(El-Fayoum 17)*

(a) 40° (b) 70° (c) 110° (d) 140°

2 **In the opposite figure :**



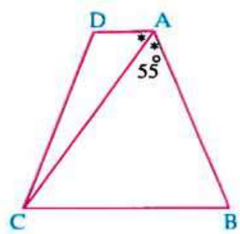
If m (∠ B) = 120°

, $\overline{BC}$ // $\overline{AD}$

, then m (∠ BCF) = ... *(North Sinai 17)*

(a) 30° (b) 60° (c) 80° (d) 120°

3 **In the opposite figure :**



ABCD is a cyclic quadrilateral

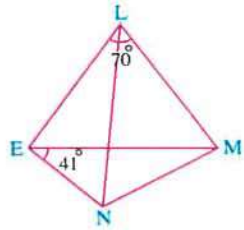
in which $\overrightarrow{AC}$ bisects ∠ BAD ,

If m (∠ BAC) = 55°, then m (∠ BCD) = ... *(Cairo 05)*

(a) 55° (b) 70° (c) 110° (d) 125°

Unit 5

4 **In the opposite figure :**



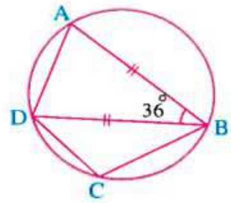
LMNE is a cyclic quadrilateral

, m (∠ MLE) = 70° , m (∠ MEN) = 41°

, then m (∠ EMN) = ...

(a) 70° (b) 41° (c) 29° (d) 110°

5 **In the opposite figure :**



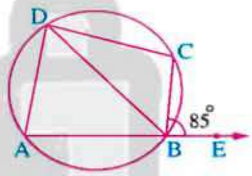
If AB = BD

and m (∠ ABD) = 36°

, then m (∠ C) = ... *(Luxor 19)*

(a) 140° (b) 70° (c) 54° (d) 108°

6 **In the opposite figure :**



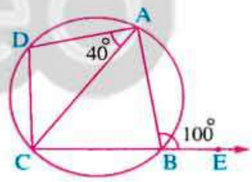
$E \in \overrightarrow{AB}$, $E \notin \overline{AB}$, m $(\overset{\frown}{AB})$ = 110°

and m (∠ CBE) = 85°

Find : m (∠ BDC)

(Souhag 19 , El-Kalyoubia 18 , Damietta 18 , El-Beheira 14 , Port Said 13) « 30° »

7 **In the opposite figure :**



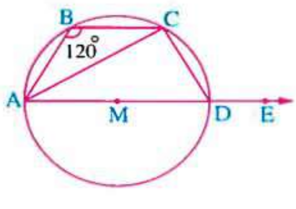
m (∠ ABE) = 100°

and m (∠ CAD) = 40°

Prove that : m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{AD})$

(Giza 19 , Red sea 18 , El-Gharbia 17 , Souhag 15)

8 **In the opposite figure :**



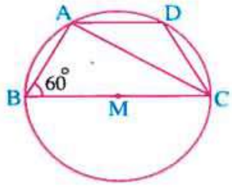
ABCD is a quadrilateral inscribed in a circle M

where m (∠ B) = 120° , $\overline{AD}$ is a diameter in the circle , $E \in \overrightarrow{AD}$

1 **Find :** m (∠ CDE) , m (∠ CAD)

2 If DC = 7 cm. , **find :** The length of $\overset{\frown}{AD}$ $\left(\pi \approx \frac{22}{7}\right)$ « 120° , 30° , 22 cm. »

9 **In the opposite figure :**

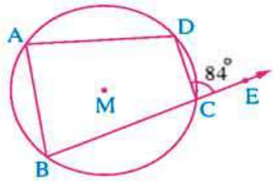


ABCD is a cyclic quadrilateral , $\overline{CB}$ is a diameter in the circle M ,

m (∠ ABC) = 60° , the length of $\overset{\frown}{AD}$ = the length of $\overset{\frown}{CD}$

Prove that : $\overrightarrow{CA}$ bisects ∠ DCB *(Monofia 08)*

10 **In the opposite figure :**



ABCD is a quadrilateral inscribed in the circle M

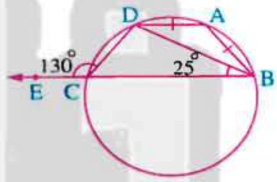
, E ∈ $\overrightarrow{BC}$, m (∠ DCE) = 84°

and m (∠ B) = $\frac{1}{2}$ m (∠ D)

Find :

1 m (∠ A) 2 m (∠ B) « 84° , 60° »

11 **In the opposite figure :**



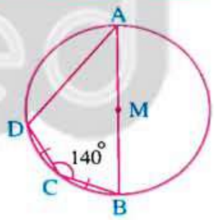
ABCD is a cyclic quadrilateral in which :

AB = AD ,

m (∠ CBD) = 25° , E ∈ $\overrightarrow{BC}$ and m (∠ ECD) = 130°

Prove that : AD = DC

12 **In the opposite figure :**



ABCD is a quadrilateral inscribed in a circle M where

M ∈ $\overline{AB}$, CB = CD and m (∠ BCD) = 140°

Find : 1 m (∠ A)

2 m (∠ D)

(Matrouh 17 , Kafr El-Sheikh 14) « 40° , 110° »

13 **With the assistance of the given figures, find with proof the measures of the angles of the figure ABCD :**

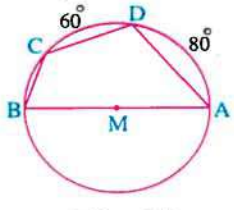


Fig. (1)

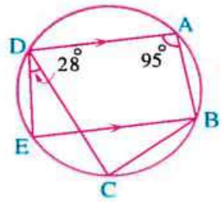


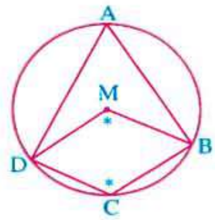
Fig. (2)

Unit 5

14 **In the opposite figure :**

m (∠ BMD) = m (∠ BCD)

Find : m (∠ A)



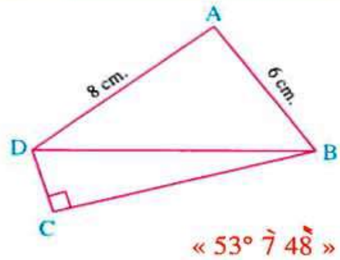
(Souhag 18) « 60° »

15 **In the opposite figure :**

ABCD is a cyclic quadrilateral in which :

m (∠ C) = 90° , AB = 6 cm. and AD = 8 cm.

Find : m (∠ ABD)



« 53° 7' 48" »

16 A is a point outside a circle , $\overrightarrow{AB}$ is drawn to cut the circle at B and C respectively , $\overrightarrow{AD}$ is drawn to cut the circle at D and E respectively. If AC = AE

Prove that :

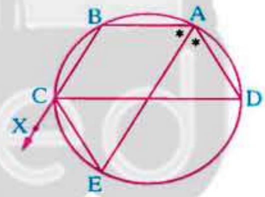
1 $\overline{BD}$ // $\overline{CE}$ 2 m $(\widehat{BC})$ = m $(\widehat{ED})$

17 **In the opposite figure :**

ABCD is a quadrilateral inscribed in a circle M where

$\overrightarrow{AE}$ bisects ∠ BAD and cuts the circle at E

Prove that : $\overrightarrow{CE}$ bisects ∠ XCD



(Assiut 12)

18 ABCD is a cyclic quadrilateral in which $\overline{AD}$ // $\overline{BC}$ and m (∠ C) = 105°

Find :

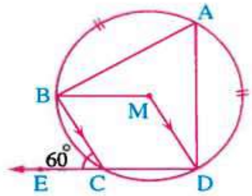
1 m (∠ A) 2 m (∠ B) « 75° , 105° »

19 ABCD is a cyclic quadrilateral , AB = AD , m (∠ C) = 124° and m (∠ BAC) = 36°

Find :

1 m (∠ ACD) 2 m (∠ ADC) « 62° , 98° »

20 **In the opposite figure :**



m (∠ BCE) = 60° , $\overline{BC}$ // $\overline{MD}$

and A is the midpoint of $\widehat{BD}$ the major

Prove that : 1 The figure BMDC is a rhombus.

2 $\overline{AC}$ is a diameter in the circle.

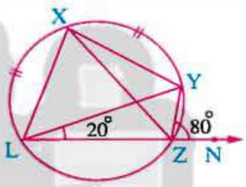
21 $\overline{BC}$ is a diameter in the circle M, $\overline{BA}$ is a chord in it, D ∈ $\widehat{AC}$ such that :

m (∠ ADC) = 118° , $\overrightarrow{AE}$ // $\overline{DC}$ and intersects the circle at E

1 **Find :** m (∠ ABC) « 62° »

2 **Prove that :** m (∠ ACD) = m (∠ CBE)

22 **In the opposite figure :**



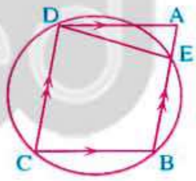
X is the midpoint of $\widehat{YL}$,

m (∠ YZN) = 80° and m (∠ YLZ) = 20°

Find :

1 m (∠ ZXL) 2 m $(\widehat{XYZ})$ « 60° , 140° »

23 **In the opposite figure :**



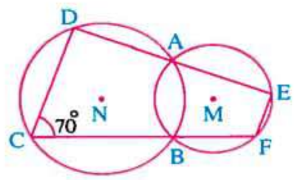
ABCD is a parallelogram ,

the circle which passes through

the points B , C and D intersects $\overline{AB}$ at E

Prove that : AD = ED *(El-Fayoum 11)*

24 **In the opposite figure :**



M and N are two intersecting circles at A and B ,

$\overleftrightarrow{AD}$ is drawn to intersect circle M at E and circle N at D ,

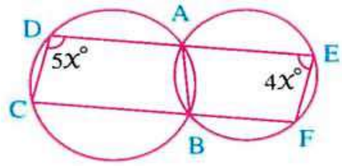
$\overleftrightarrow{BC}$ is drawn to intersect circle M at F and circle N at C

and m (∠ C) = 70°

1 **Find :** m (∠ F) 2 **Prove that :** $\overleftrightarrow{CD}$ // $\overleftrightarrow{EF}$ *(El-Monofia 17)* « 110° »

Unit 5

25 **In the opposite figure :**



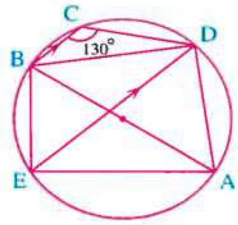
Two intersecting circles at A and B

, A ∈ $\overline{ED}$, B ∈ $\overline{FC}$, m (∠ D) = 5 x°

and m (∠ E) = 4 x°

Find with proof : m (∠ ABF) « 100° »

26 **In the opposite figure :**



ABCD is a cyclic quadrilateral ,

$\overline{AB}$ is a diameter in the circle.

Draw $\overrightarrow{DE}$ // $\overline{BC}$ to cut the circle at E

1 **Prove that :** m (∠ DBC) = m (∠ BAE)

2 If m (∠ C) = 130°

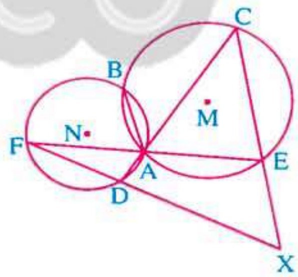
Find : m (∠ AED) « 40° »

For excellent pupils

1 ABCDE is a pentagon inscribed in a semicircle of diameter $\overline{AB}$

Prove that : m (∠ AED) + m (∠ BCD) = 270°

2 **In the opposite figure :**



$\overline{AB}$ is a common chord of the two circles M and N ,

C ∈ the circle M , F ∈ the circle N. If $\overrightarrow{CA}$ intersects the circle N at D and $\overrightarrow{FA}$ intersects the circle M at E

, $\overrightarrow{CE} \cap \overrightarrow{FD} = \{X\}$ and the figure AEXD is a cyclic quadrilateral.

Prove that : C , B and F are collinear.

Exercise 10 Cases of proving the cyclic quadrilateral

From the school book

1 **Which of the following figures is a cyclic quadrilateral ? Explain your answer :**

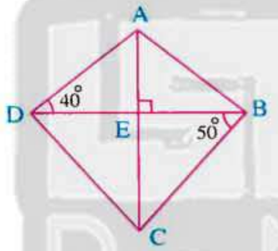


Fig. (1)

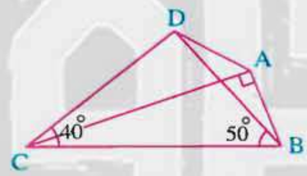


Fig. (2)

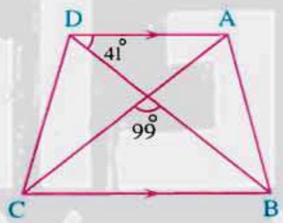


Fig. (3)

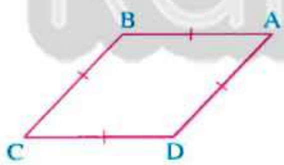


Fig. (4)

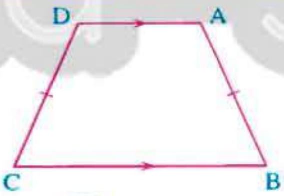


Fig. (5)

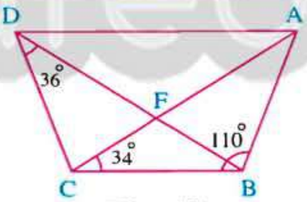
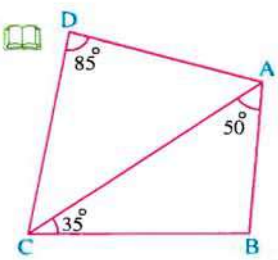


Fig. (6)

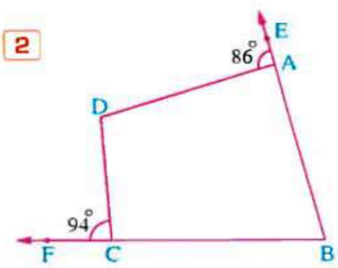
2 **In each of the following figures , prove that the figure ABCD is a cyclic quadrilateral :**

1

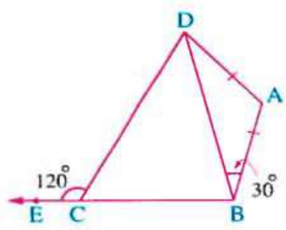


(South Sinai 16)

2

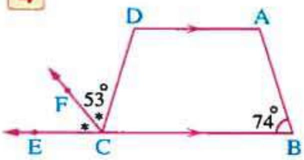


3



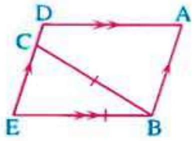
(Aswan 19 , Aswan 16 , Damietta 15)

4

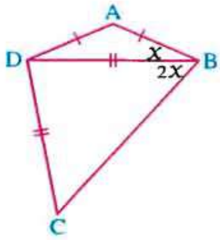


(Port Said 17 , Damietta 17)

5



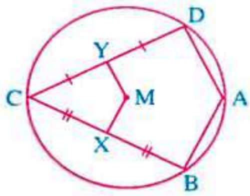
6



(El-Dakahlia 13)

3 Mention two cases in which the quadrilateral is a cyclic. *(Cairo 19 , Assiut 19 , Suez 19)*

4 **In the opposite figure :**



ABCD is a quadrilateral inscribed in a circle M

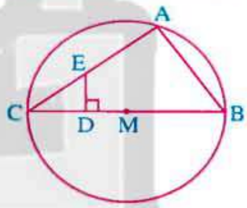
, X is the midpoint of $\overline{BC}$ and Y is the midpoint of $\overline{CD}$

Prove that :

1 The figure MXCY is a cyclic quadrilateral.

2 m (∠ XMY) = m (∠ BAD)

5 **In the opposite figure :**



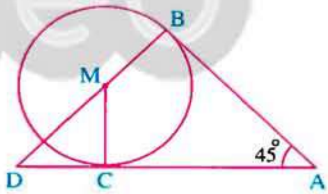
$\overline{BC}$ is a diameter in the circle M and $\overline{ED} \perp \overline{BC}$

Prove that :

1 The figure ABDE is a cyclic quadrilateral.

2 m (∠ CED) = $\frac{1}{2}$ m ($\overset{\frown}{AC}$) *(Cairo 18 , Giza 09)*

6 **In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ touch the circle M at B and C respectively

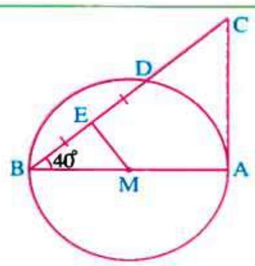
, m (∠ A) = 45°

Prove that :

1 The figure ABMC is a cyclic quadrilateral.

2 Δ MCD is an isosceles triangle. *(South Sinai 12)*

7 **In the opposite figure :**



$\overline{AB}$ is a diameter in a circle of centre M

, $\overrightarrow{AC}$ is a tangent to the circle at A

, E is the midpoint of $\overline{DB}$, m (∠ B) = 40°

1 **Prove that :** The figure AMEC is a cyclic quadrilateral.

2 **Find :** m (∠ C) *(New vally 14) « 50° »*

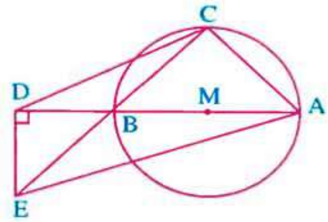
8 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

Draw $\overrightarrow{DE} \perp \overrightarrow{AB}$ and $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$

Prove that :

ACDE is a cyclic quadrilateral.



(El-Fayoum 11)

9 **In the opposite figure :**

$\overline{AC}$ is a diameter in the circle M

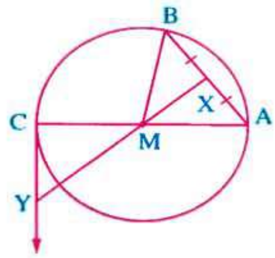
, X is the midpoint of $\overline{AB}$

and $\overrightarrow{CY}$ is a tangent to the circle cutting $\overrightarrow{XM}$ at Y

Prove that :

1 The figure AXCY is a cyclic quadrilateral.

2 m (∠ BMC) = 2 m (∠ MYC)



10 **In the opposite figure :**

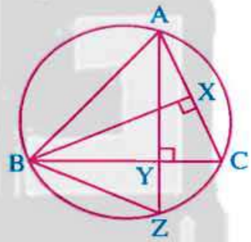
ABC is a triangle drawn in a circle , $\overrightarrow{BX} \perp \overline{AC}$, $\overrightarrow{AY} \perp \overline{BC}$

cuts it at Y and cuts the circle at Z

Prove that :

1 ABYX is a cyclic quadrilateral.

2 $\overrightarrow{BC}$ bisects ∠ XBZ



(El-Gharbia 17 , El-Beheira 17)

11 **In the opposite figure :**

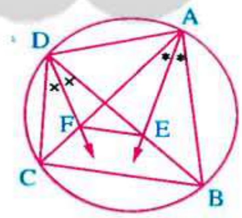
ABCD is a cyclic quadrilateral which has $\overrightarrow{AE}$ bisects ∠ BAC

and $\overrightarrow{DF}$ bisects ∠ BDC

Prove that :

1 AEFD is a cyclic quadrilateral.

2 $\overline{EF}$ // $\overline{BC}$



(El-Gharbia 18 , Luxor 16 , El-Dakahlia 13)

12 ABCD is a square , $\overrightarrow{AX}$ bisects ∠ BAC and intersects $\overline{BD}$ at X , $\overrightarrow{DY}$ bisects ∠ CDB and intersects $\overline{AC}$ at Y

Prove that :

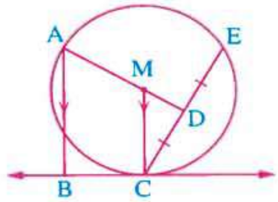
1 AXYD is a cyclic quadrilateral.

2 m (∠ AYX) = 45°

(Alexandria 16 , El-Sharkia 12)

Unit 5

13 **In the opposite figure :**



M is a circle , D is the midpoint of the chord $\overline{EC}$

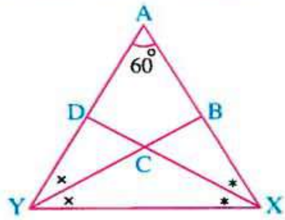
, $\overleftrightarrow{BC}$ is a tangent to the circle M at C and $\overline{AB} // \overline{MC}$

Prove that :

The figure ABCD is a cyclic quadrilateral.

(Cairo 12)

14 **In the opposite figure :**



Δ AXY in which m ($\angle$ A) = 60°

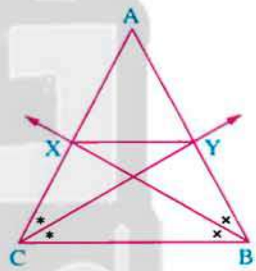
, $\overrightarrow{XD}$ bisects $\angle$ AXY , $\overrightarrow{YB}$ bisects $\angle$ AYX

Prove that :

ABCD is a cyclic quadrilateral.

(El-Beheira 16)

15 **In the opposite figure :**



ABC is a triangle in which : AB = AC ,

$\overrightarrow{BX}$ bisects $\angle$ B and intersects $\overline{AC}$ at X ,

$\overrightarrow{CY}$ bisects $\angle$ C and intersects $\overline{AB}$ at Y

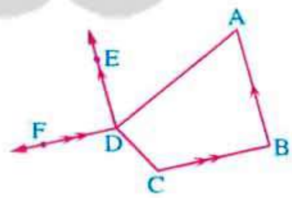
Prove that :

1 BCXY is a cyclic quadrilateral.

2 $\overleftrightarrow{XY} // \overleftrightarrow{BC}$

(El-Fayoum 17 , Assiut 11)

16 **In the opposite figure :**



$\overline{AB} // \overrightarrow{DE}$, $\overline{BC} // \overrightarrow{DF}$

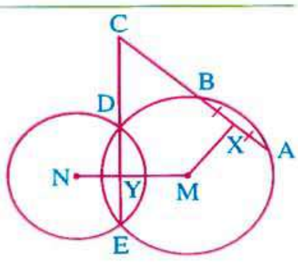
and m ($\angle$ ADE) + m ($\angle$ CDF) = 180°

Prove that :

The figure ABCD is cyclic quadrilateral.

(El-Monofia 06)

17 **In the opposite figure :**



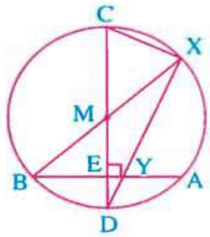
X is the midpoint of $\overline{AB}$, $\overline{MN} \cap \overline{EC} = \{Y\}$

1 **Prove that :** CXMY is a cyclic quadrilateral.

2 Find the centre of the circle which passes through the vertices of the figure CXMY

(Ismailia 17)

18 **In the opposite figure :**



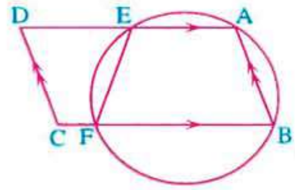
$\overline{AB}$ is a chord in the circle M and $\overline{CD}$ is a perpendicular diameter on $\overline{AB}$ and intersects it at E , $\overrightarrow{BM}$ intersects the circle at X and $\overline{XD} \cap \overline{AB} = \{Y\}$

Prove that : 1 XYEC is a cyclic quadrilateral.

2 m ($\angle$ DYB) = m ($\angle$ DBX)

(New valley 19 , Cairo 11)

19 **In the opposite figure :**



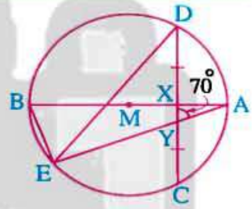
ABCD is a parallelogram.

A circle is drawn to pass through the two points A and B to cut $\overline{AD}$ at E and $\overline{BC}$ at F

Prove that : The figure CDEF is a cyclic quadrilateral.

(Luxor 19)

20 **In the opposite figure :**



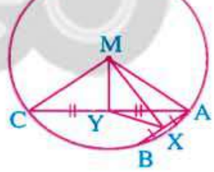
$\overline{AB}$ is a diameter in the circle M , X is the midpoint of $\overline{DC}$, m ($\angle$ AYX) = 70°

1 **Prove that :** The figure XYEB is a cyclic quadrilateral.

2 **Find :** m ($\angle$ ADE)

(Damietta 08) « 70° »

21 **In the opposite figure :**



A circle with centre M , X and Y are the two midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.

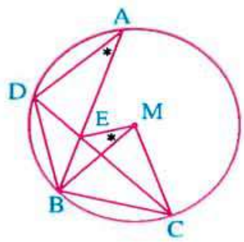
Prove that :

1 AXYM is a cyclic quadrilateral.

2 m ($\angle$ MXY) = m ($\angle$ MCY)

3 $\overline{AM}$ is a diameter in the circle passing through the points A , X , Y and M *(Red Sea 12)*

22 **In the opposite figure :**



$\overline{AB}$, $\overline{CD}$ are two chords in the circle M , $\overline{AB} \cap \overline{CD} = \{E\}$ and m ($\angle$ BAD) = m ($\angle$ BME)

Prove that :

1 The figure MCBE is a cyclic quadrilateral.

2 m ($\angle$ CEB) = 2 m ($\angle$ CDB)

Unit 5

23 ABC is a triangle inscribed in a circle , $X \in \widehat{AB}$, $Y \in \widehat{AC}$, where $m(\widehat{AX}) = m(\widehat{AY})$, $\overline{CX} \cap \overline{AB} = \{D\}$ and $\overline{BY} \cap \overline{AC} = \{E\}$

Prove that :

1 BCED is a cyclic quadrilateral.

2 $m(\angle DEB) = m(\angle XAB)$ *(El-Monofia 19 , New valley 19 , Alexandria 15)*

24 ABCD is a quadrilateral inscribed inside a circle , $F \in \overline{AB}$

Draw $\overrightarrow{FE} \parallel \overline{BC}$ to cut $\overline{CD}$ at E , $\overrightarrow{DF} \cap \overrightarrow{CB} = \{X\}$

Prove that : 1 The figure AFED is a cyclic quadrilateral.

2 $m(\angle BXF) = m(\angle EAD)$ *(Matrouh 17)*

25 ABC is a triangle. A circle of diameter $\overline{BC}$ is drawn to cut $\overline{AB}$ at D and $\overline{AC}$ at E

If $\overline{BE} \cap \overline{CD} = \{F\}$,

prove that : 1 ADFE is a cyclic quadrilateral.

2 $m(\angle DAF) = m(\angle BCD)$

26 $\overline{AB}$ is a diameter in a circle. $D \in \overline{AB}$

Draw $\overrightarrow{DE} \perp \overline{AB}$ where E is outside the circle and draw $\overrightarrow{EA}$ to cut the circle at X

Draw $\overrightarrow{XD}$ to cut the circle at Y

Prove that :

1 The figure EBDX is a cyclic quadrilateral.

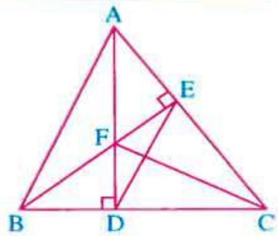
2 $\overrightarrow{BA}$ bisects $\angle EBY$

27 ABC is an inscribed triangle in a circle which has $AB > AC$ and $D \in \overline{AB}$, where $AC = AD$, $\overrightarrow{AE}$ bisects $\angle A$ and intersects $\overline{BC}$ at E and intersects the circle at F

Prove that : BDEF is a cyclic quadrilateral.

(Kafr El-Sheikh 19 , El-Menia 19 , El-Monofia 18 , El-Menia 18)

28 **In the opposite figure :**



ABC is a triangle.

$\overline{AD} \perp \overline{BC}$, $\overline{BE} \perp \overline{AC}$ and $\overline{AD} \cap \overline{BE} = \{F\}$

1 Mention two cyclic quadrilaterals. (give reasons)

2 **Prove that :** $m(\angle ECF) = m(\angle EBA)$

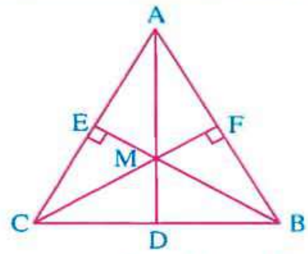
29 **In the opposite figure :**

ΔABC , $\overline{BE} \perp \overline{AC}$, $\overline{CF} \perp \overline{AB}$, $\overline{CF} \cap \overline{BE} = \{M\}$

, $\overrightarrow{AM} \cap \overline{BC} = \{D\}$

Prove that :

MDCE is a cyclic quadrilateral.

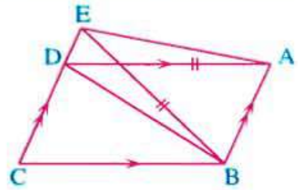


(South Sinai 17)

30 **In the opposite figure :**

ABCD is a parallelogram , $E \in \overrightarrow{CD}$, where BE = AD

Prove that : ABDE is a cyclic quadrilateral.



(El-Beheira 18 , Luxor 17 , South Sinai 13)

31 **In the opposite figure :**

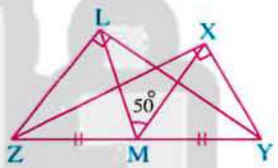
m (∠YXZ) = m (∠ YLZ) = 90° , M is the midpoint of $\overline{YZ}$ and m (∠ XML) = 50°

Find : m (∠ XYL) in degrees.

Prove that :

1 m (∠ XYL) = m (∠ XZL)

2 m (∠ XMZ) = m $(\widehat{XL})$ + m $(\widehat{LZ})$



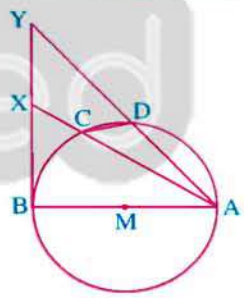
« 25° »

32 **In the opposite figure :**

$\overline{AB}$ is a diameter in circle M , $\overline{AC}$ and $\overline{AD}$ are two chords in it and in one side from $\overline{AB}$

A tangent to the circle was drawn from B and intersected $\overrightarrow{AC}$ at X and $\overrightarrow{AD}$ at Y

Prove that : XYDC is a cyclic quadrilateral.



33 **In the opposite figure :**

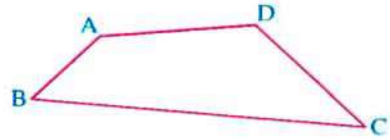
If ABCD is a quadrilateral ,

, m (∠ A) = 7 x° , m (∠ B) = 4 x° – 30°

, m (∠ C) = 2 x° and m (∠ D) = 5 x° + 30°

Prove that :

The figure ABCD is a cyclic quadrilateral.

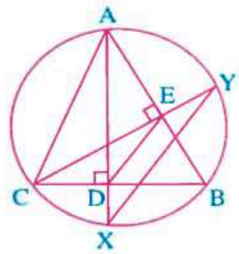


Unit 5

34 **In the opposite figure :**

ABC is a triangle inscribed in a circle , $\overline{AX} \perp \overline{BC}$ cutting it at D and $\overline{CY} \perp \overline{AB}$ cutting it at E

Prove that : $\overline{XY} \parallel \overline{DE}$



35 ABCD is a quadrilateral in which m ($\angle$ A) = 90° and the lengths of its sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ are 8 , $5\sqrt{3}$, 5 and 6 respectively.

Prove that : The figure ABCD is a cyclic quadrilateral and determine the centre of the circumcircle of it and also its radius length. « 5 cm. »

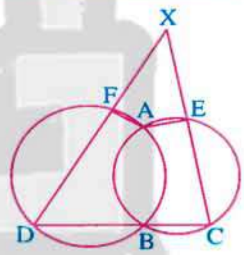
36 $\overline{AB}$ is a chord in a circle M , C is the midpoint of $\widehat{AB}$. From C the two rays $\overrightarrow{CX}$ and $\overrightarrow{CY}$ are drawn to cut $\overline{AB}$ at X and Y respectively and they cut the circle at L and Z respectively.

Prove that : XYZL is a cyclic quadrilateral.

37 **In the opposite figure :**

Two intersecting circles at A and B , $\overline{CD}$ passes through the point B and intersects the two circles at C and D , $\overrightarrow{CE} \cap \overrightarrow{DF} = \{X\}$

Prove that : AFXE is a cyclic quadrilateral.



(Damietta 17 , El-Dakahlia 15 , El-Gharbia 14 , North Sinai 13)

For excellent pupils

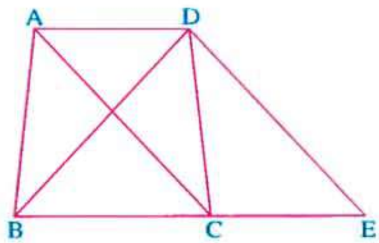
1 **In the opposite figure :**

ABCD is a quadrilateral , C $\in \overline{EB}$, Δ DCE $\sim \Delta$ BAD

Prove that :

1 The figure ABCD is a cyclic quadrilateral.

2 $\overline{ED} \parallel \overline{CA}$



2 $\overline{AB}$ is a diameter in a circle and L is a straight line touching the circle at B Taking the two points C and D on the circle in two different sides of $\overline{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ are drawn to cut the straight line L at E and F respectively.

Prove that :

The figure CDFE is a cyclic quadrilateral.

Exercise 11
The relation between the tangents of a circle

From the school book

1 **Complete the following :**

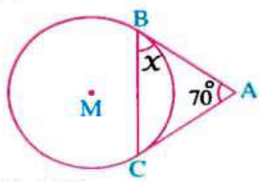
1. The two tangents drawn to the circle at the two ends of a diameter in it are .... *(El-Kalyoubia 12)*
2. The two tangent-segments drawn to a circle from a point outside it are .... *(Cairo 18 , Alex. 11)*
3. The centre of any circle inscribed in a triangle is the intersection point of ....
4. The number of common tangents of two distant circles is .... *(New Valley 12)*
5. The number of internal common tangents of the two intersecting circles is ....
6. The straight line passing through the centre of a circle and the point of intersection of two tangents to it is the axis of symmetry of ....
7. The straight line passing through the centre of the circle and the intersection point of two tangents to it bisects .... and bisects ....

2 **Choose the correct answer from those given :**

1. The number of tangents can be drawn from a point lies on a circle is .... *(El-Beheira 17)*

(a) 1 (b) 2 (c) 4 (d) infinite number

2. **In the opposite figure :**



If $\overline{AB}$, $\overline{AC}$ are two tangent segments of circle M , m (∠ A) = 70° , then x = ...

(a) 50° (b) 55° (c) 60° (d) 70°

Unit 5

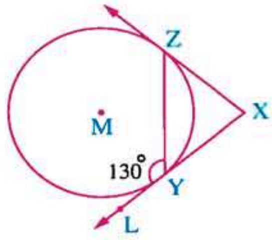
3 **In the opposite figure :**

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle at Y and Z

, m ($\angle$ LYZ) = 130° , then m ($\angle$ X) = ...

(a) 50° (b) 65°

(c) 80° (d) 100°



(Souhag 09)

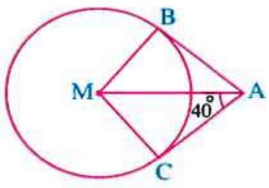
4 **In the opposite figure :**

If $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, m ($\angle$ MAC) = 40° , then m ($\angle$ CAB) = ...

(a) 80° (b) 50°

(c) 40° (d) 20°



(Damietta 04)

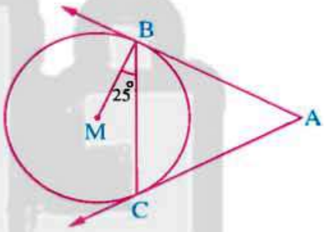
5 **In the opposite figure :**

If $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M

, m ($\angle$ CBM) = 25° , then m ($\angle$ BAC) = ...

(a) 75° (b) 50°

(c) 25° (d) $12^\circ\ 30'$



(Qena 12)

6 **In the opposite figure :**

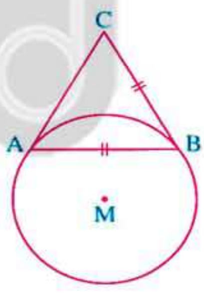
$\overline{CB}$ and $\overline{CA}$ are two tangent - segments

to the circle M and CB = BA

, then m ($\angle$ C) = ...

(a) 60° (b) 120°

(c) 90° (d) 100°



(Suez 08)

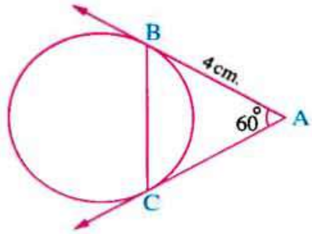
7 **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents , if AB = 4 cm.

, m ($\angle$ A) = 60° , then BC = ...

(a) 3 cm. (b) 4 cm.

(c) 5 cm. (d) 8 cm.



(Nourth Sinai 15 , Port Said 13)

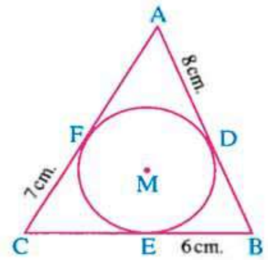
8 **In the opposite figure :**

The circle M touches the sides of Δ ABC , if AD = 8 cm. ,

BE = 6 cm. and CF = 7 cm. , then the perimeter of Δ ABC = ………

(a) 21 cm. (b) 42 cm.

(c) 48 cm. (d) 28 cm.

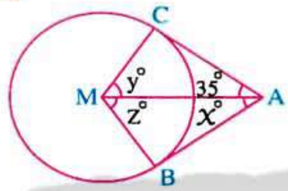


3 **In each of the following figures :**

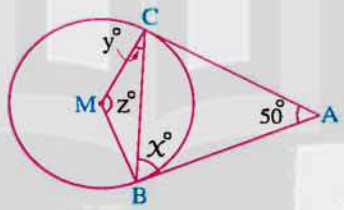
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

Find the value of the symbol used in measuring :

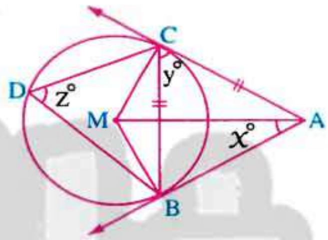
1



2

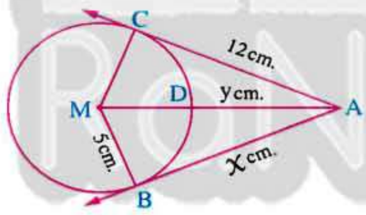


3



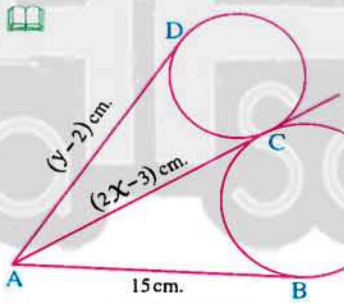
4 **Using data of each figure , find the value of the symbol used in measuring :**

1



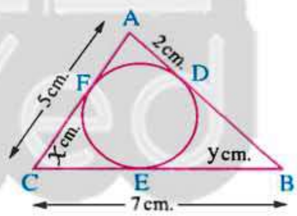
(New Valley 12)

2

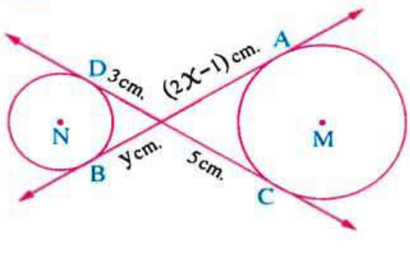


(Aswan 14)

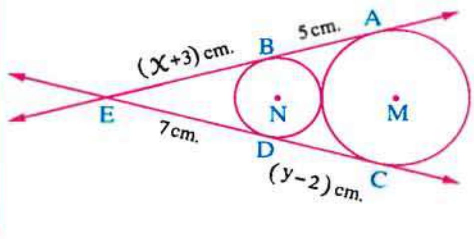
3



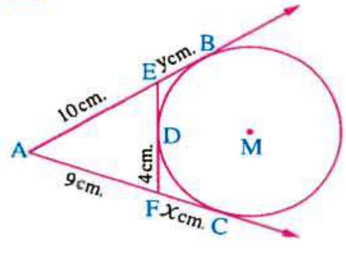
4



5



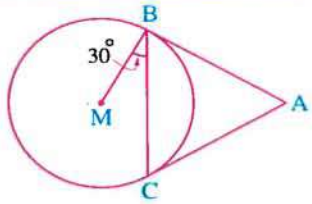
6



5 **Prove that** : the two tangent-segments drawn to a circle from a point outside it are equal in length. *(Kafr El-Sheikh 15)*

Unit 5

6 **In the opposite figure :**



If $\overline{AB}$ and $\overline{AC}$ are two tangent - segments to the circle M and m (∠ MBC) = 30°

Prove that : Δ ABC is equilateral.

(Kafr El-Sheikh 11)

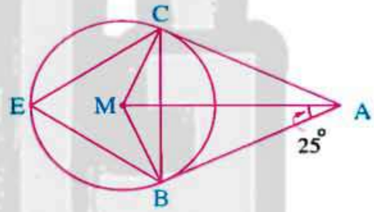
7 The two circles M and N are touching internally at B , $\overleftrightarrow{BA}$ is a tangent of the two circles. $\overleftrightarrow{AD}$ was drawn as a tangent to circle M at D and $\overleftrightarrow{AC}$ was drawn as a tangent to circle N at C

Prove that : AD = AC

8 $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M at B and C If the radius length of the circle equals 10 cm. , m (∠ BAC) = 60°

Find : The length of each of $\overline{MA}$ and $\overline{AB}$ « 20 cm. , $10\sqrt{3}$ cm. »

9 **In the opposite figure :**



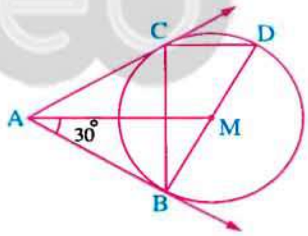
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M , m (∠ BAM) = 25° and E ∈ $\widehat{BC}$ the major

Find :

1 m (∠ ACB) 2 m (∠ BEC)

(El-Kalyoubia 18 , New valley 18) « 65° , 65° »

10 **In the opposite figure :**

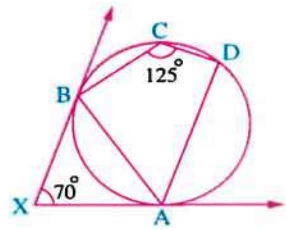


$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M , $\overline{BD}$ is a diameter in it , m (∠ MAB) = 30°

Find : m (∠ ACD)

(El-Sharkia 13) « 150° »

11 **In the opposite figure :**



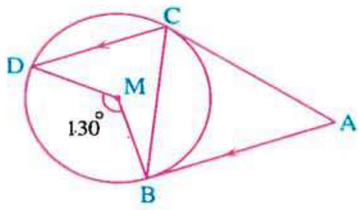
$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B , m (∠ AXB) = 70° and m (∠ DCB) = 125°

Prove that :

1 $\overrightarrow{AB}$ bisects ∠ DAX 2 $\overline{AD}$ // $\overrightarrow{XB}$

(Assiut 18 , Luxor 16 , Qena 16 , El-Beheira 11)

12 **In the opposite figure :**



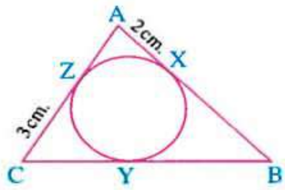
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB}$ // $\overline{CD}$ and m (∠ BMD) = 130°

1 **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD **2** **Find :** m (∠ A)

(New valley 19 , Matrouh 18 , El-Fayoum 17 , El-Gharbia 16 , El-Menia 15) « 50° »

13 **In the opposite figure :**



Δ ABC touches the circle externally at X , Y and Z

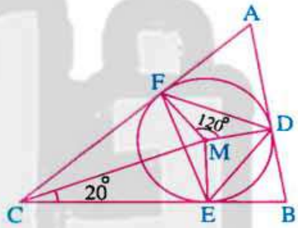
If the perimeter of Δ ABC = 18 cm.

, AX = 2 m. and CZ = 3 cm.

Calculate : The length of $\overline{BY}$

(Sharkia 03) « 4 cm. »

14 **In the opposite figure :**



If the inscribed circle M of Δ ABC

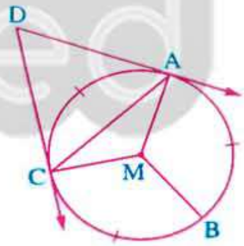
touches its sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$

at D , E and F respectively

, m (∠ DMF) = 120° and m (∠ ECM) = 20°

Find : The measures of the angles of Δ ABC « 40° , 60° , 80° »

15 **In the opposite figure :**



The circle M is divided into three arcs equal in length

, $\overrightarrow{DA}$ and $\overrightarrow{DC}$ are drawn from the point D to touch the circle.

1 **Find :** m (∠ AMB) « 120° »

2 **Prove that : First :** The figure AMCD is a cyclic quadrilateral.

Second : Δ ACD is an equilateral triangle.

16 **In each of the opposite figures :**

$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two

tangents to the

two circles M and N

Prove that : AB = CD

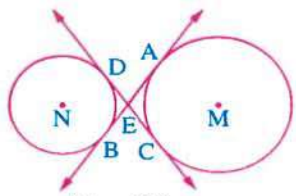


Fig. (1)

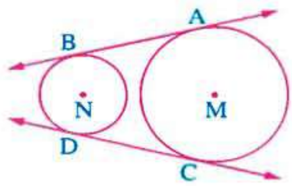


Fig. (2)

(Suez 11 , Port Said 13)

Unit 5

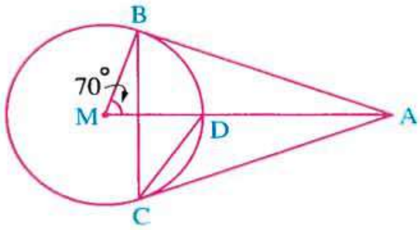
17 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments drawn from A

, m ($\angle$ AMB) = 70°

Find : 1 m ($\angle$ ABC)

2 m ($\angle$ ACD)



(El-Ismailia 17) « 70° , 35° »

18 **In the opposite figure :**

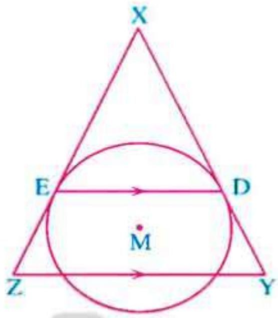
XYZ is a triangle

, $\overline{XY}$ and $\overline{XZ}$ touch the circle M at D and E

If $\overline{DE}$ // $\overline{YZ}$,

prove that :

The figure DYZE is a cyclic quadrilateral.



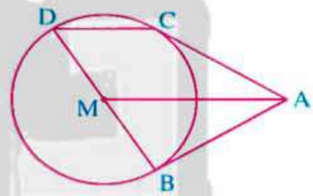
(Alex. 04)

19 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

and $\overline{BD}$ is a diameter in the circle.

Prove that : $\overline{AM}$ // $\overline{CD}$



(EL-Monofia 11)

20 **In the opposite figure :**

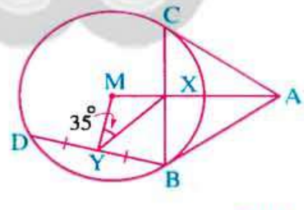
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M at B and C,

$\overleftrightarrow{AM} \cap \overline{BC} = \{X\}$, Y is the midpoint of the chord $\overline{BD}$

and m ($\angle$ XYM) = 35°

1 **Prove that :** XBYM is a cyclic quadrilateral.

2 **Find :** m ($\angle$ BAC)



« 70° »

21 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to

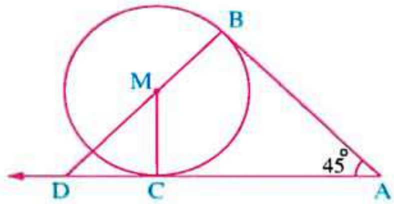
the circle M at B and C respectively , m ($\angle$ A) = 45°

, $\overrightarrow{BM} \cap \overrightarrow{AC} = \{D\}$

Prove that :

1 The figure ABMC is cyclic quadrilateral.

2 AD = AB + MB



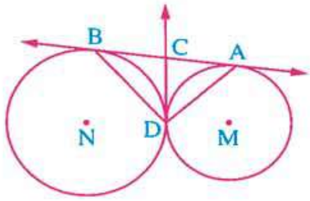
(Helwan 09)

22 **In the opposite figure :**

M and N are two circles touching externally at D and $\overleftrightarrow{AB}$ is a common tangent to them at A and B

$\overrightarrow{DC}$ is a common tangent to the two circles at D ,

where $\overrightarrow{DC} \cap \overleftrightarrow{AB} = \{C\}$



Prove that : **1** C is the midpoint of $\overline{AB}$

2 $\overline{AD} \perp \overline{BD}$ *(Alex. 14 , South Sinai 12)*

23 $\overline{AB}$ is a diameter of the circle M , AB = 10 cm. , C ∈ circle M , a tangent was drawn to the circle at C , so it intersected the two drawn tangents for it at A and B in X and Y respectively where XY = 13 cm.

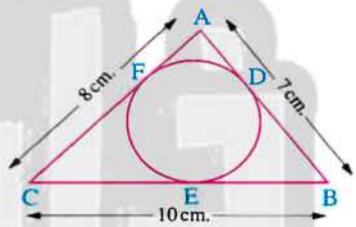
1 **Prove that :** $\overline{MX} \perp \overline{YM}$

2 **Find :** The area of the figure AXYB « 65 cm.^{2} »

24 **In the opposite figure :**

ABC is a triangle where the lengths of its sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are 7 cm. , 10 cm. and 8 cm. respectively.

If the inscribed circle of it touches the previous sides at D , E and F respectively ,



1 **Prove that :** BC + AD = AC + BD

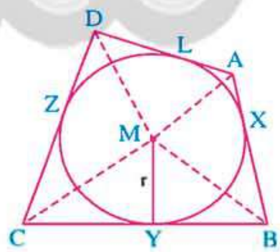
2 Find the length of each of $\overline{AD}$ and $\overline{EC}$ « 2.5 cm. , 5.5 cm. »

For excellent pupils

1 **In the opposite figure :**

M is an inscribed circle to the quadrilateral ABCD with radius length of 5 cm.

AB = 9 cm. and CD = 12 cm.



Find : The perimeter of ABCD

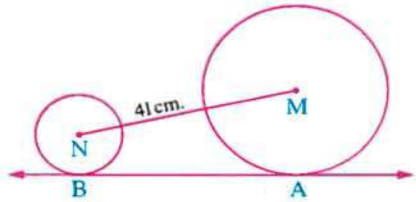
, then calculate its area. « 42 cm. , 105 cm.^{2} »

2 **In the opposite figure :**

$\overleftrightarrow{AB}$ is a common tangent to the two circles M and N externally at A and B respectively ,

their two radii lengths are 17 cm. and 8 cm. respectively.

If MN = 41 cm. ,



Find : The length of $\overline{AB}$ « 40 cm. »

Exercise 12

Angles of tangency

From the school book

First : Problems on theorem (5) and its corollary :

1 In each of the following , find the measure of the angle denoted by (?) knowing that $\overrightarrow{AC}$ touches the circle M at A :

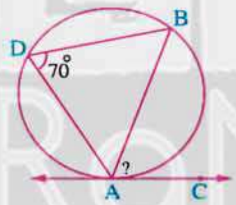


Fig. (1)

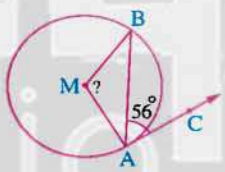


Fig. (2)

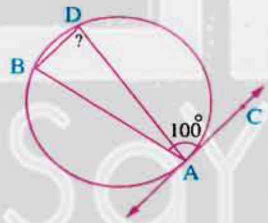


Fig. (3)

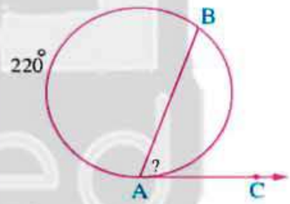


Fig. (4)

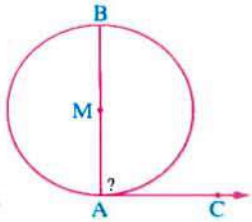


Fig. (5)

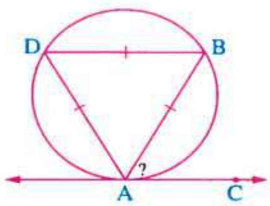


Fig. (6)

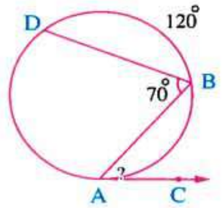


Fig. (7)

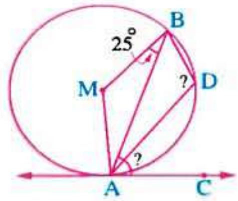


Fig. (8)

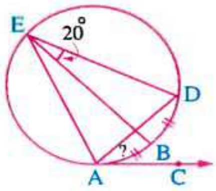


Fig. (9)

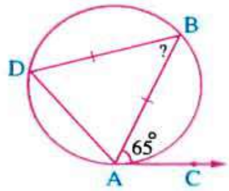


Fig. (10)

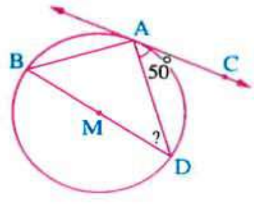


Fig. (11)

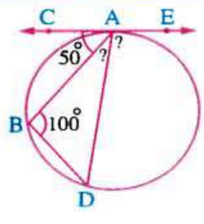


Fig. (12)

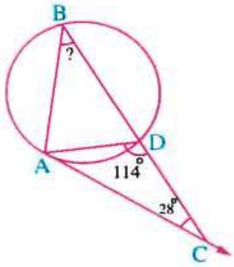


Fig. (13)

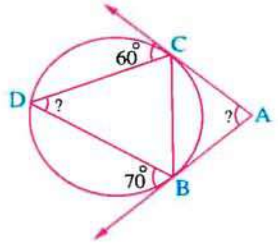


Fig. (14)

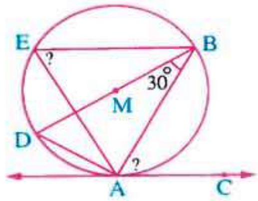


Fig. (15)

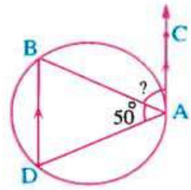


Fig. (16)

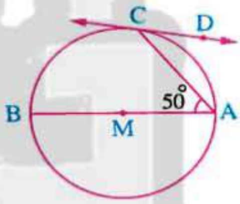
2 **Complete the following :**

1. The angle of tangency is the included angle between .... , ...

2. The measure of the angle of tangency equals the measure of .... subtended by the same arc. *(El-Dakahlia 12)*

3. The measure of the tangency angle equals half the measure of .... subtended by the same arc. *(Damietta 11)*

4. **In the opposite figure :**



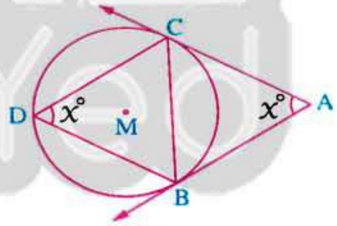
If $\overline{AB}$ is a diameter in the circle M

, $\overleftrightarrow{DC}$ is a tangent to it at C and m (∠ A) = 50° , then :

First : m $(\overset{\frown}{BC})$ = ... ° **Second :** m $(\overset{\frown}{AC})$ = ... °

Third : m (∠ ACD) = ... °

5. **In the opposite figure :**

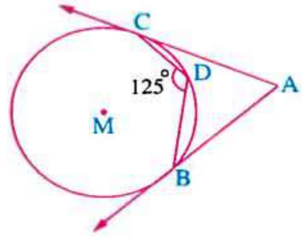


$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M ,

D lies on the circle

, then the value of x = ...° *(South Sinai 05)*

6. **In the opposite figure :**



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M ,

D is a point on the circle

such that : m (∠ CDB) = 125° , then :

m (∠ A) = ...°

3 **Choose the correct answer from those given :**

1. If the measure of an angle of tangency = 70° , then the measure of the central angle subtended by the same arc equals .... *(El-Kalyoubia 16 , Aswan 13)*

(a) 35° (b) 70° (c) 140° (d) 105°

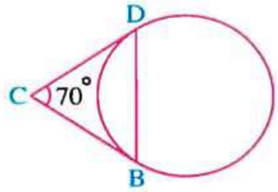
Unit 5

2 **In the opposite figure :**

$\overline{CB}$, $\overline{CD}$ are two tangent-segments at B , D

, m (∠ C) = 70°

, then m $(\widehat{DB})$ = ...



(El-Dakahlia 17)

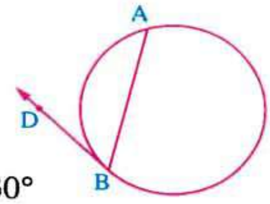
(a) 180° (b) 90° (c) 100° (d) 110°

3 **In the opposite figure :**

$\overrightarrow{BD}$ touches the circle and m $(\widehat{AB}) = \frac{1}{3}$ the measure of the circle

, then m (∠ ABD) = ...

(a) 60° (b) 90° (c) 120° (d) 30°



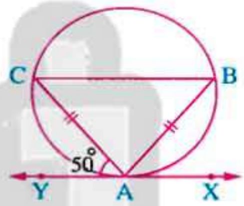
(El-Monofia 15)

4 **In the opposite figure :**

If AB = AC

and m (∠ YAC) = 50°

, then m $(\widehat{BC})$ = ...



(Cairo 04)

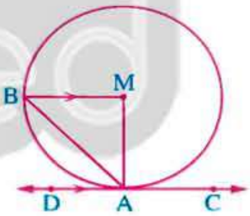
(a) 50° (b) 100° (c) 80° (d) 160°

5 **In the opposite figure :**

$\overleftrightarrow{CD}$ is a tangent to the circle M at A

and $\overline{MB}$ // $\overleftrightarrow{CD}$

, then m (∠ BAD) = ...



(Port Said 06)

(a) 30° (b) 45° (c) 60° (d) 90°

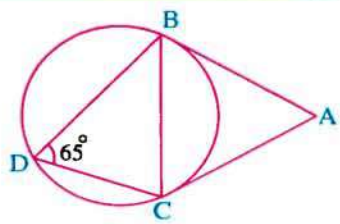
4 Prove that the measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc. *(Matrouh 17)*

5 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C

, m (∠ BDC) = 65°

Find with proof : m (∠ BAC)



(South Sinai 17 , El-Menia 16 , Beni Suef 14) « 50° »

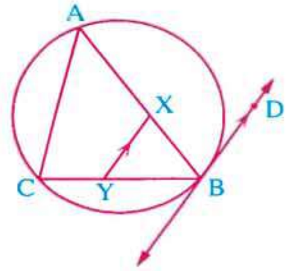
6 In the opposite figure :

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, $X \in \overline{AB}$ and $Y \in \overline{BC}$, where $\overline{XY} \parallel \overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



(Assiut 19 , Kafr El-Sheikh 18 , Cairo 17 , El-Kalyoubia 14)

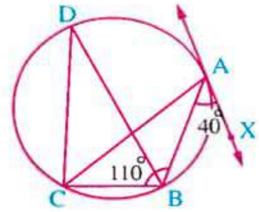
7 **In the opposite figure :**

$\overleftrightarrow{AX}$ is a tangent

, m (∠ XAB) = 40°

and m (∠ ABC) = 110°

Find : m (∠ CDB)

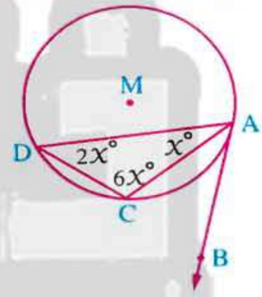


(Kafr El-Sheikh 11) « 30° »

8 **In the opposite figure :**

$\overrightarrow{AB}$ is a tangent to the circle M

Find : m (∠ BAC)



(El-Wadi El-Gedied 17) « 40° »

9 **In the opposite figure :**

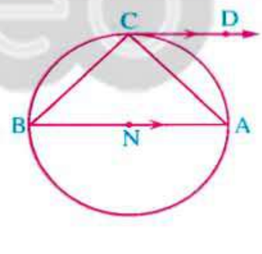
$\overline{AB}$ is a diameter in a circle N

, its circumference is 44 cm.

, $\overrightarrow{CD}$ is a tangent to it at C and $\overrightarrow{CD} \parallel \overline{BA}$

Find with proof :

1 m (∠ DCA) 2 The length of $(\widehat{AC})$



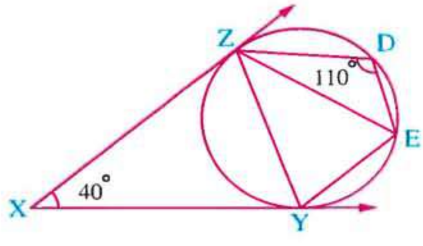
(El-Fayoum 13) « 45° , 11 cm. »

10 **In the opposite figure :**

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle from the point X

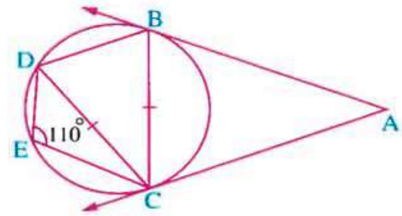
, m (∠ D) = 110° , m (∠ X) = 40°

Prove that : m $(\widehat{ZDE})$ = m $(\widehat{ZY})$



(Assiut 17 , El-Gharbia 17)

11 **In the opposite figure :**



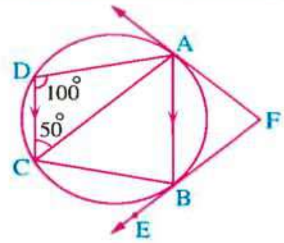
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

If CB = CD

1 **Prove that :** m (∠ ABC) = m (∠ DBC)

2 If m (∠ CED) = 110° , **find :** m (∠ A) « 40° »

12 **In the opposite figure :**



$\overrightarrow{FA}$ and $\overrightarrow{FB}$ touch the circle at A and B

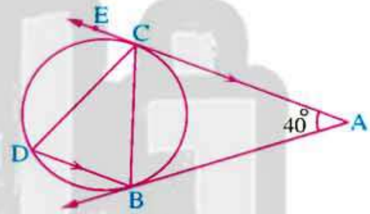
, $\overline{AB}$ // $\overline{CD}$

, m (∠ ADC) = 100° and m (∠ ACD) = 50°

Find :

1 m (∠ ABC) **2** m (∠ CBE) **3** m (∠ AFB) « 80° , 50° , 80° »

13 **In the opposite figure :**



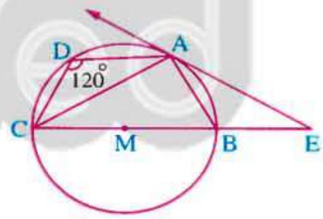
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ touch the circle at B and C

, $\overline{AC}$ // $\overline{BD}$ and m (∠ A) = 40°

1 **Find with proof :** m (∠ ACB) , m (∠ ECD)

2 **Prove that :** CB = CD *(El-Gharbia 04)* « 70° , 70° »

14 **In the opposite figure :**



$\overrightarrow{EA}$ is a tangent for the circle M at point A

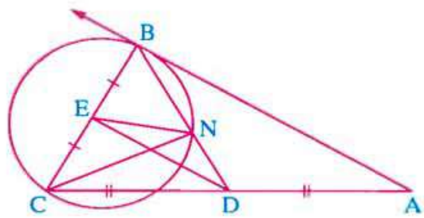
, $\overrightarrow{EM}$ is drawn and cuts the circle at B , C

and m (∠ ADC) = 120°

Prove that : **1** BA = BE

2 m (∠ ABE) = m (∠ EAC) *(Damietta 09)*

15 **In the opposite figure :**



$\overrightarrow{AB}$ is a tangent to the circle , $\overline{AC}$ is a secant to it

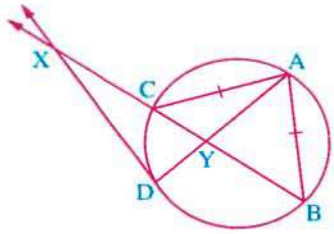
, D is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

and $\overline{BD} \cap$ the circle = {N} **Prove that :**

1 $\overline{AB}$ // $\overline{DE}$

2 The points N , D , C , E have one circle passing through them. *(Port Said 15)*

16 **In the opposite figure :**



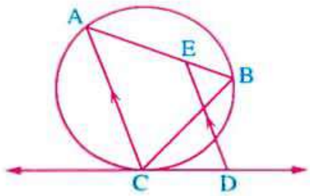
ABC is a triangle inscribed in a circle in which AB = AC ,

$D \in \widehat{BC}$, $\overrightarrow{DX}$ is drawn to be a tangent to the circle at D

where $\overrightarrow{DX} \cap \overleftrightarrow{BC} = \{X\}$ and $\overline{AD} \cap \overline{BC} = \{Y\}$

Prove that : XY = XD

17 **In the opposite figure :**



ABC is a triangle inscribed in a circle ,

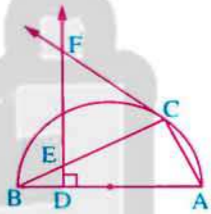
$\overleftrightarrow{CD}$ is a tangent to the circle at C

Draw $\overrightarrow{DE}$ // $\overline{AC}$ to cut $\overline{AB}$ at E

Prove that : BECD is a cyclic quadrilateral.

(El-Menia 09)

18 **In the opposite figure :**



$\overline{AB}$ is a diameter of the semicircle ,

$\overrightarrow{CF}$ is a tangent to it at C and $\overrightarrow{DF} \perp \overline{AB}$

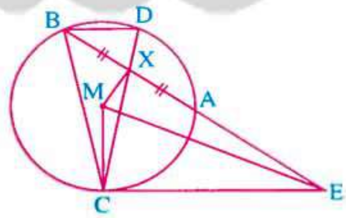
1 **Prove that :** The figure ADEC is a cyclic quadrilateral.

2 **Prove that :** Δ FCE is isosceles.

3 Determine the centre of the circle passing through the vertices of the quadrilateral ADEC

(Kafr El-Sheikh 08)

19 **In the opposite figure :**



$\overline{EC}$ is a tangent-segment to the circle M at C

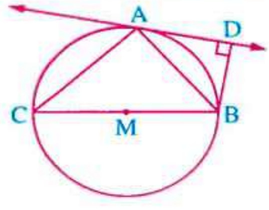
and X is the midpoint of $\overline{AB}$

Prove that :

1 The figure ECMX is a cyclic quadrilateral.

2 m ($\angle$ EMX) = m ($\angle$ CBD)

20 **In the opposite figure :**



$\overleftrightarrow{AD}$ is a tangent to the circle M at A

, $\overline{BC}$ is a diameter in the circle M

and $\overline{BD} \perp \overleftrightarrow{AD}$

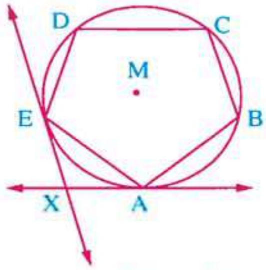
Prove that : m ($\angle$ ABD) = m ($\angle$ ABC)

(Port Said 06)

Unit 5

21 **In the opposite figure :**

ABCDE is a regular pentagon inscribed in the circle M ,
$\overleftrightarrow{AX}$ is a tangent to the circle at A , $\overleftrightarrow{EX}$ is a tangent to the circle at E where $\overleftrightarrow{AX} \cap \overleftrightarrow{EX} = \{X\}$



Find :

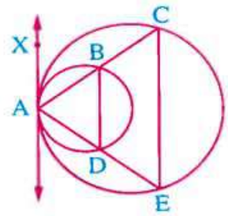
1 m $(\overset{\frown}{AE})$

2 m (∠ AXE)

(Matrouh 11) « 72° , 108° »

22 **In the opposite figure :**

Two circles are touching internally at A
, $\overleftrightarrow{AX}$ is the common tangent to them at A
, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AD}$ intersect the small circle at B , D
and the great circle at C , E

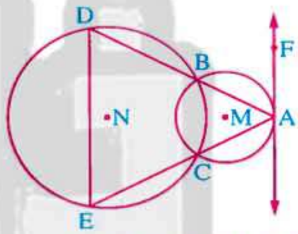


Prove that : $\overline{DB}$ // $\overline{EC}$

(El-Gharbia 15 , El-Monofia 14 , Souhag 13)

23 **In the opposite figure :**

Two circles are intersecting at B and C
, A ∈ one of the two circles ,
$\overleftrightarrow{AF}$ is drawn as a tangent to it at A
, then $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are drawn to cut the other circle at D and E

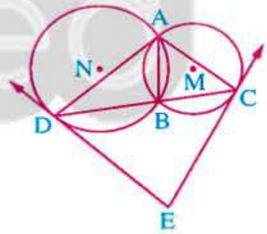


Prove that : $\overleftrightarrow{AF}$ // $\overline{DE}$

(Kafr El-Sheikh 09)

24 **In the opposite figure :**

M and N are two circles intersecting at A and B
, B ∈ $\overline{CD}$, $\overrightarrow{EC}$ and $\overrightarrow{ED}$ are two tangents.



Prove that :

1 m (∠ ECD) + m (∠ EDC) = m (∠ CAD)

2 The figure ACED is a cyclic quadrilateral.

(El-Fayoum 08)

Second : Problems on the converse of theorem (5) :

1 **In each of the following figures , prove that $\overrightarrow{AC}$ touches the circle M at A :**

1

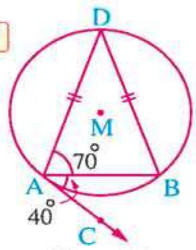


Fig. (1)

2

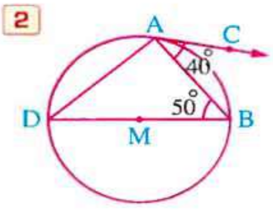


Fig. (2)

3

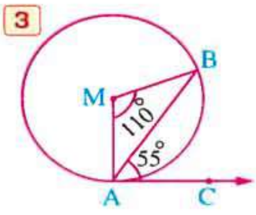


Fig. (3)

4

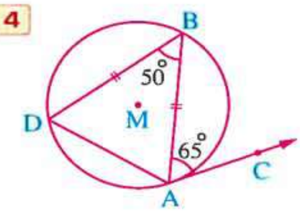
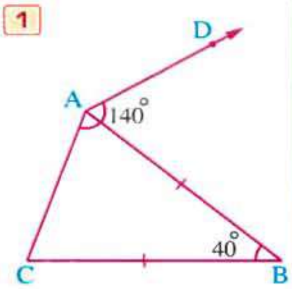


Fig. (4)

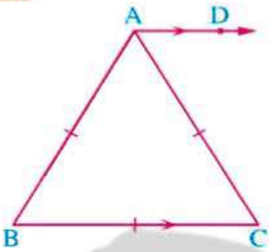
2 **In each of the following figures , prove that $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC :**

1

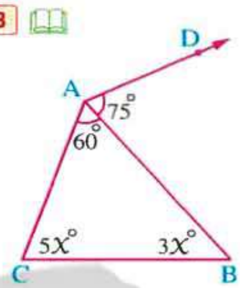


(Beni Suef 17)

2

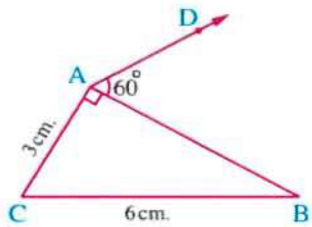


3



(El-Gharbia 16)

4



(Aswan 18 , Damietta 17)

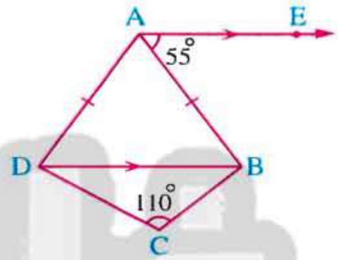
3 **In the opposite figure :**

$\overrightarrow{AE}$ // $\overline{DB}$, m ($\angle$ BAE) = 55° ,

m ($\angle$ C) = 110° and AB = AD

Prove that : **1** The figure ABCD is a cyclic quadrilateral.

2 $\overrightarrow{AE}$ is a tangent to the circumcircle of the quadrilateral ABCD



(El-Beheira 05)

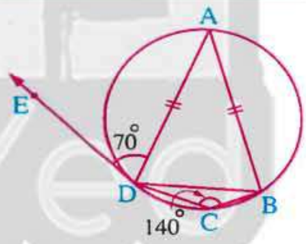
4 **In the opposite figure :**

ABCD is a quadrilateral inscribed in a circle in which

AB = AD , m ($\angle$ C) = 140° and m ($\angle$ ADE) = 70°

Prove that :

$\overrightarrow{DE}$ is a tangent to the circle at D *(El-Menia 09)*



5 ABCD is a quadrilateral inscribed in a circle , E is a point outside the circle and $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are two tangents to the circle at A and B , if m ($\angle$ AEB) = 70° and m ($\angle$ ADC) = 125° , **prove that :**

1 AB = AC

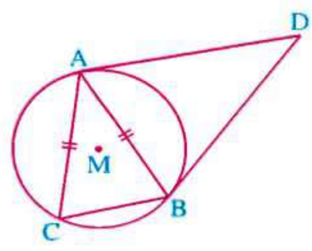
2 $\overleftrightarrow{AC}$ is a tangent to the circle passing through the points A , B and E *(Alex. 17)*

6 **In the opposite figure :**

$\overline{DA}$ and $\overline{DB}$ are two tangent-segments to the circle M at A and B

, C $\in$ the circle M such that AB = AC

Prove that : $\overleftrightarrow{AC}$ is a tangent to the circumcircle of Δ ABD



(Damietta 19 , Alex. 18 , Cairo 17 , Damietta 16 , North Sinai 14)

7 ABCD is a parallelogram in which AC = BC

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle circumscribed about the triangle ABC

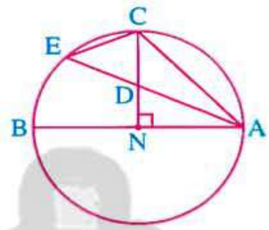
(Damietta 19 , El-Kalyoubia 18 , Port Said 17 , Ismailia 16)

8 ABC is a triangle inscribed in a circle , $\overrightarrow{AD}$ bisects $\angle$ BAC and intersects $\overline{BC}$ at D and the circle at E **Prove that :** $\overleftrightarrow{BE}$ is a tangent to the circle passing through the points A , B and D

9 $\overline{AB}$ and $\overline{AC}$ are two chords in a circle such that AB = AC , D $\in \overline{BC}$ and $\overrightarrow{AD}$ is drawn to cut the circle at E

Prove that : $\overline{AC}$ is a tangent-segment to the circumcircle of Δ CDE *(El-Fayoum 09)*

10 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle N

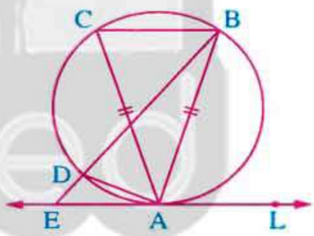
, $\overline{NC} \perp \overline{AB}$, D $\in \overline{NC}$

and $\overrightarrow{AD}$ is drawn to cut the circle at E

Prove that : $\overrightarrow{AC}$ is a tangent to the circle circumscribed about Δ CDE

(Kafr El-Sheikh 08)

11 **In the opposite figure :**



AB = AC and $\overleftrightarrow{EL}$ is a tangent to the circle at A

Prove that :

1 m ($\angle$ LAB) = m ($\angle$ ABC)

2 $\overleftrightarrow{AC}$ is a tangent to the circumcircle of Δ ADE

(South Sinai 05)

12 ABCD is a quadrilateral inscribed in a circle , its two diagonals intersect at E , $\overleftrightarrow{XY}$ is drawn to be a tangent to the circle at C where $\overleftrightarrow{XY}$ // $\overleftrightarrow{BD}$ **Prove that :**

1 $\overrightarrow{AC}$ bisects $\angle$ BAD

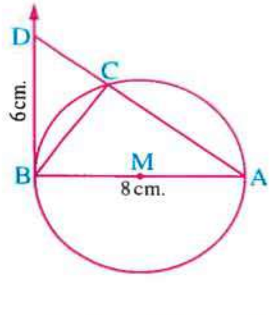
2 $\overleftrightarrow{BC}$ touches the circle passing through the vertices of the triangle ABE

13 ABC is a triangle inscribed in a circle. $\overleftrightarrow{AD}$ is a tangent to the circle at A , X $\in \overline{AB}$ and Y $\in \overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

(El-Kalyoubia 19 , New Valley 18 , El-Fayoum 17 , Alexandria 15)

14 **In the opposite figure :**



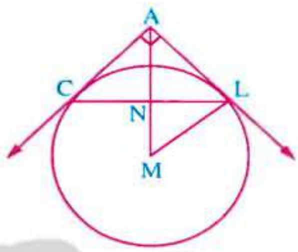
$\overline{AB}$ is a diameter in the circle M where AB = 8 cm.

, $\overline{AC}$ is a chord in it. Draw $\overrightarrow{BD}$ to be a tangent to the circle to cut $\overrightarrow{AC}$ at D. If BD = 6 cm.

1 **Prove that :** $\overleftrightarrow{AB}$ is a tangent to the circumcircle of Δ CBD

2 **Find :** The length of $\overline{BC}$ *(El-Monofia 09)* « 4.8 cm. »

15 **In the opposite figure :**



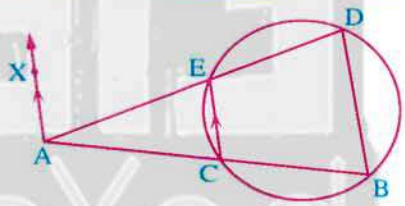
$\overline{AL}$ and $\overline{AC}$ are two tangent-segments to the circle M at L and C

, $\overline{AL} \perp \overline{AC}$, AC = 7 cm.

1 **Find with proof :** the length of $\overline{AL}$

2 **Prove that :** $\overleftrightarrow{AL}$ is a tangent to the circle passing through the vertices of the triangle ANC *(El-Sharkia 16)* « 7 cm. »

16 **In the opposite figure :**



The figure BCED is a cyclic quadrilateral

, $\overrightarrow{DE} \cap \overrightarrow{BC} = \{A\}$

and $\overrightarrow{AX}$ // $\overline{CE}$

Prove that : $\overleftrightarrow{AX}$ is a tangent to the circumcircle of Δ ABD

(Kafr El-Sheikh 13)

17 $\overline{AB}$ and $\overline{AC}$ are two chords in a circle including an acute angle where D is the midpoint of $\widehat{BC}$, $\overrightarrow{BX}$ is a tangent to the circle at B drawn to intersect $\overrightarrow{AD}$ at X and $\overrightarrow{BD} \cap \overrightarrow{AC} = \{Y\}$ **Prove that :**

1 ABXY is a cyclic quadrilateral.

2 $\overleftrightarrow{XY}$ is a tangent to the circle circumscribed about the triangle ADY

18 $\overline{AB}$ is a diameter in the circle M , D $\in \overrightarrow{AB}$, D $\notin \overline{AB}$, draw $\overrightarrow{DC}$ to be a tangent to the circle at C , draw $\overrightarrow{CB}$, E $\in \overrightarrow{CB}$ where DE = DC

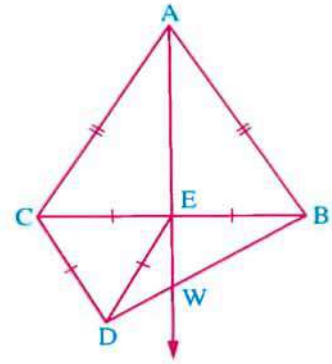
Prove that : 1 The figure ACDE is a cyclic quadrilateral.

2 $\overline{AE}$ is a diameter in the circumcircle of the figure ACDE

3 $\overleftrightarrow{ED}$ is a tangent to the circle passing through the vertices of the triangle ABE

Unit 5

19 **In the opposite figure :**



ABC and DCE are two equilateral triangles

, E is the midpoint of $\overline{BC}$, $\overrightarrow{AE} \cap \overline{BD} = \{W\}$

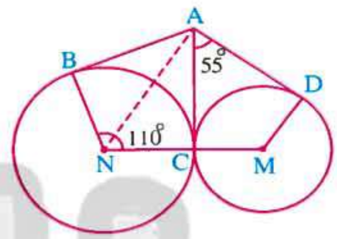
1 **Prove that :** $\overline{AC}$ is a tangent-segment to the circle which passes through the vertices of Δ CED

2 **Prove that :** CDWE is a cyclic quadrilateral.

3 **Find :** The centre of the circle which passes through the vertices of the quadrilateral CDWE

(El-Sharkia 17)

20 **In the opposite figure :**



M , N are two circles touching externally at C ,

$\overline{AD}$ touching the circle M at D ,

$\overline{AB}$ touching the circle N at B

If m ($\angle$ DAC) = 55° , m ($\angle$ CNB) = 110° ,

MN = 6 cm. , AD = 5 cm.

1 **Prove that :** AD = AC = AB

2 **Find :** The perimeter of ABNMD

3 **Prove that :** $\overrightarrow{NA}$ bisects $\angle$ CNB

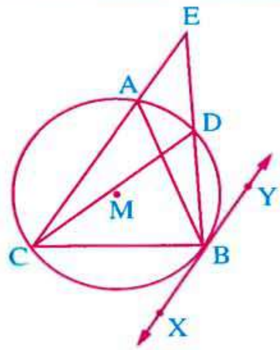
4 **Prove that :** $\overline{AD}$ is a tangent-segment to the circle passes through the points A , C and N

(Ismailia 15)

21 Δ ABC is an equilateral triangle. D and E are located on $\overleftrightarrow{BC}$, such that DB = BC = CE

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABE

22 **In the opposite figure :**



The circle M is the circumcircle of Δ ABC in which

AB = BC , $\overrightarrow{BD}$ intersects the circle at D where $\overrightarrow{BD} \cap \overrightarrow{CA} = \{E\}$

and $\overleftrightarrow{XY}$ is a tangent to the circle at B

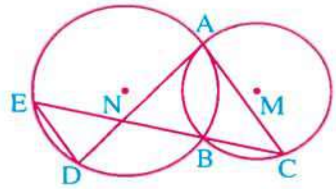
Prove that :

1 $\overrightarrow{BX}$ // $\overline{CE}$

2 $\overrightarrow{BC}$ is a tangent to the circle passing through the points C , D and E

For excellent pupils

1 **In the opposite figure :**

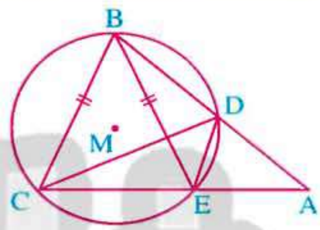


M and N are two circles intersecting at A and B

, $\overline{AC}$ is a tangent-segment to the circle N and cuts the circle M at C , $\overline{AD}$ is a tangent-segment to the circle M and cuts the circle N at D , $\overrightarrow{CB}$ cuts the circle N at E

Prove that : $\overline{AC} \parallel \overline{DE}$

2 **In the opposite figure :**



A is a point outside the circle M , $\overrightarrow{AD}$ intersects the circle at D and B , $\overrightarrow{AE}$ intersects the circle at E and C , then draw $\overline{DC}$ and $\overline{DE}$. If BE = BC ,

prove that : $\overline{BE}$ is a tangent-segment of the circle passing through the vertices of Δ ADE

Summary of the second part of Unit 5
"From lesson 4 to lesson 7"

- The cyclic quadrilateral is a quadrilateral whose vertices belong to one circle

  i.e. We can draw one circle passing through its four vertices.

  Properties of the cyclic quadrilateral :

  1. In a cyclic quadrilateral , each two angles drawn on one of its sides as a base and on one side of this side are equal in measure.
  2. In a cyclic quadrilateral , each two opposite angles are supplementry.
  3. The measure of the exterior angle at a vertex of a cyclic quadrilateral is equal to the measure of the interior angle at the opposite vertex.

- **To prove that the quadrilateral is cyclic , prove one of the following cases :**

  1. Prove that there is a point in the plane of the figure such that it is equidistant from its vertices.
  2. Prove that there are two equal angles in measure and drawn on one of its sides as a base and on one side of this side.
  3. Prove that there are two opposite supplementary angles.
  4. Prove that there is an exterior angle at any of its vertices equal in measure to the measure of the interior angle at the opposite vertex.

- If one of a cyclic quadrilateral's angles is right , then the diagonal opposite to this angle is a diameter of the circumcircle of this cyclic quadrilateral and the midpoint of this diagonal is the centre of this circle.
- The two tangents drawn at the two ends of a diameter in a circle are parallel
- The two tangents drawn at the two ends of a chord of a circle are intersecting
- The two tangent-segments drawn to a circle from a point outside it are equal in length.

- The straight line passing through the centre of the circle and the intersection point of the two tangents is an axis of symmetry to the chord of tangency of those two tangents.
- The straight line passing through the centre of the circle and the intersection point of its two tangents bisects the angle between these two tangents. It also bisects the angle between the two radii passing through the two points of tangency.
- The inscribed circle of a polygon is the circle which touches all of its sides.
- The centre of the inscribed circle of any triangle is the point of intersection of the bisectors of its interior angles.
- The two distant circles have 4 common tangents.
- The two circles touching externally have 3 common tangents.
- The two circles touching internally have one common tangent.
- The two intersecting circles have 2 common tangents.
- The two circles one inside the other have no common tangents.
- The angle of tangency is the angle which is composed of the union of two rays , one of them is a tangent to the circle and the other contains a chord of the circle passing through the point of tangency.
- The measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc.
- The measure of the angle of tangency is half the measure of the central angle subtended by the same arc.
- The angle of tangency is supplementary to the drawn inscribed angle on the chord of the angle of tangency and in one side of it.
- If a ray is drawn from one end of a chord of a circle so that the angle between this ray and the chord is equal in measure to the inscribed angle subtended by the chord in the alternate side , then this ray is a tangent to the circle.

Exams on the second part of unit Five
from lesson (4) to lesson (7)

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 The number of common tangents of two distant circles is ....

(a) 1 (b) 2 (c) 3 (d) 4

2 If ABCD is a cyclic quadrilateral , then m (∠ A) + m (∠ C) – 100° = ...

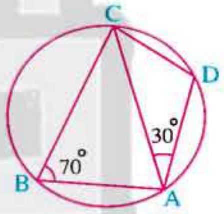
(a) 80° (b) 90° (c) 100° (d) 180°

3 The centre of any circle inscribed in a triangle is the intersection point of ....

(a) its altitudes. (b) the bisectors of its interior angles.

(c) the axes of symmetry of its sides. (d) its medians.

4 **In the opposite figure :**



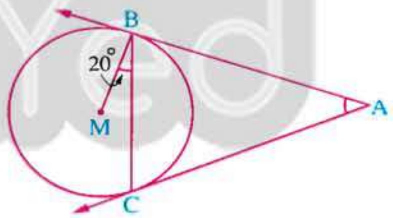
If ABCD is a cyclic quadrilateral

, then m (∠ DCA) = ...

(a) 70° (b) 30°

(c) 110° (d) 40°

5 **In the opposite figure :**



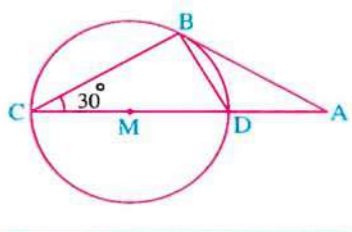
If $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M

, m (∠ CBM) = 20° , then m (∠ BAC) = ...

(a) 90° (b) 70°

(c) 40° (d) 30°

6 **In the opposite figure :**



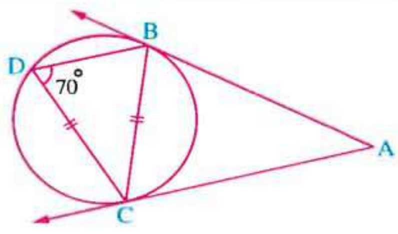
If $\overline{AB}$ is a tangent-segment to the circle M

, then m (∠ ABC) = ...

(a) 120° (b) 110°

(c) 90° (d) 30°

2 [a] In the opposite figure :



If $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ D) = 70° , CB = CD

1 **Find :** m (∠ A)

2 **Prove that :** $\overline{BD}$ // $\overline{AC}$

[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

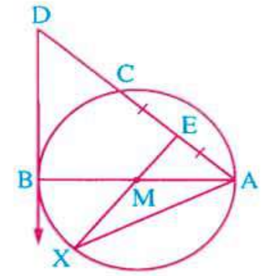
, $\overline{AC}$ is a chord in it , E is the midpoint of $\overline{AC}$

, a tangent $\overleftrightarrow{BD}$ is drawn to the circle intersecting $\overrightarrow{AC}$ at D

, $\overrightarrow{EM}$ is drawn to cut the circle at X

Prove that : 1 The figure MEDB is a cyclic quadrilateral.

2 m (∠ D) = 2 m (∠ BAX)

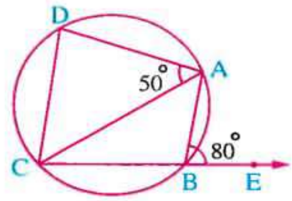


3 **[a] In the opposite figure :**

m (∠ ABE) = 80°

, m (∠ CAD) = 50°

Prove that : m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{AD})$



[b] $\overline{AB}$ is a diameter in the circle , $\overline{AC}$ is a chord of it

and m (∠ CAB) = 30° , $\overrightarrow{AC}$ intersects the tangent to the circle from B at D

Prove that : $\overrightarrow{BA}$ is a tangent to the circle passing through the vertices of Δ BCD

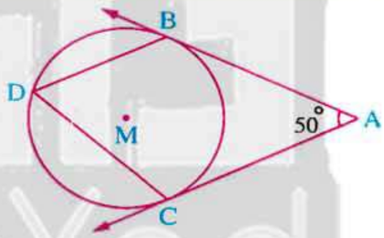
4 **[a] In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents

to the circle M at B and C respectively

, m (∠ A) = 50°

Find with proof : m (∠ D)



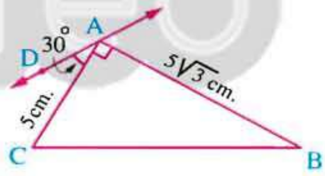
[b] In the opposite figure :

ABC is a right-angled triangle at A

, AC = 5 cm. , AB = $5\sqrt{3}$ cm.

, m (∠ DAC) = 30°

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC



5 **[a] In the opposite figure :**

Two circles are touching at B

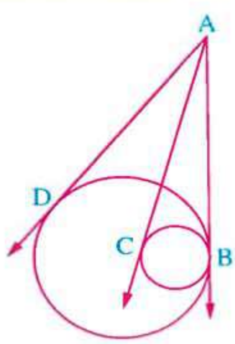
, $\overrightarrow{AB}$ is a common tangent to the two circles

, $\overrightarrow{AC}$ is a tangent to the smaller circle

, $\overrightarrow{AD}$ is a tangent to the greater circle , AC = 15 cm.

, AB = $(2x - 3)$ cm. and AD = $(y - 2)$ cm.

Find each of : x and y



Unit 5

[b] ABC is an acute-angled triangle inscribed in a circle , draw $\overrightarrow{AD} \perp \overline{BC}$ to cut $\overline{BC}$ at D and to cut the circle to E , draw $\overrightarrow{CN} \perp \overline{AB}$ to cut $\overline{AB}$ at N

Prove that :

1. The figure ANDC is a cyclic quadrilateral.
2. m (∠ BND) = m (∠ BED)

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1. If the measure of the angle of tangency is 50° , then the measure of the central angle subtended by the same arc equals ....

   (a) 25° (b) 50° (c) 100° (d) 75°

2. ABCD is a cyclic quadrilateral , if m (∠ A) = x ° , m (∠ C) = 2 x ° , then x = ...

   (a) 60° (b) 50° (c) 80° (d) 120°

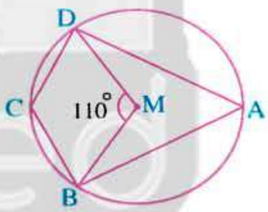
3. Number of common tangents of two circles touching externally = ...

   (a) zero (b) 1 (c) 2 (d) 3

4. **In the opposite figure :**

   In the circle M , if m (∠ M) = 110°
   , then m (∠ BCD) = ...

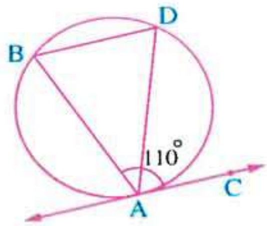
   (a) 110° (b) 70°
   (c) 55° (d) 125°



5. **In the opposite figure :**

   $\overleftrightarrow{AC}$ is a tangent to the circle at A
   , m (∠ BAC) = 110° , then m (∠ ADB) = ...

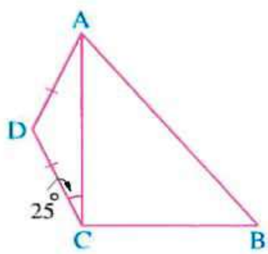
   (a) 110° (b) 70°
   (c) 55° (d) 90°



6. **In the opposite figure :**

   If ABCD is a cyclic quadrilateral
   , m (∠ ACD) = 25° , AD = DC
   , then m (∠ B) = ...

   (a) 130° (b) 65°
   (c) 50° (d) 25°



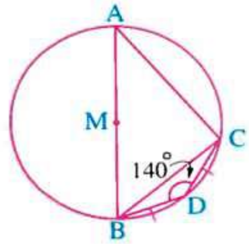
2 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m $(\widehat{BD})$ = m $(\widehat{CD})$, m (∠ BDC) = 140°

Find : 1 m (∠ ABC)

2 m $(\widehat{ABC})$



[b] In the opposite figure :

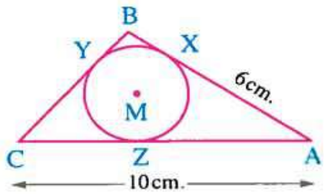
$\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are tangents to the circle M at X

, Y and Z respectivly , if AC = 10 cm.

, AX = 6 cm. , the perimeter of Δ ABC = 24 cm.

Find : 1 The length of $\overline{AB}$

2 The type of Δ ABC according to its angles.



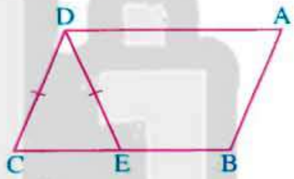
3 **[a] In the opposite figure :**

ABCD is a parallelogram , E ∈ $\overline{BC}$ where DE = DC

Prove that :

1 ABED is a cyclic quadrilateral.

2 $\overrightarrow{DA}$ is a tangent to the circle passing through the vertices of Δ DEC



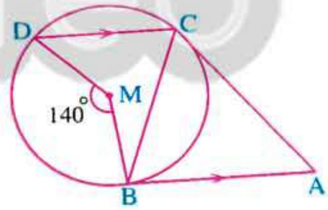
[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments of the circle M at B , C

, $\overline{AB}$ // $\overline{CD}$, m (∠ BMD) = 140°

1 **Find :** m (∠ ABC)

2 **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD



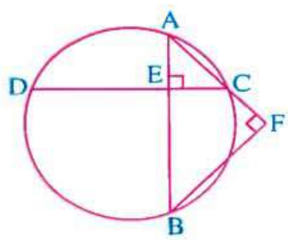
4 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two perpendicular chords in the circle

, $\overline{BF}$ ⊥ $\overline{AF}$

Prove that : 1 FCEB is a cyclic quadrilateral

2 $\overrightarrow{BA}$ bisects ∠ DBF



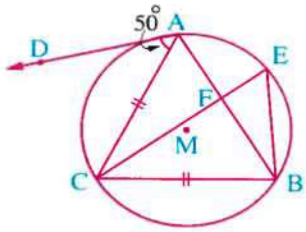
[b] In the opposite figure :

In the circle M : AC = BC

, $\overrightarrow{AD}$ is a tangent at A , m (∠ CAD) = 50°

1 **Find :** m (∠ ABC) , m (∠ BEC)

2 **Prove that :** $\overrightarrow{CB}$ is a tangnet to the circle passing through the vertices of Δ BEF

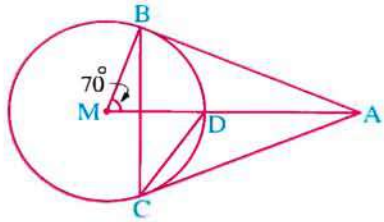


5 **[a] In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangent-segments are drawn from A

, m (∠ AMB) = 70°

Find : 1 m (∠ ABC) 2 m (∠ ACD)

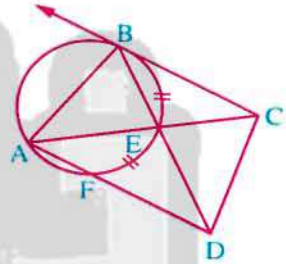


[b] In the opposite figure :

$\overrightarrow{CB}$ is a tangent to the circle

, m $(\overset{\frown}{BE})$ = m $(\overset{\frown}{EF})$

Prove that : ABCD is a cyclic quadrilateral.



Accumulative basic skills

Choose the correct answer from those given :

1 The corresponding angles of the two similar polygons are .... in measure.

(a) equal (b) different (c) proportional (d) alternate

(Alexandria 16)

2 The area of a rhombus which the lengths of its diagonals are 6 cm. and 8 cm. equals ....

(a) 2 cm^2. (b) 14 cm^2. (c) 24 cm^2. (d) 48 cm^2.

(Cairo 19)

3 The number of axes of symmetry of two congruent circles and touching externally equals .... *(El-Dakahlia 16)*

(a) 4 (b) 2 (c) 1 (d) an infinite number.

4 The medians of a triangle meet at the same point which divides each in the ratio .... : .... from the base. *(Beni Suef 17)*

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 2

5 If the projection of a line segment on a straight line is a point , then the line segment .... the straight line. *(North Sinai 17 , Alexandria 16)*

(a) // (b) $\perp$ (c) $\in$ (d) $\subset$

6 ABC is a right-angled triangle at B where AB = 6 cm. , BC = 8 cm. , then its area = ... cm^2. *(Damietta 16)*

(a) 48 (b) 14 (c) 24 (d) 7

7 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse. *(Cairo 19 , Ismailia 16)*

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

Basic skills

8 If m_1 and m_2 are two slopes of two parallel straight lines , then .... (El-Fayoum 16)

(a) $m_1 + m_2 = 0$ (b) $m_1 = m_2$ (c) $m_1 \times m_2 = -1$ (d) $m_1 - m_2 = -1$

9 The image of the point (2 , 3) by rotation R (O , 180°) is the point .... (South Sinai 17)

(a) (2 , 3) (b) (– 2 , 3) (c) (2 , – 3) (d) (– 2 , – 3)

10 If the side length of a rhombus is L cm. , then its perimeter = ... cm. (New Valley 17)

(a) L^2 (b) $2 L^2$ (c) 4 L (d) $2\sqrt{2}$ L

11 The measure of the interior angle of the regular hexagon = ... (Alexandria 17)

(a) 60° (b) 108° (c) 120° (d) 135°

12 If M is a circle of radius length r cm. , then the length of the semicircle = ... cm.

(a) $2 \pi r$ (b) $\frac{1}{4} \pi r$ (c) $\frac{1}{2} \pi r$ (d) πr

(South Sinai 17 , El-Beheira 16)

13 A square of perimeter 20 cm. , then its area = ... cm^2. (El-Menia 19 , Beni Suef 16)

(a) 20 (b) 25 (c) 50 (d) 100

14 The two diagonals are equal in length and not perpendicular in the .... (El-Menia 16)

(a) square. (b) rhombus. (c) rectangle. (d) parallelogram.

15 If $\cos 2X = \frac{1}{2}$ where X is an acute angle , then m ($\angle X$) = ... (Beni Suef 16)

(a) 15° (b) 30° (c) 45° (d) 60°

16 Δ ABC is a right-angled triangle at C , then the two angles A and B are .... (El-Menia 17)

(a) supplementary. (b) complementary.

(c) adjacent. (d) vertically opposite angles.

17 Two parallel lines to a third are .... (Luxor 16)

(a) perpendicular. (b) parallel.

(c) intersecting. (d) skew.

18 The radius length of the circle whose centre is (7 , 4) and passes through the point (3 , 1) equals .... length units. (Aswan 16)

(a) 3 (b) 4 (c) 5 (d) 6

19 The number of symmetry axes of the square is .... (El-Fayoum 17)

(a) 1 (b) 2 (c) 3 (d) 4

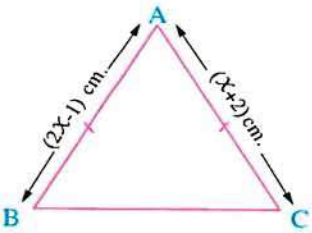
20 The numbers 5 , 4 and .... can be side lengths of a triangle. (El-Menia 16)

(a) 8 (b) 9 (c) 10 (d) 12

21 Δ XYZ is a right-angled triangle at Y , then XZ .... YZ *(North Sinai 17)*

(a) $<$ (b) $>$ (c) $=$ (d) is twice

22 **In the opposite figure :**



AB = AC , AB = $(2\mathcal{X} - 1)$ cm. and AC = $(\mathcal{X} + 2)$ cm. ,

then $\mathcal{X}$ = ... *(Cairo 16)*

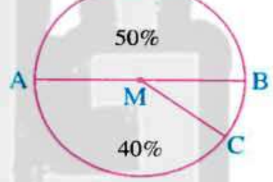
(a) 3 (b) 5

(c) 11 (d) 14

23 If Y $\in \overline{XZ}$, XY = 2 ZY , then the area of the square drawn on $\overline{XY}$ = ... the area of the square drawn on $\overline{XZ}$ *(El-Monofia 19)*

(a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) 2 (d) $\frac{1}{2}$

24 **In the opposite figure :**



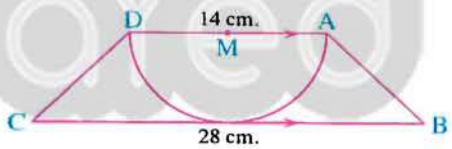
M is the centre of the circle ,

then m ($\angle$ CMB) = ... *(South Sinai 16)*

(a) 36° (b) 72°

(c) 144° (d) 180°

25 **In the opposite figure :**



ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$

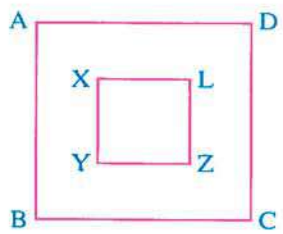
and $\overline{AD}$ is a diameter of circle M ,

then the area of the shaded region = ... *(Damietta 16)*

(a) 70 cm^2. (b) 147 cm^2.

(c) 170 cm^2. (d) 224 cm^2.

26 **In the opposite figure :**



If the side length of the square ABCD = 7 cm.

and of the square XYZL = 3 cm. ,

then the area of the shaded part = ... *(El-Monofia 17)*

(a) $(7 - 3)$ (b) $4\,(7 - 3)$

(c) $(7 - 3)^2$ (d) $(7^2 - 3^2)$

Basic skills

27 ABC is a triangle have one symmetric axis and its side lengths are 10 , 5 and x cm.
, then x = ... cm. *(Damietta 17)*

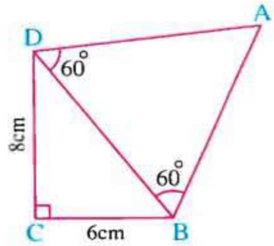
(a) 5 (b) 8 (c) 10 (d) 12

28 **In the opposite figure :**

The length of $\overline{AB}$ = ... cm.

(a) $10\sqrt{3}$ (b) 10

(c) 5 (d) $5\sqrt{3}$



29 A rectangular picture its length is 60 cm. and its width is 40 cm. We need to make a wooden frame its width is 5 cm. , then its total area = ... cm^2. *(Damietta 17)*

(a) 3050 (b) 3500 (c) 2925 (d) 3250

30 If $\overline{MA}$ and $\overline{MB}$ are two perpendicular radii in a circle M and the area of triangle AMB = 8 cm^2. , then the length of the radius of this circle = ... *(El-Monofia 17)*

(a) 8 cm. (b) 16 cm. (c) 4 cm. (d) 2 cm.

EL-MOASSER

In Mathematics
Notebook

For 3rd Prep.
Second Term

- Quizzes
- Final Revision
- Final Examinations

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By
A group of supervisors

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- 10 quizzes.
- Final revision.
- Final examinations :
  - School book examinations.
    (2 models + model for the merge students)
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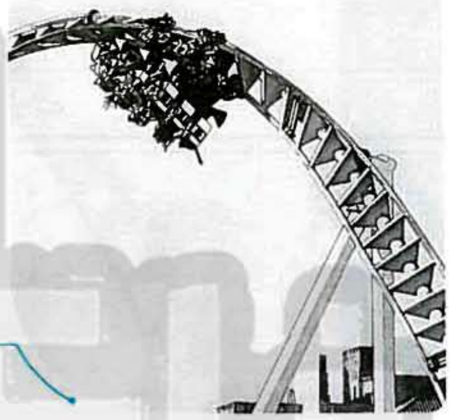

Second Geometry

- 12 quizzes.
- Final revision.
- Final examinations :
  - School book examinations.
    (2 models + model for the merge students)
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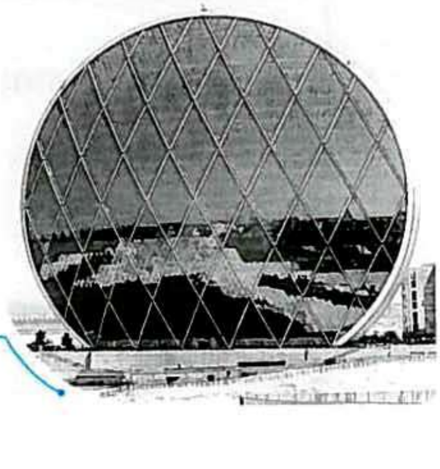
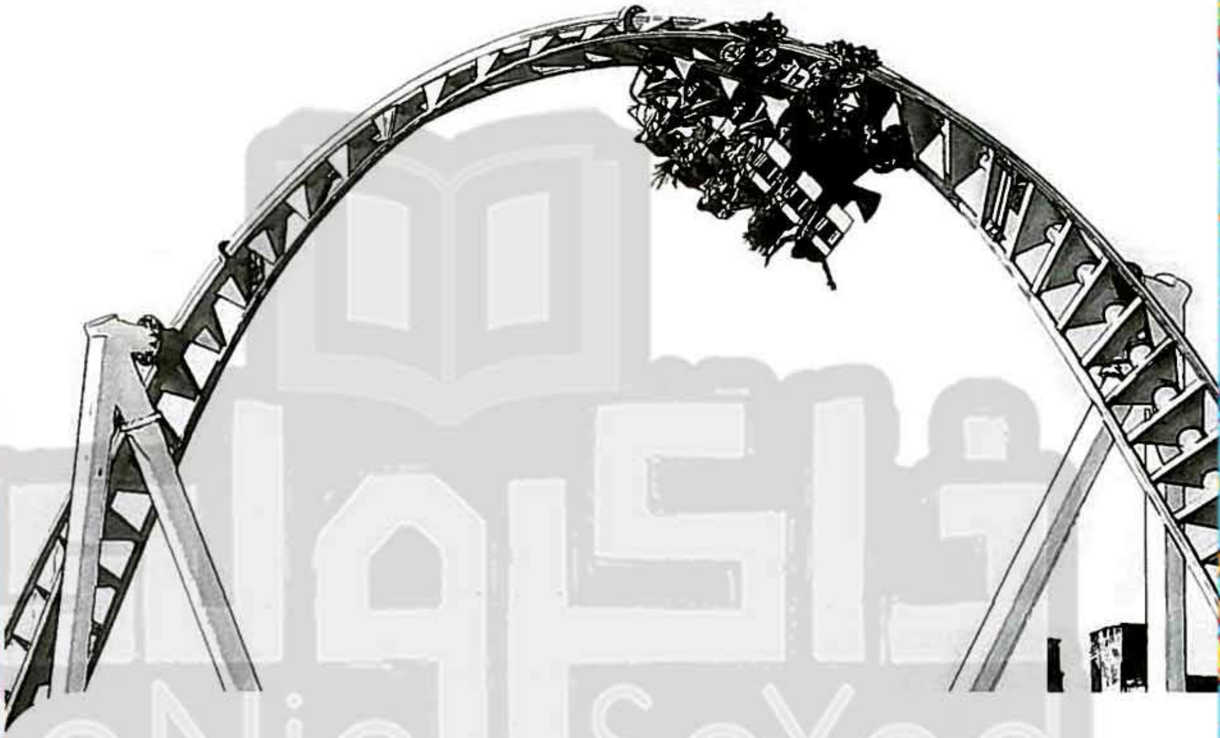


First Algebra and Probability



Quizzes

on Algebra and Probability

Quiz 1 On lesson 1 – unit 1

time 20 min.

1 Choose the correct answer from those given :

1 The solution set of the two equations : $x - y = 3$, $x + y = 7$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(6 , 3)\}$ (b) $\{(4 , 3)\}$ (c) $\{(5 , 2)\}$ (d) $\{(3 , 7)\}$

2 If the S.S. of the two equations : $x + 2 y = 5$ and $2 x + k y = 3$ in $\mathbb{R} \times \mathbb{R}$ equals $\varnothing$, then k = ...

(a) 2 (b) – 2 (c) 4 (d) – 4

3 The number of possible solutions of the two equations : $x - 2 y = 3$, $3 x - 6 y = 9$ is ....

(a) 1 (b) 2 (c) 3 (d) infinite.

2 [a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations : $x - 2 y = 0$, $2 x - y = 3$

[b] The perimeter of a rectangle is 32 cm. , its length is more than its width by 2 cm. What is its area ?

Quiz 2 Till lesson 2 – unit 1

time 20 min.

1 Choose the correct answer from those given :

1 The point of intersection of the two straight lines : $y = x$, $x + 3 = 0$ is ....

(a) (3 , 3) (b) (3 , – 3) (c) (– 3 , – 3) (d) (– 3 , 3)

2 If $x = 3$ is a root for the equation : $x^2 + m x = 3$, then m = ...

(a) – 1 (b) – 2 (c) 2 (d) 1

3 Two positive numbers , their sum is 9 and their product is 8 , then the two numbers are ....

(a) 2 , 7 (b) 3 , 6 (c) 4 , 5 (d) 1 , 8

2 [a] Find in $\mathbb{R}$ the solution set of the equation :

$x (x + 8) = - 9$ using the general formula to the nearest one decimal.

[b] **Graph the function $f : f(x) = x^2 - 4$ in the interval $[-3 , 3]$ and from the graph find :**

The two roots of the equation : $x^2 - 4 = 0$

Quiz 3 Till lesson 3 – unit 1

time 20 min.

1 Choose the correct answer from those given :

1 The S.S. of the two equations : $x = y$, $x^2 + y^2 = 18$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(3, 3)\}$ (b) $\{(-3, -3)\}$

(c) $\{(3, -3), (-3, 3)\}$ (d) $\{(3, 3), (-3, -3)\}$

2 The ordered pair which satisfies each of the following equations :

$xy = 2$, $x - y = 1$ is ....

(a) $(1, 2)$ (b) $(2, 1)$ (c) $(1, 1)$ (d) $(3, 1)$

3 If the two equations : $x + 4y = 7$ and $3x + ky = 21$ have an infinite number of solutions in $\mathbb{R} \times \mathbb{R}$, then k = ...

(a) 4 (b) 12 (c) 7 (d) 21

2 [a] The hypotenuse length of a right-angled triangle is 10 cm. and the lengths of the two sides of the right-angle are x and y cm. if its perimeter = 24 cm. Find its area.

[b] **Find by using the general formula , the S.S. in $\mathbb{R}$ of the equation :**

$3x - x^2 + 2 = 0$

Quiz 4 Till lesson 1 – unit 2

time 20 min.

1 Choose the correct answer from those given :

1 The set of zeroes of the function $f : f(x) = x^2 + 2x$ is ....

(a) $\{0\}$ (b) $\{-2\}$ (c) $\{0, -2\}$ (d) $\{0, 2\}$

2 If the set of zeroes of the function $f : f(x) = x^2 + a$ is $\{5, -5\}$, then a = ...

(a) 5 (b) – 5 (c) 25 (d) – 25

3 If the two equations : $x - y = 4$, $y - x = k$ have an infinite number of solutions in $\mathbb{R} \times \mathbb{R}$, then k = ...

(a) zero (b) 4 (c) – 4 (d) 1

2 [a] If the set of zeroes of the function $f : f(x) = a x^2 - b x - 9$ is $\{3, -1\}$

Find the values of a and b

[b] **Find graphically , then satisfy algebraically in $\mathbb{R} \times \mathbb{R}$ the S.S. of the two equations : $x - y = 4$, $3x + 2y = 7$ in $\mathbb{R} \times \mathbb{R}$**

Quiz 5 Till lesson 2 – unit 2

time 20 min.

1 Choose the correct answer from those given :

1 The common domain of the two functions n_1 and n_2 , where $n_1(x) = \frac{2}{x-1}$, $n_2(x) = x + 1$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{1, -1\}$ (d) $\mathbb{R} - \{-1\}$

2 The S.S. in $\mathbb{R} \times \mathbb{R}$ of the two equations : $x = 0$, $x^2 + xy + y^2 = 4$ is ....

(a) $\{(0, 2)\}$ (b) $\{(0, -2)\}$ (c) $\{(0, 2), (0, -2)\}$ (d) $\{(0, 0)\}$

3 The set of zeroes of the function $f : f(x) = x^3 + 4x$ is ....

(a) $\varnothing$ (b) $\{0\}$ (c) $\{0, 2\}$ (d) $\{0, 2, -2\}$

2 [a] Determine the common domain of the two functions n_1 and n_2 If :

$n_1(x) = \frac{x}{x^2+2}$, $n_2(x) = \frac{x+2}{x^2-4}$

[b] If the domain of the function n : $n(x) = \frac{5x+10}{x^2 - ax + 9}$ is $\mathbb{R} - \{3\}$

Find the value of : a

Quiz 6 Till lesson 3 – unit 2

time 20 min.

1 Choose the correct answer from those given :

1 The common domain of the two algebraic fractions : $\frac{2}{x-3}$ and $\frac{7}{2x-6}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{2, 3\}$ (d) $\mathbb{R} - \{3, -3\}$

2 If $f(x) = \frac{3-x}{x-3}$, $x \neq 3$, then $f(x)$ in its simplest form is ....

(a) 2 (b) 1 (c) – 2 (d) – 1

3 The set of zeroes of the function f where $f(x) = \frac{x-3}{x+3}$ is ....

(a) {zero} (b) $\{3\}$ (c) $\{-2\}$ (d) $\{(3, -2)\}$

2 [a] If $n_1(x) = \frac{x^2-4}{x^2+x-6}$, $n_2(x) = \frac{x^3-x^2-6x}{x^3-9x}$

Prove that : $n_1(x) = n_2(x)$ for all the values of x which belong to the common domain of the two functions n_1 and n_2 and find this domain.

[b] Find by using the general formula the S.S. in $\mathbb{R}$ of the equation :

$1 - \frac{2}{x} = \frac{2}{x^2}$ where $x \neq 0$, $\sqrt{3} \simeq 1.73$

Quiz 7 Till lesson 4 – unit 2

time 20 min.

1 Choose the correct answer from those given :

1 If $n_1(x) = \frac{x}{x^3 + 4x}$, $n_2(x) = \frac{1}{x^2 + 4}$, then $n_1(x) = n_2(x)$ for every $x \in$ ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$

(c) $\mathbb{R} - \{4\}$ (d) $\mathbb{R} - \{-2, 2, 0\}$

2 The domain of the additive inverse of the fraction : $\frac{x-3}{x+3}$ is ....

(a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{-3\}$ (c) $\mathbb{R} - \{3, -3\}$ (d) $\mathbb{R} - \{0\}$

3 If $f(x) = \frac{x^2 - 9}{x + b}$, $f(4) = 1$, then b = ...

(a) -7 (b) 7 (c) 3 (d) 1

2 [a] Reduce n (x) to the simplest form showing the domain of n if :

$$n(x) = \frac{3x - 6}{x^2 - 4} - \frac{9}{2 - x - x^2}$$

[b] If $n_1(x) = \frac{x^2 + 4x + 3}{x^2 + x - 6}$, $n_2(x) = \frac{x^2 - 6x - 7}{x^2 - 9x + 14}$, is $n_1 = n_2$? Why ?

Quiz 8 Till lesson 5 – unit 2

time 20 min.

1 Choose the correct answer from those given :

1 If $n(x) = \frac{x}{x-5}$, then the domain of : n^{-1} is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0, 5\}$ (c) $\mathbb{R} - \{5\}$ (d) $\{0, 5\}$

2 $\frac{3}{x-3} + \frac{3}{3-x} =$ .... where $x \neq 3$

(a) $\frac{6}{x-3}$ (b) $\frac{3}{3-x}$ (c) 1 (d) zero

3 The function n in the simplest form where $n(x) = \frac{x}{x-3} \div \frac{3x}{x^2 - 9}$ is .... where $x \notin \{3, 0, -3\}$

(a) $\frac{3}{x-3}$ (b) $\frac{3}{x+3}$ (c) $\frac{x+3}{3}$ (d) $\frac{x-3}{3}$

2 [a] Find n (x) in the simplest form showing the domain of n :

$$n(x) = \frac{x^3 - 8}{x^2 - 5x + 6} \div \frac{x^2 + 2x + 4}{x - 3}$$

[b] If the domain of the function n where : $n(x) = \frac{b}{x} + \frac{9}{x + a}$ is $\mathbb{R} - \{0, 4\}$, $n(5) = 2$
Find the value of each of : a , b

Quiz 9 Till lesson 1 – unit 3

time 20 min.

1 Choose the correct answer from those given :

1 If A and B are two mutually exclusive events , then $P(A \cap B) =$ ....

(a) zero (b) 1 (c) $P(A)$ (d) $P(A \cup B)$

2 If $P(A) = 0.2$, $P(B) = 0.6$, $P(A \cap B) = 0.3$, then $P(A \cup B) =$ ....

(a) 0.5 (b) 0.62 (c) 5 (d) 0.13

3 If the two straight lines representing the two equations : $x + 2y = 4$, $2x + ky = 11$ are parallel , then $k =$ ....

(a) 4 (b) 1 (c) – 1 (d) 2

2 [a] If A and B are two mutually exclusive events from the sample space such that the probability of occuring the event A equals twice the probability of occuring the event B and the probability of occuring one of the two events at least is 0.66

Find :

1 The probability of occuring A 2 The probability of occuring B

[b] Reduce n (x) to the simplest form showing the domain of n :

$$n(x) = \frac{x^2 - 12x + 36}{x^2 - 6x} \times \frac{4x + 24}{36 - x^2}$$

Quiz 10 Till lesson 2 – unit 3

time 20 min.

1 Choose the correct answer from those given :

1 If A and B are two mutually exclusive events , then $P(A - B) =$ ....

(a) $P(A)$ (b) $P(B)$ (c) zero (d) 1

2 If $n_1(x) = \frac{x^2 - 4}{x - 2}$, $n_2(x) = x + 2$, then $n_1 = n_2$ when they have the common domain which is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\mathbb{R} - \{2, -2\}$

3 If A , B are events of the sample space of a random experiment and $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{3}$, then probability of occurrence of A or B = ...

(a) $\frac{5}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

2 [a] In the experiment of drawing one card randomly from 10 identical and well mixed cards and numbered from 1 to 10 , if A is the event that the card carries an even number and B is the event that the card carries a prime number.

Find : 1 $P(A)$ 2 $P(B)$ 3 $P(A \cup B)$

4 The probability of occuring one of the two events but not the other.

[b] Reduce n (x) to the simplest form showing the domain of n where :

$$n(x) = \frac{x^2 - 4}{x^2 - x - 2} - \frac{x^2 - 3x}{x^2 - 2x - 3}$$

Final Revision

on Algebra and Probability

Final revision on algebra and probability

Remember How to solve two equations of the first degree in two variables

It is possible to solve the two equations $2x - y = 5$ and $x + 3y + 1 = 0$ simultaneously

- Algebraically
  - Substituting method
  - Omitting method
- Graphically

First Graphically

Draw in the Cartesian plane the two straight lines L_1 and L_2 that represent the two equations , then the S.S. is the points of intersection of the two straight lines.

$\because L_1 : y = 2x - 5$

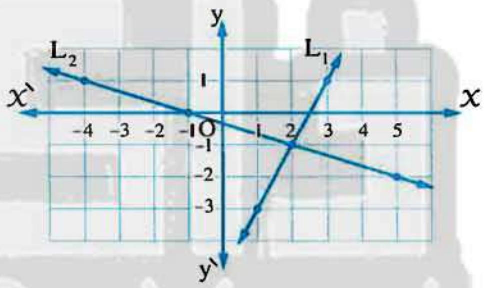
$\therefore$

x	1	2	3
y	-3	-1	1

$\because L_2 : x = -3y - 1$

$\therefore$

x	-1	-4	5
y	0	1	-2



From the graph : The solution set in $\mathbb{R} \times \mathbb{R} = \{(2 , -1)\}$

Notice that

- If L_1 and L_2 are coincidents , then there are infinite number of solutions.
- If L_1 and L_2 are parallel , then S.S. = $\varnothing$

Second Algebraically

1 Using substituting method :

1 Make one of the two variables in one of the two equations in one hand side of the equation with coefficient one. $\Rightarrow$ $\because 2x - y = 5$ $\therefore y = 2x - 5$

② Substitute by the value of the chosen variable y in the other equation to get the value of x	$x + 3(2x - 5) + 1 = 0$ $\therefore x + 6x - 15 + 1 = 0$ $\therefore 7x = 14 \quad \therefore x = 2$
③ Substitute by the value of x in the resulted equation in step (1) to get the value of y	$y = 2 \times 2 - 5$ $\therefore y = -1$

$\therefore$ The S.S. in $\mathbb{R} \times \mathbb{R} = \{(2, -1)\}$

2 Using omitting method :

① Put each of the two equations in the form : $a x + b y = c$	$2x - y = 5 \quad (1)$ $x + 3y = -1 \quad (2)$
② Make the coefficient of one of the two variables x or y in one of the two equations as an additive inverse of the coefficient of the same variable in the other equation.	Multiplying equation (2) by -2 : $\therefore -2x - 6y = 2 \quad (3)$
③ Add the two equations (1) , (3) , to find the value of y	$2x - y = 5 \quad (1)$ $-2x - 6y = 2 \quad (3)$ $-7y = 7$ $\therefore y = -1$
④ Substitut by the value of y in one of the two equations to find the value of x	Substituting in equation (1) $\therefore 2x - (-1) = 5$ $\therefore 2x + 1 = 5 \quad \therefore 2x = 4$ $\therefore x = 2$

$\therefore$ The S.S. in $\mathbb{R} \times \mathbb{R} = \{(2, -1)\}$

Remember **How to solve two equations in two variables one of them is of first degree and the other is of second degree**

To solve the two equations : $y - x = 3$, $x^2 + y^2 - xy = 13$ in $\mathbb{R} \times \mathbb{R}$, we follow the substituting method as the following :

1 From the first degree equation make one of the two variables in one hand side.	$\because y - x = 3$ $\therefore y = x + 3$
2 Substitute by the value of y in the equation of the second degree to get an equation of second degree in one unknown.	$\therefore x^2 + (x+3)^2 - x(x+3) = 13$ $\therefore x^2 + x^2 + 6x + 9 - x^2 - 3x - 13 = 0$ $\therefore x^2 + 3x - 4 = 0$
3 Solve the equation that you get using factorization or general formula to get the value of one of the two variables.	$\therefore (x + 4)(x - 1) = 0$ $\therefore x + 4 = 0$, then $x = -4$ or $x - 1 = 0$, then $x = 1$
4 Find the values of the other variable by substituting in the first degree equation in step (1).	at $x = -4$ $\therefore y = -4 + 3$ $\therefore y = -1$, at $x = 1$ $\therefore y = 1 + 3$ $\therefore y = 4$

$\therefore$ The S.S. $= \{(-4 , -1) , (1 , 4)\}$

Remember Solving an equation of the second degree in one unknown

First Graphically

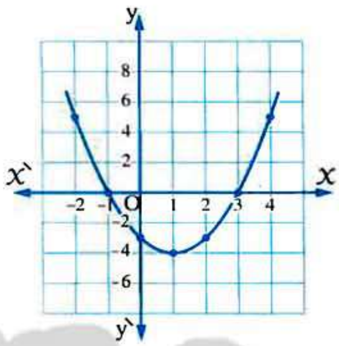
Put the equation in the form : $a x^2 + b x + c = 0$

, then draw the curve of the function which is related to the equation , then the solution set is the x-coordinates of the points of intersection of the function curve with x-axis.

For example :

To find the solution set of the equation : $x^2 - 2x = 3$

graphically in $\mathbb{R}$ on the interval $[-2 , 4]$



- Put the equation in the form : $x^2 - 2x - 3 = 0$
- Assume that : $f(x) = x^2 - 2x - 3$

$\therefore$

x	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

From the graph : The S.S. $= \{3 , -1\}$

Note that

- If the curve touches x-axis at one point , then the equation has a unique solution in $\mathbb{R}$
- If the curve does not intersect x-axis , then the S.S. of the equation is $\varnothing$

Second By using the general rule (general formula)

If $a x^2 + b x + c = 0$ where a , b and c are real numbers , $a \neq 0$

, then $\boxed{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

For example :

To find the S.S. of the equation : $x^2 - 6x + 7 = 0$ in $\mathbb{R}$

, then $a = 1$, $b = -6$ and $c = 7$

$\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{6 \pm \sqrt{36 - 4 \times 1 \times 7}}{2 \times 1} = \dfrac{6 \pm \sqrt{8}}{2} = \dfrac{6 \pm 2\sqrt{2}}{2} = 3 \pm \sqrt{2}$

$\therefore x = 3 + \sqrt{2}$ or $x = 3 - \sqrt{2}$ $\therefore$ The S.S. $= \{3 + \sqrt{2} , 3 - \sqrt{2}\}$

Remember **How to find zeroes of the function , domain and common domain of the algebraic fractions**

- To get zeroes of the polynomial function , put $f(x) = 0$ and solve the resulting equation.
- The set of zeroes of the algebraic fractional function
= the set of zeroes of the numerator – the set of zeroes of the denominator
- Domain of the algebraic fractional function = $\mathbb{R}$ – the set of zeroes of the denominator
- Common domain for two algebraic fractions or more = $\mathbb{R}$ – the set of zeroes of the denominators of these fractions

Example

Find in $\mathbb{R}$ the set of zeroes of the functions that are defined by the following rules :

1. $f(x) = 3x^2 - 15x$
2. $f(x) = x^2 + 4$
3. $f(x) = \dfrac{x^2 - 9}{x^2 - x - 6}$, then find the domain of f
4. $f(x) = \text{zero}$, $n(x) = 5$

Solution

1. Let $3x^2 - 15x = 0$
$\therefore 3x(x-5) = 0$
$\therefore x = 0$ or $x = 5$
$\therefore z(f) = \{0 , 5\}$

2. Let $x^2 + 4 = 0$
$\therefore x^2 = -4 \quad \therefore x = \pm\sqrt{-4}$
$, \because \sqrt{-4} \notin \mathbb{R}$
$\therefore z(f) = \varnothing$

3. $\because f(x) = \dfrac{(x-3)(x+3)}{(x-3)(x+2)}$
$\therefore$ The domain of $f = \mathbb{R} - \{3 , -2\}$, $z(f) = \{3 , -3\} - \{3 , -2\} = \{-3\}$

4. $z(f) = \mathbb{R}$, $z(n) = \varnothing$

Remember **How to simplify the algebraic fraction**

1. Factorize each of the numerator and the denominator perfectly.
2. Determine the domain.
3. Omit the common factors of the numerator and the denominator.

Example

If $n(x) = \dfrac{x^2 - 3x - 10}{x^2 - 25}$, then find : $n(x)$ in its simplest form showing the domain of n

Solution

$n(x) = \dfrac{(x-5)(x+2)}{(x-5)(x+5)}$, the domain of $n = \mathbb{R} - \{5 , -5\}$, $n(x) = \dfrac{x+2}{x+5}$

Remember **Equality of two algebraic fractions**

Two functions n_1 and n_2 are said to be equal if the two following conditions satisfied :

1. The domain of n_1 = the domain of n_2
2. $n_1(X) = n_2(X)$ for each $X \in$ the common domain.

Example

If $n_1(X) = \frac{3X}{3X + 12}$, $n_2(X) = \frac{X^2 + 4X}{X^2 \times 8X + 16}$ **Prove that :** $n_1 = n_2$

Solution

$\because n_1(X) = \frac{3X}{3(X+4)}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{-4\}$, $n_1(X) = \frac{X}{X+4}$

, $\because n_2(X) = \frac{X(X+4)}{(X+4)^2}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{-4\}$

, $n_2(X) = \frac{X}{X+4}$ $\therefore$ The domain of n_1 = the domain of n_2

, $n_1(X) = n_2(X)$ $\therefore n_1 = n_2$

Remark

For any two functions n_1 and n_2 if $n_1(X) = n_2(X)$ while the domain of $n_1 \neq$ the domain of n_2 , then $n_1 = n_2$ only in the common domain of the two functions

i.e. The domain in which the two functions are equal is the common domain of these two functions.

For example :

If $n_1(X) = \frac{X^2 + 2X}{X^2 - 4}$, $n_2(X) = \frac{X^2 - X}{X^2 - 3X + 2}$

, then $n_1(X) = \frac{X(X+2)}{(X+2)(X-2)} = \frac{X}{X-2}$

, the domain of $n_1 = \mathbb{R} - \{2 , -2\}$, $n_2(X) = \frac{X(X-1)}{(X-2)(X-1)} = \frac{X}{X-2}$

, the domain of $n_2 = \mathbb{R} - \{2 , 1\}$

i.e. $n_1(X) = n_2(X)$ while the domain of $n_1 \neq$ the domain of n_2

Therefore $n_1 = n_2$ only in the common domain which is $\mathbb{R} - \{2 , -2 , 1\}$

Remember steps of performing operations on the algebraic fractions

Addition and subtraction | **Multiplication** | **Division**

(1) Arrange the terms of the numerator and the denominator of each fraction ascendingly (or descendingly) according to the powers of any variable in it.

(2) Factorize the numerator and the denominator of each fraction if possible.

Addition and subtraction

(3) Find the common domain.

(4) Simplify each fraction

(5) Make the denominator common

(6) Perform adding or subtracting terms of numerators

(7) Simplify the result

Multiplication

(3) Find the common domain

(4) Cancel the common factors between numerator and denominator of any of the two fractions

(5) Perform the multiplication operation

(6) Simplify the result

Division

(3) Find the common domain between divident and multiplicative inverse of the divisor

(4) Change division into multiplication by using reciprocal of the divisor

(5) Cancel the common factors between numerators and denominators of the two fractions

(6) Perform multiplication operation

(7) Simplify the result

Remarks

(1) The domain of each of $(n_1 + n_2)$ or $(n_1 - n_2)$ or $(n_1 \times n_2)$ is the common domain of the two fractions n_1 , n_2

(2) The domain of $(n_1 \div n_2)$ is the common domain of the fractions : n_1 , n_2^{-1}

(3) The number "zero" is the additive neutral for any algebraic fraction and the number "one" is the multiplicative neutral for any algebraic fraction.

(4) The domain of an algebraic fraction = the domain of its additive inverse

"To find the additve inverse of an algebraic fraction change the sign of its numerator or denominator".

For example :

Additive inverse of the fraction $\frac{2}{x-1}$ is $-\frac{2}{x-1} = \frac{-2}{x-1} = \frac{2}{1-x}$

(5) If $n(x) = \frac{p(x)}{k(x)} \neq 0$, then the multiplicative inverse of the fraction n is n^{-1}

where $n^{-1}(x) = \frac{k(x)}{p(x)}$

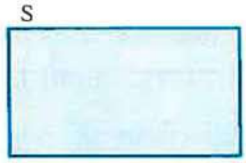
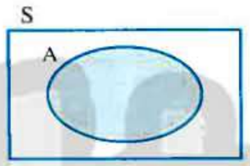
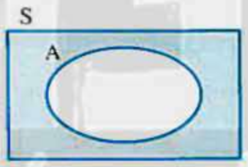
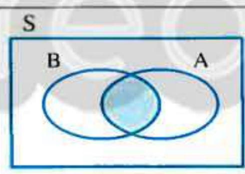
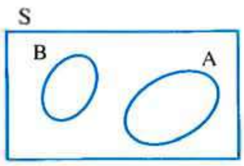
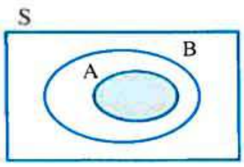
and the domain of n^{-1} is $\mathbb{R}$ – the set of zeroes of each of the numerator and the denominator of any of the two fractions.

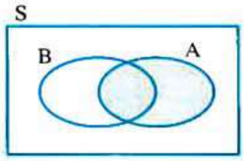
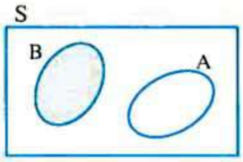
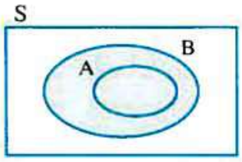
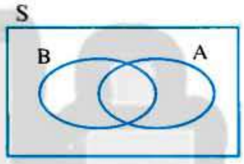
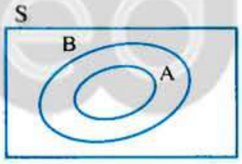
For example :

If $n(x) = \frac{x+1}{x-5}$, then $n^{-1}(x) = \frac{x-5}{x+1}$

Where the domain of $n = \mathbb{R} - \{5\}$, the domain of $n^{-1} = \mathbb{R} - \{5 , -1\}$

Remember **Probability of some events and operations on them**

Words representation of the event	Probability of the event	Representing event by Venn diagram
Probability of occuring the certain event = 1	$P(S) = 1$	
Probability of occuring the impossible event = zero	$P(\varnothing) = \text{zero}$	
Probability of occuring the event A	$P(A) = \frac{n(A)}{n(S)}$	
The complementary event probability of occuring the complementary event of the event A or probability of non occuring event A	$P(\grave{A}) = \frac{n(\grave{A})}{n(S)} = 1 - P(A)$	
Intersecting of two events $(A \cap B)$ Probability of occuring A and B together	$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$	
	* If A and B are mutually exclusive events , then $P(A \cap B) = \text{zero}$	
	* If $A \subset B$, then $P(A \cap B) = P(A)$	

Union of two events $(A \cup B)$ * Probability of occuring the events A or B or both of them. * Probability of occuring one of the two events at least. * Probability of occuring any of the two events.	$P(A \cup B) = \frac{n(A \cup B)}{n(S)}$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
	* If A and B are two mutually exclusive events , then : $P(A \cup B) = P(A) + P(B)$	
	* If $A \subset B$, then $P(A \cup B) = P(B)$	
The difference between events $(A - B)$ * Probability of occuring the event A and non occuring of event B * Probability of occuring the event A only.	$P(A - B) = \frac{n(A - B)}{n(S)}$, $P(A - B) = P(A) - P(A \cap B)$	
	* If A and B are mutually exclusive events , then $P(A - B) = P(A)$	
	* If $A \subset B$, then $P(A - B) = P(\varnothing) = \text{zero}$	

Remarks

(1) $P(A \cap \grave{A}) = \text{zero}$

(2) If $P(A) = P(\grave{A})$, then $P(A) = \frac{1}{2}$, $P(\grave{A}) = \frac{1}{2}$

(3) Probability of non occuring the two events A and B together $= P(\grave{(A \cap B)}) = 1 - P(A \cap B)$

(4) Probability of non occuring any of the two events A or B $= P(\grave{(A \cup B)}) = 1 - P(A \cup B)$

(5) Probability of occuring one of the two event with non occuring of the other (Probability of occuring only one of the two events) $= P(A - B) + P(B - A)$

Final Examinations

of Algebra and Probability

Model Examinations of the School Book on Algebra and Probability

Model 1

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 The domain of the function n : n $(x) = \frac{x}{x-1}$ is ....

(a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{0, 1\}$ (d) $\mathbb{R} - \{-1\}$

2 The number of solutions of the two equations : $x + y = 2$ and $y + x = 3$ together in $\mathbb{R} \times \mathbb{R}$ is ....

(a) zero (b) 1 (c) 2 (d) 3

3 If $x \neq 0$, then $\frac{5x}{x^2+1} \div \frac{x}{x^2+1} =$ ....

(a) -5 (b) -1 (c) 1 (d) 5

4 If the ratio between the perimeters of two squares is 1 : 2 , then the ratio between their areas is ....

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

5 The equation of the symmetric axis of the curve of the function f where $f(x) = x^2 - 4$ is ....

(a) $x = -4$ (b) $x = 0$ (c) $y = 0$ (d) $y = -4$

6 If $A \subset S$ of random experiment and $P(\grave{A}) = 2\,P(A)$, then $P(A) =$ ....

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1

2 [a] **By using the general formula , find in $\mathbb{R}$ the solution set of the equation :**

$2x^2 - 5x + 1 = 0$ "approximate the result to the nearest one decimal".

[b] **Find n (x) in the simplest form showing the domain where :**

$$n(x) = \frac{x-3}{x^2-7x+12} - \frac{4}{x^2-4x}$$

3 [a] **Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :**

$x - y = 0$ and $x^2 + xy + y^2 = 27$

[b] **Find n (x) in the simplest form showing the domain where :**

$n(x) = \frac{x^2+4x+3}{x^3-27} \div \frac{x+3}{x^2+3x+9}$ then find n (2) , n (– 3) if possible.

4 **[a]** A rectangle with a length more than its width by 4 cm. If the perimeter of the rectangle is 28 cm. , find the area of the rectangle.

[b] If $n(x) = \dfrac{x^2 - 2x}{x^2 - 3x + 2}$,

1. Find $n^{-1}(x)$ in the simplest form showing the domain of n^{-1}
2. If $n^{-1}(x) = 3$, then find the value of x

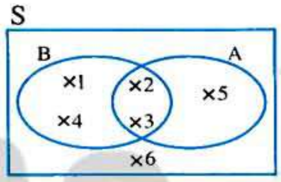
5 **[a]** If $n_1(x) = \dfrac{x^2}{x^3 - x^2}$ and $n_2(x) = \dfrac{x^3 + x^2 + x}{x^4 - x}$ **, then prove that :** $n_1 = n_2$

[b] In the opposite figure :

If A and B are two events in a sample space S of a random experiment **, then find :**

1. $P(A \cap B)$
2. $P(A - B)$
3. The probability of non-occurrence of the event A

S



Model 2

Answer the following questions :

1 **Choose the correct answer :**

1. The solution set of the two equations : $x = 3$, $y = 4$ in $\mathbb{R} \times \mathbb{R}$ is ....
   (a) $\{(3, 4)\}$ (b) $\{(4, 3)\}$ (c) $\mathbb{R}$ (d) $\varnothing$
2. The set of zeroes of the function f where $f(x) = x^2 + 4$ in $\mathbb{R}$ is ....
   (a) $\{2\}$ (b) $\{2, -2\}$ (c) $\mathbb{R}$ (d) $\varnothing$
3. If A and B are two mutually exclusive events of a random experiment , then $P(A \cap B) =$ ....
   (a) 0 (b) 1 (c) 0.5 (d) $\varnothing$
4. The domain of the multiplicative inverse of the function $f : f(x) = \dfrac{x+2}{x-3}$ is ....
   (a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{-2, 3\}$ (c) $\mathbb{R} - \{-3\}$ (d) $\mathbb{R}$
5. The two straight lines : $3x + 5y = 0$, $5x - 3y = 0$ are intersect in ....
   (a) first quadrant. (b) second quadrant. (c) the origin point. (d) fourth quadrant.
6. If $P(A) = 0.6$, then $P(\grave{A}) =$ ....
   (a) 0.4 (b) 0.6 (c) 0.5 (d) 1

2 [a] **Find in $\mathbb{R}$ the solution set of the equation : $3x^2 - 5x + 1 = 0$**

by using the formula "approximate the result to the nearest two decimal places".

[b] **Simplify :**

$n(x) = \frac{x^3 - 8}{x^2 + x - 6} \times \frac{x + 3}{x^2 + 2x + 4}$, showing the domain of n.

3 [a] **Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x - y = 1$, $x^2 + y^2 = 25$

[b] If A and B are two events of a random experiment and

$P(A) = 0.3$, $P(B) = 0.6$, $P(A \cap B) = 0.2$

Find : 1 $P(A \cup B)$ 2 $P(A - B)$

4 [a] **Solve the following two equations in $\mathbb{R} \times \mathbb{R}$:** $2x - y = 3$, $x + 2y = 4$

[b] **Simplify :**

$n(x) = \frac{x^2 + 3x}{x^2 - 9} \div \frac{2x}{x + 3}$, showing the domain of n.

5 [a] **Simplify :**

$n(x) = \frac{x^2 + 2x}{x^2 - 4} + \frac{x + 3}{x^2 - 5x + 6}$, showing the domain of n.

[b] Graph the function f where $f(x) = x^2 - 1$, $x \in [-3, 3]$, from the graph find in $\mathbb{R}$ the solution set of the equation : $x^2 - 1 = 0$

Governorates' Examinations on Algebra and Probability

1 Cairo Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1. If the two equations $x + 3y = 6$, $2x + my = 12$ have an infinite number of solutions , then m = ...
   (a) 1 (b) 2 (c) 3 (d) 6
2. If $2^{k-3} = 1$, then k = ...
   (a) -3 (b) zero (c) 3 (d) 8
3. The set of zeroes of the function $f : f(x) =$ zero is ....
   (a) $\mathbb{R} - \{0\}$ (b) $\varnothing$ (c) $\{0\}$ (d) $\mathbb{R}$
4. If $x^2 + ax - 4 = (x+2)(x-2)$, then a = ...
   (a) -2 (b) zero (c) 2 (d) 4
5. If the two events A , B are mutually exclusive events from the sample space of a random experiment , then $P(A \cap B) =$ ....
   (a) 1 (b) $\frac{1}{2}$ (c) $\varnothing$ (d) zero
6. If $|x| = 7$, then $x =$ ....
   (a) 7 (b) -7 (c) ± 7 (d) 14

2 **[a]** Two real numbers their sum is 40 , and the difference between them is 10 , find the two numbers.

[b] Find n (x) in the simplest form , showing the domain where : $n(x) = \frac{x}{x-2} - \frac{2x+4}{x^2-4}$

3 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations together :**

$x - 3 = 0$, $x^2 + y^2 = 25$

[b] If $n_1(x) = \frac{x^2}{x^3 - x^2}$, $n_2(x) = \frac{x^2 + x + 1}{x^3 - 1}$

, prove that : $n_1(x) = n_2(x)$ for all the values of x which belong to the common domain and find this domain.

4 **[a] Find n (x) in the simplest form , showing the domain where :**

$n(x) = \frac{x^3 - 8}{x^2 + x - 6} \times \frac{x + 3}{x^2 + 2x + 4}$

[b] Find algebraically in $\mathbb{R}$ the solution set of the equation : $2x^2 + 5x - 6 = 0$, approximating the results to the nearest one decimal place.

5 [a] If A , B are two events of the sample space of a random experiment and $P(A) = 0.7$, $P(B) = 0.5$, $P(A \cap B) = 0.3$

, find : 1 $P(A \cup B)$ 2 $P(A - B)$

[b] If $n(x) = \dfrac{x}{x+3}$

1 Find $n^{-1}(x)$, showing the domain of n^{-1} 2 If $n^{-1}(x) = 4$, find the value of x

2 Giza Governorate

Answer the following questions :

1 Choose the correct answer from the given ones :

1 If the perimeter of a square is 16 cm. , then its area = ... cm^2.

(a) 4 (b) 8 (c) 16 (d) 64

2 The domain of the function n : $n(x) = \dfrac{x}{x^2 - 1}$ is ....

(a) $\{-1\}$ (b) $\mathbb{R} - \{1\}$ (c) $\{1, -1\}$ (d) $\mathbb{R} - \{1, -1\}$

3 If $\frac{1}{3}x = 2$, then $\frac{1}{2}x =$ ....

(a) 2 (b) 3 (c) 6 (d) 8

4 The number of solutions of the two equations $x + y = 1$, $x + y = 2$ together in $\mathbb{R} \times \mathbb{R}$ is ....

(a) zero (b) 1 (c) 2 (d) 3

5 If $x^2 + kx + 81$ is a perfect square , then k = ...

(a) ± 6 (b) ± 9 (c) ± 18 (d) ± 81

6 If $A \subset S$ of a random experiment , $P(A) + P(\grave{A}) = 2k$, then k = ...

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

2 [a] **By using the formula find in $\mathbb{R}$ the solution set of the equation :**

$2x^2 - 5x + 1 = 0$ rounding the results to two decimal places.

[b] **Find n (x) in its simplest form where :**

$n(x) = \dfrac{x^2 - 4}{x^3 - 8} \div \dfrac{x^2 - x - 6}{x^2 + 2x + 4}$, showing the domain.

3 **[a]** A right-angled triangle of hypotenuse length 10 cm. and its perimeter is 24 cm. Find the lengths of the other two sides.

[b] If A , B are two mutually exclusive events of a random experiment , P (A) = 0.2 , P (B) = 0.5 , **find :** P (A $\cup$ B) and P (A – B)

4 **[a]** If $n(x) = \dfrac{x^2 - 3x}{x^2 - 5x + 6}$

, **find :** **1** $n^{-1}(x)$ in the simplest form , showing the domain of n^{-1}

2 The value of x if $n^{-1}(x) = 2$

[b] Find the solution set for the following equations algebraically in $\mathbb{R} \times \mathbb{R}$:

$x + 2y = 4$, $3x - y = 5$

5 **[a]** If $n(x) = \dfrac{x^2}{x-1} + \dfrac{x}{1-x}$, then find $n(x)$ in the simplest form , showing the domain.

[b] If $n_1(x) = \dfrac{x^2 + x - 6}{x^2 - 4}$, $n_2(x) = \dfrac{x^2 - 9}{x^2 - x - 6}$, then show whether $n_1 = n_2$ or not and why.

3 Alexandria Governorate

Answer the following questions : ***(Calculators are allowed)***

1 **Choose the correct answer from those given :**

1 The set of zeroes of the function f where $f(x) = x + 4$ in $\mathbb{R}$ is ……………

(a) $\{4 , -4\}$ (b) $\{-4\}$ (c) $\mathbb{R}$ (d) $\varnothing$

2 If $x^3 y^{-3} = 8$, then $\dfrac{y}{x} =$ ……………

(a) $\frac{1}{512}$ (b) $\frac{1}{8}$ (c) 2 (d) $\frac{1}{2}$

3 The equation of the symmetric axis of the curve of the function f where $f(x) = x^2 - 4$ is ……………

(a) $x = -4$ (b) $x =$ zero (c) y = zero (d) y = – 4

4 The solution set of the equation : $x^2 = 9$ in $\mathbb{Q}$ is ……………

(a) $\{-3\}$ (b) $\{3\}$ (c) $\varnothing$ (d) $\{-3 , 3\}$

5 If A $\subset$ S of a random experiment and P $(\acute{A})$ = 2 P (A) , then P (A) = ……………

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1

6 $\dfrac{5^{x+2}}{5^{x+1}} =$ ……………

(a) 5 (b) 10 (c) 15 (d) 20

2 **[a] Find the solution set of the two equations :**

$x - y = 0$ and $x^2 + xy + y^2 = 27$ in $\mathbb{R} \times \mathbb{R}$

[b] Find the common domain for which $n_1(x)$ and $n_2(x)$ are equal , where :

$n_1(x) = \frac{x^2 + x - 12}{x^2 + 5x + 4}$, $n_2(x) = \frac{x^2 - 2x - 3}{x^2 + 2x + 1}$

3 **[a] By using the general formula , find in $\mathbb{R}$ the solution set of the equation :**

$2x^2 + 5x = 0$

[b] Find $n(x)$ in the simplest form , showing the domain where :

$n(x) = \frac{x^3 - 1}{x^2 - x} \times \frac{x + 3}{x^2 + x + 1}$

4 **[a] Find algebraically the solution set of the two equations :**

$2x + y = 1$, $x + 2y = 5$ in $\mathbb{R} \times \mathbb{R}$

[b] Find $n(x)$ in the simplest form , showing the domain where :

$n(x) = \frac{x^2 - x}{x^2 - 1} + \frac{x + 5}{x^2 + 6x + 5}$

5 [a] If $n(x) = \frac{x^2 - 2x}{(x - 2)(x^2 + 2)}$

1 Find $n^{-1}(x)$ in the simplest form , showing the domain on n^{-1}

2 If $n^{-1}(x) = 3$, then find the value of x

[b] If A and B are two mutually exclusive events of a random experiment and

$P(A) = \frac{1}{3}$, $P(A \cup B) = \frac{7}{12}$, **find :** $P(B)$

4 El-Kalyoubia Governorate

Answer the following questions :

1 **Choose the correct answer :**

1 If $x^2 + kx - 21 = (x - 3)(x + 7)$, then k = ...

(a) -2 (b) 4 (c) 8 (d) 20

2 One of the solutions for the two equations : $x - y = 2$, $x^2 + y^2 = 20$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $(-4 , 2)$ (b) $(2 , -4)$ (c) $(3 , 1)$ (d) $(4 , 2)$

3 If $5^{x-3} = 1$, then $2x^2 =$ ....

(a) 36 (b) 9 (c) 18 (d) 3

4 If $A \cap B = \varnothing$, then $P(A - B) =$ ....

(a) $P(A)$ (b) $P(B)$ (c) $P(B - A)$ (d) 1

5 If the width of the rectangle is 3 cm. , and its diagonal length is 5 cm. , then its length is .... cm.

(a) 2 (b) $\frac{5}{3}$ (c) 4 (d) $\frac{3}{5}$

6 If $\frac{1}{x} + \frac{1}{y} + \frac{1}{xy} = \frac{k}{xy}$, then k = ...

(a) 2 (b) 3 (c) $x + y + 1$ (d) $x + y$

2 [a] If A and B are two events from the sample space of a random experiment and $P(A) = 0.8$, $P(B) = 0.7$, $P(A \cap B) = 0.6$

, find : 1 $P(A \cup B)$ 2 The probability of non-occurrence of the event A

[b] A rectangle with a length more than its width by 4 cm. If the perimeter of the rectangle is 28 cm. , find the area of the rectangle.

3 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x - y = 0$, $x^2 + xy + y^2 = 27$

[b] Find n (x) in the simplest form , showing the domain : $n(x) = \frac{x^2 + 2x}{x^3 - 27} \div \frac{x + 2}{x^2 + 3x + 9}$

4 **[a] Find in $\mathbb{R}$ the solution set of the equation :** $2x^2 - 4x + 1 = 0$ approximating the results to one decimal place. (using the general rule)

[b] If $n_1(x) = \frac{2x}{2x + 4}$, $n_2(x) = \frac{x^2 + 2x}{x^2 + 4x + 4}$, **prove that :** $n_1 = n_2$

5 **[a] Find n (x) in the simplest form , showing the domain :**

$$n(x) = \frac{x^2 + 2x + 4}{x^3 - 8} + \frac{x^2 - 9}{x^2 + x - 6}$$

[b] If the domain of the function f where $f(x) = \frac{x}{x^2 - 5x + m}$ is $\mathbb{R} - \{2 , k\}$, then find the value of each of m and k

5 El-Sharkia Governorate

Answer the following questions : ***(Calculators are allowed)***

1 **Choose the correct answer from the given ones :**

1 If the domain of the fractional function n (x) is $\mathbb{R} - \{2 , 3 , 4\}$, then n (3) = ...

(a) 3 (b) 2 (c) 4 (d) not exist

2 If $x^2 + y^2 = 5$, $xy = 2$ where $x \in \mathbb{R}$, $y \in \mathbb{R}$, then $(x + y)^2 =$ ....

(a) 7 (b) 9 (c) 5 (d) 13

3 The point (2 , – 1) does not belong to the straight line whose equation is ....

(a) $x + y = 1$ (b) $x - y = 3$ (c) $x = 2$ (d) $y = 5$

4 If $n(x) = \frac{x}{x-1}$, then the domain of n^{-1} is ....

(a) $\mathbb{R} - \{1, 0\}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R} - \{1\}$ (d) $\{1, 0\}$

5 The two straight lines $L_1 : 3x + 7y = 0$ and $L_2 : 5x + 9y = 0$ are intersecting in the ....

(a) third quadrant. (b) fourth quadrant. (c) first quadrant. (d) origin point.

6 If A , B are two events from the sample space of a random experiment and $A \subset B$, which of the following expressions is false ?

(a) $P(A \cup B) = P(B)$ (b) $P(A \cap B) = P(A)$

(c) $P(A - B) =$ zero (d) $P(A - B) = P(B)$

2 [a] **By using the general formula , find in $\mathbb{R}$ the solution set of the equation :** $x(x-2) = 1$

[b] If $n(x) = \frac{x^3 + x}{x^2 + 1} + \frac{x^2 + 2x + 4}{x^3 - 8}$, find $n(x)$ in the simplest form , showing the domain.

3 [a] **Find the solution set in $\mathbb{R} \times \mathbb{R}$ of the two equations :** $2x - y = 3$, $x + 2y = 4$

[b] If $n(x) = \frac{x^2 - 2x - 15}{x^2 - 9} \div \frac{10 - 2x}{x^2 - 6x + 9}$

, find $n(x)$ in the simplest form , showing the domain.

4 [a] **Find the solution set of the following two equations in $\mathbb{R} \times \mathbb{R}$:**

$x + 2y = 2$, $x^2 + 2xy = 2$

[b] If $n_1(x) = 1 - \frac{1}{x}$, $n_2(x) = \frac{1 - x}{x}$, show whether $n_1 = n_2$ or not.

5 [a] In a random experiment , a regular dice is rolled once and observing the upper face.

If : **A :** The event of getting an even number.

B : The event of getting a prime number.

, find : $P(A)$, $P(B)$, $P(A \cup B)$

[b] If $n(x) = \frac{k}{x} + \frac{9}{x + m}$ where the domain of n is $\mathbb{R} - \{0, 4\}$, and $n(5) = 2$

, find the value of each of : m , k

6 El-Monofia Governorate

Answer the following questions : ***(Using calculator is permitted)***

1 Choose the correct answer from those given :

1 $4^{15} + 4^{15} = \dots\dots$

(a) 4^{30} (b) 4^{zero} (c) 8^{15} (d) 2^{31}

2 The necessary numbers to complete the pattern :

$\frac{1}{5}$, 0.4 , $\frac{3}{5}$, ... , ... , ... , $\frac{7}{5}$ is ....

(a) 0.8 , $\frac{6}{5}$, 1.2 (b) 0.8 , 1 , 1.2 (c) 0.6 , 0.8 , 1 (d) 0.8 , 1 , 4.1

3 The multiplicative inverse of the number $1 - \sqrt{2}$ is ....

(a) $1 + \sqrt{2}$ (b) $\sqrt{2} - 1$ (c) $-\left(1 + \sqrt{2}\right)$ (d) $\frac{1+\sqrt{2}}{2}$

4 The domain of the function $n^{-1}(x) = \frac{x+4}{x-4}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{4\}$ (c) $\mathbb{R} - \{-4\}$ (d) $\mathbb{R} - \{4 , -4\}$

5 The two straight lines : $3x - 5y = 0$, $5x + 3y = 0$ intersect at the ....

(a) 1^{st} quadrant. (b) 3^{rd} quadrant. (c) origin point. (d) 4^{th} quadrant.

6 If $P(A) = 3P(\grave{A})$, then $P(A) = \dots\dots$

(a) $\frac{3}{4}$ (b) 1 (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

2 [a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations :

$2x - y = 3$, $x + 2y = 4$

[b] Find in $\mathbb{R}$ by using the general formula the solution set of the equation :

$3x^2 = 5x - 1$ rounding the result to the nearest two decimal digits.

3 [a] If the set of zeroes of the function $f : f(x) = \frac{x^2 - ax + 9}{bx + 4}$ is $\{3\}$ and its domain is $\mathbb{R} - \{2\}$ **, find the value of each of a and b**

[b] If $n(x) = \frac{x^3 - 8}{x^2 - 3x + 2} \div \frac{x^3 + 2x^2 + 4x}{2x^2 + x - 3}$, find $n(x)$ in the simplest form , showing the domain.

4 [a] If $n(x) = \frac{x-3}{x^2 - 7x + 12} - \frac{4}{x^2 - 4x}$, find $n(x)$ in the simplest form , showing the domain , then find $n(4)$ if it is possible.

[b] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations : $x + y = 4$, $\frac{1}{x} + \frac{1}{y} = 1$, where $x \neq 0$, $y \neq 0$

5 [a] If $n_1(x) = \frac{x^2+5x+6}{x^2+x-2}$ and $n_2(x) = \frac{x^2-2x-15}{x^2-6x+5}$, is $n_1 = n_2$? and why ?

[b] If A and B are two events of the sample space of a random experiment , and $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{5}{8}$, **find each of the following :**

1 $P(A \cap B)$ 2 $P(B - A)$ 3 $P(A \cup B)'$

7 El-Gharbia Governorate

Answer the following questions :

1 Choose the correct answer :

1 If $2^{x+1} = 1$, then $x \in$ ....

(a) $\{0\}$ (b) $\{0 , -1\}$ (c) $\{-1\}$ (d) $\mathbb{R} - \{-1\}$

2 The number of solutions of the equation $x - y = 0$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) 1 (b) 2 (c) 3 (d) infinite

3 In the experiment of tossing a piece of coin once , if A is the event of appearance of a head , B is the event of appearance of a tail , then $P(A \cup B) =$ ....

(a) $\frac{1}{2}$ (b) 1 (c) zero (d) $\varnothing$

4 The set of zeroes of f : $f(x) = \frac{-3}{x-2}$ is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{3\}$ (c) $\{2\}$ (d) $\varnothing$

5 If the curve of the quadratic function f passes through the points $(-1 , 0)$, $(0 , -4)$, $(4 , 0)$, then the solution set of the equation $f(x) = 0$ in $\mathbb{R}$ is ....

(a) $\{-1 , 0\}$ (b) $\{-4 , 0\}$ (c) $\{-1 , 4\}$ (d) $\{4 , -4\}$

6 If $\sqrt{x^2} = 25$, then $x =$ ....

(a) 5 (b) ± 5 (c) 25 (d) ± 25

2 [a] If A and B are two events in the sample space of a random experiment and $P(A) = 0.5$, $P(A \cup B) = 0.8$, $P(B) = x$, $P(A \cap B) = 0.1$

Find the value of : x and $P(A - B)$

[b] If $n(x) = x + \frac{x}{x-2}$, find $n^{-1}(x)$ in the simplest form , showing the domain of n^{-1}

3 [a] **Find n (x) in the simplest form , showing the domain of n where :**

$n(x) = \frac{x}{x-2} - \frac{x}{x+2}$

[b] **Find algebraically in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations :**

$x - y = 3$, $y^2 - xy = 21$

4 **[a] By using the general rule and without using the calculator , find in $\mathbb{R}$ the solution set of the equation :** $x^2 + 2x - 4 = 0$ in the simplest form.

[b] If $n_1(x) = \frac{x^2 - 4}{x^2 + x - 6}$, $n_2(x) = \frac{x^2 - x - 6}{x^2 - 9}$, is $n_1 = n_2$? With the reason.

5 **[a] Find n (x) in the simplest form , showing the domain of n where :**

$n(x) = \frac{x^3 - 1}{x^2 - x} \div \frac{x^2 + x + 1}{x + 3}$

[b] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations algebraically or graphically : $y = x + 4$, $x + y = 4$

8 El-Dakahlia Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **[a] Choose the correct answer from the given ones :**

1. The solution set of the two equations $x - 3 = 0$, $y = 4$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{3 , 4\}$ (b) $\{(3 , 4)\}$ (c) $\{(4 , 3)\}$ (d) $\varnothing$

2. If A , B are two events in a random experiment , $A \subset B$, then $P(A \cup B) =$ ....

(a) P (B) (b) P (A) (c) $P(A \cap B)$ (d) 0

3. If $3^y \times 5^y = 225$, then y = ...

(a) 2 (b) 15 (c) 0 (d) 20

[b] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the equations : $3x - y = 5$ and $x + 2y = 4$

2 **[a] Choose the correct answer from the given ones :**

1. The domain of the additive inverse of the function n : $n(x) = \frac{x + 2}{x - 3}$ is ....

(a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{-2\}$ (c) $\mathbb{R} - \{-2 , 3\}$ (d) $\mathbb{R}$

2. The set of zeroes of the function f : $f(x) = x^2 + 9$ in $\mathbb{R}$ is ....

(a) $\mathbb{R}$ (b) $\{3\}$ (c) $\{3 , -3\}$ (d) $\varnothing$

3. The curve $y = a x^2 + b x + c$ cuts y-axis at the point ....

(a) (0 , b) (b) (b , 0) (c) (c , 0) (d) (0 , c)

[b] Find n (x) in the simplest form , showing the domain : $n(x) = \frac{x^2 + x}{x^2 - 1} - \frac{5 - x}{x^2 - 6x + 5}$

3 **[a]** If A , B are two events in a random experiment and P (A) = 0.6 , P (B) = 0.5 , $P(A \cap B) = 0.3$, **find :** $P(A \cup B)$, $P(\acute{B})$

[b] Simplify to the simplest form , showing the domain :

$$n(x) = \frac{x^3 - 1}{x^2 - 2x + 1} \times \frac{2x - 2}{x^2 + x + 1}$$

4 **[a]** If $n_1(x) = \frac{x^2 - x}{x^3 - 2x^2}$, $n_2(x) = \frac{x^2 - 3x + 2}{x^3 - 4x^2 + 4x}$ **, prove that :** $n_1 = n_2$

[b] By using the general rule , find the solution set of the equation :
$2x^2 - 4x + 1 = 0$ in $\mathbb{R}$, rounding the results to two decimal places.

5 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x - y = 0$ and $x = \frac{4}{y}$ algebraically.

[b] If $n(x) = \frac{x^2 - 2x}{(x - 2)(x^2 + 2)}$

1. Find : $n^{-1}(x)$ and identify the domain of n^{-1}
2. If $n^{-1}(x) = 3$, what is the value of x ?

9 Ismailia Governorate

Answer the following questions : (Calculators are allowed)

1 **Choose the correct answer from those given :**

1. If x is the additive identity element , y is the multiplicative identity element , then $2^x + 3^y =$ ....
   (a) 2 (b) 3 (c) 4 (d) 5
2. The set of zeroes of the function $f : f(x) = 2x - 6$ is ....
   (a) $\{1\}$ (b) $\{3\}$ (c) $\{5\}$ (d) $\{7\}$
3. If $\sqrt{x} = 2$, then $\frac{1}{2} x =$ ....
   (a) 8 (b) 6 (c) 4 (d) 2
4. The number of solutions of the two equations : $2x - y = 3$, $x + 2y = 4$ in $\mathbb{R} \times \mathbb{R}$ is ....
   (a) 1 (b) zero (c) 2 (d) infinite.
5. If A , B are two mutually exclusive events of a random experiment , then $P(A \cap B) =$ ....
   (a) $\varnothing$ (b) 1 (c) zero (d) 0.5
6. If $x - y = 3$ and $x + y = 5$, then $x^2 - y^2 + 2 =$ ....
   (a) 15 (b) 16 (c) 17 (d) 18

2 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations together :**
$2x + y = 1$, $x + 2y = 5$

[b] If $n_1(x) = \frac{x^2 - 3x + 9}{x^3 + 27}$, $n_2(x) = \frac{2}{2x + 6}$ **, prove that :** $n_1 = n_2$

3 **[a] By using the general formula , find in $\mathbb{R}$ the solution set of the equation :**
$3x^2 - 6x = -1$ (approximating the result to the nearest two decimals)

[b] If the domain of the function n is $\mathbb{R} - \{3\}$ where $n(x) = \frac{x-1}{x^2 - ax + 9}$, find the value of a

4 **[a]** Two numbers , their product is 10 and the difference between them is 3
Find the two numbers.

[b] Find n (x) in the simplest form , showing the domain of n where :
$n(x) = \frac{x^2 + 4x - 5}{x^3 - 8} \div \frac{x + 5}{x^2 + 2x + 4}$ **, then find :** n (3) , n (2) if it is possible.

5 **[a] Find n (x) in the simplest form , showing the domain of n where :**
$n(x) = \frac{x^2 - 3x}{x^2 - 9} + \frac{x - 1}{x^2 + 2x - 3}$

[b] If A and B are two events in the sample space of a random experiment and
$P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$

, find : 1 $P(A \cup B)$ 2 $P(A - B)$

10 Suez Governorate

Answer the following questions : ***(Calculators are allowed)***

1 **Choose the correct answer from the given ones :**

1 The set of zeroes of f where $f(x) = x - 5$ is ....
(a) $\mathbb{R}$ (b) $\{-5\}$ (c) $\{5\}$ (d) $\varnothing$

2 If $A \subset S$ of a random experiment , $P(\grave{A}) = P(A)$, then $P(A) =$ ....
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1

3 The solution set in $\mathbb{R} \times \mathbb{R}$ of the two equations : $x = 3$, $y = 4$ is ....
(a) $\{(3 , 4)\}$ (b) $\{(4 , 3)\}$ (c) $\mathbb{R}$ (d) $\varnothing$

4 If the ratio between the perimeters of two squares is 1 : 2 , then the ratio between their areas is ....
(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

5 If $n(x) = \frac{x-1}{x+1}$, then the domain of n^{-1} = ...
(a) $\{-1\}$ (b) $\mathbb{R} - \{-1 , 1\}$ (c) $\mathbb{R} - \{-1\}$ (d) $\mathbb{R}$

6 If $a - b = -3$, then $(a - b)^2 =$ ....
(a) -9 (b) 12 (c) 9 (d) 18

2 [a] **Find the solution set in $\mathbb{R} \times \mathbb{R}$ of the equations :** $x - y = 3$, $2x + y = 9$

(Explain your answer , showing the steps of the solution)

[b] **Find n (x) in the simplest form , showing the domain of n where :**

$$n(x) = \frac{x^2 - 2x}{x^2 - 4} + \frac{2x + 6}{x^2 + 5x + 6}$$

3 [a] **Find in $\mathbb{R} \times \mathbb{R}$ algebraically the solution set of the two equations :** $x - y = 0$, $xy = 9$

[b] **Find n (x) in the simplest form , showing the domain of n where :**

$$n(x) = \frac{x^2 + 2x - 3}{x + 3} \times \frac{x + 1}{x^2 - 1}$$

4 [a] A and B are two events from the sample space of a random experiment and

$P(A) = 0.3$, $P(B) = 0.6$, $P(A \cap B) = 0.2$

Find : [1] $P(A \cup B)$ [2] $P(\grave{A})$

[b] **Find n (x) in the simplest form , showing the domain of n where :**

$$n(x) = \frac{x^2 - 2x + 1}{x^3 - 1} \div \frac{x - 1}{x^2 + x + 1}$$

5 [a] **Find the solution set for the following equation by using the formula in $\mathbb{R}$:**

$x^2 - 2x - 6 = 0$ (Rounding the results to two decimal places)

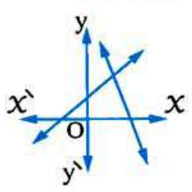
[b] If $n_1(x) = \frac{2x}{2x + 4}$, $n_2(x) = \frac{x^2 + 2x}{x^2 + 4x + 4}$, **prove that :** $n_1 = n_2$

11 Port Said Governorate

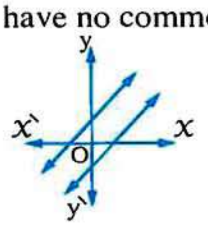
Answer the following questions :

1 **Choose the correct answer from those given :**

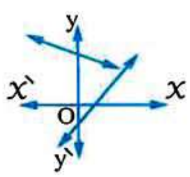
[1] Which of the following graphs represents two equations of the first degree in two variables which have no common solution ?



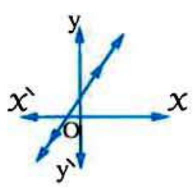
(a)



(b)



(c)



(d)

[2] The set of zeroes of the function $f : f(x) = x^2 + x + 1$ is ……………

(a) $\{1\}$ (b) $\{-1\}$ (c) $\varnothing$ (d) $\{-1 , 1\}$

3 If the ratio between the perimeters of two squares is 3 : 4 , then the ratio between their areas is ....

(a) 3 : 4 (b) 9 : 16 (c) 16 : 9 (d) 4 : 3

4 If $A \subset S$ of a random experiment , $P(\acute{A}) = 2\,P(A)$, then $P(A) =$ ....

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

5 If $n(x) = \frac{x-2}{x+5}$, then the domain of the function n^{-1} is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{5\}$ (d) $\mathbb{R} - \{2 , -5\}$

6 If a fair die is rolled once , then the probability of getting an even number and a prime number together equals ....

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) zero (d) 1

2 [a] If the domain of the function $n : n(x) = \frac{x-1}{x^2 - a\,x + 9}$ is $\mathbb{R} - \{3\}$, then find the value of a

[b] A rectangle is of perimeter 22 cm. and area 24 cm². Find its two dimensions.

3 **[a] Find in $\mathbb{R}$ by using the general formula the solution set of the equation :** $x^2 - 2\,x - 1 = 0$ approximating the results to the nearest one decimal digit.

[b] Find n (x) in the simplest form , showing the domain where :

$$n(x) = \frac{x^2 + x + 1}{x} \div \frac{x^3 - 1}{x^2 - x}$$

4 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x + 3\,y = 7$, $5\,x - y = 3$

[b] Find n (x) in the simplest form , showing the domain where :

$$n(x) = \frac{x^2 + 2\,x}{x^2 - 4} + \frac{x - 3}{x^2 - 5\,x + 6}$$

5 [a] A set of cards numbered from 1 to 20 and well mixed. If a card is drawn randomly **, find the probability that the drawn card is carrying :**

1 A number multiple of 4

2 A number multiple of 5

3 A number multiple of 4 or 5

[b] If $n_1(x) = \frac{x+3}{x^2 - 9}$, $n_2(x) = \frac{2}{2\,x - 6}$

, prove that : $n_1(x) = n_2(x)$ for the value of x which belong to the common domain and find the domain.

12 Damietta Governorate

Answer the following questions : (Calculators are allowed)

1 Choose the correct answer from the given ones :

1. If there are an infinite number of solutions of the two equations : $x + 4y = 7$, $x + (k - 1)y = 7$ in $\mathbb{R} \times \mathbb{R}$, then k = ...

(a) 5 (b) 7 (c) 12 (d) 13

2. If $B \subset A$, then $P(A \cup B) =$ ....

(a) 1 (b) P (A) (c) P (B) (d) 2 P (B)

3. If $x = 2$, $y = 3$, then $(y - 2x)^{10} =$ ....

(a) -1 (b) zero (c) 5 (d) 1

4. If $ab = 3$, $ab^2 = 12$, then b = ...

(a) 4 (b) 2 (c) -2 (d) ± 2

5. If 3 is one of zeroes of the function f where $f(x) = x^2 - 3x + c$, then c = ...

(a) 6 (b) 0 (c) -6 (d) 3

6. If a , b , c are three rational numbers where a < b and c is a negative number , then ac .... bc

(a) > (b) = (c) $\leq$ (d) <

2 **[a] By using the general formula , find in $\mathbb{R}$ the solution set of the equation :** $x + \frac{4}{x} = 6$, rounding the results to one decimal digit.

[b] Simplify : $n(x) = \frac{2x}{x-3} \div \frac{x^2 + 2x}{x^2 - 9}$, showing the domain.

3 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations graphically :** $x + 2y = 4$, $2x - y = 3$

[b] Simplify : $n(x) = \frac{x^2 - 2x + 4}{x^3 + 8} + \frac{x^2 - 1}{x^2 + x - 2}$, showing the domain.

4 **[a]** If $n_1(x) = \frac{x^2 + 2x}{x^2 + 4x + 4}$, $n_2(x) = \frac{2x}{2x + 4}$,

then prove that : $n_1 = n_2$

[b] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations : $x - y = 2$, $x^2 + y^2 = 20$

5 **[a]** If the domain of the function n : $n(x) = \frac{x + 1}{x^2 - ax + 25}$ is $\mathbb{R} - \{5\}$,

then find the value of a

[b] If A and B are two events from the sample space of a random experiment ,

$P(A) = 0.8$, $P(B) = 0.7$, $P(A \cap B) = 0.6$

, **find :** 1 $P(A \cup B)$ 2 The probability of non-occurrence of the event A

13 Kafr El-Sheikh Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **[a] Choose the correct answer :**

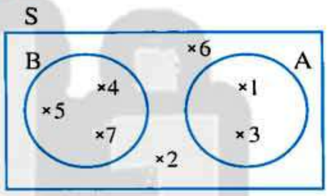
1 If there is only one solution for the two equations $x + 4y = 5$ and $3x + ky = 15$, then k can't equal ....

(a) -4 (b) 4 (c) 12 (d) -12

2 If $\sqrt{100-36} = 10 - a$, then a = ...

(a) 2 (b) 6 (c) 4 (d) 3

3 **In the opposite figure :**



If A and B are two events in the sample space S of a random experiment ,

then $P(B - A) =$ ....

(a) $\frac{1}{2}$ (b) $\frac{5}{7}$ (c) $\frac{2}{7}$ (d) $\frac{3}{7}$

[b] Find n (x) in the simplest form , showing the domain of n where :

$$n(x) = \frac{2x^2 - x - 6}{x^2 - 3x} \div \frac{4x^2 - 9}{2x^2 - 3x}$$

2 **[a] Choose the correct answer :**

1 If the domain of the function n : $n(x) = \frac{x + 2}{4x^2 + kx + 9}$ is $\mathbb{R} - \left\{\frac{-3}{2}\right\}$, then the value of k = ...

(a) 15 (b) -15 (c) 12 (d) -12

2 If $6^{x} = 12$, then $6^{x+1} =$ ....

(a) 66 (b) 13 (c) 27 (d) 72

3 The S.S. of the inequality : $-x < 3$ in $\mathbb{R}$ is ....

(a) $[3 , \infty[$ (b) $]3 , \infty[$ (c) $]-3 , \infty[$ (d) $[-3 , \infty[$

[b] If $n_1(x) = \frac{x}{x^2 - x}$, $n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$, **prove that :** $n_1 = n_2$

3 **[a] Find in $\mathbb{R}$ the solution set of the equation :** $3x^2 + 1 = 5x$, rounding the results to two decimal places.

[b] If $n_1(x) = \dfrac{x^2 - 2x - 15}{x^2 - 9}$, $n_2(x) = \dfrac{6 - a x}{x^2 - 6x + 9}$, where the set of zeroes of n_2 is $\{-3\}$

1. Find the value of a
2. Find $n(x)$ where $n(x) = n_1(x) - n_2(x)$ in the simplest form , showing the domain of n

4 **[a] Find algebraically in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations :**

$3x + 2y = 4$, $x - 3y = 5$

[b] If A and B are two events from the sample space S of a random experiment , $P(A) = \frac{1}{2}$, $2P(B) = P(\grave{B})$ **, then find $P(A \cup B)$ in each of the following cases :**

1. $P(A \cap B) = \frac{1}{6}$
2. A , B are mutually exclusive events.

5 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations :**

$x - 2y - 1 = 0$, $x^2 - xy = 0$

[b] If $n(x) = \dfrac{x^2 - 3x}{(x-3)(x^2+2)}$ **, then find :** $n^{-1}(x)$ and identify the domain of n^{-1}

14 El-Beheira Governorate

Answer the following questions : (Calculator is permitted)

1 **Choose the correct answer from the given ones :**

1. If $x^2 - y^2 = 12$, $x - y = 3$, then $x + y =$ ……………
   (a) 3 (b) 4 (c) 12 (d) 15
2. If $3a = \sqrt{4}\, b$, then $\frac{a}{b} =$ ……………
   (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$
3. If $5x = 5^3$, then $\frac{4}{5}x =$ ……………
   (a) 10 (b) 15 (c) 20 (d) 25
4. The number of solution of the two equations $x + y = 1$ and $y + x = 2$ together in $\mathbb{R} \times \mathbb{R}$ is ……………
   (a) zero (b) 1 (c) 2 (d) 3
5. The common domain of the functions n_1 , n_2 where $n_1(x) = \dfrac{x+2}{x^2-4}$, $n_2(x) = \dfrac{1}{x+1}$ is ……………
   (a) $\{-2, -1, 2\}$ (b) $\mathbb{R} - \{-1, 2\}$
   (c) $\mathbb{R} - \{-2, -1, 2\}$ (d) $\mathbb{R}$
6. If $A \subset B$, then $P(A \cup B) =$ ……………
   (a) zero (b) P(A) (c) P(B) (d) $P(A \cap B)$

2 **[a] Find the solution set of the following two equations together in $\mathbb{R} \times \mathbb{R}$:**

$y - x = 2 \quad , \quad x^2 + x y - 4 = 0$

[b] Find n (x) in the simplest form , showing the domain of n where :

$n(x) = \frac{x^3 - 1}{x^2 - 2x + 1} \times \frac{2x - 2}{x^2 + x + 1}$

3 **[a]** Two acute angles in a right-angled triangle. The difference between their measures is 50° Find the measure of each angle.

[b] If $n(x) = \frac{x^2 - 2x}{(x - 2)(x^2 + 2)}$ **, find :**

1 $n^{-1}(x)$ in the simplest form , showing the domain of n^{-1}

2 The value of x if $n^{-1}(x) = 3$

4 **[a] By using the general formula , find the solution set of the following equation in $\mathbb{R}$:**

$3x^2 = 5x - 1$ (rounding the results to two decimal places).

[b] If $n_1(x) = \frac{2x}{2x + 4}$, $n_2(x) = \frac{x^2 + 2x}{x^2 + 4x + 4}$ **, then prove that :** $n_1 = n_2$

5 **[a] Find n (x) in the simplest form , showing the domain where :**

$n(x) = \frac{x - 3}{x^2 - 7x + 12} - \frac{4}{x^2 - 4x}$

[b] If A and B are two events from the sample space of a random experiment and

$P(A) = 0.8 \quad , \quad P(B) = 0.7 \quad , \quad P(A \cap B) = 0.6$

, then find : 1 $P(\grave{A})$ 2 $P(A \cup B)$

15 El-Fayoum Governorate

Answer the following questions : (Using calculators is allowed)

1 **Choose the correct answer :**

1 In the equation : $a x^2 + b x + c = 0$, if : $b^2 - 4ac > 0$, then the equation has .... roots in $\mathbb{R}$

(a) 1 (b) 2 (c) zero (d) ∞

2 If $3^x = 4$, $4^y = 12$, then $\frac{x y}{x + 1} =$ ....

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

3 If $n(x) = \frac{x^2 - 2x}{(x - 2)(x^2 + 2)}$, then the domain of n^{-1} is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{0\}$ (d) $\mathbb{R} - \{0 , 2\}$

4 If $2^7 \times 3^7 = 6^k$, then k = ...

(a) 14 (b) 7 (c) 6 (d) 5

5 If A and B are two mutually exclusive events from the sample space S of a random experiment , then P (A – B) = ...

(a) P (A) (b) P (À) (c) P (B) (d) P (B̀)

6 The rule which describes the pattern $\left(\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\right)$ where $n \in \mathbb{Z}_+$ is ....

(a) $\frac{2}{n+1}$ (b) $n + \frac{1}{2}$ (c) $\frac{n}{n+1}$ (d) $\frac{2n-1}{n+1}$

2 [a] **Find graphically in $\mathbb{R} \times \mathbb{R}$ the solution set of the following pair of equations :**

$3x - y + 4 = 0$, $y = 2x + 3$

[b] Reduce $n(x) = \frac{x^2 - x}{x^2 - 1} + \frac{x - 5}{x^2 - 6x + 5}$ to the simplest form , showing the domain of n

3 [a] **By using the general formula , find in $\mathbb{R}$ the solution set of the equation :**

$x^2 + 3x + 5 = 0$

[b] If $n(x) = \frac{x^2 - 49}{x^3 - 8} \div \frac{x + 7}{x - 2}$, find the simplest form of n (x) , showing the domain , then find n (1)

4 [a] If $n_1(x) = \frac{x^2 - 4}{x^2 + x - 6}$, $n_2(x) = \frac{x^3 - x^2 - 6x}{x^3 - 9x}$, show whether $n_1 = n_2$ or not. (give a reason)

[b] The sum of two real numbers is 9 , and the difference between their squares equals 45 , find the two numbers.

5 [a] If the set of zeroes of the function $f : f(x) = a x^2 + b x + 15$ is $\{3, 5\}$, find the values of a and b

[b] If A and B are two events of the sample space of a random experiment , P (A) = P (À) , $P(A \cap B) = \frac{1}{16}$, $P(B) = \frac{5}{8} P(A)$

, find : 1 P (B) 2 $P(A \cup B)$

16 Beni Suef Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 If a coin is tossed once , then the probability of appearing a tail equals ....

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1

2 The set of zeroes of the function f where $f(x) = \frac{x-3}{x-2}$ is ....

(a) {zero} (b) {2} (c) {3} (d) {2 , 3}

3 The equation $3x + 4y + x^2 y = 5$ is of the .... degree.

(a) zero (b) first (c) second (d) third

4 The domain of the function f where $f(x) = \frac{x-3}{2}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-2\}$ (c) $\mathbb{R} - \{3\}$ (d) $\mathbb{R} - \{-2 , 3\}$

5 If $x + y = xy = 10$, then $x^2 y + x y^2 =$ ....

(a) 10 (b) 20 (c) 30 (d) 100

6 The solution set of the two equations : $y = 4$, $x + y = 7$ together in $\mathbb{R} \times \mathbb{R}$ is ....

(a) (3 , 4) (b) (4 , 3) (c) {(3 , 4)} (d) {(4 , 3)}

2 [a] **Find in $\mathbb{R}$ by using the general formula , the solution set of the equation :**

$x^2 - 2(x + 1) = 0$

[b] If $n_1(x) = \frac{5x}{5x + 25}$, $n_2(x) = \frac{x^2 + 5x}{x^2 + 10x + 25}$ **, then prove that : $n_1 = n_2$**

3 [a] **Find algebraically in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :**

$x + y = 7$, $x^2 + y^2 = 25$

[b] **Find n (x) in its simplest form , showing the domain where :**

$n(x) = \frac{x^2}{x^2 - 3x} \div \frac{3x}{x^2 - 9}$

4 [a] If A , B are two events from the sample space of a random experiment and

$P(A) = 0.7$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$

, find : $P(\acute{A})$, $P(A - B)$ and $P(A \cup B)$

[b] If the set of zeroes of the function f where $f(x) = x^2 - 10x + a$ is {5}

, then find the value of a

5 [a] **Find graphically in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations :**

$3x + y = 3$, $2x - y = 7$

[b] **Find n (x) in its simplest form , showing the domain where :**

$n(x) = \frac{x^2 + x + 1}{x^3 - 1} + \frac{x^2 - x - 2}{x^2 - 1}$

17 El-Menia Governorate

Answer the following questions : ***(Calculators are allowed)***

1 Choose the correct answer from those given :

1 If k < zero , which of the following quantities is the greatest in the numerical value ?
(a) $5 - k$ (b) $5 + k$ (c) $5k$ (d) $\frac{5}{k}$

2 If $a + b = 3$, $a^2 - ab + b^2 = 5$, then $a^3 + b^3 =$ ....
(a) 8 (b) 9 (c) 15 (d) 25

3 Half the number $4^6 =$ ....
(a) 2^3 (b) 2^6 (c) 4^3 (d) 2^{11}

4 The S.S of the two equations $x = 3$, $y = 4$ in $\mathbb{R} \times \mathbb{R}$ is ....
(a) $\{(3, 4)\}$ (b) $\{(4, 3)\}$ (c) $\mathbb{R}$ (d) $\varnothing$

5 If A , B are two mutually exclusive events from the sample space of a random experiment , then $P(A \cap B) =$ ....
(a) $\varnothing$ (b) zero (c) 0.5 (d) 1

6 The simplest form of the function $f : f(x) = \frac{2x}{x+1} + \frac{x}{x+1}$ is ....
(a) $\frac{3x}{x+1}$ (b) 3 (c) 2 (d) $\frac{2}{x+1}$

2 [a] **Find the S.S. in $\mathbb{R}$ for the equation :** $3x^2 - 5x + 1 = 0$, using the general rule , rounding the result to one decimal place.

[b] **Find n (x) in the simplest form , showing the domain :**

$$n(x) = \frac{x^3 - 8}{x^2 - 5x + 6} \div \frac{x^2 + 2x + 4}{x - 3}$$

3 [a] **Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of the two equations :** $2x + y = 1$, $x + 2y = 5$ algebraically.

[b] **Find n (x) in the simplest form showing the domain where :**

$$n(x) = \frac{x^2 - 2x - 15}{x^2 - 9} - \frac{10 - 2x}{x^2 - 8x + 15}$$

4 [a] **Find the S.S. in $\mathbb{R}^2$ of the two equations :** $x + y = 2$, $\frac{1}{x} + \frac{1}{y} = 2$

[b] If $n_1(x) = \frac{x^2}{x^3 - x^2}$, $n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$,

prove that : $n_1 = n_2$

5 **[a]** If n $(x) = \frac{x^2 - 2x}{(x-2)(x^2+2)}$, **find :** $n^{-1}(x)$, showing the domain.

[b] If A , B are two events from the sample space of a random experiment , P (A) = 0.3 , P (B) = 0.6 , P $(A \cap B)$ = 0.2

find : **1** P $(A \cup B)$ **2** P (A – B)

18 Assiut Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer :**

1 If $\frac{1}{3}x = 8$, then $\frac{1}{6}x = \ldots\ldots\ldots$

(a) $\frac{4}{3}$ (b) 4 (c) 48 (d) 16

2 If there are an infinite number of solutions of the equations $x + 6y = 3$, $2x + ky = 6$ in $\mathbb{R} \times \mathbb{R}$, then k = ...

(a) 4 (b) 6 (c) 12 (d) 21

3 The set of zeroes of the function f where $f(x) = x^2 - 3$ is ....

(a) $\{\sqrt{3}\}$ (b) $\{-\sqrt{3}\}$ (c) $\{3\}$ (d) $\{-\sqrt{3}, \sqrt{3}\}$

4 $\frac{3}{\sqrt{5}+\sqrt{2}} = \ldots\ldots\ldots$

(a) $3\sqrt{5}$ (b) $2\sqrt{5}$ (c) $\sqrt{5}-\sqrt{2}$ (d) $\sqrt{5}+\sqrt{2}$

5 If the curve of the function f where $f(x) = x^2 - m$ passes through the point (3 , 0) , then m = ...

(a) 3 (b) – 3 (c) 6 (d) 9

6 If $X \subset S$ and $\grave{X}$ is the complementary event to event X , then $P(X \cap \grave{X}) = \ldots\ldots\ldots$

(a) zero (b) S (c) $\varnothing$ (d) 1

2 **[a] Find the solution set of the two following equations algebraically in $\mathbb{R} \times \mathbb{R}$:**

$3x - y + 4 = 0$, $y = 2x + 3$

[b] If n $(x) = \frac{x^2 - 49}{x^3 - 8} \div \frac{x+7}{x-2}$, then find n (x) in the simplest form and identify the domain and find n (1)

3 **[a] By using the general formula , find in $\mathbb{R}$ the solution set of the equation :**

$x(x-1) = 5$, rounding the results to one decimal place.

[b] If $n_1(x) = \dfrac{x^2-4}{x^2+x-6}$, $n_2(x) = \dfrac{x^3-x^2-6x}{x^3-9x}$

, prove that : $n_1(x) = n_2(x)$ for the values of x which belong to the common domain and find this domain.

4 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x - y = 2$, $x^2 + y^2 = 20$

[b] If $Z(f) = \{5\}$, $f(x) = x^3 - 3x^2 + a$ **, find the value of :** a

5 **[a] Find n (x) in the simplest form , showing the domain of n :**

$n(x) = \dfrac{x-3}{x^2-7x+12} - \dfrac{x-3}{3-x}$

[b] If $S = \{2, 3, 4, 5, 6, 7, 8\}$, $A = \{2, 4, 6, 8\}$, $B = \{2, 3, 5, 7\}$

, find : 1 $P(A)$, $P(\grave{B})$ 2 $P(A \cup B)$

19 Souhag Governorate

Answer the following questions : ***(Calculators are allowed)***

1 **Choose the correct answer :**

1 If $x \neq 0$, then $\dfrac{5x}{x^2+1} \div \dfrac{x}{x^2+1} = \dots\dots$

(a) -5 (b) -1 (c) 1 (d) 5

2 $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = a x^2 + b x + c$, where a , b , c $\in \mathbb{R}$, a = 0 , b $\neq$ 0 is a polynomial function of the .... degree in x

(a) second (b) third (c) first (d) zero

3 If $2^x = \dfrac{1}{4}$, then $x = \dots\dots$

(a) 2 (b) -2 (c) 1 (d) -1

4 $\sqrt[3]{3\frac{3}{8}}$ .... $\sqrt{2\frac{1}{4}}$

(a) = (b) > (c) < (d) $\neq$

5 If there are an infinite number of solutions in $\mathbb{R} \times \mathbb{R}$ of the two equations :

$x + 4y = 7$, $3x + ky = 21$, then k = ...

(a) 4 (b) 7 (c) 21 (d) 12

6 If $A \subset S$ of a random experiment and $P(\grave{A}) = 2P(A)$, then $P(A) = \dots\dots$

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) 1

2 [a] By using the general formula (rounding the results to one decimal digit), find in $\mathbb{R}$ the solution set of the equation : $x(x-1)=4$

[b] If $n_1(x)=\frac{x^2}{x^3-x^2}$, $n_2(x)=\frac{x^3+x^2+x}{x^4-x}$, **prove that :** $n_1=n_2$

3 [a] **Find the solution set of the following equations in $\mathbb{R}\times\mathbb{R}$:**

$x-y=0$, $x^2+xy+y^2=27$

[b] If $n(x)=\frac{x^2-2x}{x^2-3x+2}$, **then find :** $n^{-1}(x)$ in the simplest form showing the domain of n^{-1}

4 [a] **Solve in $\mathbb{R}\times\mathbb{R}$:** $2x-y=5$, $x+y=4$

[b] **Simplify :** $n(x)=\frac{x^2+2x}{x^2-4}-\frac{2x-6}{x^2-5x+6}$, showing the domain.

5 [a] **Simplify :** $n(x)=\frac{x^3-8}{x^2+x-6}\times\frac{x+3}{x^2+2x+4}$, showing the domain.

[b] If A , B are two mutually exclusive events of a random experiment and $P(A)=0.3$, $P(B)=0.6$, $P(A\cap B)=0.2$

, **find :** $P(\grave{A})$, $P(A\cup B)$

20 Qena Governorate

Answer the following questions : ***(Calculators are permitted)***

1 **Choose the correct answer :**

1. The domain of the function f where $f(x)=\frac{x-2}{x^2+1}$ is ....
   (a) $\mathbb{R}-\{-1\}$ (b) $\mathbb{R}-\{1,-1\}$ (c) $\mathbb{R}-\{1\}$ (d) $\mathbb{R}$
2. $10+(10)^2+(10)^3=$ ....
   (a) 1000 (b) 3000 (c) 1110 (d) 1010
3. The two straight lines : $x-y=0$, $3x+2y=0$ intersect at the point ....
   (a) (0 , 0) (b) (1 , 1) (c) (3 , 0) (d) (0 , 2)
4. $\sqrt{64+36}=8+$ ....
   (a) 9 (b) 2 (c) 6 (d) 10
5. If $P(A)=3P(\grave{A})$, then $P(A)=$ ....
   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$
6. If $ab=3$, $ab^2=12$, then $b=$ ....
   (a) 4 (b) 2 (c) -2 (d) ± 2

2 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :**

$x - 2 = 0$, $y^2 - 3xy + 5 = 0$

[b] Find n (x) in the simplest form , showing the domain where : $n(x) = \frac{5}{x-3} + \frac{4}{3-x}$

3 [a] Graph the function f where $f(x) = x^2 - 2x + 3$ over the interval $[-1, 3]$, then from the graph find in $\mathbb{R}$ the solution set of the equation $x^2 - 2x + 3 = 0$

[b] If $n(x) = \frac{x^2 + x - 12}{x^2 + 5x + 4}$, find $n^{-1}(x)$, showing the domain of n^{-1} , then find $n^{-1}(0)$

4 **[a] Find in $\mathbb{R}$ the solution set of the equation :**

$2x^2 - 5x + 1 = 0$, approximating the results to two decimals.

[b] If $n_1(x) = \frac{x^3 + 1}{x^3 - x^2 + x}$, $n_2(x) = \frac{x^3 + x^2 + x + 1}{x^3 + x}$, **prove that :** $n_1 = n_2$

5 [a] If A and B are two events from the sample space S , P (A) = 0.8 , P (B) = 0.7 , $P(A \cap B) = 0.6$, **find :**

1 $P(\grave{A})$ 2 $P(A \cup B)$ 3 $P(A - B)$

[b] Find n (x) in the simplest form , showing the domain where :

$n(x) = \frac{x^2 + 2x}{x^3 - 27} \div \frac{x + 2}{x^2 + 3x + 9}$

21 Luxor Governorate

محافظة الأقصر

Answer the following questions :

1 **Choose the correct answer :**

1 If $f(x) = 9$, then $3f(-x) =$ ....

(a) -3 (b) 6 (c) -12 (d) 27

2 The set of zeroes of f : $f(x) =$ zero is ....

(a) $\varnothing$ (b) $\mathbb{R}$ (c) $\mathbb{R} - \{0\}$ (d) zero

3 If $xy = 4$, $xz = 4$, $yz = 4$, where $x , y , z \in \mathbb{R}^+$, then $xyz =$ ....

(a) 64 (b) 12 (c) 8 (d) 4

4 If A , B are two events of the sample space of a random experiment , $A \subset B$, P (A) = 0.2 and P (B) = 0.6 , then P (B – A) = ...

(a) 0.2 (b) 0.4 (c) 0.6 (d) 0.8

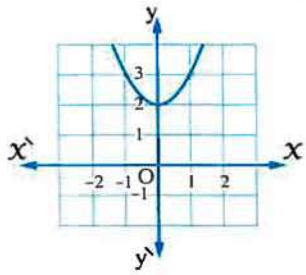
5 $\frac{1}{3}$ the number $(27)^3$ is ....

(a) 3^3 (b) 3^4 (c) 3^6 (d) 3^8

6 **From the opposite figure :**

The S.S. of $f(x) = 0$

in $\mathbb{R}$ is ……………



(a) $\varnothing$ (b) $\{2\}$

(c) $\{0\}$ (d) $\{(0, 2)\}$

2 [a] **Find the common domain of the functions defined by the following rules :**

$$\frac{x-4}{x^2-5x+6} \quad , \quad \frac{2x}{x^3-9x}$$

[b] **Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $y + 2x = 7$, $2x^2 + x + 3y = 19$

3 [a] **Find n (x) in the simplest form and state the domain :**

$$n(x) = \frac{x-3}{x^2-7x+12} - \frac{x-3}{3-x}$$

[b] A class has 40 students , 30 of them succeeded in math , 24 succeeded in science and 20 of them succeeded in both math and science. **If one student is chosen at random , find the probability that the student :**

1 Succeeded in math.

2 Succeeded in science only.

3 Succeeded in one of them at least.

4 [a] **Find in $\mathbb{R}$ the solution set of :** $2x^2 - x - 2 = 0$ by using the general rule where $(\sqrt{17} \simeq 4.12)$

[b] If $n_1(x) = \frac{x}{x^2-1}$, $n_2(x) = \frac{5x}{5x^2-5}$, **prove that :** $n_1 = n_2$

5 [a] **Find n (x) in the simplest form and state the domain if :**

$$n(x) = \frac{x^2-3x}{2x^2-x-6} \div \frac{2x^2-3x}{4x^2-9}$$

[b] **Find graphically in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :**

$x + 2y = 8$, $3x + y = 9$

22 Aswan Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer :**

1 The solution set of the two equations $x + y = 0$, $y - 5 = 0$ in $\mathbb{R} \times \mathbb{R}$ is ……………

(a) $\varnothing$ (b) $\mathbb{R}$ (c) $\{(-5, 5)\}$ (d) $\{(5, -5)\}$

2 If $2^3 \times 5^3 = 10^x$, then $x =$ ....

(a) zero (b) 3 (c) 6 (d) 9

3 If $a^2 - b^2 = 6$, $a - b = \sqrt{3}$, then $(a + b)^2 =$ ....

(a) 3 (b) 6 (c) 9 (d) 12

4 If $(5 , x - 4) = (y , 2)$, then $x + y =$ ....

(a) 6 (b) 8 (c) 11 (d) 25

5 If $f(x) = x^2 + x + a$ and the set of zeroes of the function f is $\{1 , -2\}$, then $a =$ ....

(a) 2 (b) 1 (c) -1 (d) -2

6 If $A \subset B$, then $P(A \cup B) =$ ....

(a) zero (b) P (A) (c) P (B) (d) $P(A \cap B)$

2 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :**

$3x - y = -4$, $y - 2x = 3$

[b] Find n (x) in the simplest form , showing the domain of n where :

$$n(x) = \frac{x^2 + 4x + 3}{x^3 - 27} \div \frac{x + 3}{x^2 + 3x + 9}$$

3 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :**

$x - y = 1$, $x^2 + y^2 = 25$

[b] If $n(x) = \frac{x^2 - 2x}{x^2 - 3x + 2}$

, find : $n^{-1}(x)$ in the simplest form , showing the domain of n^{-1}

4 **[a] Using the general rule , find the solution set of the following equation in $\mathbb{R}$:**

$2x^2 - 5x + 1 = 0$

[b] Find n (x) in the simplest form , showing the domain of n where :

$$n(x) = \frac{x^2 + 2x}{x^2 - 4} - \frac{2x - 6}{x^2 - 5x + 6}$$

5 **[a]** If $n_1(x) = \frac{2x}{2x + 8}$, $n_2(x) = \frac{x^2 + 4x}{x^2 + 8x + 16}$ **, prove that :** $n_1 = n_2$

[b] If A , B are two mutually exclusive events and $P(A) = \frac{1}{3}$, $P(A \cup B) = \frac{7}{12}$

, find : P (B)

23 New Valley Governorate

إيمان - عمل - تنمية

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 The degree of the function $f : f(x) = x + x^2 - 5$ is the ....

(a) first (b) second (c) third (d) fourth

2 The set of zeroes of the function $f : f(x) = 7$ is ....

(a) $\varnothing$ (b) $\{7\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{7\}$

3 If a + b = 3 and (a + b) (a + 1) = 15 , then ab = ...

(a) – 4 (b) 4 (c) – 6 (d) 6

4 The number of solutions of the equation : $x = 3$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) zero (b) 1 (c) 2 (d) an infinite number.

5 If A and B are two mutually exclusive events of a random experiment , then : $P(A \cap B) =$ ....

(a) P (A) (b) $\varnothing$ (c) zero (d) P (B)

6 If n_1 and n_2 are two algebraic fractions , the domain of $n_1 = \mathbb{R} - X_1$ (where X_1 is the set of zeroes of the denominator of n_1) and the domain of $n_2 = \mathbb{R} - X_2$ (where X_2 is the set of zeroes of the denominator of n_2) , then the common domain of n_1 and n_2 equals ....

(a) $X_1 \cup X_2$ (b) $X_1 \cap X_2$

(c) $(\mathbb{R} - X_1) \cup (\mathbb{R} - X_2)$ (d) $(\mathbb{R} - X_1) \cap (\mathbb{R} - X_2)$

2 [a] Find n (x) in its simplest form, showing the domain of n :

$$n(x) = \frac{x^2 - 4}{x^2 + 5x + 6}$$

[b] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two following equations :

$x^2 + y^2 = 17$, $y - x = 3$

3 [a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two following equations algebraically :

$3x - 2y = 4$, $x + 3y = 5$

[b] Find n (x) in its simplest form, showing the domain of n :

$$n(x) = \frac{x}{x + 2} \div \frac{x^2 - 2x}{\frac{1}{2}x^2 - 2}$$

4 [a] If $n_1(x) = \frac{x^3 - 1}{x^3 + x^2 + x}$, $n_2(x) = \frac{x^3 - x^2 + x - 1}{x^3 + x}$

, then prove that : $n_1 = n_2$

[b] **Find n (x) in the simplest form, showing the domain of n :**

$n(x) = \frac{3x}{x^2 - 3x} - \frac{x}{x - 3}$

5 [a] If A and B are two events from the sample space of a random experiment, and $P(A) = \frac{1}{5}$, $P(B) = \frac{3}{5}$, $P(A \cap B) = \frac{1}{10}$ **, then find :**

1 $P(\acute{A})$ 2 $P(A \cup B)$ 3 $P(B - A)$

[b] Draw the graph of the function $f : f(x) = x^2 - 2x + 1$ in the interval $[-2, 4]$, then from the graph find in $\mathbb{R}$ the solution set of the equation : $x^2 - 2x + 1 = 0$

24 South Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If $\frac{x}{y} = \frac{3}{4}$, then $\frac{4x}{3y} =$ ....

(a) 1 (b) $\frac{4}{3}$ (c) $\frac{9}{16}$ (d) $\frac{16}{9}$

2 If $x^2 = 25$, then $x =$ ....

(a) -5 (b) ± 5 (c) 5 (d) 10

3 If $x + 3y = 7$, then $x + 3(y + 5) =$ ....

(a) 3 (b) 7 (c) 22 (d) 21

4 The probability of the impossible event equals ....

(a) 1 (b) $\frac{1}{2}$ (c) -1 (d) zero

5 The domain of $f : f(x) = \frac{x + 5}{x^2 - 4}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-2, 2\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\mathbb{R} - \{2\}$

6 If A and B are mutually exclusive events, then $P(A \cap B) =$ ....

(a) $\varnothing$ (b) zero (c) 0.56 (d) 1

2 [a] **Find in $\mathbb{R}$ the solution set of the equation :** $x^2 - 2x - 6 = 0$ by using the formula , approximating the result to the nearest two decimal places.

[b] Find n (x) in the simplest form , showing the domain of n :

$$n(x) = \frac{x}{x+2} + \frac{2x^3}{x^3 + 2x^2}$$

3 **[a] Find n (x) in the simplest form , showing the domain of n :**

$$n(x) = \frac{x^2 + 2x}{x^3 - 8} \times \frac{x^2 + 2x + 4}{x + 2}$$

[b] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations algebraically :

$2x - y = 3$, $x + 2y = 4$

4 [a] If $n_1(x) = \frac{x}{x^2 + x}$, $n_2(x) = \frac{x^4 - x^3 + x^2}{x^5 + x^2}$ **, prove that :** $n_1 = n_2$

[b] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :

$x - y = 7$, $xy = 60$

5 **[a] Find n (x) in the simplest form , showing the domain of n where :**

$$n(x) = \frac{x+1}{x^2 + 3x + 2} - \frac{x+2}{x^2 - 4}$$

[b] If A and B are mutually exclusive events of the sample space of a random experiment and $P(A) = \frac{1}{4}$, $P(A \cup B) = \frac{5}{12}$ **, find :** P (B)

25 North Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 One of the solutions of the inequality : $2x - 3 > 3$ where $x \in \mathbb{Z}$ is ....

(a) $x = 3$ (b) $x = -3$ (c) $x = 7$ (d) $x = -7$

2 If $x - y = 3$, $x + y = 9$, then y = ...

(a) 6 (b) – 6 (c) 3 (d) – 3

3 If $a = \sqrt{3}$, $b = \frac{1}{\sqrt{3}}$, then $a^{50} \times b^{51}$ = ...

(a) 3 (b) $\frac{1}{3}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

4 If $n(x) = \frac{x}{x+5}$, then the domain of n^{-1} = ...

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R} - \{5\}$ (d) $\mathbb{R} - \{0, -5\}$

5 If $x^2 - y^2 = 15$, $x - y = 3$, then $x + y$ = ...

(a) 5 (b) 13 (c) 18 (d) 45

6 If a regular die is tossed once , the probability of appearance of a number less than 3 equals ....

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

2 **[a]** If A , B are two events of a random experiment and

$P(A) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{5}$, $P(B) = \frac{2}{5}$

, **find :** 1 $P(A \cup B)$ 2 $P(A - B)$

[b] Find the common domain of n_1 , n_2 : if $n_1(x) = \frac{-1}{x^2 - 9}$, $n_2(x) = \frac{7}{x}$

3 **[a] By using the general rule , find in $\mathbb{R}$ the solution set of the equation :** $x^2 - 2x = 4$, rounding the results to two decimal places.

[b] Find n (x) in the simplest form , showing the domain :

$n(x) = \frac{x^2 - 2x}{x^2 - 4} + \frac{2x + 6}{x^2 + 5x + 6}$

4 **[a] Find the solution set of the following two equations in $\mathbb{R} \times \mathbb{R}$:**

$x - y = 0$, $xy = 16$

[b] If $n_1(x) = \frac{x^2}{x^3 - x^2}$, $n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$, **prove that :** $n_1 = n_2$

5 **[a]** If $n(x) = \frac{x^2 - 2x + 1}{x^3 - 1} \div \frac{x - 1}{x^2 + x + 1}$

, **find :** n (x) in the simplest form , showing the domain of n

[b] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations graphically :

$x + y = 4$, $2x - y = 2$

26 Red Sea Governorate

Answer the following questions : ***(Calculators are allowed)***

1 **Choose the correct answer from those given :**

1 The solution set of the two equations : $x + 2 = 0$, $y = 3$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(2 , 3)\}$ (b) $\{(3 , 2)\}$ (c) $\{(-2 , 3)\}$ (d) $\{(3 , -2)\}$

2 If $2^5 \times 3^5 = 6^m$, then m = ...

(a) 10 (b) 5 (c) 6 (d) 25

3 If $A \subset S$ of a random experiment, $P(\acute{A}) = 2P(A)$, then P (A) ....

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1

4 If $(5 , x - 4) = (y , 3)$, then $x + y =$ ....

(a) 25 (b) 12 (c) 8 (d) 6

5 The set of zeroes of f where $f(x) =$ zero is ....

(a) $\varnothing$ (b) zero (c) $\mathbb{R}$ (d) $\mathbb{R} - \{0\}$

6 $(-1)^{15} + (-1)^{14} =$ ....

(a) 1 (b) 2 (c) -2 (d) zero

2 [a] **Find the S.S of the following two equations in $\mathbb{R} \times \mathbb{R}$:**

$2x - y = 3$, $x + 2y = 4$

[b] **Find n (x) in the simplest form , showing the domain :** $n(x) = \dfrac{x^2}{x - 1} + \dfrac{x}{1 - x}$

3 [a] **By using the general rule, solve the equation :** $x^2 - x = 4$ in $\mathbb{R}$

, approximating the result to the nearest two decimals

[b] **Prove that $n_1 = n_2$ if :** $n_1(x) = \dfrac{x^3 + 1}{x^3 - x^2 + x}$, $n_2(x) = \dfrac{x^3 + x^2 + x + 1}{x^3 + x}$

4 [a] **Find the S.S. in $\mathbb{R} \times \mathbb{R}$ of the two equations :** $x - y = 1$, $x^2 + y^2 = 25$

[b] If $n(x) = \dfrac{x^2 - 2x}{x^2 - 5x + 6}$

1 **Find :** $n^{-1}(x)$ and identify the domain of n^{-1}

2 If $n^{-1}(x) = 2$, what is the value of x ?

5 [a] **Find n (x) in the simplest form , showing the domain where :**

$n(x) = \dfrac{x^3 - 8}{x^2 + x - 6} \times \dfrac{x + 3}{x^2 + 2x + 4}$

[b] If A and B are two events from the sample space of a random experiment and

$P(A) = 0.3$, $P(B) = 0.6$, $P(A \cap B) = 0.2$

, **find :** 1 $P(A \cup B)$ 2 $P(A - B)$

27 Matrouh Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 The two straight lines : $x + 2y = 1$, $2x + 4y = 6$ are ....

(a) parallel. (b) intersecting.

(c) perpendicular. (d) intersecting and perpendicular.

2 The solution set of the equation : $x^2 = 2x$ in $\mathbb{Z}$ is ....

(a) $\{2\}$ (b) $(0, 2)$ (c) $\{0, 2\}$ (d) $\{(0, 2)\}$

3 The intersection point of the two straight lines : $x = 1$ and $y - 2 = 0$ lies on the .... quadrant.

(a) first. (b) second. (c) third. (d) fourth.

4 If $A \subset B$, then $P(A \cup B) =$ ....

(a) $P(A)$ (b) $P(B)$ (c) $P(A \cap B)$ (d) zero

5 If x is a negative number, then the largest number from the following is ....

(a) $5 + x$ (b) $5x$ (c) $5 - x$ (d) $\frac{5}{x}$

6 The set of zeroes of the function f where $f(x) = 4$ is ....

(a) zero (b) $\{4\}$ (c) $\{0, 4\}$ (d) $\varnothing$

2 **[a] By using the general formula, find in $\mathbb{R}$ the solution set of the equation :**

$x + \frac{1}{x} + 3 = 0$ where $x \neq 0$, rounding the results to two decimal places.

[b] If $n(x) = \frac{x^2 - 1}{x^2 - x}$, then reduce $n(x)$ to the simplest form, showing the domain of n

3 **[a] Simplify :** $n(x) = \frac{x - 1}{x^2 - 1} \div \frac{x^2 - 5x}{x^2 - 4x - 5}$, showing the domain.

[b] If the sum of two positive numbers is 9, and the difference between their squares is 27, find the two numbers.

4 **[a]** If A, B are two events from the sample space of a random experiment and

$P(A) = 0.3$, $P(B) = 0.6$, $P(A \cap B) = 0.2$

, find : 1 $P(A \cup B)$ 2 $P(A - B)$

[b] If $n_1(x) = \frac{x^2}{x^3 - x^2}$, $n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$, **then prove that :** $n_1 = n_2$

5 **[a] Find n (x) in the simplest form, showing the domain where :**

$n(x) = \frac{3x}{x^2 - x - 2} + \frac{x - 1}{1 - x^2}$

[b] Find the solution set of the following two equations graphically in $\mathbb{R} \times \mathbb{R}$:

$y = x + 4$, $x + y = 4$

Second Geometry

Quizzes

on Geometry

Quiz 1 On lesson 1 – unit 4

time 20 min.

1 Choose the correct answer from those given :

1 A chord of length 8 cm. is drawn in a circle whose diameter length 10 cm. , then the distance between this chord and the centre of the circle equals ....

(a) 2 cm. (b) 3 cm. (c) 4 cm. (d) 10 cm.

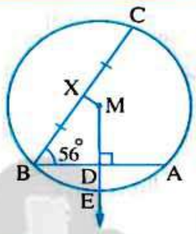
2 The number of axis of symmetry of the circle equals ....

(a) 1 (b) 2
(c) 3 (d) an infinite number.

3 If $\overline{AB}$ is a chord in the circle M and m ($\angle$ MAB) = 30° , then m ($\angle$ AMB) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

2 In the opposite figure :



$\overline{AB}$ and $\overline{BC}$ are two chords of the circle M whose radius length = 5 cm.
, $\overrightarrow{MD} \perp \overline{AB}$ and cuts $\overline{AB}$ at D and cuts the circle M at E
, X is the midpoint of $\overline{BC}$, AB = 8 cm. and m ($\angle$ ABC) = 56°

Find : 1 m ($\angle$ DMX) 2 The length of $\overline{DE}$ 3 sin ($\angle$ DBM)

Quiz 2 Till lesson 2 – unit 4

time 20 min.

1 Choose the correct answer from those given :

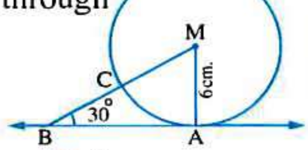
1 The circumference of a circle = 6 π cm. , the straight line L is far from its centre by 3 cm. , then the straight line L is ....

(a) a tangent to the circle. (b) a secant to the circle.
(c) outside the circle. (d) a diameter of the circle.

2 If the length of the perpendicular line segment from the centre of the circle M on the straight line L equals 6 cm. and radius length of the circle = 3 cm. , then L .... the circle.

(a) is a secant to (b) is a tangent to
(c) is outside (d) is passing through

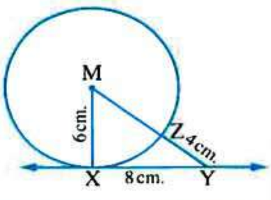
3 **In the opposite figure :**



$\overleftrightarrow{AB}$ is a tangent to the circle M
, m ($\angle$ B) = 30° , then CB = ... cm.

(a) 3 (b) 6 (c) 4 (d) 5

2 In the opposite figure :



M is a circle whose radius length is 6 cm. ,
XY = 8 cm. , $\overline{MY} \cap$ the circle M = {Z} , ZY = 4 cm.
Prove that : $\overleftrightarrow{XY}$ is a tangent to the circle at X

Quiz 3 Till lesson 3 – unit 4

time 20 min.

1 Choose the correct answer from those given :

1 M and N are two intersecting circles whose radii lengths are 6 cm. and 4 cm. , then MN ∈ ....

(a) $]10 , \infty[$ (b) $]2 , 10[$ (c) $]0 , 2[$ (d) $]4 , 6[$

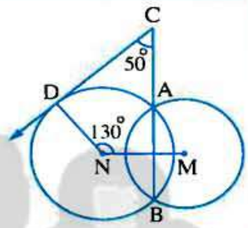
2 The two tangents drawn at the two ends of a diameter in a circle are ....

(a) parallel. (b) intersecting. (c) perpendicular. (d) coincide.

3 The point .... belongs to the circle whose centre is the origin point and the length of its radius is 3 units.

(a) $(1 , 2)$ (b) $(\sqrt{5} , -2)$ (c) $(\sqrt{2} , 1)$ (d) $(\sqrt{3} , 1)$

2 In the opposite figure :



M and N are two intersecting circles at A and B , $C \in \overrightarrow{BA}$, D ∈ the circle N , m (∠ MND) = 130° , m (∠ BCD) = 50°

Prove that : $\overrightarrow{CD}$ is a tangent to the circle N at D

Quiz 4 Till lesson 4 – unit 4

time 20 min.

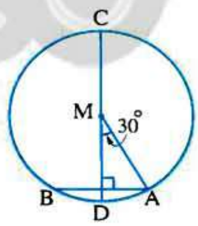
1 Choose the correct answer from those given :

1 If the radius length of the circle M is 5 cm. , and the radius length of the circle N is 3 cm. , MN = 8 cm. , then the two circles M and N are ....

(a) distant. (b) intersecting.

(c) touching externally. (d) touching internally.

2 **In the opposite figure :**



If $\overline{AB}$ is a chord in the circle M , $\overline{CD} \perp \overline{AB}$, $M \in \overline{CD}$, AB = 4 cm. and m (∠ AMD) = 30° , then the length of $\overline{CD}$ = ...

(a) 4 cm. (b) 8 cm. (c) 12 cm. (d) 16 cm.

3 If AB = 8 cm. , then the number of circles each of radius length 5 cm. and passing through the two points A , B is ....

(a) 1 (b) 2

(c) zero (d) an infinite number

2 Draw Δ ABC where AB = 3 cm. , BC = 5 cm. and AC = 7 cm. , then draw a circle passing through its vertices. How many circles can be drawn ?

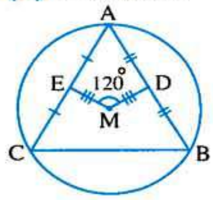
Quiz 5 — Till lesson 5 – unit 4

time 20 min.

1 Choose the correct answer from those given :

1. It is impossible to draw a circle passing through the vertices of a ....
   (a) rectangle. (b) triangle. (c) square. (d) rhombus.

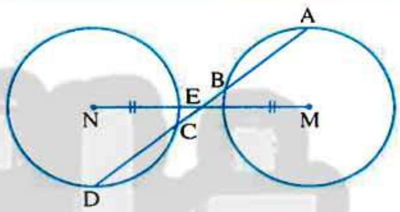
2. **In the opposite figure :**



   ABC is a triangle inscribed in the circle M
   , D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$
   , MD = ME and m ($\angle$ M) = 120°
   , then m ($\angle$ C) = ...
   (a) 30° (b) 60° (c) 90° (d) 120°

3. If M is a circle of circumference 8 π cm. , A is a point on the circle , then MA = ...
   (a) 5 cm. (b) 7 cm. (c) 4 cm. (d) 6 cm.

2 In the opposite figure :



M and N are two distant congruent circles , E is the midpoint of $\overline{MN}$ Draw $\overrightarrow{AE}$ to cut the circle M at A and B and cut the circle N at C and D

Prove that : 1 AB = CD

2 E is the midpoint of $\overline{AD}$

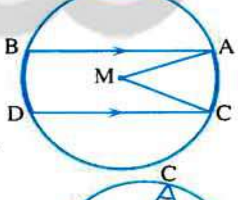
Quiz 6 — Till lesson 1 – unit 5

time 20 min.

1 Choose the correct answer from those given :

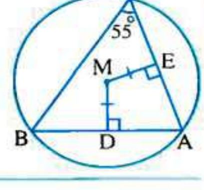
1. The length of the arc opposite to the central angle whose measure is 90° in a circle of radius length 7 cm. equals .... ($\pi = \frac{22}{7}$)
   (a) 11 cm. (b) 22 cm. (c) 44 cm. (d) 33 cm.

2. **In the opposite figure :**



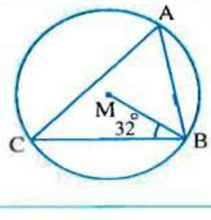
   If $\overline{AB}$ // $\overline{CD}$, m ($\widehat{AC}$) = $(2 \mathcal{X})°$, m ($\widehat{BD}$) = $(\mathcal{X} + 20)°$
   , then m ($\angle$ AMC) = ...
   (a) 60° (b) 40° (c) 20° (d) 100°

3. **In the opposite figure :**



   m ($\angle$ C) = 55° , ME = MD , $\overline{ME} \perp \overline{AC}$, $\overline{MD} \perp \overline{AB}$
   , then m ($\angle$ A) = ...
   (a) 55° (b) 70° (c) 60° (d) 90°

2 In the opposite figure :



M is a circle
, m ($\angle$ MBC) = 32°
Find with proof : m ($\widehat{BC}$)

Quiz 7 Till lesson 2 – unit 5

time 20 min.

1 Choose the correct answer from those given :

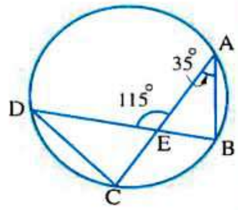
1 The inscribed angle subtended by a minor arc in the circle is ....

(a) acute. (b) obtuse. (c) right. (d) reflex.

2 If the straight line L is a tangent to the circle which its diameter length is 6 cm. , then the distance between L and the centre of the circle is .... cm.

(a) 3 (b) 4 (c) 5 (d) 6

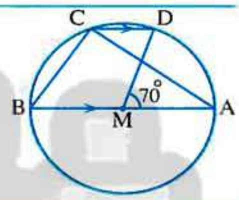
3 **In the opposite figure :**



$\overline{AC}$ and $\overline{BD}$ are two chords in a circle intersecting at E , m (∠ A) = 35° , m (∠ AED) = 115° , then m ($\overset{\frown}{AD}$) = ...

(a) 70° (b) 80° (c) 115° (d) 160°

2 In the opposite figure :



$\overline{AB}$ is a diameter in the circle M , $\overline{DC}$ // $\overline{AB}$, m (∠ AMD) = 70°

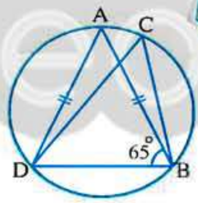
Find : 1 m (∠ ACD) 2 m (∠ ABC)

Quiz 8 Till lesson 3 – unit 5

time 20 min.

1 Choose the correct answer from those given :

1 **In the opposite figure :**



AB = AD and m (∠ ABD) = 65° , then m (∠ C) = ...

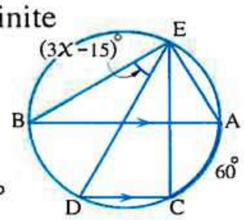
(a) 65° (b) 25°

(c) 50° (d) 30°

2 The number of circles passing through three collinear points is ....

(a) zero (b) one (c) two (d) infinite

3 **In the opposite figure :**

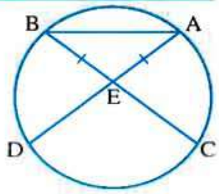


If m ($\overset{\frown}{AC}$) = 60°, then :

The value of X = ...

(a) 60° (b) 25° (c) 15° (d) 75°

2 In the opposite figure :



$\overline{AD} \cap \overline{BC} = \{E\}$ and EA = EB

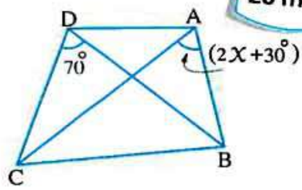
Prove that : AD = BC

Quiz 9 Till lesson 4 – unit 5

time 20 min.

1 Choose the correct answer from those given :

1 In the opposite figure :



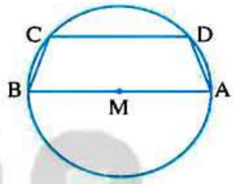
If the figure ABCD is a cyclic quadrilateral , then $X =$ ....

(a) 20° (b) 70° (c) 40° (d) 10°

2 Two circles M , N , the lengths of their radii are 10 cm. , 6 cm. and MN = 4 cm. , then the two circles are ....

(a) disjoint. (b) intersecting. (c) touching internally. (d) touching externally.

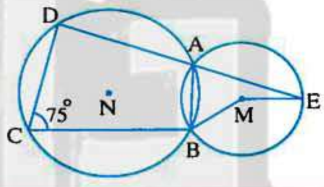
3 In the opposite figure :



M is a circle , m $(\overset{\frown}{BC}) = 70°$, then m (∠ D) = ...

(a) 110° (b) 100° (c) 135° (d) 125°

2 In the opposite figure :



M and N are two intersecting circles at A and B , $\overrightarrow{DA}$ intersects the circle M at E , if : m (∠ C) = 75°

Find with proof : m (∠ BME)

Quiz 10 Till lesson 5 – unit 5

time 20 min.

1 Choose the correct answer from those given :

1 Which of the following figures is a cyclic quadrilateral ?

(a) The rhombus. (b) The parallelogram. (c) The rectangle. (d) The trapezium.

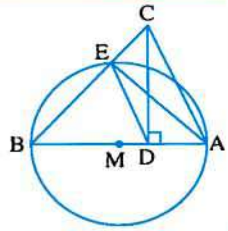
2 In the cyclic quadrilateral ABCD , if m (∠ A) : m (∠ C) = 1 : 2 , then m (∠ A) = ...

(a) 60° (b) 90° (c) 100° (d) 120°

3 If A and B are two points in the plane where AB = 4 cm. , then the length of the radius of the smallest circle can be drawn passing through A and B equals .... cm.

(a) 2 (b) 3 (c) 4 (d) 8

2 In the opposite figure :



$\overline{AB}$ is a diameter in the circle M , $\overline{CD} \perp \overline{AB}$

Prove that : 1 The figure ADEC is a cyclic quadrilateral.

2 m (∠ DCE) $= \frac{1}{2}$ m $(\overset{\frown}{EB})$

Quiz 11 Till lesson 6 – unit 5

time 20 min.

1 Choose the correct answer from those given :

1 The number of common tangents drawn to two distant circles is ....

(a) four. (b) three.

(c) two. (d) an infinite number.

2 The centre of the inscribed circle of any triangle is the point of intersection of ....

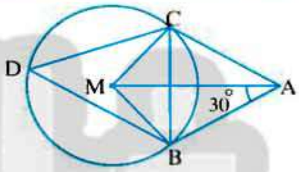
(a) its altitudes. (b) the bisectors of its interior angles.

(c) the axes of symmetry of its sides. (d) its medians.

3 The central angle whose measure = 60° is opposite to an arc of length equals .... the circumference of the circle.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

2 In the opposite figure :



$\overline{AB}$, $\overline{AC}$ are two tangent segments to the circle M

, m (∠ BAM) = 30° , D ∈ $\widehat{BC}$ (the major)

Find : 1 m (∠ ACB) 2 m (∠ BDC)

Quiz 12 Till lesson 7 – unit 5

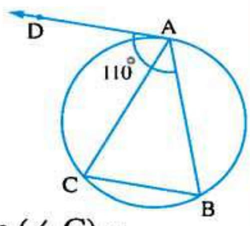
time 20 min.

1 Choose the correct answer from those given :

1 The measure of the tangency angle is .... the measure of the central angle subtended by the same arc.

(a) twice (b) half (c) quarter (d) equal to

2 **In the opposite figure :**



$\overrightarrow{AD}$ is a tangent to the circle at A

, if m (∠ DAB) = 110° , then m (∠ ACB) = ...

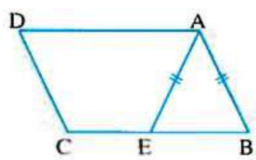
(a) 70° (b) 60°

(c) 55° (d) 35°

3 In the cyclic quadrilateral ABCD , if m (∠ A) = $\frac{1}{3}$ m (∠ C) , then m (∠ C) = ...

(a) 45° (b) 90° (c) 135° (d) 60°

2 In the opposite figure :



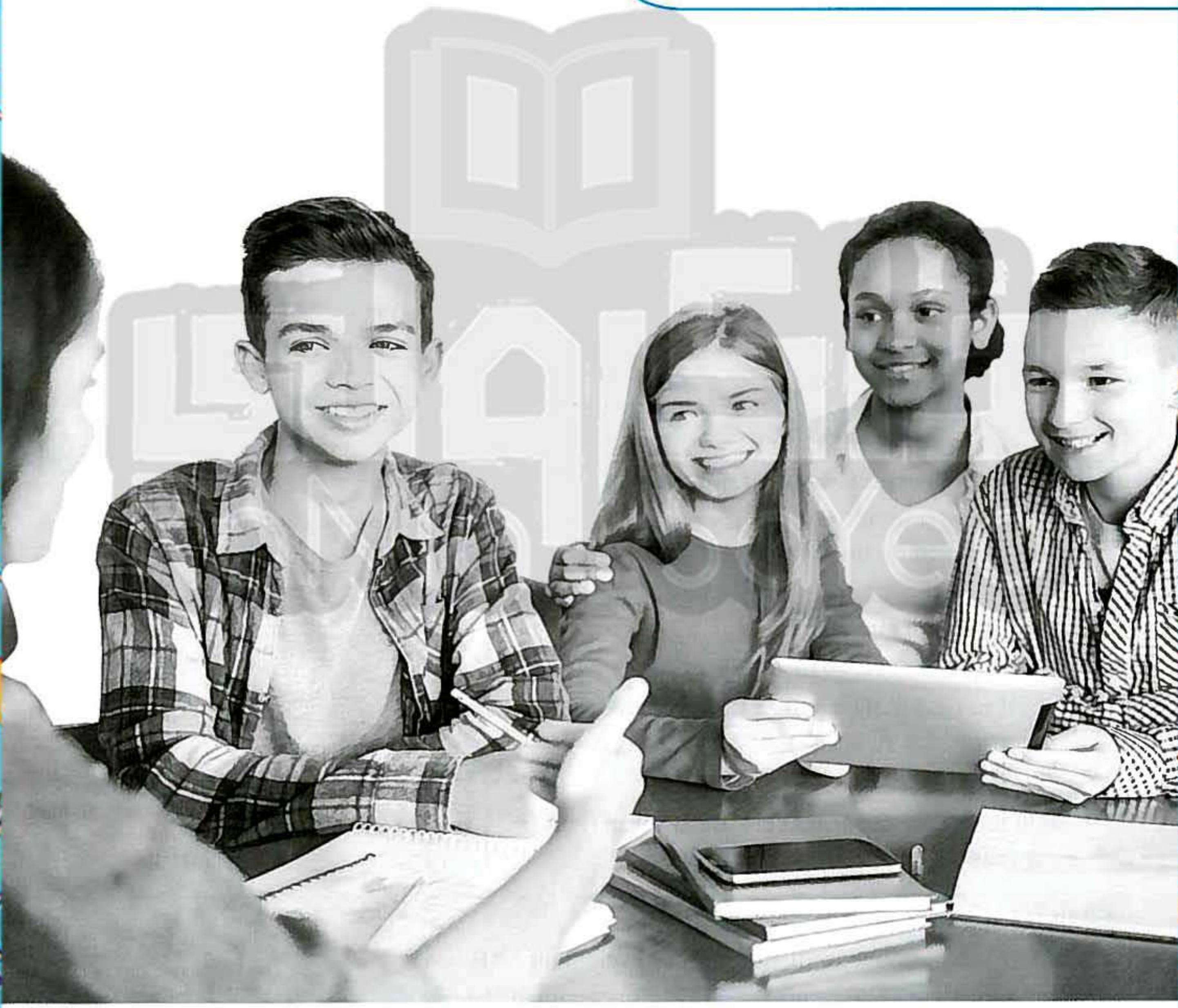
ABCD is a parallelogram and E ∈ $\overline{BC}$ such that : AB = AE

Prove that : 1 The figure AECD is a cyclic quadrilateral.

2 $\overrightarrow{AD}$ is a tangent to the circumcircle of Δ ABE

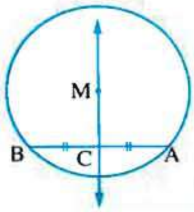
Final Revision

on Geometry



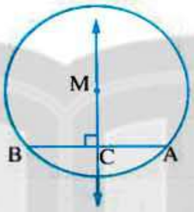
Final revision on geometry

The straight line passing through the centre of the circle and the midpoint of any chord of it (not passing through the centre) is perpendicular to this chord.



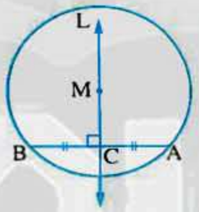
If $\overline{AB}$ is a chord of the circle M and C is the midpoint of $\overline{AB}$, then : $\overleftrightarrow{MC} \perp \overline{AB}$

The straight line passing through the centre of the circle and perpendicular to any chord of it bisects this chord.



If $\overline{AB}$ is a chord of the circle M and $\overleftrightarrow{MC} \perp \overline{AB}$, where $C \in \overline{AB}$, then : C is the midpoint of $\overline{AB}$

The perpendicular bisector to any chord of a circle passes through the centre of the circle.

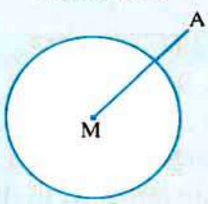


If $\overline{AB}$ is a chord of the circle M, C is the midpoint of $\overline{AB}$ and the straight line $L \perp \overline{AB}$ from the point C, then $M \in$ the straight line L

Position of a point with respect to a given circle

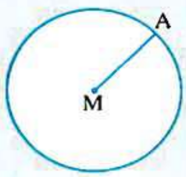
A is outside the circle M

If $MA > r$



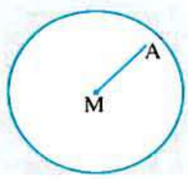
A is on the circle M

If $MA = r$

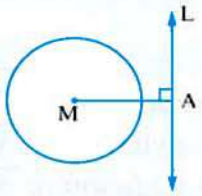
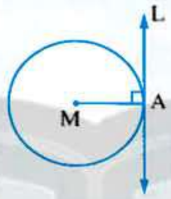
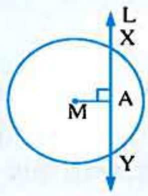


A is inside the circle M

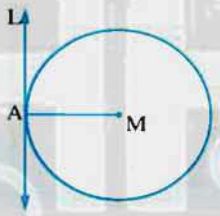
If $MA < r$



Position of a straight line L with respect to a circle M which at distance MA from its centre

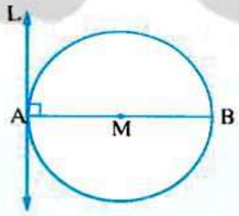
L lies outside the circle M	L touches the circle M	L is a secant to the circle M
If $MA > r$	If $MA = r$	If $MA < r$
		
• $L \cap$ the circle $M = \varnothing$ • $L \cap$ the surface of the circle $M = \varnothing$	• $L \cap$ the circle $M = \{A\}$ • $L \cap$ the surface of the circle $M = \{A\}$	• $L \cap$ the circle $M = \{X , Y\}$ • $L \cap$ the surface of the circle $M = \overline{XY}$ $\overline{XY}$ is called the chord of intersection

The tangent to a circle is perpendicular to the radius drawn from the point of tangency.



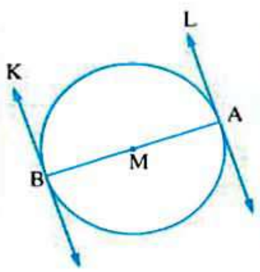
If L is a tangent to the circle M at the point A , then $\overline{MA} \perp L$

The straight line which is perpendicular to the diameter of a circle at one of its endpoints is a tangent to the circle.



If $\overline{AB}$ is a diameter of the circle M , $L \perp \overline{AB}$ at the point A , then L is a tangent to the circle M at the point A

The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.



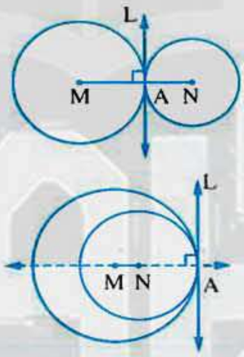
If $\overline{AB}$ is a diameter in the circle M , L and K are tangents to the circle M at A , B , then L // K

Position of the circle M with respect to the circle N

To determine the position of the circle M (with radius length r_1) with respect to the circle N (with radius length r_2) , find $r_1 - r_2$, $r_1 + r_2$, then use the following diagram to determine the position (where $r_1 > r_2$)

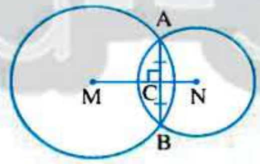
- $MN = \text{zero}$: **Concentric**
- $MN < r_1 - r_2$: One inside the other
- $MN = r_1 - r_2$: **Touching internally**
- $r_1 - r_2 < MN < r_1 + r_2$: Intersecting
- $MN = r_1 + r_2$: **Touching externally**
- $MN > r_1 + r_2$: Distant

The line of centres of two touching circles passes through the point of tangency and is perpendicular to the common tangent at this point.



If the two circles M and N are touching at A , L is a common tangent to them at A , then $\overleftrightarrow{MN} \perp L$

The line of centres of two intersecting circles is perpendicular to the common chord and bisects it.



If M and N are intersecting circles at A and B , then $\overleftrightarrow{MN} \perp \overline{AB}$, $AC = BC$ ($\overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$)

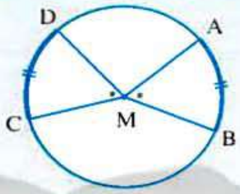
Remarks on identifying the circle

- It is possible to draw an infinite number of circles passing through a given point.
- There is an infinite number of circles that can be drawn to pass through the two points A and B and all their centres lie on the axis of symmetry of $\overline{AB}$
- The smallest circle passes through the two points A , B is the circle in which $\overline{AB}$ is a diameter in it and its centre is the midpoint of $\overline{AB}$ and length of its radius $= \frac{1}{2}$ AB
- It is impossible to draw a circle passing through three collinear points.

- There is a unique circle passing through three points as A , B and C which are not collinear and the centre of this circle is the point of intersection of any two axes of symmetry of the axes of the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$

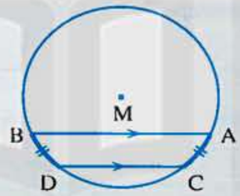
Equality of arcs in measure and length

In the same circle (or in congruent circles) , if the measures of arcs are equal , then the lengths of the arcs are equal and vice versa.



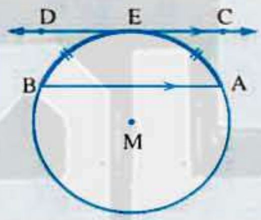
If $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$, then the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$ and vice versa

If two parallel chords are drawn in a circle , then the measures of the two arcs between them are equal.



If $\overline{AB} \parallel \overline{CD}$, then $m(\overset{\frown}{AC}) = m(\overset{\frown}{BD})$

If a chord is parallel to a tangent of a circle , then the measures of the two arcs between them are equal.



If $\overleftrightarrow{CD} \parallel \overline{AB}$, then $m(\overset{\frown}{EA}) = m(\overset{\frown}{EB})$

Notice that

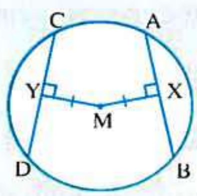
$$\frac{\text{The measure of the arc}}{360^\circ} = \frac{\text{The length of the arc}}{2\pi r}$$

Equality of two chords in length

In the same circle (or in congruent circles) , if the measures of arcs are equal , then their chords are equal in length and vice versa.

If $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$, then AB = CD and vice versa

In the same circle (or in congruent circles) , if chords of a circle are equal in length , then they are equidistant from the centre and vice versa.

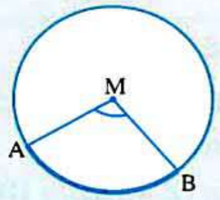


If AB = CD , $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$, then MX = MY and vice versa

Central angle , inscribed angle and angle of tangency and relation between them

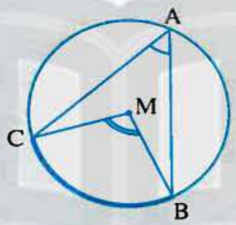
The measure of central angle

Equals the measure of subtended arc



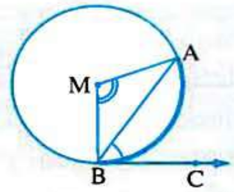
$m(\angle M) = m(\widehat{AB})$

Equals twice the measure of the inscribed angle subtended by the same arc



$m(\angle M) = 2\, m(\angle A)$

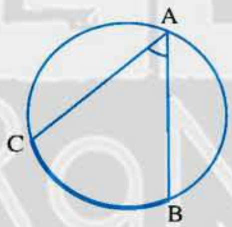
Equals twice the measure of the angle of tangency subtended by the same arc



$m(\angle M) = 2\, m(\angle ABC)$

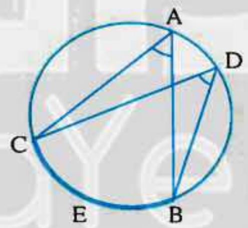
The measure of the inscribed angle

Equals half the measure of the subtended arc



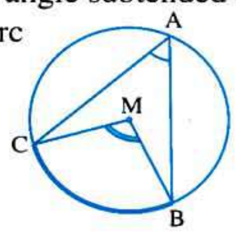
$m(\angle A) = \frac{1}{2}\, m(\widehat{BC})$

Equals the measure of the inscribed angle subtended by the same arc



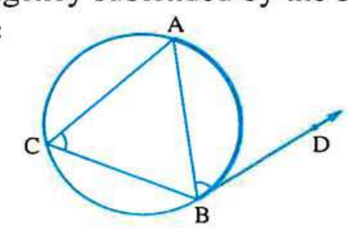
$m(\angle A) = m(\angle D)$

Equals half the measure of the central angle subtended by the same arc

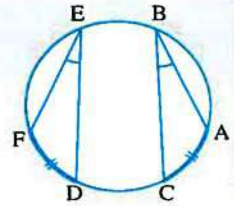
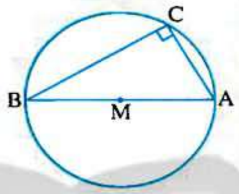
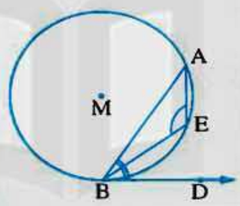


$m(\angle A) = \frac{1}{2}\, m(\angle M)$

Equals the measure of the angle of tangency subtended by the same arc



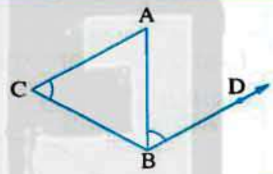
$m(\angle C) = m(\angle ABD)$

In the same circle (or in any number of circles) , the measures of the inscribed angles subtended by arcs of equal measures are equal and vice versa.		If m $(\widehat{AC})$ = m $(\widehat{DF})$, then m (∠ B) = m (∠ E) and vice versa
The inscribed angle in a semicircle is a right angle		If $\overline{AB}$ is a diameter , then m (∠ C) = 90°
The angle of tangency is supplementary to the drawn inscribed angle on the chord of the angle of tangency and in one side of it.		If ∠ AEB is inscribed drawn on $\overline{AB}$, ∠ ABD is angle of tangences , then m (∠ ABD) + m (∠ AEB) = 180°

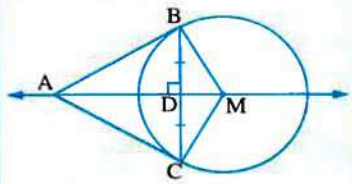
Notice that

To prove that $\overleftrightarrow{BD}$ is a tangent to the circumcircle of Δ ABC

Prove that : m (∠ ABD) = m (∠ ACB)



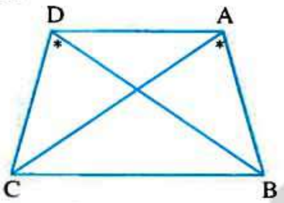
Relation between tangents of the circle

The two tangent-segments drawn to a circle from a point outside it are equal in length		If $\overline{AB}$ and $\overline{AC}$ are tangent segments to the circle M , then • AB = AC • $\overleftrightarrow{AM}$ bisects ∠ BAC and ∠ BMC • $\overleftrightarrow{AM} \perp \overline{BC}$ and bisects it.

Cyclic quadrilateral

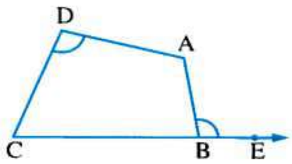
When is the quadrilateral cyclic ?

If there are two equal angles in measure and drawn on one of its sides as a base and on one side of this side.



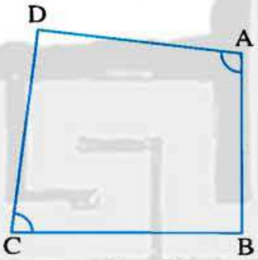
If m (∠ BAC) = m (∠ BDC)

If there is an exterior angle at any of its vertices equal in measure to the measure of the interior angle at the opposite vertex.



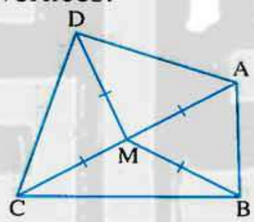
If m (∠ ABE) = m (∠ D)

If there are two opposite supplementary angles



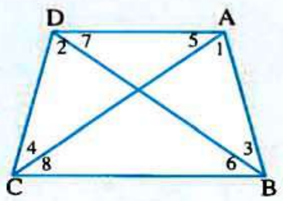
If m (∠ A) + m (∠ C) = 180°

If there is a point in the plane of the figure such that it is equidistant from its vertices.

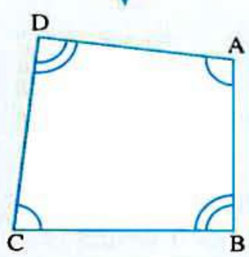


If MA = MB = MC = MD

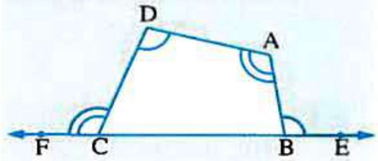
Properties of cyclic quadrilateral



- m (∠ 1) = m (∠ 2)
- m (∠ 3) = m (∠ 4)
- m (∠ 5) = m (∠ 6)
- m (∠ 7) = m (∠ 8)



- m (∠ A) + m (∠ C) = 180°
- m (∠ B) + m (∠ D) = 180°

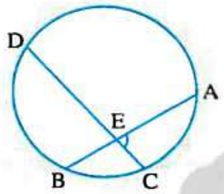


- m (∠ ABE) = m (∠ D)
- m (∠ DCF) = m (∠ A)

Well known problems

Well known problem (1)

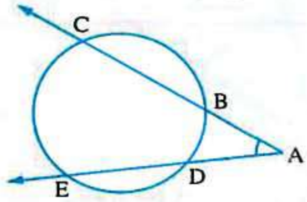
If $\overline{AB}$, $\overline{CD}$ are two chords in a circle intersecting at the point E , then :



$m(\angle AEC) = \frac{1}{2}[m(\widehat{AC}) + m(\widehat{BD})]$

Well known problem (2)

If $\overline{CB}$ and $\overline{ED}$ are two chords in a circle , where $\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$, then :



$m(\angle A) = \frac{1}{2}[m(\widehat{CE}) - m(\widehat{BD})]$

Circumcircle and inscribed a circle of the triangle

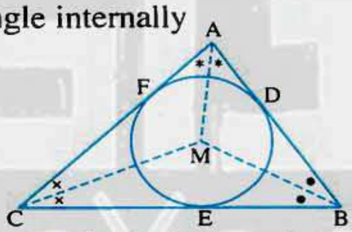
The circumcircle of the triangle

Is the circle that passes through vertices of the triangle

and its centre is the point of intersection of the perpendicular bisectors of its sides

The inscribed circle of the triangle

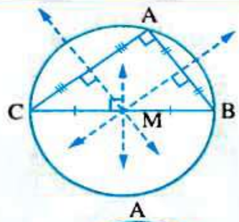
Is the circle is that touches all sides of the triangle internally



and its centre is the intersection point of the bisectors of its interior angles

Notice that

1. Centre of the circumcircle of the right-angled triangle is the midpoint of its hypotenuse.



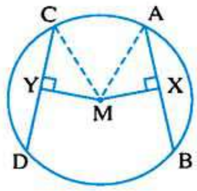
2. Centre of the circumcircle of equilateral triangle is the same centre of the inscribed circle to it which is the point of intersection of axes of its sides and the point of intersection of its median and the point of intersection of the bisectors of its interior angles and also point of intersection of its altitudes.

3. It's possible to draw a circumcircle to a rectangle , a square and an isosceles trapezium while it's impossible to draw a circumcircle to a parallelogram , a rhombus and not isosceles trapezium.

Important theorems and their proofs

Theorem 1

If chords of a circle are equal in length, then they are equidistant from the centre.



Given $AB = CD$, $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{CD}$

R.T.P. $MX = MY$

Construction Draw $\overline{MA}$ and $\overline{MC}$

Proof

$\because \overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$ $\therefore AX = \frac{1}{2} AB$

$\because \overline{MY} \perp \overline{CD}$ $\therefore$ Y is the midpoint of $\overline{CD}$

$\because AB = CD$ (given) $\therefore CY = \frac{1}{2} CD$ $\therefore AX = CY$

$\because \Delta\Delta$ AXM and CYM , both have $\begin{cases} AX = CY \text{ (by proof)} \\ MA = MC = r \\ m(\angle AXM) = m(\angle CYM) = 90^\circ \end{cases}$

$\therefore \Delta AXM \equiv \Delta CYM$, then we get : $MX = MY$ (Q.E.D.)

Theorem 2

The measure of the inscribed angle is half the measure of the central angle , subtended by the same arc.

Given In the circle M : $\angle ACB$ is an inscribed angle ,

$\angle AMB$ is a central angle

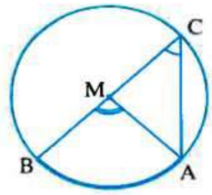
R.T.P. $m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

Proof

$\because \angle AMB$ is an exterior angle of ΔAMC

$\therefore m(\angle AMB) = m(\angle A) + m(\angle C)$ (1)

, $\because MA = MC$ (two radii lengths)



$\therefore m(\angle A) = m(\angle C)$ (2)

From (1) and (2) we get : $m(\angle AMB) = 2\, m(\angle ACB)$

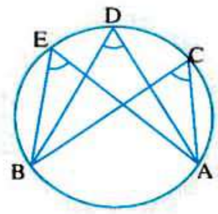
$\therefore m(\angle ACB) = \frac{1}{2} m(\angle AMB)$ (Q.E.D.)

Theorem 3

In the same circle, the measures of all inscribed angles subtended by the same arc are equal.

Given $\angle C$, $\angle D$ and $\angle E$ are inscribed angles subtended by $\overset{\frown}{AB}$

R.T.P. $m(\angle C) = m(\angle D) = m(\angle E)$



Proof $\because m(\angle C) = \frac{1}{2} m(\overset{\frown}{AB})$

$, m(\angle D) = \frac{1}{2} m(\overset{\frown}{AB})$

$, m(\angle E) = \frac{1}{2} m(\overset{\frown}{AB})$

$\therefore m(\angle C) = m(\angle D) = m(\angle E)$ (Q.E.D.)

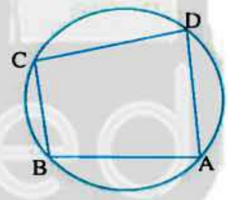
Theorem 4

In a cyclic quadrilateral , each two opposite angles are supplementary.

Given ABCD is a cyclic quadrilateral

R.T.P. ① $m(\angle A) + m(\angle C) = 180°$

② $m(\angle B) + m(\angle D) = 180°$



Proof $\because m(\angle A) = \frac{1}{2} m(\overset{\frown}{BCD})$ and $m(\angle C) = \frac{1}{2} m(\overset{\frown}{BAD})$

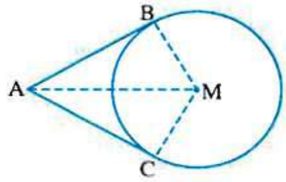
$, \because m(\angle A) + m(\angle C) = \frac{1}{2} [m(\overset{\frown}{BCD}) + m(\overset{\frown}{BAD})]$

$= \frac{1}{2}$ the measure of the circle $= \frac{1}{2} \times 360° = 180°$

Similarly : $m(\angle B) + m(\angle D) = 180°$ (Q.E.D.)

Theorem 5

The two tangent-segments drawn to a circle from a point outside it are equal in length.



Given A is a point outside the circle M

, $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C respectively.

R.T.P. AB = AC

Construction Draw $\overline{MB}$, $\overline{MC}$, $\overline{MA}$

Proof $\because$ $\overleftrightarrow{AB}$ is a tangent to the circle M $\therefore$ m ($\angle$ ABM) = 90°

, $\because$ $\overleftrightarrow{AC}$ is a tangent to the circle M $\therefore$ m ($\angle$ ACM) = 90°

In $\Delta\Delta$ ABM , ACM : $\begin{cases} MB = MC \text{ (the lengths of two radii)} \\ \overline{AM} \text{ is a common side.} \\ m(\angle ABM) = m(\angle ACM) = 90° \text{ (proved)} \end{cases}$

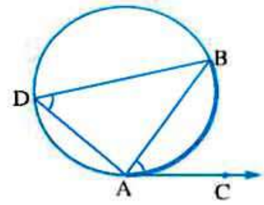
$\therefore$ Δ ABM $\equiv$ Δ ACM and we deduce that : AB = AC (Q.E.D.)

Theorem 6

The measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc.

Given $\angle$ BAC is an angle of tangency and $\angle$ D is an inscribed angle.

R.T.P. m ($\angle$ BAC) = m ($\angle$ D)



Proof $\because$ $\angle$ BAC is an angle of tangency.

$\therefore$ m ($\angle$ BAC) = $\frac{1}{2}$ m ($\overset{\frown}{AB}$) (1)

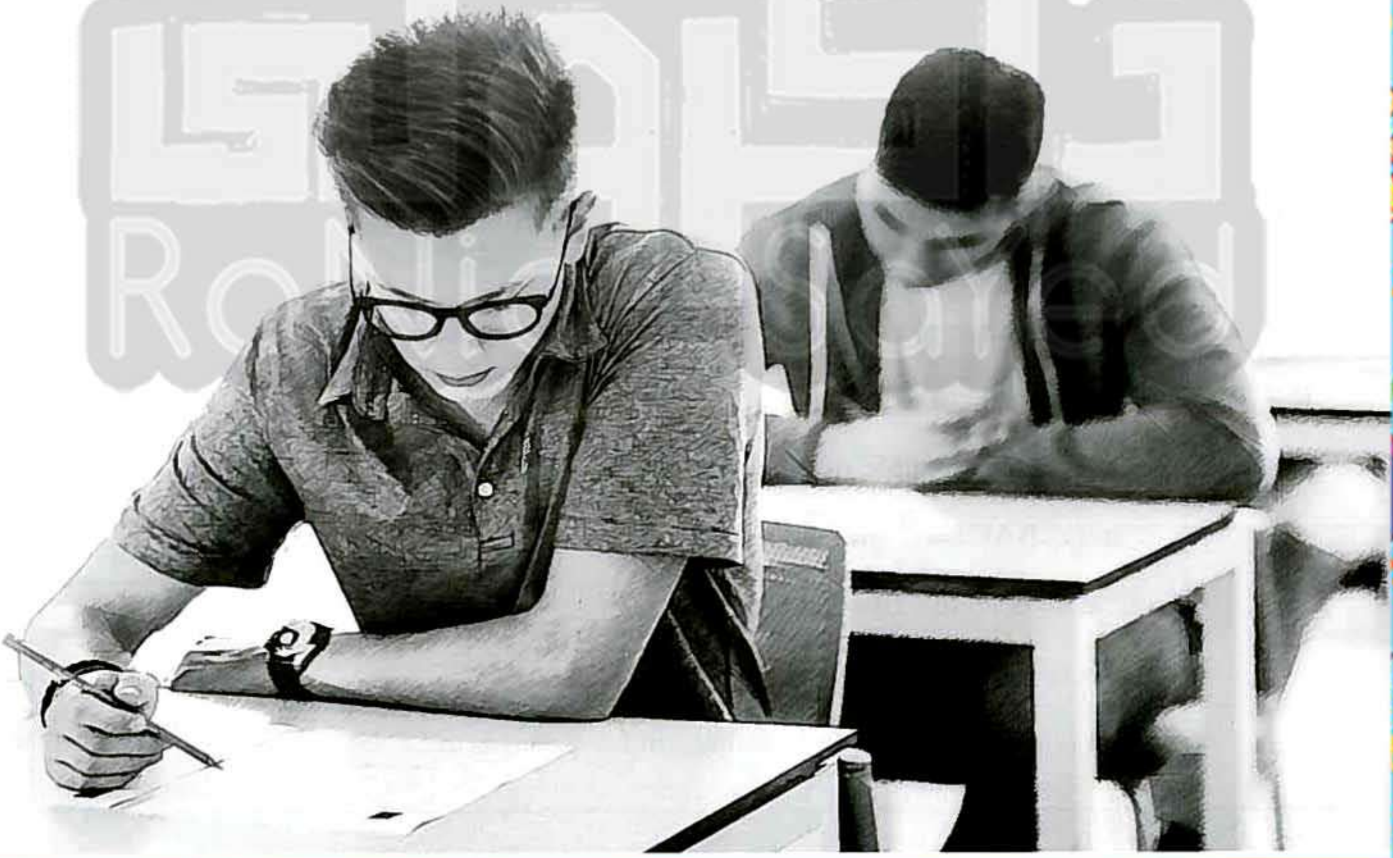
, $\because$ $\angle$ D is an inscribed angle

$\therefore$ m ($\angle$ D) = $\frac{1}{2}$ m ($\overset{\frown}{AB}$) (2)

From (1) and (2) , we deduce that : m ($\angle$ BAC) = m ($\angle$ D) (Q.E.D.)

Final Examinations

of Geometry



Model Examinations of the School Book on Geometry

Model 1

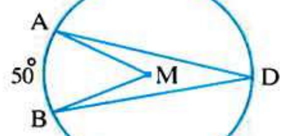
Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 The inscribed angle drawn in a semicircle is ....

(a) an acute. (b) an obtuse. (c) a straight. (d) a right.

2 **In the opposite figure :**



Circle of centre M

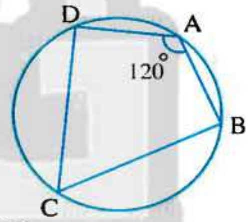
If m $(\widehat{AB}) = 50°$, then m (∠ ADB) = ...

(a) 25° (b) 50° (c) 100° (d) 150°

3 The number of symmetric axes of any circle is ....

(a) zero (b) 1 (c) 2 (d) an infinite number.

4 **In the opposite figure :**



If m (∠ A) = 120° , then m (∠ C) = ...

(a) 60° (b) 90°

(c) 120° (d) 180°

5 If the straight line L is a tangent to the circle M of diameter length 8 cm.
, then the distance between L and the centre of the circle equals .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

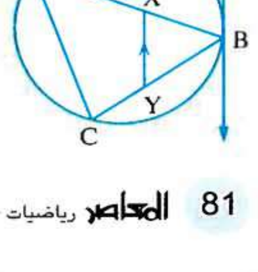
6 The surface of the circle M ∩ the surface of the circle N = {A} and the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle equals .... cm.

(a) 5 (b) 6 (c) 11 (d) 16

2 [a] Complete and prove that :

In a cyclic quadrilateral , each two opposite angles are ....

[b] In the opposite figure :



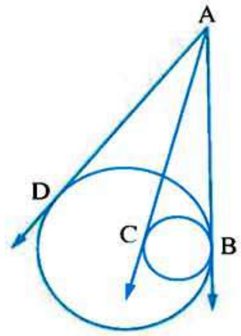
ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, X ∈ $\overline{AB}$, Y ∈ $\overline{BC}$ where $\overline{XY}$ // $\overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.

3 **[a] In the opposite figure :**



Two circles are touching internally at B

, $\overrightarrow{AB}$ is a common tangent

, $\overrightarrow{AC}$ is a tangent to the smaller circle at C

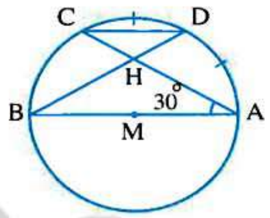
, $\overrightarrow{AD}$ is a tangent to the greater circle at D

, AC = 15 cm. , AB = $(2x - 3)$ cm.

and AD = $(y - 2)$ cm.

Find : The value of each of x and y

[b] In the opposite figure :



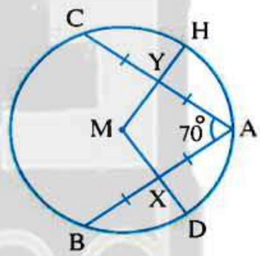
$\overline{AB}$ is a diameter in the circle M

, C $\in$ the circle M , m ($\angle$ CAB) = 30°

, D is midpoint of $\widehat{AC}$, $\overline{DB} \cap \overline{AC} = \{H\}$

1 **Find :** m ($\angle$ BDC) and m ($\widehat{AD}$) **2** **Prove that :** $\overline{AB}$ // $\overline{DC}$

4 **[a] In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in circle M

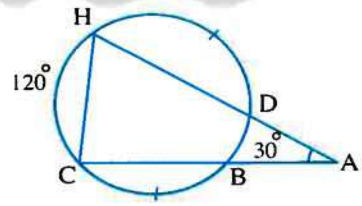
, X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$

, m ($\angle$ CAB) = 70°

1 **Calculate :** m ($\angle$ DMH)

2 **Prove that :** XD = YH

[b] In the opposite figure :



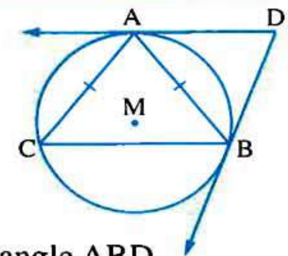
m ($\angle$ A) = 30° , m ($\widehat{HC}$) = 120°

, m ($\widehat{BC}$) = m ($\widehat{DH}$)

1 **Find :** m ($\widehat{BD}$ the minor)

2 **Prove that :** AB = AD

5 **[a] In the opposite figure :**



$\overrightarrow{DA}$ and $\overrightarrow{DB}$ are two tangents of the circle M

and AB = AC

Prove that :

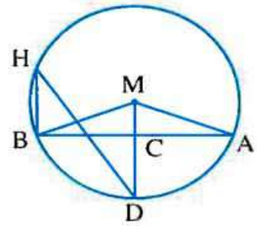
$\overline{AC}$ is a tangent to the circle passing through the vertices of the triangle ABD

[b] In the opposite figure :

C is the midpoint of $\overline{AB}$, $\overrightarrow{MC} \cap$ the circle M = {D}

, m (∠ MAB) = 20°

Find : m (∠ BHD) and m ($\overset{\frown}{ADB}$)



Model 2

1 Choose the correct answer from those given :

1 The measure of the arc which equals half the measure of the circle equals ....

(a) 360° (b) 180° (c) 120° (d) 90°

2 The number of common tangents of two touching circles externally equals ....

(a) 0 (b) 1 (c) 2 (d) 3

3 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 45° (b) 90° (c) 120° (d) 80°

4 The angle of tangency is included between ....

(a) two chords. (b) two tangents.

(c) a chord and a tangent. (d) a chord and a diameter.

5 ABCD is a cyclic quadrilateral , m (∠ A) = 60° , then m (∠ C) = ...

(a) 60° (b) 30° (c) 90° (d) 120°

6 If M , N are two touching circles internally , their radii lengths are 5 cm. , 9 cm.

, then MN = ... cm.

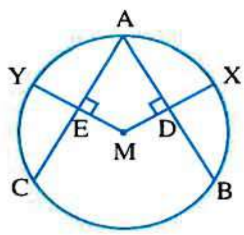
(a) 14 (b) 4 (c) 5 (d) 9

2 [a] In the opposite figure :

AB = AC , $\overline{MD} \perp \overline{AB}$,

$\overline{ME} \perp \overline{AC}$

Prove that : XD = YE



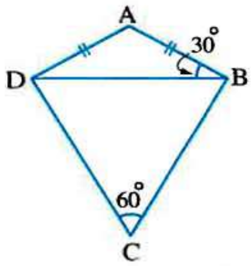
[b] In the opposite figure :

ABCD is a quadrilateral in which AB = AD ,

m (∠ ABD) = 30° ,

m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.



3 [a] State two cases of a cyclic quadrilateral.

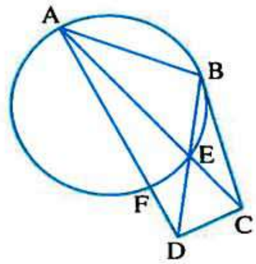
[b] In the opposite figure :

$\overline{BC}$ is a tangent at B ,

E is the midpoint of $\overset{\frown}{BF}$

Prove that :

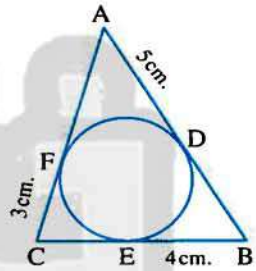
ABCD is a cyclic quadrilateral.



4 **[a] In the opposite figure :**

A circle is drawn touches the sides of a triangle ABC , $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D , E , F , AD = 5 cm , BE = 4 cm. , CF = 3 cm.

Find the perimeter of Δ ABC

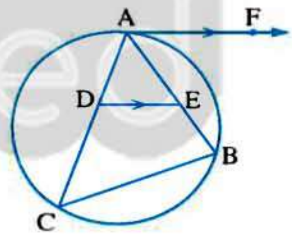


[b] In the opposite figure :

$\overrightarrow{AF}$ is a tangent to the circle at A , $\overrightarrow{AF}$ // $\overrightarrow{DE}$

Prove that :

DEBC is a cyclic quadrilateral.



5 **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

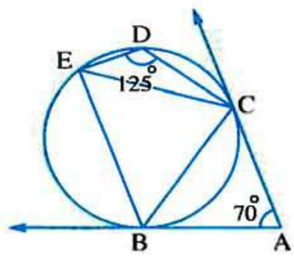
, m (∠ A) = 70° ,

m (∠ CDE) = 125°

Prove that :

1 CB = CE

2 $\overrightarrow{AC}$ // $\overline{BE}$

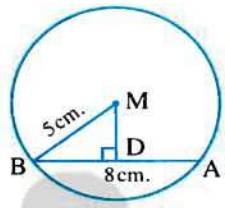


Model examination for the merge students

Answer the following questions in the same paper : (Calculator is allowed)

1 Complete each of the following :

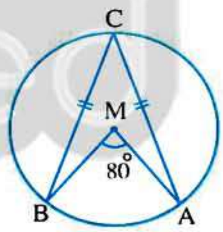
1. The longest chord in the circle is called ....
2. The straight line passing through the center of the circle and the midpoint of any chord is ....
3. The two tangent-segments drawn to a circle from a point outside it are .... in length.
4. **In the opposite figure :**
   The length of $\overline{MD}$ = ... cm.



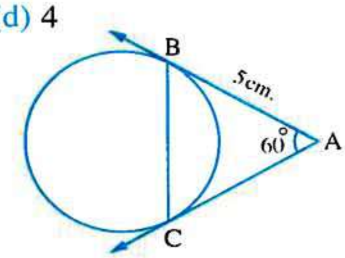
5. The number of symmetry axes of a circle is ....
6. If $\overline{AC}$ is a diameter in a circle M , then m $(\widehat{AC})$ = ... °

2 Choose the correct answer from those given :

1. If A $\in$ the circle M of diameter length 6 cm. , then MA = ... cm.
   (a) 3 (b) 4
   (c) 5 (d) 6
2. **In the opposite figure :**
   m ($\angle$ ACB) = ...
   (a) 40° (b) 80°
   (c) 90° (d) 180°

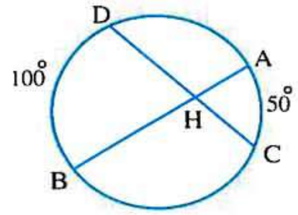


3. The number of the common tangents of two distant circles is ....
   (a) 1 (b) 2 (c) 3 (d) 4
4. **In the opposite figure :**
   The length of $\overline{BC}$ = ... cm.
   (a) 3 (b) 4
   (c) 5 (d) 6



5. The number of circles which can be drawn passing through the endpoints of a line segment $\overline{AB}$ equals ....
   (a) 1 (b) 2 (c) 3 (d) an infinite number.

6 In the opposite figure :



m (∠ AHC) = ...

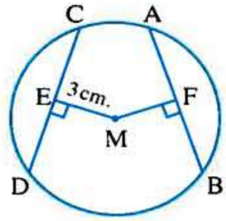
(a) 25° (b) 50°

(c) 75° (d) 100°

3 Put (✓) for the correct statement , (✗) for the incorrect statement :

1 If M , N are two touching externally circles with radii lengths are $r_1 = 5$ cm. , $r_2 = 3$ cm. , then MN = 15 cm. ( )

2 In the opposite figure :



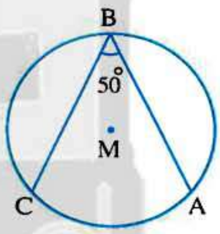
If AB = CD ,

ME = 3 cm. , then

MF = 3 cm. ( )

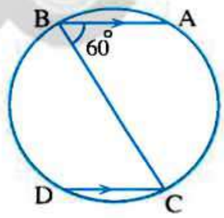
3 The quadrilateral ABCD is a cyclic quadrilateral if m (∠ A) + m (∠ C) = 90° ( )

4 In the opposite figure :



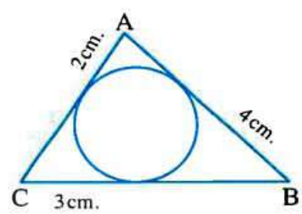
m $(\overset{\frown}{AC})$ = 100° ( )

5 In the opposite figure :



m $(\overset{\frown}{AB})$ + m $(\overset{\frown}{CD})$ = 300° ( )

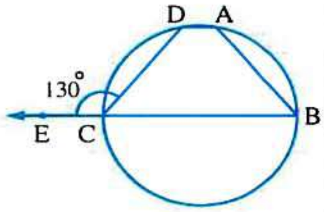
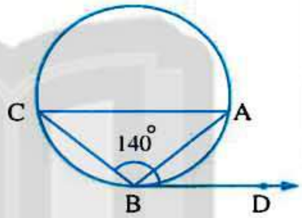
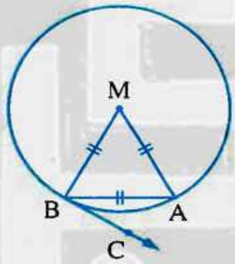
6 In the opposite figure :



The perimeter of

Δ ABC = 9 cm. ( )

4 **Join from the column (A) to the suitable one of the column (B) :**

(A)	(B)
1 The measure of the inscribed angle which is drawn in a semicircle equals	• 130°
2 **In the opposite figure :** m (∠ A) = ... 	• 40°
3 **In the opposite figure :** $\overrightarrow{BD}$ is a tangent at B , m (∠ DBC) = 140° , then m (∠ A) = ... 	• 90°
4 The radius of the circumcircle of the vertices of right-angled triangle of hypotenuse length 10 cm. equals .... cm.	• 30°
5 **In the opposite figure :** Δ MAB is an equilateral triangle , $\overrightarrow{BC}$ is a tangent at B , , then m (∠ ABC) = ... 	• 2 : 1
6 The ratio between the measures of the central angle and inscribed angle subtended by the same arc is	• 5

Governorates' Examinations on Geometry

1 Cairo Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1 The area of the rhombus with diagonal lengths 6 cm. , 8 cm. is .... cm^2

(a) 2 (b) 14 (c) 24 (d) 48

2 Two distant circles M and N with radii lengths 6 cm and 8 cm respectively , then MN .... 14 cm.

(a) < (b) > (c) = (d) ≥

3 The measure of the inscribed angle is .... the measure of the central angle subtended by the same arc.

(a) half (b) twice (c) quarter (d) third

4 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

5 In the cyclic quad. ABCD , if $m(\angle A) = \frac{1}{2} m(\angle C)$, then $m(\angle A) =$ ....°

(a) 20 (b) 30 (c) 60 (d) 120

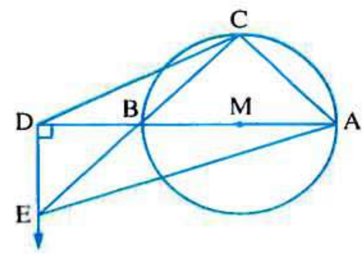
6 The angle of measure 40° is the complemented angle of the angle of measure .... °

(a) 320 (b) 140 (c) 60 (d) 50

2 [a] Mention two cases of the cyclic quadrilateral.

[b] **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M , $D \in \overrightarrow{AB}$, $D \notin \overline{AB}$, $\overrightarrow{DE} \perp \overrightarrow{AB}$, $C \in \widehat{AB}$, $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$

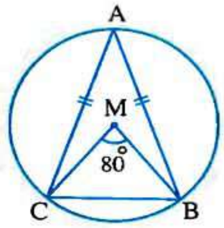


1 **Find :** $m(\angle ACB)$

2 **Prove that :** The figure ACDE is a cyclic quadrilateral.

3 [a] Find the measure of the arc which represents $\frac{1}{3}$ of the measure of the circle.

[b] **In the opposite figure :**



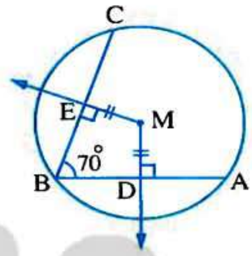
Δ ABC is drawn inside the circle M

, AB = AC , m ($\angle$ BMC) = 80°

Find : 1 m ($\angle$ ABC)

2 The measure of the major arc $\overset{\frown}{BC}$

4 [a] **In the opposite figure :**



$\overline{AB}$ and $\overline{BC}$ are two chords in the circle M , $\overrightarrow{MD} \perp \overline{AB}$

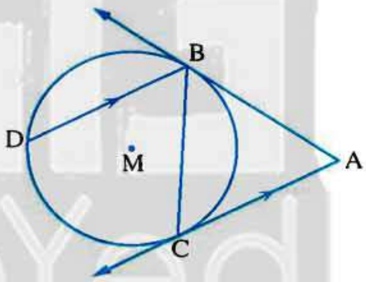
, $\overrightarrow{ME} \perp \overline{CB}$, MD = ME

, m ($\angle$ ABC) = 70°

1 **Find :** m ($\angle$ DME)

2 **Prove that :** AB = CB

[b] **In the opposite figure :**



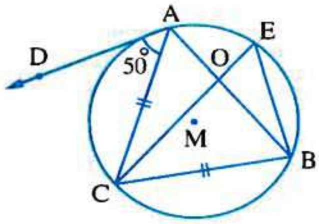
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M at B and C respectively

, $\overrightarrow{BD}$ // $\overrightarrow{AC}$

Prove that : $\overrightarrow{BC}$ bisects $\angle$ ABD

5 [a] Using the geometric tools , draw $\overline{AB}$ with length 6 cm , and then draw a circle passing through the two points A , B with radius length 4 cm. What is the length of the radius of the smallest circle passing through the two points A and B ?

[b] **In the opposite figure :**



A circle M , AC = BC

, $\overrightarrow{AD}$ is a tangent to the circle at A , m ($\angle$ CAD) = 50°

1 **Find :** m ($\angle$ ABC) , m ($\angle$ BEC)

2 **Prove that :**

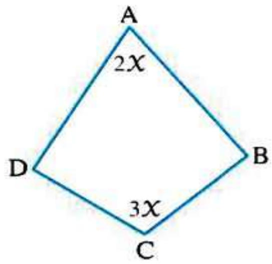
$\overrightarrow{BC}$ is a tangent to the circle passing through the vertices of the triangle BEO

2 Giza Governorate

Answer the following questions :

1 Choose the correct answer :

1 In the opposite figure :



ABCD is a cyclic quadrilateral

, m (∠ A) = 2 X , m (∠ C) = 3 X

, then the value of X = ...°

(a) 20 (b) 30

(c) 32 (d) 36

2 If the ratio between the perimeters of two squares is 1 : 2 , then the ratio between their areas equals ....

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

3 The measure of the inscribed angle in a semicircle equals ....°

(a) 45 (b) 90 (c) 120 (d) 180

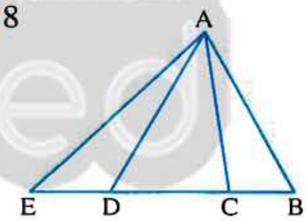
4 The median of the triangle divides its surface into two triangles ....

(a) congruent. (b) equal in area. (c) isosceles. (d) right-angled.

5 If the two circles M , N are touching internally , their radii lengths are 3 cm. , 5 cm. , then MN = ... cm.

(a) 3 (b) 5 (c) 2 (d) 8

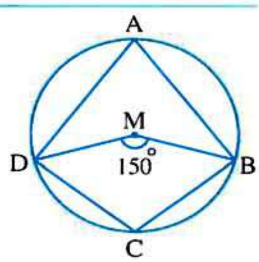
6 The number of triangles in the opposite figure equals ....



(a) 3 (b) 4

(c) 5 (d) 6

2 [a] In the opposite figure :

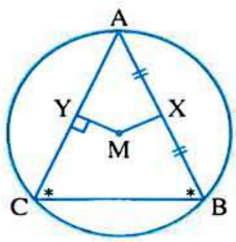


A circle of centre M

, m (∠ BMD) = 150°

Find with proof : m (∠ C)

[b] In the opposite figure :



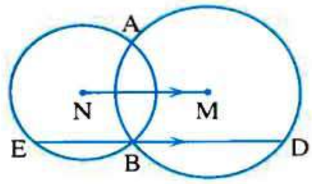
ABC is an inscribed triangle in a circle M

in which m (∠ B) = m (∠ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

3 **[a] In the opposite figure :**

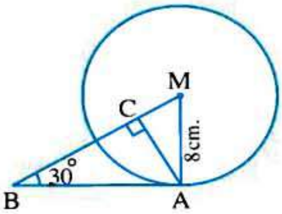


M , N are two intersecting circles at A , B

, $\overleftrightarrow{BD}$ // $\overleftrightarrow{MN}$ and intersects the two circles at D , E

Prove that : DE = 2 MN

[b] In the opposite figure :

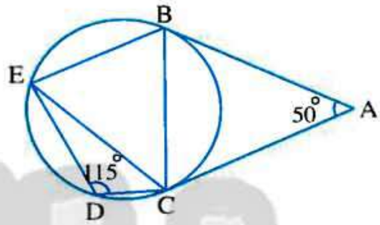


$\overrightarrow{AB}$ is a tangent to the circle M at A

, MA = 8 cm. , m (∠ ABM) = 30° , $\overline{AC} \perp \overline{MB}$

Find : The length of each of $\overline{AB}$, $\overline{AC}$

4 **[a] In the opposite figure :**



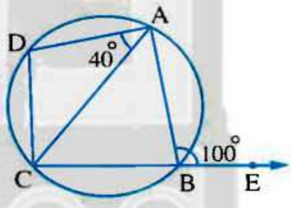
$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle at B , C

, m (∠ A) = 50° , m (∠ CDE) = 115°

Prove that : 1 $\overrightarrow{BC}$ bisects ∠ ABE

2 CB = CE

[b] In the opposite figure :

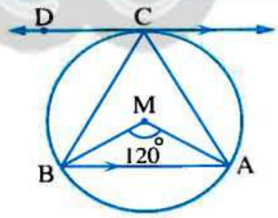


m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{AD})$

5 **[a] In the opposite figure :**



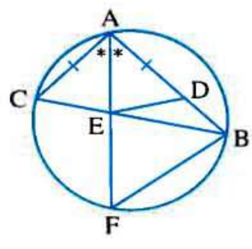
$\overleftrightarrow{CD}$ is a tangent to the circle M at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$

, m (∠ AMB) = 120°

Prove that : Δ CAB is an equilateral triangle.

[b] In the opposite figure :



AC = AD , $\overrightarrow{AE}$ bisects ∠ BAC

and cuts $\overline{BC}$ at E and the circle at F

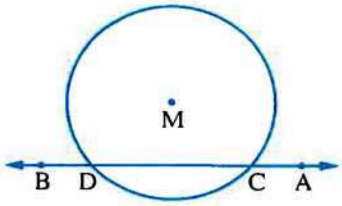
Prove that : BDEF is a cyclic quadrilateral.

3 Alexandria Governorate

Answer the following questions : ***(Calculators are permitted)***

1 Choose the correct answer from those given :

1 In the opposite figure :



$\overleftrightarrow{AB} \cap$ the surface of the circle M = ...

(a) $\{C , D\}$ (b) $\overline{CD}$
(c) $\overleftrightarrow{CD}$ (d) $\varnothing$

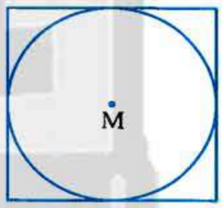
2 $\angle$ A and $\angle$ B are two complementary angles , $\angle$ B and $\angle$ C are two supplementary angles , m ($\angle$ A) = 30° , then m ($\angle$ C) = ...°

(a) 30 (b) 60 (c) 90 (d) 120

3 If the surface of the circle M $\cap$ the surface of the circle N = $\{A\}$ and the radius length of one of them equals 3 cm and MN = 8 cm. , then the radius length of the other circle equals .... cm.

(a) 5 (b) 6 (c) 11 (d) 16

4 In the opposite figure :



If the side length of the square = 10 cm.
, then the surface area of the circle = ... cm^2

(a) 100 π (b) 25 π
(c) 50 π (d) 40 π

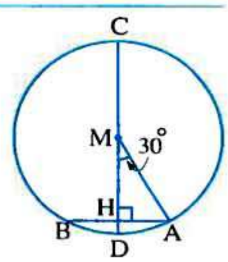
5 A circle can be drawn passing through the vertices of a ....

(a) rhombus (b) parallelogram (c) trapezium (d) rectangle

6 The rhombus whose two diagonal lengths are 12 cm. and 16 cm. , then its side length equals .... cm.

(a) 6 (b) 8 (c) 10 (d) 20

2 [a] In the opposite figure :



$\overline{CD}$ is a diameter in the circle M
, AB = 10 cm. , $\overline{MH} \perp \overline{AB}$
, m ($\angle$ AMD) = 30°
Find : The length of $\overline{CD}$

[b] ABCD is a quadrilateral inscribed in a circle , E is a point outside the circle , $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are two tangents to the circle at A and B , if m ($\angle$ AEB) = 70° and m ($\angle$ ADC) = 125° , **prove that :** AB = AC

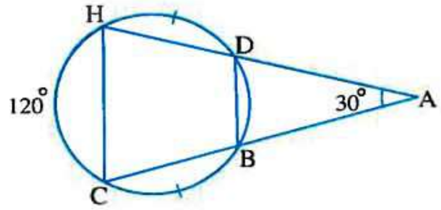
3 **[a] In the opposite figure :**

m (∠ A) = 30° , m $(\overset{\frown}{HC})$ = 120°

, m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{DH})$

1 **Find :** m $(\overset{\frown}{BD})$ «the minor arc»

2 **Prove that :** AB = AD



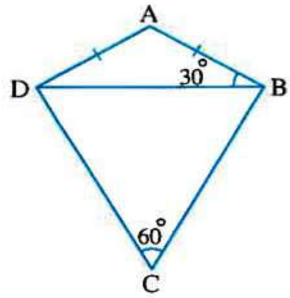
[b] In the opposite figure :

ABCD is a quadrilateral , AB = AD

, m (∠ ABD) = 30°

, m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.

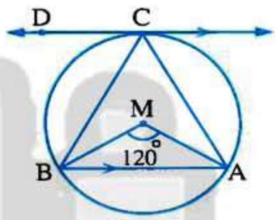


4 **[a] In the opposite figure :**

$\overleftrightarrow{CD}$ is a tangent to the circle at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$, m (∠ AMB) = 120°

Prove that : The triangle CAB is an equilateral triangle.



[b] In the opposite figure :

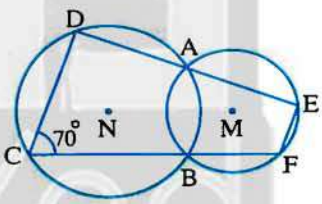
M and N are two intersecting circles at A and B

, $\overleftrightarrow{AD}$ is drawn to intersect the circle M at E and the circle N at D

, $\overleftrightarrow{BC}$ is drawn to intersect the circle M at F and the circle N at C

, m (∠ C) = 70°

Prove that : $\overline{CD}$ // $\overline{EF}$

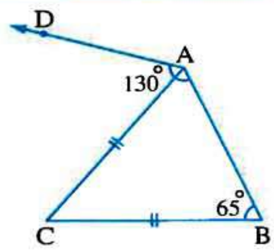


5 **[a] In the opposite figure :**

AC = BC , m (∠ ABC) = 65°

, m (∠ DAB) = 130°

Prove that : $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC



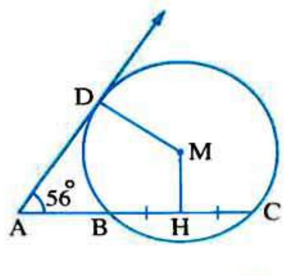
[b] In the opposite figure :

$\overrightarrow{AD}$ is a tangent to the circle M

, $\overrightarrow{AC}$ intersects the circle M at B , C

, m (∠ A) = 56° and H is the midpoint of $\overline{BC}$

Find with proof : m (∠ DMH)



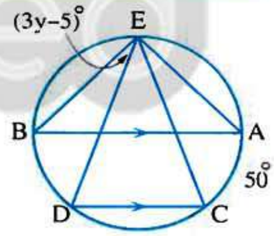
4 El-Kalyoubia Governorate

Answer the following questions :

1 Choose the correct answer :

1. ABC is a triangle in which : $(AB)^2 > (BC)^2 + (AC)^2$, then $\angle$ C is ....
   (a) acute. (b) right. (c) obtuse. (d) straight.
2. If M and N are two intersecting circles whose radii length are 5 cm and 2 cm. , then MN $\in$ ....
   (a) $]3 , 7[$ (b) $[3 , 7[$ (c) $]3 , 7]$ (d) $[3 , 7]$
3. If Δ ABC $\sim$ Δ XYZ , m ($\angle$ A) = 50° , m ($\angle$ B) = 60° , then m ($\angle$ Z) = ...°
   (a) 90 (b) 110 (c) 10 (d) 70
4. The measure of the central angle which is opposite to an arc of length $\frac{1}{3}\pi r$ equals ....°
   (a) 30 (b) 60 (c) 120 (d) 240
5. ABC is a right-angled triangle at B , $\overline{BD} \perp \overline{AC}$ where $\overline{BD} \cap \overline{AC} = \{D\}$, then the projection of $\overline{BD}$ on $\overleftrightarrow{AC}$ is ....
   (a) A (b) B (c) C (d) D
6. If ABCD is a cyclic quadrilateral , then m ($\angle$ BAC) = m ($\angle$ ....)
   (a) BCA (b) DBA (c) BDC (d) ACD

2 [a] **In the opposite figure :**

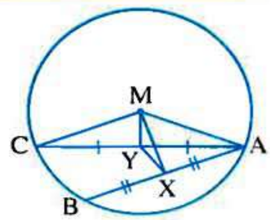


$\overline{AB}$ // $\overline{CD}$, m ($\overset{\frown}{AC}$) = 50°
, m ($\angle$ BED) = (3 y − 5)°

Find : The value of y

[b] Using your geometric tools , draw $\overline{AB}$ with length 4 cm , then draw a circle passing through the two points A and B whose diameter length is 5 cm.
How many circles can be drawn ? (Don't erase the arcs).

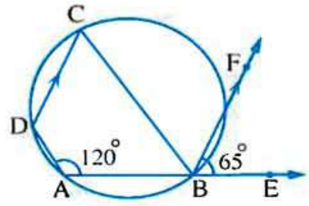
3 [a] **In the opposite figure :**



A circle with centre M
, X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.

Prove that : 1 AXYM is a cyclic quadrilateral.
2 m ($\angle$ MXY) = m ($\angle$ MCY)

[b] In the opposite figure :



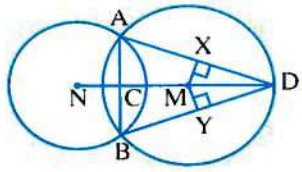
$m(\angle A) = 120°$, $m(\angle EBF) = 65°$

, $\overline{DC} \parallel \overrightarrow{BF}$

Find with proof :

1 $m(\angle C)$ 2 $m(\angle D)$

4 **[a] In the opposite figure :**



Circle M $\cap$ circle N = $\{A , B\}$

, $\overleftrightarrow{AB} \cap \overleftrightarrow{MN} = \{C\}$, $D \in \overleftrightarrow{MN}$

, $\overline{MX} \perp \overline{AD}$, $\overline{MY} \perp \overline{BD}$

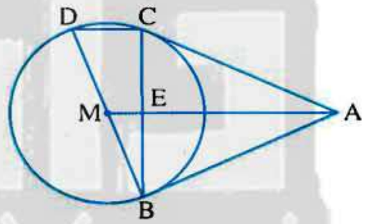
Prove that : MX = MY

[b] ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A

, $X \in \overline{AB}$, $Y \in \overline{AC}$, where $\overline{XY} \parallel \overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

5 **[a] In the opposite figure :**



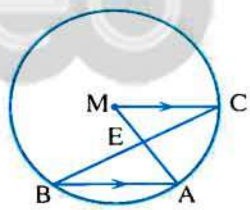
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AM} \cap \overline{CB} = \{E\}$

and $\overline{BD}$ is a diameter of the circle.

Prove that : $\overline{AM} \parallel \overline{CD}$

[b] In the opposite figure :



$\overline{AB}$ is a chord in the circle M

, $\overline{CM} \parallel \overline{AB}$

, $\overline{BC} \cap \overline{AM} = \{E\}$

Prove that : BE > AE

5 El-Sharkia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from the given ones :**

1 A circle can be drawn passing through the vertices of a ....

(a) rhombus. (b) rectangle. (c) trapezium. (d) parallelogram.

2 A circle with diameter length 10 cm. , the straight line L is distant from its centre by 5 cm. , then the straight line L is ....

(a) a tangent. (b) a secant.

(c) outside the circle. (d) a diameter of the circle.

3 The number of common tangents of two touching circles externally equals ....

(a) zero (b) 1 (c) 2 (d) 3

4 If M , N are two touching circles externally , the lengths of their radii are 2 cm. , 4 cm. respectively , then the area of the circle with diameter $\overline{MN}$ equals .... cm^2

(a) 36π (b) 9π (c) 16π (d) 4π

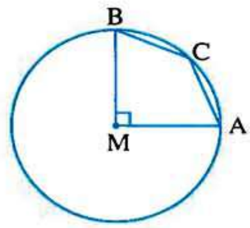
5 **In the opposite figure :**

A circle M , $\overline{MA} \perp \overline{MB}$

, then m ($\angle$ ACB) = ...

(a) 45° (b) 90°

(c) 145° (d) 135°



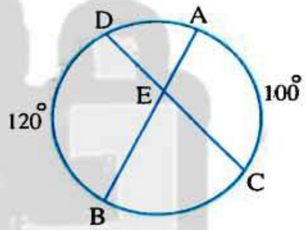
6 **In the opposite figure :**

m ($\widehat{AC}$) = 100° , m ($\widehat{DB}$) = 120°

, then m ($\angle$ AEC) = ...

(a) 110° (b) 55°

(c) 70° (d) 100°



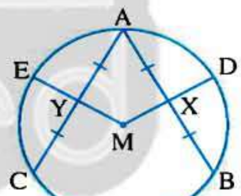
2 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two equal chords in circle M

, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$

Prove that : XD = YE

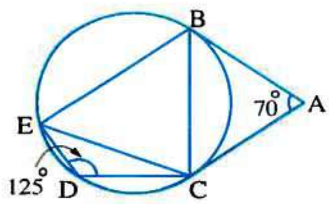


[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C

, m ($\angle$ A) = 70° , m ($\angle$ CDE) = 125°

Prove that : $\overrightarrow{BC}$ bisects $\angle$ ABE



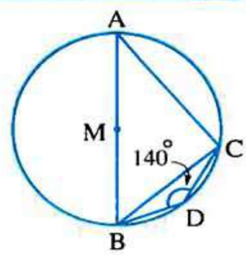
3 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

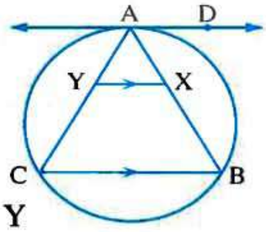
, m ($\widehat{BD}$) = m ($\widehat{DC}$) , m ($\angle$ BDC) = 140°

Find with proof : 1 m ($\angle$ ABC)

2 m ($\widehat{ABD}$)



[b] In the opposite figure :



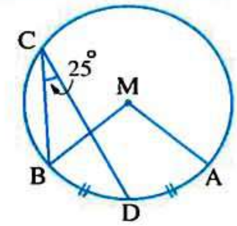
$\overleftrightarrow{AD}$ is a tangent to the circle at A , X $\in \overline{AB}$

, Y $\in \overline{AC}$ and $\overline{XY}$ // $\overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle which passes through the points A , X and Y

4 **[a] In the opposite figure :**

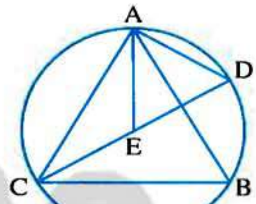


A circle M , D is the midpoint of $\widehat{AB}$

, m ($\angle$ DCB) = 25°

Find : m ($\angle$ AMB)

[b] In the opposite figure :



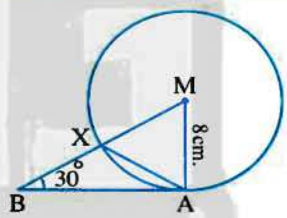
ABC is an equilateral triangle drawn in the circle

, D $\in \widehat{AB}$, E $\in \overline{DC}$, where AD = DE

Prove that : 1 Δ ADE is an equilateral triangle.

2 m ($\angle$ DAB) = m ($\angle$ EAC)

5 **[a] In the opposite figure :**



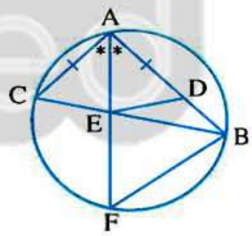
$\overline{AB}$ is a tangent-segment to the circle M at A

, AM = 8 cm. , m ($\angle$ ABM) = 30°

1 **Find :** The length of $\overline{AB}$

2 **Prove that :** Δ XAB is an isosceles triangle.

[b] In the opposite figure :



AD = AC

, $\overrightarrow{AF}$ bisects $\angle$ BAC

Prove that : DBFE is a cyclic quadrilateral.

6 El-Monofia Governorate

Answer the following questions : ***(Calculators are permitted)***

1 **Choose the correct answer from those given :**

1 The axis of symmetry of a circle is ....

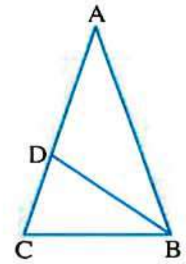
(a) the diameter. (b) the chord.

(c) the straight line passing through the center. (d) the tangent.

2 XYZ is a triangle. If $(XY)^2 - (YZ)^2 > (XZ)^2$, then $\angle$ Y is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

3 **In the opposite figure :**



If AB = AC , BC = BD = AD

, then m ($\angle$ A) = ...°

(a) 30 (b) 36

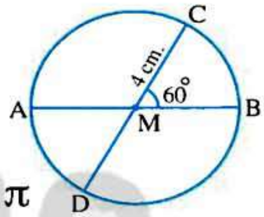
(c) 45 (d) 72

4 ABCD is a cyclic quadrilateral in which m ($\angle$ A) = 2 m ($\angle$ C)

, then m ($\angle$ C) = ...°

(a) 30 (b) 60 (c) 90 (d) 120

5 **In the opposite figure :**



A circle M , MC = 4 cm.

, m ($\angle$ CMB) = 60°

, then the length of $\widehat{BD}$ = ... cm.

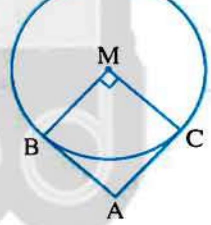
(a) 4π (b) 8π (c) $\frac{8}{3}\pi$ (d) 16π

6 If Y $\in \overline{XZ}$ and XY = 2 YZ , then the area of the square drawn on $\overline{XY}$ = ...

The area of the square drawn on $\overline{XZ}$

(a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) 2 (d) $\frac{1}{2}$

2 [a] **In the opposite figure :**

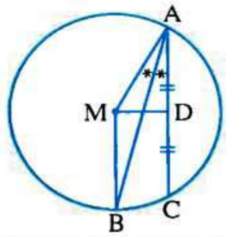


$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, m ($\angle$ BMC) = 90°

Prove that : ABMC is a square.

[b] **In the opposite figure :**



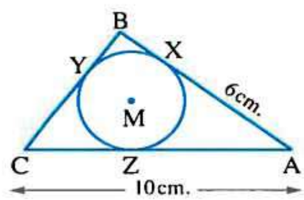
$\overline{AC}$ is a chord in the circle M

, $\overrightarrow{AB}$ bisects $\angle$ CAM

, D is the midpoint of $\overline{AC}$

Prove that : $\overline{DM} \perp \overline{MB}$

3 [a] **In the opposite figure :**



$\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ are tangents to the circle M at X

, Y and Z respectively , AC = 10 cm.

, AX = 6 cm. and the perimeter of Δ ABC = 24 cm.

1 **Find :** The length of $\overline{AB}$

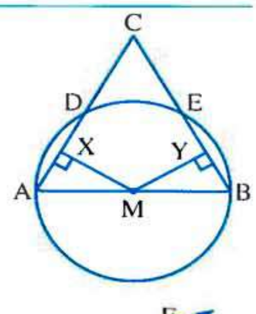
2 Determine the type of Δ ABC according to the measures of its angles.

[b] ABC is a triangle inscribed in a circle , $X \in \overset{\frown}{AB}$, $Y \in \overset{\frown}{AC}$ where $m(\overset{\frown}{AX}) = m(\overset{\frown}{AY})$, $\overline{CX} \cap \overline{AB} = \{D\}$ and $\overline{BY} \cap \overline{AC} = \{E\}$

Prove that : 1 The figure BCED is a cyclic quadrilateral.

2 $m(\angle DEB) = m(\angle XAB)$

4 **[a] In the opposite figure :**



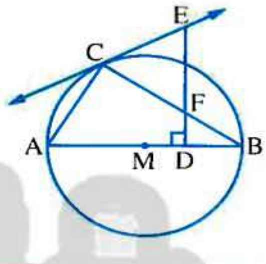
$\overline{AB}$ is a diameter in the circle M

, CA = CB , $\overline{MX} \perp \overline{DA}$

, $\overline{MY} \perp \overline{EB}$

Prove that : CD = CE

[b] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M

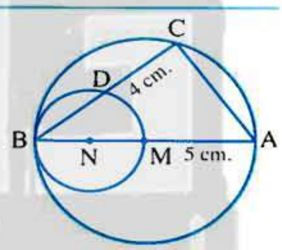
, $\overleftrightarrow{EC}$ is a tangent to the circle M at C , $\overline{ED} \perp \overline{AB}$

, where $\overline{ED} \cap \overline{CB} = \{F\}$

Prove that : 1 The figure ADFC is a cyclic quadrilateral.

2 Δ ECF is an isosceles triangle.

5 **[a] In the opposite figure :**

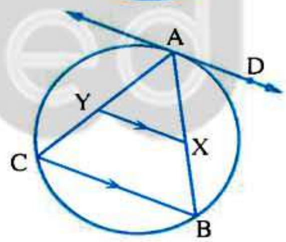


M , N are two circles touching internally at B

, AM = 5 cm. , CD = 4 cm.

Find with proof : The length of $\overline{AC}$

[b] In the opposite figure :



ABC is a triangle inscribed in a circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A , $\overline{XY} \parallel \overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

7 El-Gharbia Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 A square whose diagonal length is 10 cm. , then its surface area equals .... cm^2.

(a) 40 (b) 50 (c) 80 (d) 100

2 ABC is a triangle in which $(AC)^2 > (AB)^2 + (BC)^2$, then $\angle$ BAC is ....

(a) acute. (b) obtuse. (c) right. (d) straight.

3 M and N are two intersecting circles at two points and the two radii lengths are 3 cm. and 5 cm. , then MN ∈ ....

(a) $]8 , \infty[$ (b) $]2 , \infty[$ (c) $]0 , 2[$ (d) $]2 , 8[$

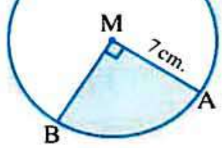
4 ABCD is a cyclic quadrilateral in which m (∠ A) = 3 m (∠ C) , then m (∠ A) = ...°

(a) 90 (b) 45 (c) 135 (d) 120

5 **In the opposite figure :**

$\overline{MA}$, $\overline{MB}$ are two radii perpendicular in the circle M whose radius length is 7 cm.

, then the perimeter of the shaded part = ... cm. $\left(\pi = \frac{22}{7}\right)$



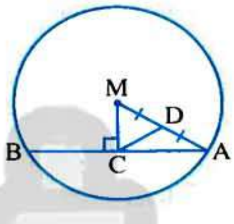
(a) 14 (b) 11 (c) $38\frac{1}{2}$ (d) 25

6 **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$, D is the midpoint of $\overline{MA}$, CD = 3 cm.

, then the surface area of the circle M = ... π cm².



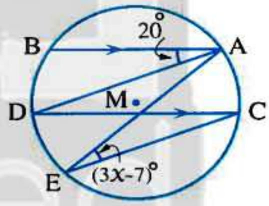
(a) 3 (b) 6 (c) 9 (d) 36

2 **[a] In the opposite figure :**

$\overline{AB} // \overline{CD}$, m (∠ BAD) = 20°

, m (∠ AEC) = $(3x - 7)°$

What is the value of x ?

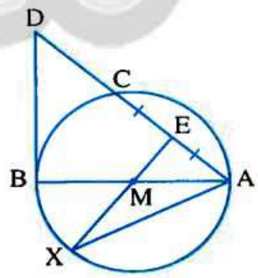


[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M , $\overline{BD}$ is a tangent-segment to the circle M at B , E is the midpoint of $\overline{AC}$ and $\overrightarrow{EM}$ intersects the circle M at X

Prove that : 1 The figure MEDB is a cyclic quadrilateral.

2 m (∠ BAX) = $\frac{1}{2}$ m (∠ D)

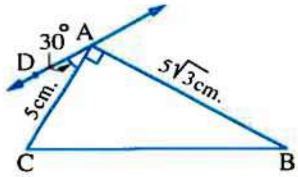


3 **[a] In the opposite figure :**

ABC is a right-angled triangle at A

, AC = 5 cm. , AB = $5\sqrt{3}$ cm.

, m (∠ DAC) = 30°



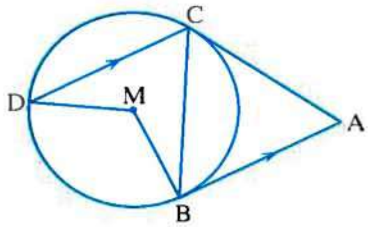
Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC

[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M at B and C

, $\overline{AB}$ // $\overline{CD}$

Prove that : $\overrightarrow{CB}$ bisects $\angle$ ACD



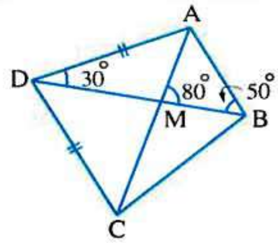
4 **[a] In the opposite figure :**

ABCD is a quadrilateral in which $\overline{AC} \cap \overline{BD} = \{M\}$, DA = DC

, m ($\angle$ ADM) = 30° , m ($\angle$ AMB) = 80°

, m ($\angle$ ABD) = 50°

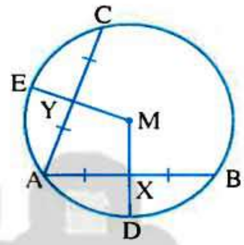
Prove that : The figure ABCD is a cyclic quadrilateral.



[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two chords equal in length in the circle M , X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , $\overrightarrow{MX}$ intersects the circle M at D , $\overrightarrow{MY}$ intersects the circle M at E

Prove that : XD = YE



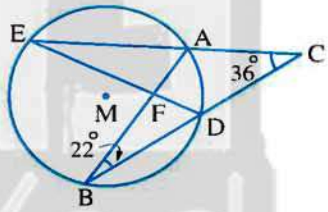
5 **[a] In the opposite figure :**

$\overrightarrow{EA} \cap \overrightarrow{BD} = \{C\}$

, m ($\angle$ C) = 36°

, m ($\angle$ ABD) = 22°

Find with the proof : m ($\overset{\frown}{BE}$)



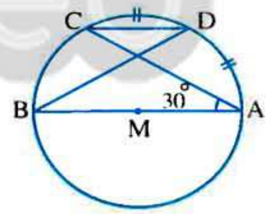
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, m ($\angle$ CAB) = 30° , m ($\overset{\frown}{AD}$) = m ($\overset{\frown}{DC}$)

1 **Find with the proof :** m ($\angle$ CDB)

2 **Prove that :** $\overline{DC}$ // $\overline{AB}$



8 El-Dakahlia Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **[a] Choose the correct answer from the given ones :**

1 A circle with greatest chord with length = 12 cm. , then the circumference of the circle = ... cm.

(a) 12 π (b) 6 π (c) 24 π (d) 10 π

2 M and N are two circles whose radii lengths are 6 cm. , 8 cm. and MN = 14 cm. , then the two circles are ....

(a) intersecting. (b) distant.

(c) one inside the other. (d) touching externally.

3 The inscribed angle drawn in a semicircle is .... angle.

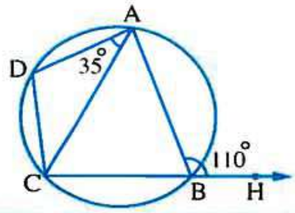
(a) an acute (b) a straight (c) a right (d) an obtuse

[b] In the opposite figure :

m (∠ ABH) = 110°

, m (∠ CAD) = 35°

Prove that : m $(\widehat{CD})$ = m $(\widehat{AD})$



2 **[a] Choose the correct answer from the given ones :**

1 A chord is of length 8 cm. in a circle of diameter length 10 cm. , then the chord is at .... from the center of the circle.

(a) 2 cm. (b) 4 cm. (c) 3 cm. (d) 6 cm.

2 The number of common tangents of two circles touching internally is ....

(a) 1 (b) 3 (c) 2 (d) 0

3 ABCD is a cyclic quadrilateral , m (∠ A) = 2 m (∠ C) , then m (∠ A) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

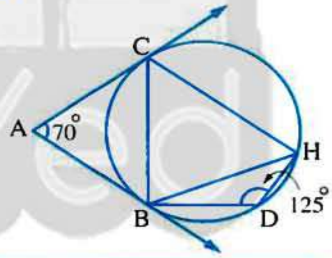
[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m (∠ A) = 70° , m (∠ D) = 125°

1 **Find :** m (∠ ABC)

2 **Prove that :** CB = BH



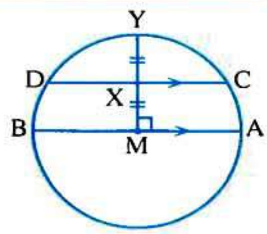
3 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M , $\overline{CD}$ // $\overline{AB}$

, X is the midpoint of $\overline{MY}$

, $\overline{MY} \perp \overline{AB}$

Find : m $(\widehat{AC})$, m $(\widehat{CY})$



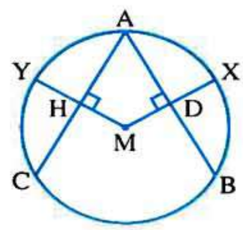
[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two equal chords in the circle M

, $\overrightarrow{MD} \perp \overline{AB}$ and cuts the circle at X

, $\overrightarrow{MH} \perp \overline{AC}$ and cuts the circle at Y

Prove that : XD = HY

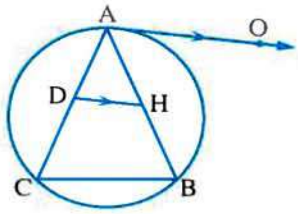


4 [a] **In the opposite figure :**

$\overrightarrow{AO}$ is a tangent to the circle at A

, $\overrightarrow{AO} \parallel \overline{DH}$

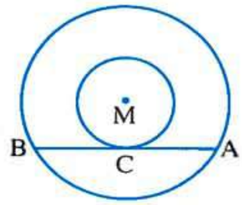
Prove that : DHBC is a cyclic quadrilateral.



[b] **In the opposite figure :**

$\overline{AB}$ is a chord in the greater circle M and touches the smaller circle at C , if AB = 14 cm.

, find the area of the part included between the two circles.



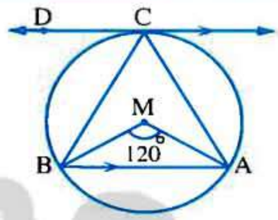
5 [a] **In the opposite figure :**

The circle M passes through the vertices of the triangle ABC

, m (∠ AMB) = 120°

, $\overleftrightarrow{CD}$ is a tangent to the circle M at C , $\overleftrightarrow{CD} \parallel \overline{AB}$

Prove that : Δ ABC is equilateral.

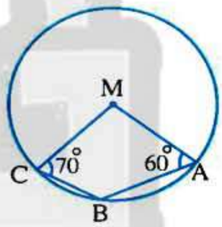


[b] **In the opposite figure :**

m (∠ MAB) = 60°

, m (∠ MCB) = 70°

Find : m (∠ AMC)



9 Ismailia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The least number of acute angles at any triangle equals ....

(a) zero (b) 1 (c) 2 (d) 3

2 The measure of the central angle drawn in $\frac{1}{3}$ circle equals ....°

(a) 240 (b) 120 (c) 60 (d) 30

3 ABC is a triangle in which : $(AC)^2 = (AB)^2 + (BC)^2 + 5$, then ∠ B is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

4 Which of the following figures is a cyclic quadrilateral ?

(a) The square. (b) The rhombus.

(c) The parallelogram. (d) The trapezium.

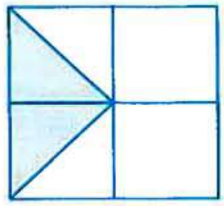
5 If AB = 8 cm. , then the length of the radius of the smallest circle can be drawn passing through the two points A and B equals .... cm.

(a) 1 (b) 2 (c) 3 (d) 4

6 **In the opposite figure :**

A square consists of congruent squares , then the area of the shaded part = ... the figure area.

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$



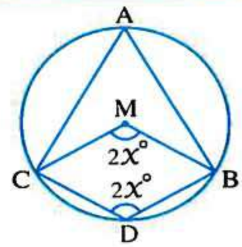
2 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords of the circle M

, $D \in \widehat{BC}$

, m (∠ BMC) = m (∠ BDC) = $(2\,x)°$

Find with proof : m (∠ A)

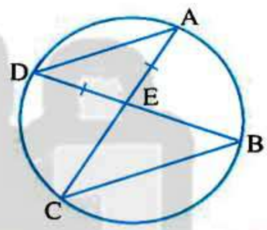


[b] In the opposite figure :

$\overline{AC} \cap \overline{BD} = \{E\}$

, EA = ED

Prove that : EB = EC



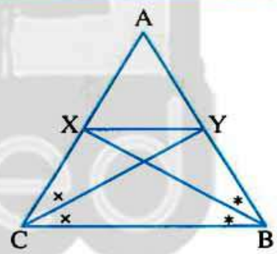
3 **[a] In the opposite figure :**

ABC is a triangle in which AB = AC

, $\overrightarrow{BX}$ bisects ∠ ABC and intersects $\overline{AC}$ at X

, $\overrightarrow{CY}$ bisects ∠ ACB and intersects $\overline{AB}$ at Y

Prove that : BCXY is a cyclic quadrilateral.



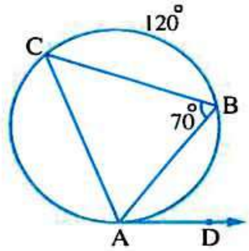
[b] In the opposite figure :

$\overrightarrow{AD}$ is a tangent to the circle at A

, m (∠ B) = 70°

, m $(\widehat{BC})$ = 120°

Find : m (∠ DAB)

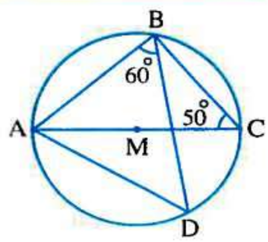


4 **[a] In the opposite figure :**

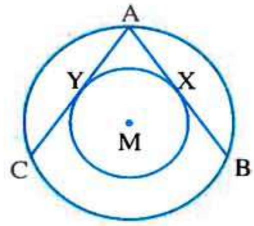
$\overline{AC}$ is a diameter of the circle M

, m (∠ C) = 50° , m (∠ ABD) = 60°

Find : m (∠ CBD) , m (∠ BAD)



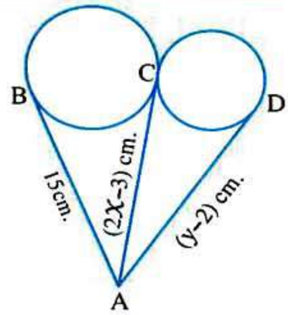
[b] In the opposite figure :



Two concentric circles at M , $\overline{AB}$ and $\overline{AC}$ are two chords in the greater circle and two tangents to the smaller circle at X and Y respectively.

Prove that : AB = AC

5 **[a] In the opposite figure :**



Two circles are touching externally at C

, $\overline{AD}$ is a tangent-segment to the smaller circle at D

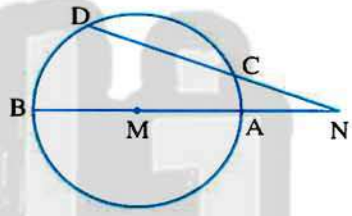
, $\overline{AB}$ is a tangent-segment to the greater circle at B

If AD = (y – 2) cm.

, AC = (2 x – 3) cm. , AB = 15 cm.

Find with proof : The value of each of x and y

[b] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{BA} \cap \overrightarrow{DC} = \{N\}$

Prove that : NB > ND

10 Suez Governorate

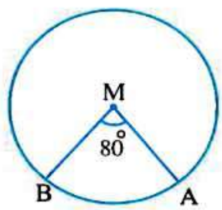
Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

1 The inscribed angle drawn in a semicircle is ....

(a) reflex. (b) right. (c) obtuse. (d) acute.

2 **In the opposite figure :**



If M is a circle , m (∠ AMB) = 80°

then m ($\widehat{AB}$) = ...°

(a) 40 (b) 80

(c) 160 (d) 90

3 If the two circles M , N are touching externally , the length of the radius of one of them is 3 cm. , MN = 8 cm. , then the length of the radius of the other circle is .... cm.

(a) 5 (b) 6 (c) 11 (d) 16

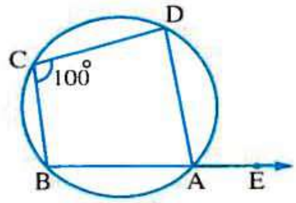
4 **In the opposite figure :**

$E \in \overrightarrow{BA}$, m (∠ C) = 100°

, then m (∠ DAE) = ... °

(a) 80 (b) 60

(c) 100 (d) 200



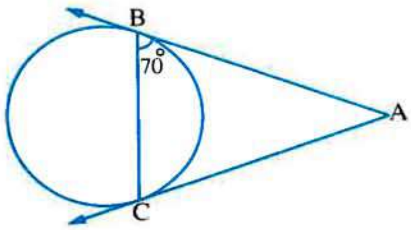
5 **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C , m (∠ ABC) = 70°

, then m (∠ A) = ... °

(a) 80 (b) 70

(c) 60 (d) 40



6 The area of the circle = ...

(a) 2 π r (b) π r^2 (c) 2 π r^2 (d) π r

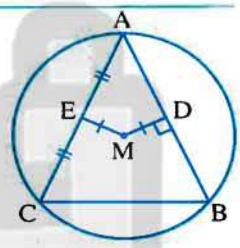
2 **[a] In the opposite figure :**

If M is a circle , $\overline{MD} \perp \overline{AB}$

, E is the midpoint of $\overline{AC}$

, MD = ME

, **prove that :** AB = AC

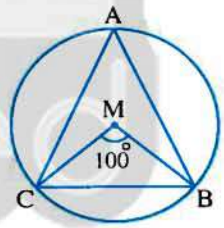


[b] In the opposite figure :

If M is a circle , m (∠ BMC) = 100°

, **find :** **1** m (∠ A)

2 m (∠ MBC)



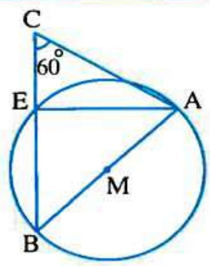
3 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M

, $C \in \overrightarrow{BE}$, m (∠ ACE) = 60°

Find : **1** m (∠ AEB)

2 m (∠ CAE)



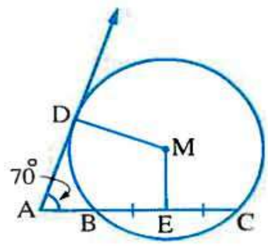
[b] In the opposite figure :

$\overrightarrow{AD}$ is a tangent to the circle M

, $\overrightarrow{AC}$ intersects the circle M at B , C

, E is the midpoint of $\overline{BC}$, m (∠ A) = 70°

Find : m (∠ DME)



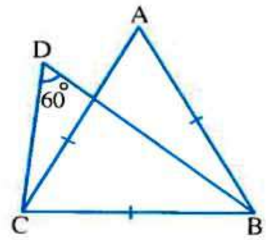
4 **[a]** State two cases of the cyclic quadrilateral.

[b] In the opposite figure :

ABC is an equilateral triangle

, m (∠ D) = 60°

Prove that : ABCD is a cyclic quadrilateral.

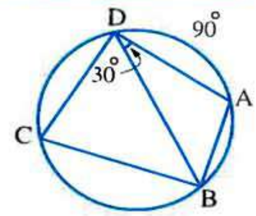


5 **[a] In the opposite figure :**

In the circle , m (∠ ADB) = 30°

, m $(\overset{\frown}{AD})$ = 90°

Find : **1** m $(\overset{\frown}{AB})$ **2** m (∠ DCB)



[b] In the opposite figure :

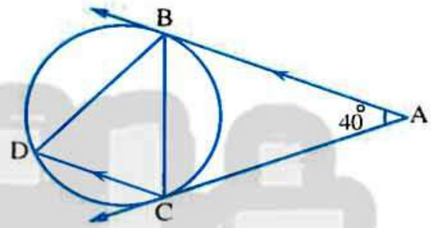
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents

to the circle at B and C

, $\overrightarrow{AB}$ // $\overline{CD}$, m (∠ A) = 40°

1 **Find :** m (∠ ABC)

2 **Prove that :** BC = BD



11 Port Said Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 M and N are two intersecting circles. The two radii lengths are 3 cm. and 5 cm. respectively , then MN ∈ ....

(a)]8 , ∞[(b)]2 , ∞[(c)]0 , 2[(d)]2 , 8[

2 If the straight line L is a tangent to the circle M of diameter length 10 cm. , then the distance between L and the center of the circle equals .... cm.

(a) 3 (b) 4 (c) 5 (d) 10

3 The longest chord in the circle is called a ....

(a) chord. (b) diameter. (c) tangent. (d) radius.

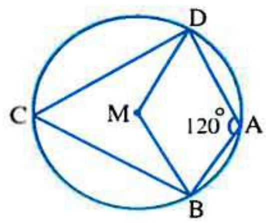
4 **In the opposite figure :**

If m (∠ A) = 120°

, then m (∠ DMB) = ...

(a) 180° (b) 120°

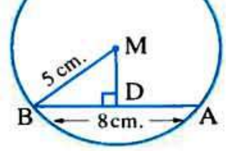
(c) 90° (d) 60°



5 The ratio between the measure of the central angle and the measure of the inscribed angle subtended by the same arc is ……………

(a) 4 : 2 (b) 2 : 4 (c) 3 : 2 (d) 2 : 3

6 **In the opposite figure :**



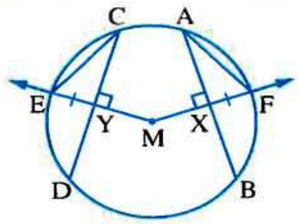
AB = 8 cm. , MB = 5 cm.

, then MD = ……………

(a) 5 cm. (b) 3 cm.

(c) 4 cm. (d) 2 cm.

2 [a] **In the opposite figure :**



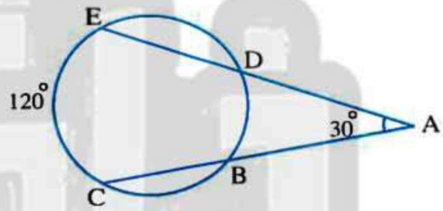
$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle at F

, $\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E , FX = EY

Prove that : 1 AB = CD 2 AF = CE

[b] **In the opposite figure :**



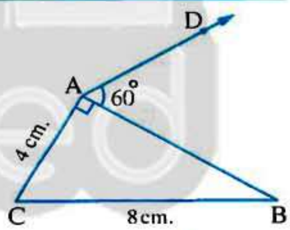
$\overrightarrow{ED} \cap \overrightarrow{CB} = \{A\}$

, m $(\overset{\frown}{CE})$ = 120°

, m (∠ A) = 30°

Find : m $(\overset{\frown}{BD})$

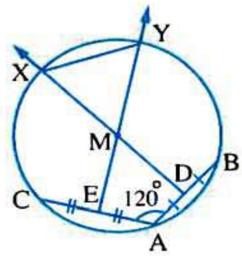
3 [a] **Using the given data , prove that :**



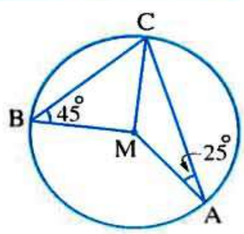
$\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC

[b] **Using the given data , prove that :**

The triangle XYM is an equilateral triangle.



4 [a] **In the opposite figure :**



A circle with center M

, m (∠ MAC) = 25°

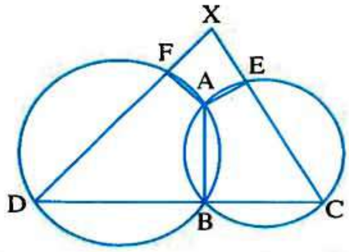
, m (∠ MBC) = 45°

Find : m (∠ AMB)

[b] In the opposite figure :

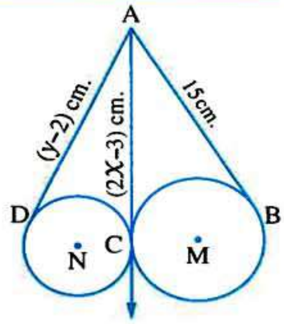
Two intersecting circles at A and B , $\overline{CD}$ passes through the point B and intersects the two circles at C and D , $\overrightarrow{CE} \cap \overrightarrow{DF} = \{X\}$

Prove that : The figure AFXE is a cyclic quadrilateral.



5 **[a] Using the given data , find :**

The values of the symbols x and y

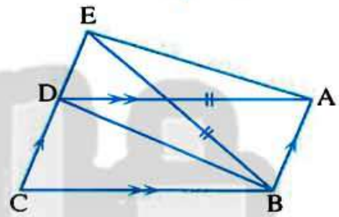


[b] In the opposite figure :

ABCD is a parallelogram

, $E \in \overrightarrow{CD}$ where BE = AD

Prove that : The figure ABDE is a cyclic quadrilateral.



12 Damietta Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer :**

1 The length of the projection of a line segment on a given straight line .... the length of the line segment.

(a) > (b) ≤ (c) ≥ (d) <

2 The number of symmetry axes of any circle is ....

(a) zero (b) 1 (c) 2 (d) an infinite number

3 If a square is of side length 6 cm. , then the square of its diagonal length is .... cm^2

(a) 36 (b) 12 (c) 72 (d) $6\sqrt{2}$

4 If the straight line L is a tangent to the circle M of diameter length 10 cm. , then the distance between L and the center of the circle equals .... cm.

(a) 3 (b) 5 (c) 6 (d) 10

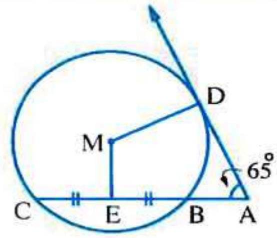
5 If M , N are two touching circles internally , their radii lengths are 7 cm. , 10 cm. , then MN = ... cm.

(a) 3 (b) 17 (c) 7 (d) 10

6 If $\Delta XYZ \sim \Delta ABC$, m ($\angle$ Y) = 60° and m ($\angle$ C) = 40° , then m ($\angle$ X) = ...°

(a) 40 (b) 80 (c) 100 (d) 120

2 [a] **In the opposite figure :**



$\overrightarrow{AD}$ is a tangent to the circle M , $\overline{AC}$ intersects the circle at B , C

, E is the midpoint of $\overline{BC}$

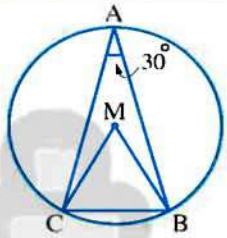
, m ($\angle$ A) = 65°

Find : m ($\angle$ DME)

[b] If the length of $\overline{AB}$ = 6 cm. , draw a circle of radius length 4 cm. that passes through A , B

How many circles can be drawn ? (Don't remove the arcs).

3 [a] **In the opposite figure :**

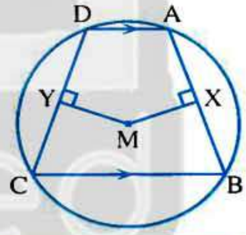


A circle M , m ($\angle$ A) = 30°

1 **Find :** m ($\angle$ BMC)

2 **Prove that :** MBC is an equilateral triangle.

[b] **In the opposite figure :**



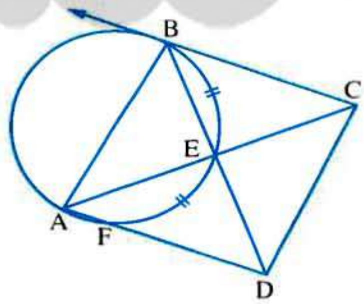
A circle M , $\overline{AD}$ // $\overline{BC}$

, $\overline{MX} \perp \overline{AB}$

, $\overline{MY} \perp \overline{DC}$

Prove that : MX = MY

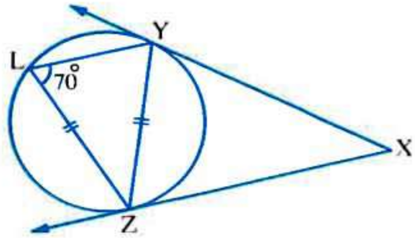
4 [a] **In the opposite figure :**



$\overrightarrow{CB}$ is a tangent , m ($\overset{\frown}{BE}$) = m ($\overset{\frown}{EF}$)

Prove that : ABCD is a cyclic quadrilateral.

[b] **In the opposite figure :**



$\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle at Y , Z

, YZ = LZ , m ($\angle$ L) = 70°

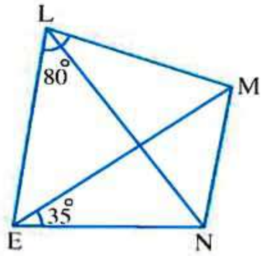
1 **Find with proof :** m ($\angle$ X)

2 **Prove that :** $\overline{XZ}$ // $\overline{YL}$

5 **[a]** ABCD is a parallelogram in which AC = BC

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle circumscribed about the triangle ABC

[b] In the opposite figure :



LMNE is a cyclic quadrilateral , m (∠ MEN) = 35°

, m (∠ MLE) = 80°

Find with proof :

1 m (∠ MLN)

2 m (∠ EMN)

13 Kafr El-Sheikh Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

1 The triangle contains two ·············· angles at least.

(a) acute (b) obtuse (c) right (d) reflex

2 ABCD is a rhombus in which m (∠ ACB) = 32° , then m (∠ D) = ··············

(a) 32° (b) 64° (c) 116° (d) 26°

3 A tangent to a circle of diameter length 6 cm. is at a distance of ·············· cm. from its center.

(a) 6 (b) 12 (c) 3 (d) 2

4 If M , N are two touching circles internally their radii lengths are 8 cm. , 3 cm. , then MN = ·············· cm.

(a) 3 (b) 5 (c) 7 (d) 11

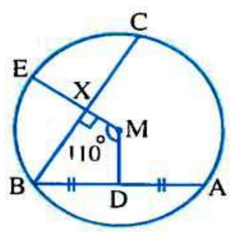
5 The triangle whose side lengths are 5 cm. , 7 cm. and 8 cm. is ·············· triangle.

(a) obtuse-angled. (b) acute-angled. (c) right-angled. (d) equilateral.

6 The number of common tangents to two touching circles externally is ··············

(a) 0 (b) 1 (c) 2 (d) 3

2 **[a] In the opposite figure :**



$\overline{AB}$ and $\overline{BC}$ are two chords in the circle M

, which has radius length of 10 cm.

, $\overrightarrow{MX} \perp \overline{BC}$ intersecting $\overline{BC}$ at X and intersecting the circle M at E

, D is the midpoint of $\overline{AB}$, BC = 16 cm.

, m (∠ DMX) = 110°

Find : 1 The length of $\overline{XE}$ 2 m (∠ ABC)

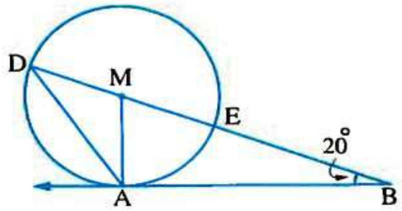
[b] In the opposite figure :

B is a point outside the circle M

, $\overrightarrow{BA}$ is a tangent to the circle M at A

, $\overrightarrow{BM}$ intersects the circle at E and D , m (∠ B) = 20°

Find with proof : m (∠ ADB)

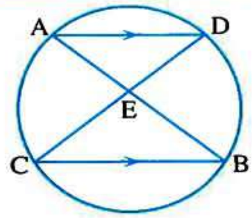


3 **[a] In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{E\}$

, $\overline{AD} \parallel \overline{CB}$

Prove that : EA = ED



[b] In the opposite figure :

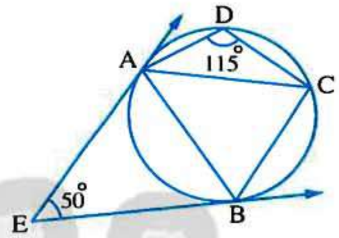
$\overrightarrow{EA}$ and $\overrightarrow{EB}$ are two tangents to the circle at A , B

, m (∠ AEB) = 50°

, m (∠ ADC) = 115°

Prove that :

$\overleftrightarrow{AC}$ is a tangent to the circle passing through the points A , B and E

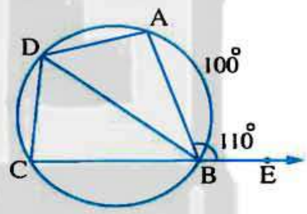


4 **[a] In the opposite figure :**

$E \in \overrightarrow{CB}$, m $(\widehat{AB})$ = 100°

, m (∠ ABE) = 110°

Find with proof : m (∠ BDC)



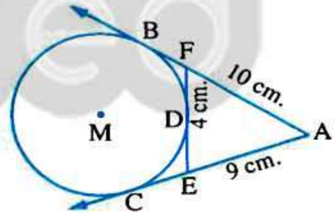
[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M at B , C

, $\overline{FE}$ is a tangent-segment at D , DF = 4 cm.

, AF = 10 cm. , AE = 9 cm.

Find with proof : The length of $\overline{EC}$



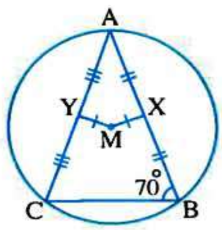
5 **[a] In the opposite figure :**

ABC is an inscribed triangle inside the circle M

, MX = MY , X and Y are the midpoints of $\overline{AB}$

, $\overline{AC}$ respectively , m (∠ B) = 70°

Find with proof : m (∠ A)



[b] ABC is an inscribed triangle in a circle where AB > AC and $D \in \overline{AB}$ where AC = AD , $\overrightarrow{AE}$ bisects ∠ A and intersects $\overline{BC}$ at E and intersects the circle at F

Prove that : BDEF is a cyclic quadrilateral.

14 El-Beheira Governorate

Answer the following questions : ***(Calculator is permitted)***

1 Choose the correct answer from the given ones :

1 M and N are two intersecting circles , their radii lengths are 3 cm. and 5 cm. , then MN $\in$ ....

(a)]8 , ∞[(b)]2 , ∞[(c)]0 , 2[(d)]2 , 8[

2 ABCD is a cyclic quadrilateral , m (∠ A) = 70° , then m (∠ C) equals ....

(a) 25° (b) 20° (c) 110° (d) 100°

3 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 130° (b) 90° (c) 50° (d) 180°

4 The slope of the straight line $3x + 2y = 1$ is ....

(a) $\frac{2}{3}$ (b) $-\frac{3}{2}$ (c) $-\frac{2}{3}$ (d) $\frac{3}{2}$

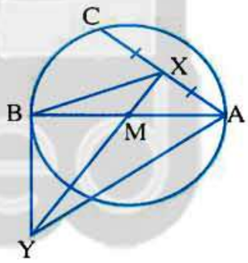
5 The measurement of any angle of the regular hexagon is ....

(a) 90° (b) 108° (c) 120° (d) 135°

6 In Δ ABC , if $(AB)^2 = (AC)^2 + (BC)^2$, then ∠ B is ....

(a) acute. (b) obtuse. (c) right. (d) reflex.

2 [a] In the opposite figure :

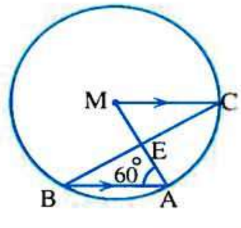


$\overline{AB}$ is a diameter in the circle M

, X is the midpoint of $\overline{AC}$ and $\overrightarrow{XM}$ intersects the tangent to the circle at B in Y

Prove that : The figure AXBY is a cyclic quadrilateral.

[b] In the opposite figure :



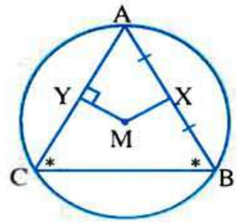
$\overline{AB}$ is a chord in the circle M

, $\overline{CM}$ // $\overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$

, m (∠ A) = 60°

Find : m (∠ B)

3 [a] In the opposite figure :



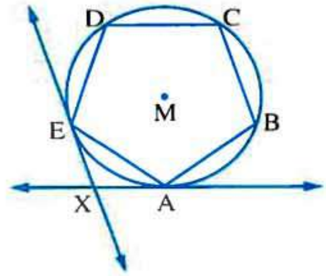
The triangle ABC is inscribed in the circle M

, in which : m (∠ B) = m (∠ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

[b] In the opposite figure :



ABCDE is a regular pentagon inscribed in a circle M

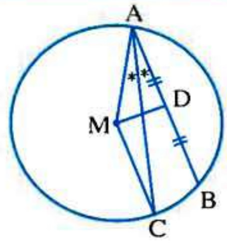
, $\overleftrightarrow{AX}$ is a tangent to the circle at A

, $\overleftrightarrow{EX}$ is a tangent to the circle at E

where $\overleftrightarrow{AX} \cap \overleftrightarrow{EX} = \{X\}$

Find : 1 m $(\widehat{AE})$ 2 m $(\angle AXE)$

4 **[a] In the opposite figure :**



$\overline{AB}$ is a chord in the circle M

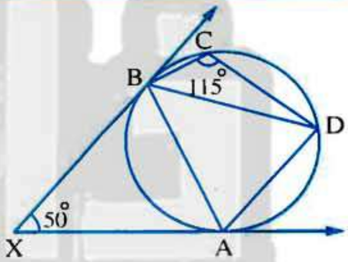
, $\overrightarrow{AC}$ bisects $\angle$ BAM and intersects the circle M at C

If D is the midpoint of $\overline{AB}$

, prove that : $\overline{DM} \perp \overline{CM}$

[b] $\overline{AB}$ is a diameter in the circle M , $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are two tangents to the circle M , $\overrightarrow{CM}$ intersects the circle M at X and Y and intersects $\overleftrightarrow{BD}$ at E **Prove that :** CX = YE

5 **[a] In the opposite figure :**



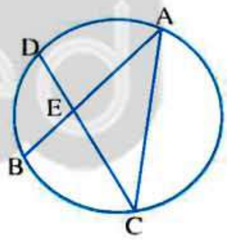
$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

, m $(\angle AXB) = 50°$, m $(\angle DCB) = 115°$

Prove that : 1 $\overrightarrow{AB}$ bisects $\angle$ DAX

2 BD = BA

[b] In the opposite figure :



$\overline{AB}$ and $\overline{CD}$ are two equal chords in length in the circle

, $\overline{AB} \cap \overline{CD} = \{E\}$

Prove that : The triangle ACE is an isosceles triangle.

15 El-Fayoum Governorate

Answer the following questions : ***(Using calculators is allowed)***

1 **Choose the correct answer :**

1 If M is a circle of diameter length 8 cm. , the straight line L is far from the centre M of the circle by 4 cm. , then the straight line L is ....

(a) a secant to the circle in two points. (b) outside the circle.

(c) a tangent to the circle. (d) an axis of symmetry of the circle.

2 If m_1 , m_2 are the slopes of two perpendicular straight lines , then ....

(a) $m_1 = m_2$ (b) $m_1 \times m_2 = -1$ (c) $m_1 \times m_2 = 1$ (d) $m_1 + m_2 = -1$

3 The centre of the circle that passes through the vertices of the triangle is the intersection point of ....

(a) the bisectors of its interior angles. (b) the bisectors of its exterior angles.

(c) its altitudes. (d) the axes of its sides.

4 ABC is a right-angled triangle at B , m ($\angle$ C) = 30° , AC = 12 cm.

, then AB = ... cm.

(a) 24 (b) $12\sqrt{3}$ (c) $6\sqrt{3}$ (d) 6

5 Which of the following figures is a cyclic quadrilateral ?

(a) The rectangle. (b) The trapezium. (c) The rhombus. (d) The parallelogram.

6 A trapezium in which the lengths of the two parallel bases are 4 cm. and 12 cm. and its height is 9 cm. , then its area = ... cm^2.

(a) 25 (b) 36 (c) 72 (d) 144

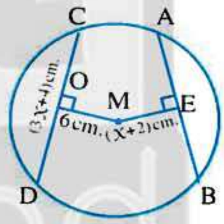
2 [a] **In the opposite figure :**

AB = CD , MO = 6 cm.

, ME = ($\mathcal{X}$ + 2) cm.

, CD = (3 $\mathcal{X}$ + 4) cm.

Find : The value of $\mathcal{X}$, CD



[b] ABC is a triangle drawn inside a circle M , m ($\angle$ AMB) = 90° , m ($\angle$ BMC) = 130°

Find : The measures of the angles of Δ ABC

3 [a] A is a point outside the circle M , $\overrightarrow{AB}$ is a tangent to the circle at B , $\overrightarrow{AM}$ intersects the circle M at C and D respectively , m ($\angle$ A) = 40°

Find with proof : m ($\angle$ BDC)

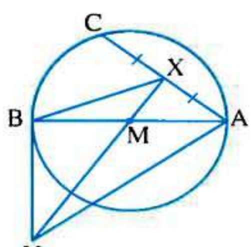
[b] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

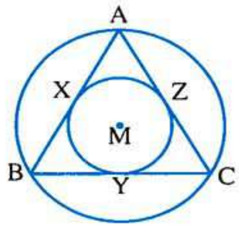
, X is the midpoint of $\overline{AC}$

and $\overrightarrow{XM}$ intersects the tangent to the circle at B at Y

Prove that : The figure AXBY is a cyclic quadrilateral.



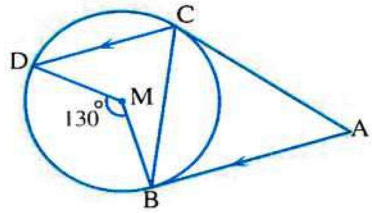
4 **[a] In the opposite figure :**



Two concentric circles with centre M
, the radii lengths of them are 4 cm. and 2 cm.
, Δ ABC is an inscribed triangle inside the greater circle
, and its sides touch the smaller circle at X , Y , Z

Prove that : Δ ABC is an equilateral triangle , and calculate its area.

[b] In the opposite figure :



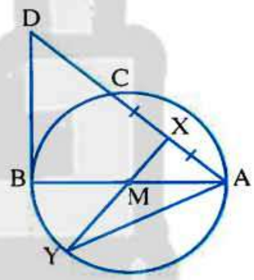
$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M
, $\overline{AB} \mathbin{/\mkern-6mu/} \overline{CD}$
, m ($\angle$ BMD) = 130°

Prove that : $\overrightarrow{CB}$ bisects $\angle$ ACD

5 **[a]** Δ ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A , X $\in \overline{AB}$ and Y $\in \overline{AC}$, where $\overline{XY} \mathbin{/\mkern-6mu/} \overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

[b] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M ,
X is the midpoint of $\overline{AC}$, $\overrightarrow{BD}$ is a tangent to the circle at B , $\overrightarrow{XM}$ intersects the circle at Y

Prove that : **1** XMBD is a cyclic quadrilateral.

2 m ($\angle$ BAY) = $\frac{1}{2}$ m ($\angle$ D)

16 Beni Suef Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The symmetry axis of the common chord $\overline{AB}$ of the two intersecting circles M , N is ....

(a) $\overleftrightarrow{MA}$ (b) $\overleftrightarrow{MB}$ (c) $\overleftrightarrow{MN}$ (d) $\overleftrightarrow{NA}$

2 ABC is a triangle in which : $(AC)^2 > (AB)^2 + (BC)^2$, then $\angle$ B is ....

(a) acute. (b) obtuse. (c) right. (d) straight.

3 In the cyclic quadrilateral , each two opposite angles are ....

(a) equal in measure. (b) complementary.
(c) supplementary. (d) alternate.

4 The area of a triangle is 35 cm^2 and its height is 7 cm. , then the length of its base equals .... cm.

(a) 5 (b) 7 (c) 10 (d) 20

5 The measure of the inscribed angle which is drawn in a semicircle equals ....

(a) 45° (b) 90° (c) 120° (d) 180°

6 The area of a square is 100 cm^2 , then its perimeter = ... cm.

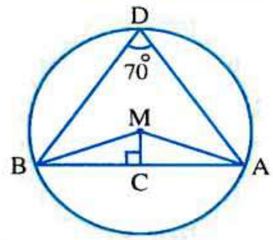
(a) 10 (b) 30 (c) 40 (d) 50

2 **[a] In the opposite figure :**

$\overline{AB}$ is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$, m ($\angle$ ADB) = 70°

Find : m ($\angle$ AMC)

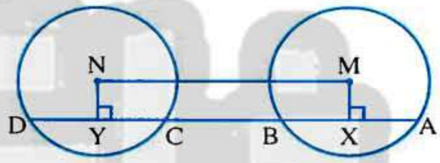


[b] In the opposite figure :

M and N are two congruent circles

, AB = CD , $\overline{MX} \perp \overline{AB}$ and $\overline{NY} \perp \overline{CD}$

Prove that : The figure MXYN is a rectangle.



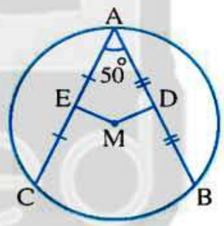
3 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords

in the circle M , D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$ and m ($\angle$ BAC) = 50°

Find : m ($\angle$ DME)



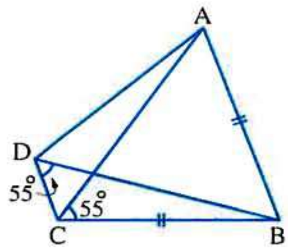
[b] In the opposite figure :

AB = BC

, m ($\angle$ ACB) = 55°

and m ($\angle$ BDC) = 55°

Prove that : The figure ABCD is a cyclic quadrilateral.

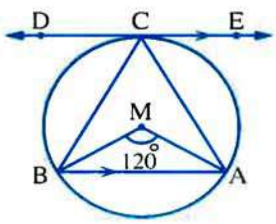


4 **[a] In the opposite figure :**

$\overleftrightarrow{ED}$ is a tangent to the circle M at C

, $\overleftrightarrow{ED}$ // $\overline{AB}$ and m ($\angle$ AMB) = 120°

Prove that : The triangle CAB is an equilateral triangle.

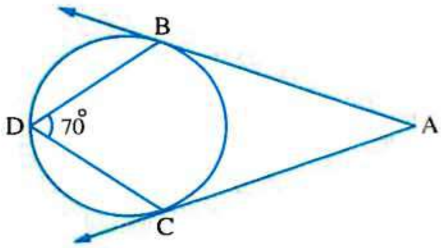


[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m ($\angle$ BDC) = 70°

Find : m ($\angle$ A)

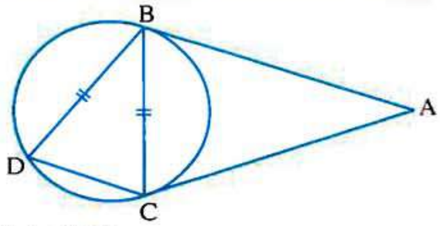


5 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C , BC = BD

Prove that :

$\overleftrightarrow{BD}$ is a tangent to the circle passing through the vertices of Δ ABC

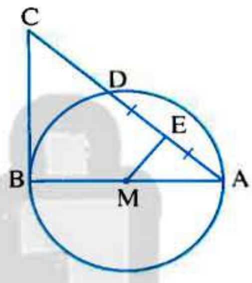


[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overleftrightarrow{BC}$ is a tangent to the circle at B and E is the midpoint of $\overline{AD}$

Prove that : The figure EMBC is a cyclic quadrilateral.



17 El-Menia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 It is possible to draw a circle passing through the vertices of a ....

(a) rhombus. (b) rectangle. (c) right trapezium. (d) parallelogram.

2 The inscribed angle drawn in a semicircle is ....

(a) acute. (b) obtuse. (c) straight. (d) right.

3 The number of rectangles in the opposite figure is ....

(a) 3 (b) 6 (c) 7 (d) 10

4 If the perimeter of a square is 20 cm. , then its surface area is .... cm^2.

(a) 20 (b) 25 (c) 50 (d) 100

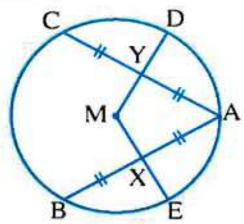
5 The measure of the exterior angle of an equilateral triangle equals .... °

(a) 60 (b) 108 (c) 120 (d) 135

6 If ABCD is a cyclic quadrilateral , 2 m (∠ A) = 120° , then m (∠ C) = …………… °

(a) 120 (b) 45 (c) 60 (d) 90

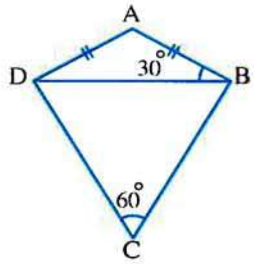
2 [a] **In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$ and Y is the midpoint of $\overline{AC}$

Prove that : XE = YD

[b] **In the opposite figure :**



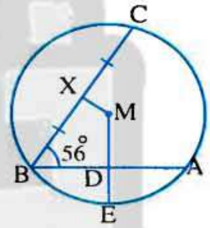
ABCD is a quadrilateral , AB = AD

, m (∠ ABD) = 30°

, m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.

3 [a] **In the opposite figure :**



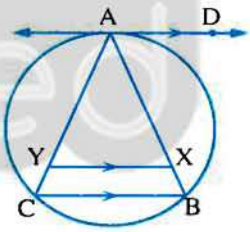
$\overline{AB}$ and $\overline{BC}$ are two chords in the circle M which has radius length of 5 cm. , $\overline{MD} \perp \overline{AB}$

, X is the midpoint of $\overline{BC}$

, AB = 8 cm. , m (∠ B) = 56°

Find : 1 m (∠ DMX) 2 The length of $\overline{DE}$

[b] **In the opposite figure :**



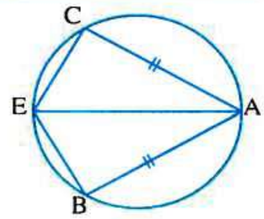
$\overleftrightarrow{AD}$ is a tangent to the circle at A

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

4 [a] **In the opposite figure :**

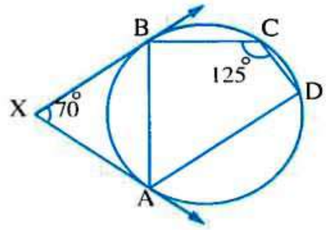


AB = AC

, E ∈ $\overset{\frown}{BC}$

Prove that : m (∠ AEB) = m (∠ AEC)

[b] **In the opposite figure :**



$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

, m (∠ AXB) = 70°

, m (∠ DCB) = 125°

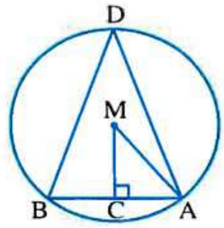
Prove that : m (∠ DAB) = m (∠ XAB)

5 **[a] In the opposite figure :**

$\overline{AB}$ is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$

Prove that : m (∠ AMC) = m (∠ ADB)



[b] ABC is an inscribed triangle in a circle M where AB > AC and D ∈ $\overline{AB}$ where AC = AD , $\overrightarrow{AE}$ bisects ∠ A and intersects $\overline{BC}$ at E and intersects the circle at F

Prove that : BDEF is a cyclic quadrilateral.

18 Assiut Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **Choose the correct answer :**

1. XYZ is a triangle in which : D is the midpoint of $\overline{XY}$, E is the midpoint of $\overline{XZ}$, then DE = ... YZ

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 2

2. The diameter is a .... passing through the center of the circle.

(a) straight line (b) ray (c) tangent (d) chord

3. If the circumference of a circle is 18 π cm. , then its radius length = ... cm.

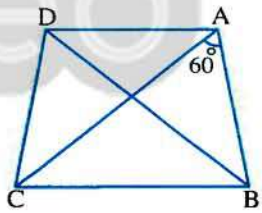
(a) 7 (b) 9 (c) 3 (d) 6

4. **In the opposite figure :**

ABCD is a cyclic quadrilateral

, m (∠ BAC) = 60°

, then m (∠ BDC) = ...



(a) 300° (b) 120°

(c) 60° (d) 30°

5. The area of the triangle which the length of its base is 9 cm. , its height is 12 cm. equals .... cm^2.

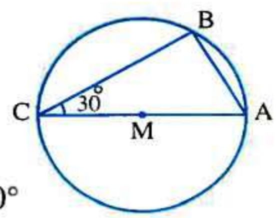
(a) 48 (b) 24 (c) 36 (d) 54

6. **In the opposite figure :**

$\overline{AC}$ is a diameter in the circle M

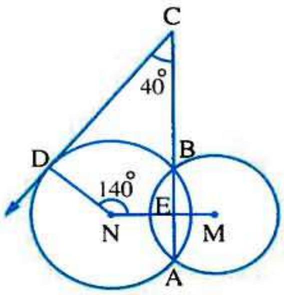
, m (∠ C) = 30°

, then m (∠ A) = ...



(a) 120° (b) 60° (c) 90° (d) 40°

2 [a] **In the opposite figure :**



M and N are two intersecting circles at A and B

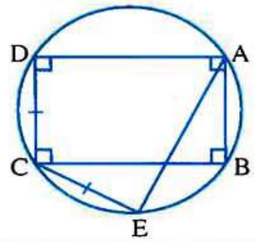
, $C \in \overrightarrow{AB}$, $\overline{AC} \cap \overline{MN} = \{E\}$

, D $\in$ the circle N , m ($\angle$ DNM) = 140°

and m ($\angle$ C) = 40°

Prove that : $\overrightarrow{CD}$ is a tangent to the circle N at D

[b] **In the opposite figure :**



ABCD is a rectangle inscribed in a circle

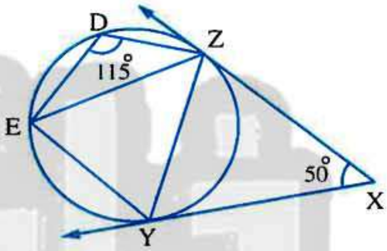
, the chord $\overline{CE}$ is drawn

where CE = CD

Prove that : AE = BC

3 [a] State two cases of the cyclic quadrilateral.

[b] **In the opposite figure :**



$\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle at Y , Z

, m ($\angle$ D) = 115°

and m ($\angle$ X) = 50°

Prove that : ZE = ZY

4 [a] **In the opposite figure :**

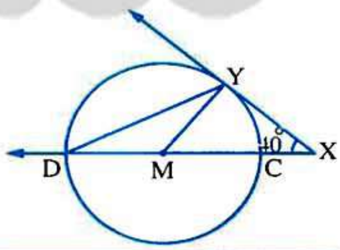
ABC is a triangle inscribed in the circle M

, in which m ($\angle$ B) = m ($\angle$ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

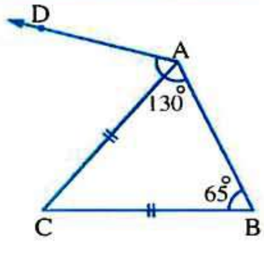
[b] **In the opposite figure :**



X is a point outside the circle M , $\overrightarrow{XY}$ is a tangent to the circle at Y , $\overrightarrow{XM}$ intersects the circle M at C and D respectively , m ($\angle$ X) = 40°

Find : m ($\angle$ YDC)

5 [a] **In the opposite figure :**



ABC is a triangle , CB = AC

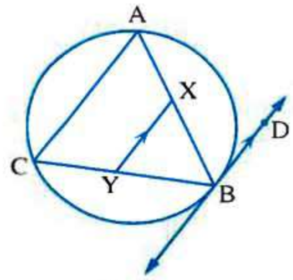
, m ($\angle$ DAB) = 130°

, m ($\angle$ B) = 65°

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC

[b] In the opposite figure :



ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, $X \in \overline{AB}$, $Y \in \overline{BC}$

where $\overline{XY} // \overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.

19 Souhag Governorate

Answer the following questions : ***(Calculator is permitted)***

1 Choose the correct answer :

1 If the straight line L is a tangent to the circle M of diameter length 8 cm. , then the distance between L and the center of the circle equals .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

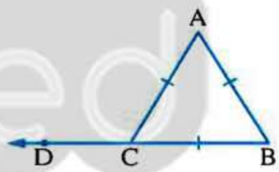
2 The area of the rhombus = ... of the product of the lengths of its diagonals.

(a) 2 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 3

3 The number of symmetry axes of any circle is ....

(a) zero (b) 1 (c) 2 (d) an infinite number.

4 **In the opposite figure :**



The triangle ABC is an equilateral triangle

, then m (∠ ACD) = ...°

(a) 45 (b) 60

(c) 120 (d) 135

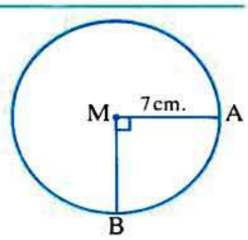
5 If the lengths of two sides of an isosceles triangle are 2 cm. and $(x + 3)$ cm. , and the length of the third side is 5 cm. , then x = ... cm.

(a) 1 (b) 2 (c) 3 (d) 4

6 If M , N are two touching circles internally , their radii lengths are 5 cm. , 9 cm. , then MN = ... cm.

(a) 14 (b) 4 (c) 5 (d) 9

2 [a] In the opposite figure :



M is a circle with radius length 7 cm.

, m (∠ AMB) = 90°

Find : The length of $\widehat{AB}$ $\left(\pi = \frac{22}{7}\right)$

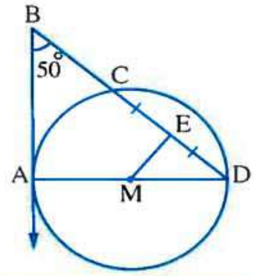
[b] In the opposite figure :

$\overline{AD}$ is a diameter in the circle M

, $\overleftrightarrow{AB}$ is a tangent , m ($\angle$ B) = 50°

, E is the midpoint of $\overline{DC}$

Find : m ($\angle$ EMA)



3 [a] State two cases of the cyclic quadrilateral.

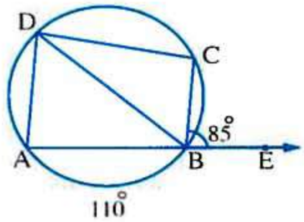
[b] In the opposite figure :

$E \in \overrightarrow{AB}$, $E \notin \overline{AB}$

, m ($\overset{\frown}{AB}$) = 110°

, m ($\angle$ CBE) = 85°

Find : m ($\angle$ BDC)

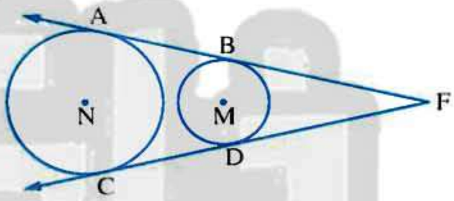


4 **[a] In the opposite figure :**

$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ are common external tangents

to the two circles M and N , $\overrightarrow{AB} \cap \overrightarrow{CD} = \{F\}$

Prove that : AB = CD



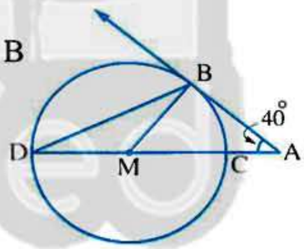
[b] In the opposite figure :

A is a point outside the circle M , $\overrightarrow{AB}$ is a tangent to the circle at B

, $\overrightarrow{AM}$ intersects the circle M at C and D respectively

, m ($\angle$ A) = 40°

Find with proof : m ($\angle$ BDC)

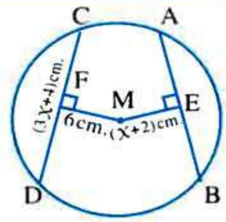


5 **[a] In the opposite figure :**

AB = CD

Find : **1** The value of x

2 The length of $\overline{CD}$



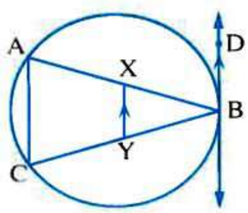
[b] In the opposite figure :

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B , $X \in \overline{AB}$

, $Y \in \overline{CB}$ where $\overline{YX} \parallel \overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



20 Qena Governorate

Answer the following questions : ***(Calculators are permitted)***

1 Choose the correct answer :

1 The measure of the inscribed angle in a semicircle is .... °

(a) 45 (b) 90 (c) 135 (d) 180

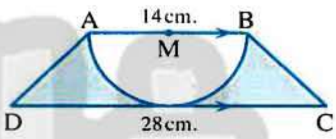
2 The perimeter of a rhombus is 12 cm. , then the length of its side = ... cm.

(a) 3 (b) 4 (c) 6 (d) 8

3 If A and B are two points in the plane , AB = 7 cm. , then the length of the diameter of the smallest circle passing through the two points A and B equals .... cm.

(a) 3 (b) 3.5 (c) 7 (d) 14

4 **In the opposite figure :**



$\overline{AB}$ is a diameter of the circle M , $\overline{CD}$ is a tangent

, AB = 14 cm. , CD = 28 cm.

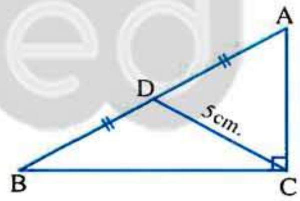
, then the area of the shaded part = ... cm^2.

(a) 70 (b) 147 (c) 170 (d) 224

5 It is possible to draw a circle passing through the vertices of a ....

(a) rhombus. (b) rectangle. (c) trapezium. (d) parallelogram.

6 **In the opposite figure :**



Δ ABC is right-angled at C

, $\overline{CD}$ is a median , CD = 5 cm.

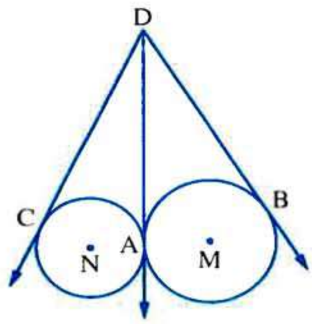
, then AB = ... cm.

(a) 4 (b) 6

(c) 8 (d) 10

2 **[a]** Find the length of the arc and its measure , which is opposite to an inscribed angle of measure 45° in a circle the length of its radius is 7 cm.

[b] In the opposite figure :



M and N are two circles touching externally at A

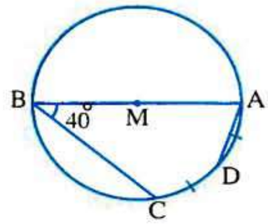
, $\overrightarrow{DA}$ is a common tangent to the circles

, $\overrightarrow{DB}$ is a tangent to the circle M at B

, $\overrightarrow{DC}$ is a tangent to the circle N at C

Prove that : DB = DC

3 **[a] In the opposite figure :**



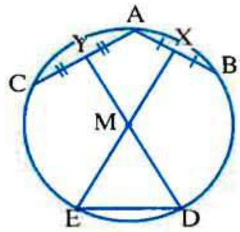
$\overline{AB}$ is a diameter of the circle M

, D is the midpoint of $\overset{\frown}{AC}$

, m ($\angle$ ABC) = 40°

Find : 1 m ($\angle$ DAB) 2 m ($\angle$ DCB)

[b] In the opposite figure :



$\overline{AB}$, $\overline{AC}$ are two chords in the circle M

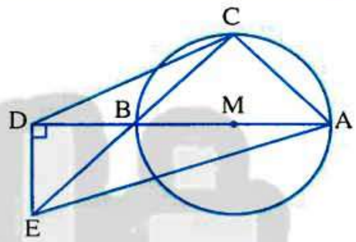
, X and Y are the two midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, $\overrightarrow{YM}$ and $\overrightarrow{XM}$ intersect the circle at D and E

If DE = r where r is the radius length of M

, **find by proof :** m ($\angle$ BAC)

4 **[a] In the opposite figure :**



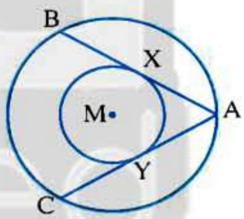
$\overline{AB}$ is a diameter in the circle M

, D $\in \overrightarrow{AB}$, D $\notin \overline{AB}$, $\overrightarrow{DE} \perp \overrightarrow{AB}$

, C $\in \overset{\frown}{AB}$, $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$

Prove that : ACDE is a cyclic quadrilateral

[b] In the opposite figure :

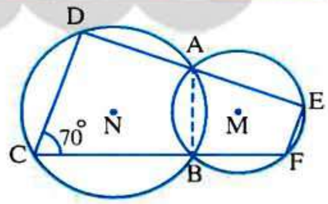


Two concentric circles of center M

, $\overline{AB}$ and $\overline{AC}$ are two chords in the greater circle and tangents to the smaller circle at X and Y respectively.

Prove that : AB = AC

5 **[a] In the opposite figure :**

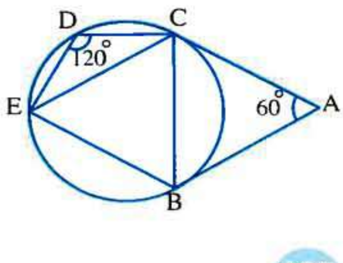


M and N are two intersecting circles at A and B

, $\overleftrightarrow{AD}$ is drawn to intersect the circle M at E and the circle N at D , $\overleftrightarrow{AB}$ is drawn to intersect the circle M at F and the circle N at C , m ($\angle$ BCD) = 70°

1 **Find :** m ($\angle$ EFB) 2 **Prove that :** $\overline{CD}$ // $\overline{EF}$

[b] In the opposite figure :



$\overline{AB}$ and $\overline{AC}$ are tangent-segments to the circle at B and C

, m ($\angle$ BAC) = 60° , m ($\angle$ CDE) = 120°

Prove that : 1 Δ BCE is an equilateral triangle.

2 $\overline{AC}$ // $\overline{BE}$

21 Luxor Governorate

محافظة الأقصر

Answer the following questions :

1 Choose the correct answer :

1 The number of axes of symmetry of the rectangle is ....

(a) 1 (b) 2 (c) 3 (d) 4

2 If M , N are two circles whose radii lengths are r_1 , r_2 and if $r_1 - r_2 < MN < r_1 + r_2$, then the two circles are ....

(a) distant. (b) concentric. (c) intersecting. (d) touching.

3 The length of the median drawn from the vertex of the right angle in the right-angled triangle equals .... the length of the hypotenuse.

(a) quarter (b) twice (c) half (d) three quarters

4 The length of the arc subtending a central angle of measure 60° in a circle whose circumference is 24 cm. equals .... cm.

(a) 4 (b) 8 (c) 12 (d) 16

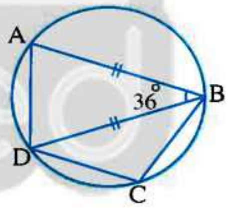
5 The measure of the exterior angle of the equilateral triangle is ....°

(a) 30 (b) 60 (c) 90 (d) 120

6 **In the opposite figure :**

AB = BD , m (∠ ABD) = 36°

, then m (∠ C) = ... °



(a) 140 (b) 108

(c) 70 (d) 54

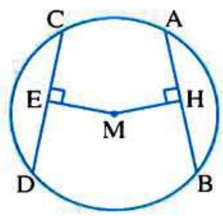
2 [a] **In the opposite figure :**

AB = CD , $\overline{MH} \perp \overline{AB}$, $\overline{ME} \perp \overline{CD}$

If ME = 6 cm. , MH = (x + 2) cm.

and CD = (3 x + 4) cm.

, **find :** The value of x and the length of $\overline{AB}$

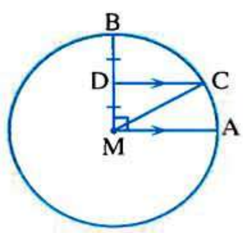


[b] **In the opposite figure :**

$\overline{AM} \parallel \overline{CD}$

, MD = DB , m (∠ AMB) = 90°

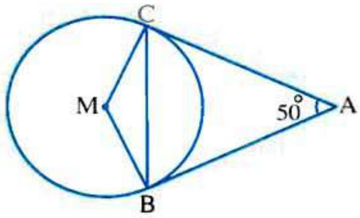
Find : m $(\widehat{AC})$



3 **[a] In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangent segments drawn to the circle from A at B , C respectively , m (∠ A) = 50°

Find : m (∠ ACB) , m (∠ BCM)



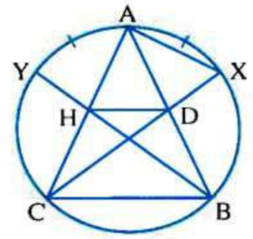
[b] In the opposite figure :

m ($\widehat{AX}$) = m ($\widehat{AY}$)

Prove that :

1 DBCH is a cyclic quadrilateral.

2 m (∠ DHB) = m (∠ XAB)



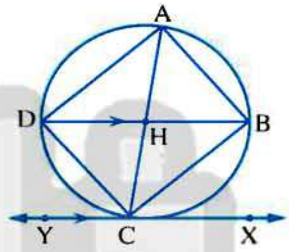
4 **[a]** Draw $\overline{AB}$ of length 3 cm. , then draw a circle passing by the two points A , B whose radius length is 2 cm. How many possible solutions are there ?

[b] In the opposite figure :

$\overline{BD}$ // $\overleftrightarrow{XY}$

Prove that : 1 $\overrightarrow{AC}$ bisects ∠ BAD

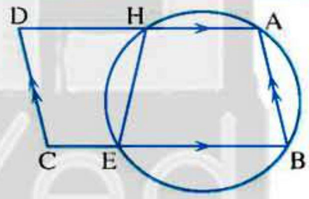
2 $\overleftrightarrow{BC}$ is a tangent to the circle passing by the vertices of Δ ABH



5 **[a] In the opposite figure :**

ABCD is a parallelogram

Prove that : HDCE is a cyclic quadrilateral

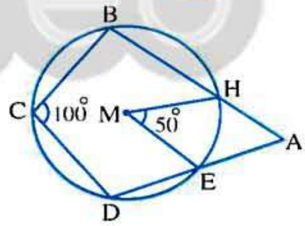


[b] In the opposite figure :

m (∠ M) = 50°

, m (∠ C) = 100°

Find : m (∠ A)



22 Aswan Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 45° (b) 180° (c) 120° (d) 90°

2 The number of symmetry axes of the isosceles triangle is ....

(a) zero (b) 1 (c) 2 (d) 3

3 The surface of the circle M $\cap$ the surface of the circle N = $\{A\}$ and the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle equals .... cm.

(a) 5 (b) 6 (c) 11 (d) 16

4 The measure of the exterior angle of the equilateral triangle equals ....

(a) 30° (b) 60° (c) 120° (d) 180°

5 The line segment joining the two midpoints of two sides of the triangle is .... the third side.

(a) perpendicular to (b) parallel to (c) bisecting (d) equal to

6 If ABCD is a cyclic quadrilateral , then m (∠ A) + m (∠ C) – 80° = ...

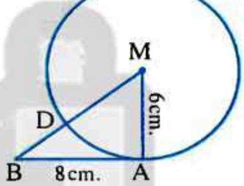
(a) 60° (b) 80° (c) 100° (d) 120°

2 **[a] In the opposite figure :**

$\overleftrightarrow{AB}$ is a tangent to the circle M at A

, MA = 6 cm. , AB = 8 cm.

Find : The length of $\overline{BD}$



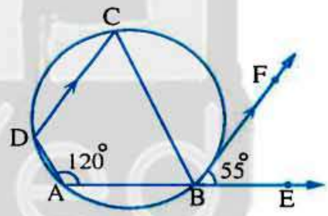
[b] In the opposite figure :

ABCD is a cyclic quadrilateral

, $\overrightarrow{BF}$ // $\overline{DC}$, m (∠ BAD) = 120°

, m (∠ EBF) = 55°

Find : m (∠ BCD) , m (∠ ADC)



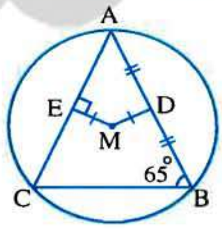
3 **[a] In the opposite figure :**

In the circle M

, MD = ME , D is the midpoint of $\overline{AB}$

, $\overline{ME} \perp \overline{AC}$, m (∠ ABC) = 65°

Find : m (∠ BAC)

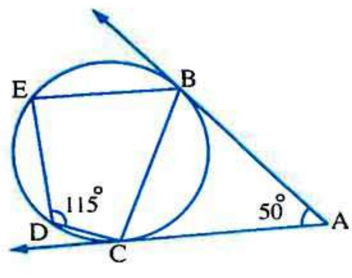


[b] In the opposite figure :

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 50° , m (∠ CDE) = 115°

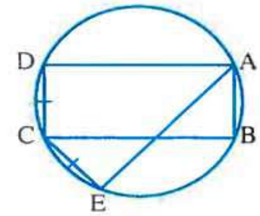
Prove that : $\overrightarrow{BC}$ bisects ∠ ABE



4 **[a] In the opposite figure :**

ABCD is a rectangle inscribed in a circle , the chord $\overline{CE}$ is drawn where CE = CD

Prove that : AE = BC



[b] In the opposite figure :

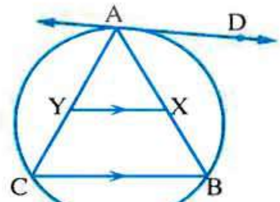
ABC is a triangle inscribed in a circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A

, $X \in \overline{AB}$, $Y \in \overline{AC}$, $\overline{XY} \parallel \overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ AXY

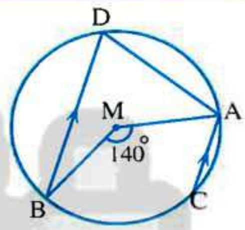


5 **[a] In the opposite figure :**

$\overline{AC}$, $\overline{DB}$ are two parallel chords in the circle M

, m ($\angle$ AMB) = 140°

Find : m ($\angle$ D) , m ($\angle$ DAC)

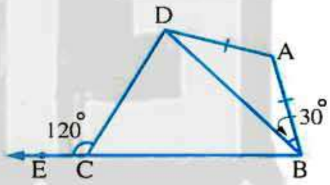


[b] In the opposite figure :

AB = AD , m ($\angle$ ABD) = 30°

, m ($\angle$ DCE) = 120°

Prove that : ABCD is a cyclic quadrilateral.



23 New Valley Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from those given :**

1 If two polygons are similar and the ratio between the lengths of two corresponding sides is 1 : 3 and the perimeter of the smaller polygon is 15 cm. , then the perimeter of the greater polygon is .... cm.

(a) 30 (b) 45 (c) 60 (d) 75

2 The inscribed angle drawn in a semicircle is ....

(a) acute. (b) obtuse. (c) straight. (d) right.

3 ABC is a right-angled triangle at B , $\overline{BD} \perp \overline{AC}$, then the projection of $\overline{BD}$ on $\overleftarrow{AC}$ is ....

(a) A (b) B (c) C (d) D

4 A tangent to a circle of diameter length 6 cm. is at a distance of .... cm. from its center.

(a) 6 (b) 12 (c) 3 (d) 2

5 **In the opposite figure :**

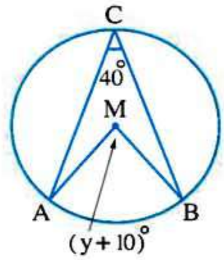
If m (∠ AMB) = (y + 10)°

, m (∠ C) = 40°

, then y = ...

(a) 70° (b) 80°

(c) 100° (d) 180°



6 **In the opposite figure :**

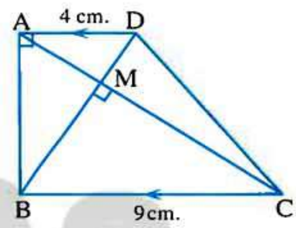
$\overline{AD}$ // $\overline{BC}$, m (∠ BAD) = m (∠ BMC) = 90°

, AD = 4 cm. , BC = 9 cm.

, then the area of the trapezium ABCD = ... cm^2

(a) 26 (b) 39

(c) 52 (d) 65

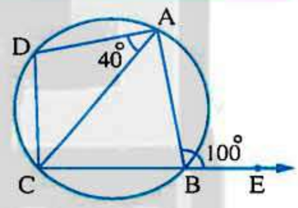


2 **[a] In the opposite figure :**

m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{AD})$



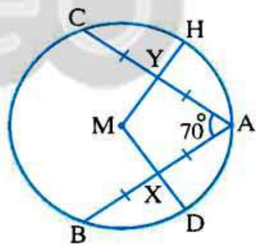
[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$, m (∠ CAB) = 70°

1 **Calculate :** m (∠ DMH)

2 **Prove that :** XD = YH



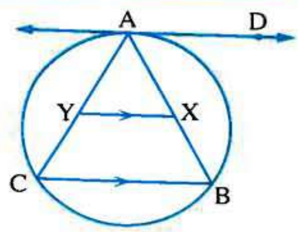
3 **[a] In the opposite figure :**

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y



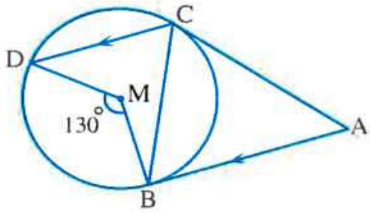
[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB} // \overline{CD}$, m ($\angle$ BMD) = 130°

1 **Prove that :** $\overrightarrow{CB}$ bisects $\angle$ ACD

2 **Find :** m ($\angle$ A)

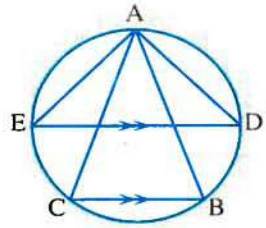


4 **[a] In the opposite figure :**

ABC is an inscribed triangle inside a circle

, $\overline{DE} // \overline{BC}$

Prove that : m ($\angle$ DAC) = m ($\angle$ BAE)



[b] ABC is a triangle inscribed in a circle , X $\in \widehat{AB}$, Y $\in \widehat{AC}$

where m ($\widehat{AX}$) = m ($\widehat{AY}$) , $\overline{CX} \cap \overline{AB} = \{D\}$, $\overline{BY} \cap \overline{AC} = \{E\}$

Prove that : 1 BCED is a cyclic quadrilateral.

2 m ($\angle$ DEB) = m ($\angle$ XAB)

5 **[a]** State two cases of the cyclic quadrilateral.

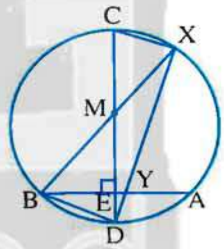
[b] In the opposite figure :

$\overline{AB}$ is a chord in the circle M and $\overline{CD}$ is the perpendicular diameter on $\overline{AB}$ and intersects it at E

, $\overrightarrow{BM}$ intersects the circle at X and $\overline{XD} \cap \overline{AB} = \{Y\}$

Prove that : 1 XYEC is a cyclic quadrilateral.

2 m ($\angle$ DYB) = m ($\angle$ DBX)



24 South Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 90° (b) 45° (c) 180° (d) 120°

2 A rhombus whose two diagonals lengths are 6 cm. , 8 cm. , then its area is .... cm^2

(a) 14 (b) 24 (c) 48 (d) 12

3 If ABCD is a cyclic quadrilateral , then m ($\angle$ A) + m ($\angle$ C) – 90° = ...

(a) 180° (b) 100° (c) 90° (d) 120°

4 In the triangle ABC , where $(AB)^2 + (BC)^2 < (AC)^2$, then $\angle$ B is ....

(a) right. (b) acute. (c) straight. (d) obtuse.

5 The sum of measures of the interior angles of the triangle equals ....

(a) 180° (b) 90° (c) 100° (d) 360°

6 The number of axes of symmery of the circle is ....

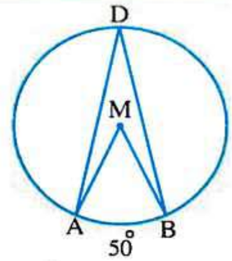
(a) zero (b) an inifinite number

(c) 2 (d) 3

2 **[a] In the opposite figure :**

$m(\widehat{AB}) = 50°$

Find : 1 m ($\angle$ D) 2 m ($\angle$ AMB)

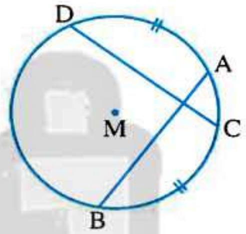


[b] In the opposite figure :

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $m(\widehat{AD}) = m(\widehat{BC})$

Prove that : AB = CD



3 **[a]** If the radius length of the circle M is 5 cm. and the radius length of the circle N is 3 cm. , MN = 8 cm. , show the position of the two circles.

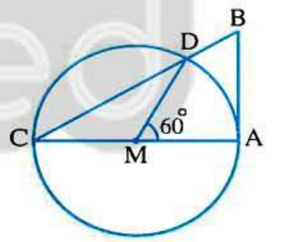
[b] In the opposite figure :

$\overline{AB}$ is a tangent-segment to the circle M

, $\overline{AC}$ is a diameter of it and m ($\angle$ AMD) = 60°

1 **Find :** m ($\angle$ ABC)

2 **Prove that :** $AB = \frac{1}{2}$ BC



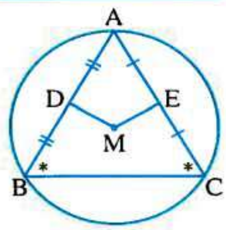
4 **[a] In the opposite figure :**

m ($\angle$ B) = m ($\angle$ C)

, D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

Prove that : MD = ME

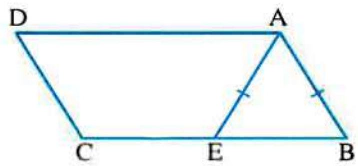


[b] In the opposite figure :

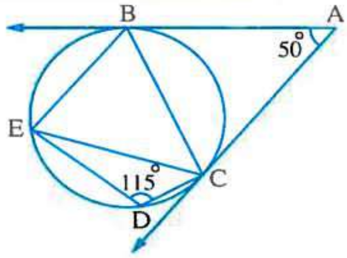
ABCD is a parallelogram

and E $\in \overline{BC}$, such that : AB = AE

Prove that : The figure AECD is a cyclic quadrilateral.



5 **[a] In the opposite figure :**



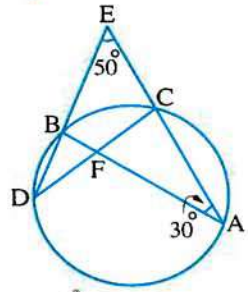
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m ($\angle$ A) = 50° , m ($\angle$ EDC) = 115°

Prove that : **1** $\overrightarrow{BC}$ bisects $\angle$ ABE

2 CB = CE

[b] In the opposite figure :



$\overline{AB} \cap \overline{CD} = \{F\}$, $\overrightarrow{AC} \cap \overrightarrow{DB} = \{E\}$

, m ($\angle$ A) = 30°

, m ($\angle$ E) = 50°

Find : **1** m ($\widehat{AD}$)

2 m ($\angle$ AFD)

25 North Sinai Governorate

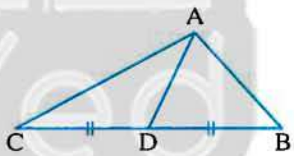
Answer the following questions :

1 **Choose the correct answer from those given :**

1 If the surface of the circle M $\cap$ the surface of the circle N = {A}
, then M , N are ....

(a) distant. (b) concentric. (c) touching externally. (d) intersecting.

2 **In the opposite figure :**



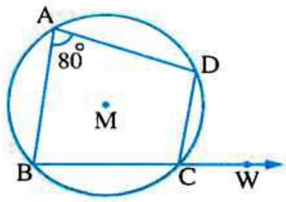
$\overline{AD}$ is a median in the triangle ABC

, the area of the triangle ABD = 20 cm^2

, then the area of the triangle ACD = ... cm^2

(a) 20 (b) 40 (c) 60 (d) 80

3 **In the opposite figure :**



If m ($\angle$ BAD) = 80°

, then m ($\angle$ DCW) = ...°

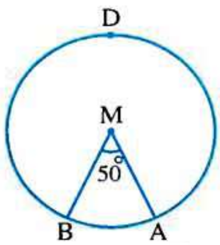
(a) 30 (b) 80

(c) 60 (d) 120

4 The area of the square whose diagonal length is 4 cm. equals .... cm^2

(a) 4 (b) 8 (c) 16 (d) 16 π

5 **In the opposite figure :**



m ($\angle$ AMB) = 50°

, then m ($\widehat{ADB}$) = ...°

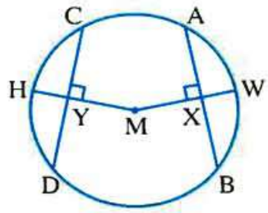
(a) 50 (b) 100

(c) 310 (d) 350

6 A triangle having one symmetry line and its side lengths are 8 , 4 , x cm.
, then x = ...

(a) 2 (b) 4 (c) 8 (d) 12

2 [a] **In the opposite figure :**



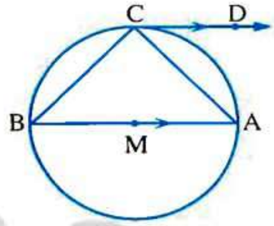
If AB = CD

, $\overline{MW} \perp \overline{AB}$

, $\overline{MH} \perp \overline{CD}$

Prove that : WX = HY

[b] **In the opposite figure :**



$\overrightarrow{CD}$ is a tangent to the circle M at C

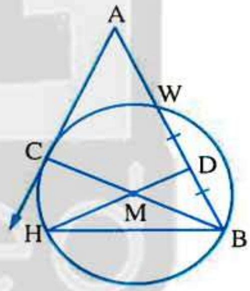
, $\overrightarrow{CD}$ // $\overline{BA}$ and M $\in \overline{AB}$

1 **Prove that :** AC = BC

2 **Find :** m (∠ B)

3 [a] State two cases in which the figure is a cyclic quadrilateral.

[b] **In the opposite figure :**



$\overline{BC}$ is a diameter in the circle M

, $\overrightarrow{AC}$ is a tangent to the circle M at C

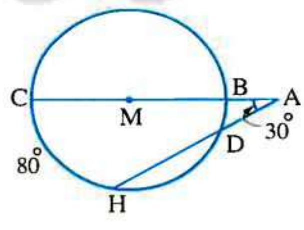
, D is the midpoint of $\overline{BW}$

Prove that :

1 The figure ADMC is a cyclic quadrilateral.

2 m (∠ CBH) = $\frac{1}{2}$ m (∠ A)

4 [a] **In the opposite figure :**



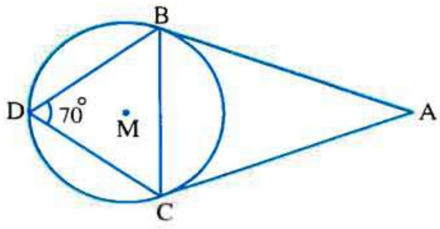
$\overline{BC}$ is a diameter in the circle M

, $\overrightarrow{CA} \cap \overrightarrow{HA} = \{A\}$, m (∠ A) = 30°

and m ($\overset{\frown}{CH}$) = 80°

Find : m ($\overset{\frown}{DH}$)

[b] **In the opposite figure :**



$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle at B and C

and m (∠ BDC) = 70°

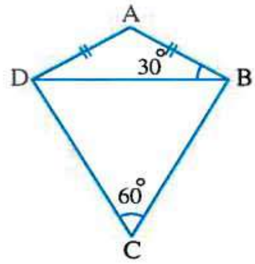
Find : m (∠ BAC)

5 **[a] In the opposite figure :**

AB = AD , m ($\angle$ ABD) = 30°

and m ($\angle$ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.



[b] By using geometric instruments , draw Δ ABC where

AB = 3 cm. , BC = 4 cm. , AC = 5 cm. , then draw a circle passing through the vertices of Δ ABC

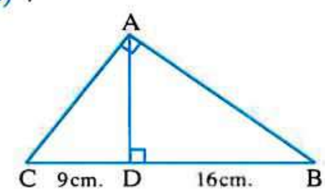
How many circles are there ?

26 Red Sea Governorate

Answer the following questions :

1 **Choose the correct answer from the given answers :**

1. The angle of tangency is included between ....
   (a) two chords. (b) two tangents.
   (c) a chord and a tangent. (d) a chord and a diameter.
2. The number of symmetry axes of the semicircle is ....
   (a) zero (b) 1 (c) 3 (d) an infinite number.
3. A circle of circumference 6 π cm. and a straight line L is at 3 cm. distant from its centre , then L is ....
   (a) a tangent. (b) a secant.
   (c) outside the circle. (d) a diameter of the circle.
4. The inscribed angle in a semicircle is .... angle.
   (a) an acute (b) an obtuse
   (c) a straight (d) a right
5. The radius length of the circle whose centre is the point of origin and passes through (– 3 , 4) equals .... length unit.
   (a) 3 (b) 4 (c) 5 (d) 7
6. **In the opposite figure :**
   ABC is a right-angled triangle at A
   , $\overline{AD} \perp \overline{BC}$, BD = 16 cm.
   , CD = 9 cm. , then AB = ... cm.



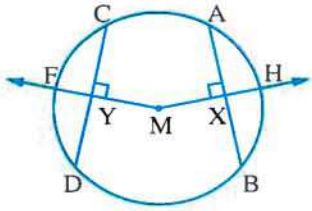
   (a) 5 (b) 7 (c) 20 (d) 25

2 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords equal in length in the circle M

, $\overrightarrow{MX} \perp \overline{AB}$, $\overrightarrow{MY} \perp \overline{CD}$

Prove that : HX = FY

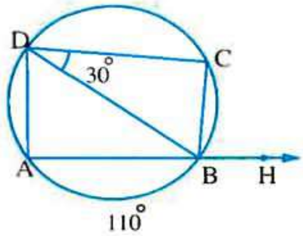


[b] In the opposite figure :

H $\in \overrightarrow{AB}$, m $(\widehat{AB}) = 110°$

, m (∠ CDB) = 30°

Find : m (∠ HBC)

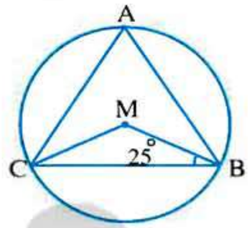


3 **[a] In the opposite figure :**

ABC is a triangle drawn in the circle M

, m (∠ MBC) = 25°

Find : m (∠ BAC)

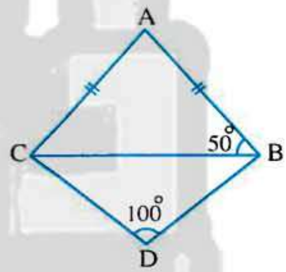


[b] In the opposite figure :

AB = AC , m (∠ D) = 100°

, m (∠ ABC) = 50°

Prove that : ABDC is a cyclic quadrilateral.



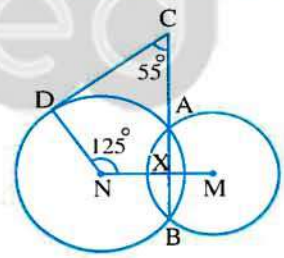
4 **[a] In the opposite figure :**

M and N are two intersecting circles at A and B

, C $\in \overrightarrow{BA}$, D $\in$ the circle N , m (∠ MND) = 125°

, m (∠ C) = 55°

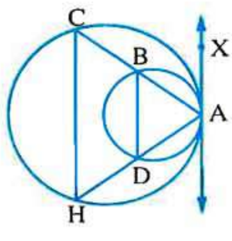
Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle N at D



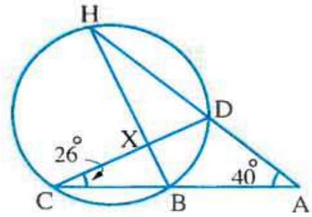
[b] In the opposite figure :

$\overleftrightarrow{AX}$ is a common tangent for the two circles touching internally at A

Prove that : $\overline{BD}$ // $\overline{CH}$



5 **[a] In the opposite figure :**



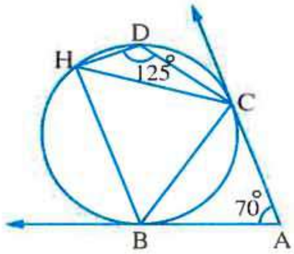
$\overrightarrow{CB} \cap \overrightarrow{HD} = \{A\}$, m (∠ A) = 40°

, $\overline{CD} \cap \overline{BH} = \{X\}$

, m (∠ DCB) = 26°

Find : m $(\overset{\frown}{CH})$, m (∠ HXC)

[b] In the opposite figure :



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 70°

, m (∠ CDH) = 125°

Prove that : CB = CH

27 Matrouh Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1. In the cyclic quadrilateral , each two opposite angles are ....
   (a) equal in measure. (b) complementary.
   (c) supplementary. (d) alternate.
2. A square is of perimeter 20 cm. , then its area equals ....
   (a) 50 cm². (b) 50 cm. (c) 25 cm². (d) 25 cm.
3. Δ ABC is right-angled at B , if BC = 8 cm. , AB = 6 cm. , then sin C = ...
   (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{5}{3}$ (d) 0.6
4. The ratio between the measure of the central angle and the measure of the inscribed angle subtended by the same arc equals ....
   (a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 1 : 4
5. The measure of the angle of the regular pentagon is equal to ....
   (a) 72° (b) 180° (c) 108° (d) 120°
6. A chord with length 8 cm. in a circle with circumference 10 π cm. , then it is distant from its center by ....
   (a) 2 cm. (b) 3 cm. (c) 4 cm. (d) 5 cm.

2 **[a]** $\overline{AB}$ and $\overline{AC}$ are two chords equal in length in a circle M , X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively, m (∠ MXY) = 30°

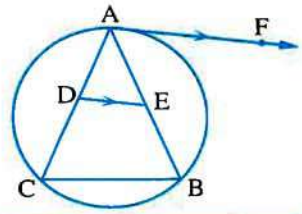
Prove that : Δ MXY is an isosceles triangle.

[b] In the opposite figure :

$\overrightarrow{AF}$ is a tangent to the circle at A

, $\overrightarrow{AF}$ // $\overline{DE}$

Prove that : DEBC is a cyclic quadrilateral.



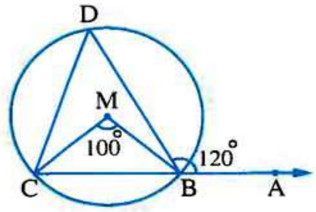
3 **[a] In the opposite figure :**

A circle of center M

, m (∠ BMC) = 100°

, m (∠ ABD) = 120°

Find : m (∠ DCB)



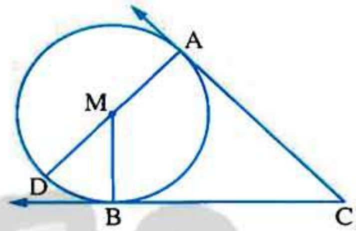
[b] In the opposite figure :

$\overline{AD}$ is a diameter in the circle M

, $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are two tangents to the circle M

, touching it at A and B respectively.

Prove that : m (∠ DMB) = m (∠ ACB)



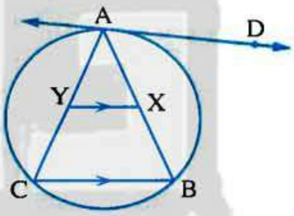
4 **[a] In the opposite figure :**

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

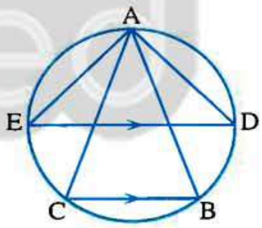


[b] In the opposite figure :

ABC is an inscribed triangle inside a circle

, $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)



5 **[a] Prove that :** In the same circle , the measures of all inscribed angles subtended by the same arc are equal.

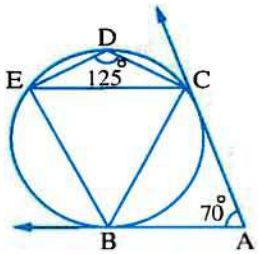
[b] In the opposite figure :

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m (∠ A) = 70° , m (∠ CDE) = 125°

Prove that : 1 CB = CE

2 $\overrightarrow{AC}$ // $\overline{BE}$



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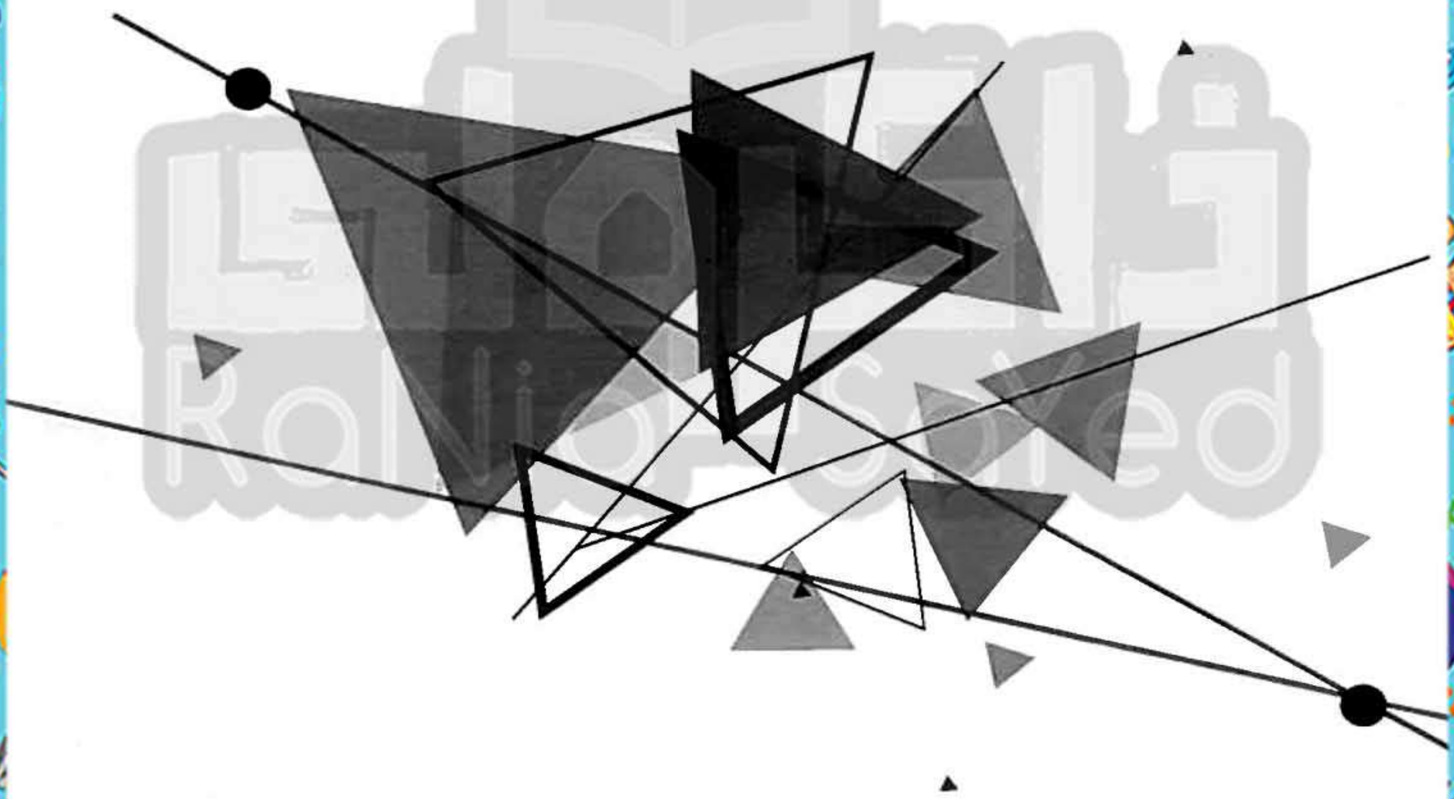
In Mathematics
Guide Answers

For 3rd Prep.
Second Term



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Guide Answers

of Algebra and Probability Exercises

Answers of Revision Exercise

1. $(5x-3y)(5x+3y)$
2. $2x^2(x^3+27)=2x^2(x+3)(x^2-3x+9)$
3. $(2y+3)(y+1)$
4. $2(x^4-9)=2(x^2-3)(x^2+3)$
5. $2(x^2-10x+24)=2(x-6)(x-4)$
6. $(x+4)^2$
7. $(2x+3)(4x^2-6x+9)$
8. $(y-51)(y+1)$
9. $(5x-3)^2$
10. $(x-9)(x+9)$
11. $y(y^4-1)=y(y^2-1)(y^2+1)$
    $=y(y-1)(y+1)(y^2+1)$
12. $(3x-2)(x+3)$
13. $(x-6)(x-2)$
14. $3x(x^2+4)+2(x^2+4)=(x^2+4)(3x+2)$
15. $(x-5)(x^2+5x+25)$
16. $(2x-3)^2$
17. $a^2(a+3)-9(a+3)=(a^2-9)(a+3)$
    $=(a-3)(a+3)(a+3)$
18. $-(2x^2+15x+7)=-(2x+1)(x+7)$
19. $(x-5)(x-2)$
20. $(3x^2-4y^2)(3x^2+4y^2)$
21. $(x^2-4)(x^2-5)=(x-2)(x+2)(x^2-5)$
22. $(1-2x)(1+2x)$
23. $(5x+2)(x-1)$
24. $3x^2(x^2-5x+4)=3x^2(x-4)(x-1)$
25. $(3x-1)(x-6)$
26. $(2x+7y)^2$
27. $(x^3-8y^3)(x^3+8y^3)$
    $=(x-2y)(x^2+2xy+4y^2)(x+2y)$
    $\times(x^2-2xy+4y^2)$
28. $2y^3(y-2)+7(y-2)=(y-2)(2y^3+7)$
29. $(x^2+3)(x^2-8)$
30. $(9x^2-4y^2)(x^2-y^2)$
    $=(3x-2y)(3x+2y)(x-y)(x+y)$

Answers of unit one

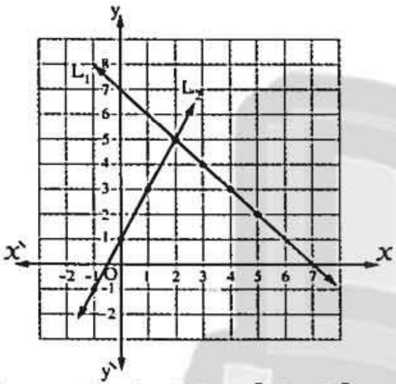
Answers of Exercise 1

1

1 $y = 7 - x$, $y = 2x + 1$

x	3	4	5
y	4	3	2

x	−1	0	1
y	−1	1	3

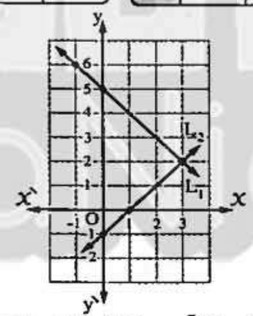


from the graph , the S.S. = $\{(2, 5)\}$

2 $y = 5 - x$, $y = x - 1$

x	0	−1	3
y	5	6	2

x	0	1	3
y	−1	0	2

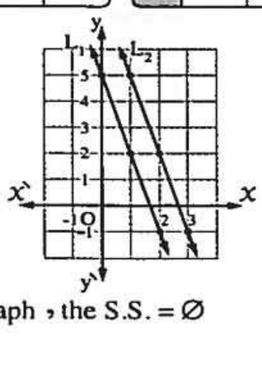


from the graph , the S.S. = $\{(3, 2)\}$

3 $y = 5 - 3x$, $y = 8 - 3x$

x	0	1	2
y	5	2	−1

x	1	2	3
y	5	2	−1

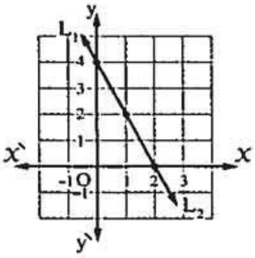


from the graph , the S.S. = $\varnothing$

4 $y = 4 - 2x$, $y = 4 - 2x$

x	0	1	2
y	4	2	0

x	0	1	2
y	4	2	0



from the graph ,

the S.S. = $\{(x, y) : y = 4 - 2x , (x, y) \in \mathbb{R} \times \mathbb{R}\}$

5 $y = -2x$, $x = 3 - 2y$

x	−1	0	1
y	2	0	−2

x	−1	1	3
y	2	1	0

Draw by yourself

from the graph , the S.S. = $\{(-1, 2)\}$

6 $y = -3$, $y = x - 5$

x	−2	0	2
y	−3	−3	−3

x	1	2	3
y	−4	−3	−2

Draw by yourself

from the graph , the S.S. = $\{(2, -3)\}$

2

1 $\because x = y$ (1) , $x + 3y = 8$ (2)

Substituting from (1) in (2) : $\therefore y + 3y = 8$

$\therefore 4y = 8$ $\therefore y = 2$

Substituting in (1) : $\therefore x = 2$

$\therefore$ The S.S. = $\{(2, 2)\}$

2 Adding the two equations we find that $2x = 6$

$\therefore x = 3$

Substituting in the second equation :

$\therefore 3 + y = 4$ $\therefore y = 1$

$\therefore$ The S.S. = $\{(3, 1)\}$

3 $\because y = x - 1$ (1) , $y + 2x = 5$ (2)

Substituting from (1) in (2) :

$\therefore x - 1 + 2x = 5$ $\therefore 3x = 6$ $\therefore x = 2$

Substituting in (1) : $\therefore y = 1$

$\therefore$ The S.S. = $\{(2, 1)\}$

4 Adding the two equations we find that : $3x = 15$

$\therefore x = 5$

Substituting in the first equation :

$\therefore 5 + 5y = 4$ $\therefore 5y = -1$

$\therefore y = \frac{-1}{5}$

$\therefore$ The S.S. = $\{(5, \frac{-1}{5})\}$

5 Substituting from the first equation in the second equation :

$\therefore 3(y + 4) + 4y = 5$ $\therefore 3y + 12 + 4y = 5$

$\therefore 7y = -7$ $\therefore y = -1$

Substituting in the first equation :

$\therefore x = -1 + 4$ $\therefore x = 3$

$\therefore$ The S.S. = $\{(3, -1)\}$

6 $\because 2x - y = 3$, multiplying by 2

$\therefore 4x - 2y = 6$ (1)

, $\because x + 2y = 4$ (2)

Adding (1) , (2) : $\therefore 5x = 10$ $\therefore x = 2$

Substituting in (2) :

$\therefore 2 + 2y = 4$ $\therefore y = 1$

$\therefore$ The S.S. = $\{(2, 1)\}$

7 $\because x - 3y = 5$, multiplying by -3

$\therefore -3x + 9y = -15$ (1)

, $\because 3x + 2y = 4$ (2)

Adding (1) , (2) : $\therefore 11y = -11$ $\therefore y = -1$

Substituting in (2) :

$\therefore 3x - 2 = 4$ $\therefore x = 2$

$\therefore$ The S.S. = $\{(2, -1)\}$

8 $\because 3x + 4y = 24$ (1)

, $x - 2y = -2$, multiplying by 2

$\therefore 2x - 4y = -4$ (2)

Adding (1) , (2) : $\therefore 5x = 20$ $\therefore x = 4$

Substituting in (1) :

$\therefore 12 + 4y = 24$ $\therefore y = 3$

$\therefore$ The S.S. = $\{(4, 3)\}$

9 Adding the two equations we find that :

$x = -1$

Substituting in the second equation :

$\therefore y + 2 = 3$ $\therefore y = 1$

$\therefore$ The S.S. = $\{(-1, 1)\}$

10 From the second equation :

$\because 3x = y + 8$ $\therefore y = 3x - 8$ (1)

Substituting in the first equation :

$\therefore x + 2(3x - 8) = 5$ $\therefore x + 6x - 16 = 5$

$\therefore 7x = 21$ $\therefore x = 3$

Substituting in (1) : $\therefore y = 3 \times 3 - 8$

$\therefore y = 1$ $\therefore$ The S.S. = $\{(3, 1)\}$

11 $\because x + 5y = 2$, multiplying by -2

$\therefore -2x - 10y = -4$ (1) , $2x - 3y = -9$ (2)

Adding (1) and (2) : $\therefore -13y = -13$ $\therefore y = 1$

Substituting in (1) : $\therefore x = -3$

$\therefore$ The S.S. = $\{(-3, 1)\}$

12 Multiplying the first equation by 2 and multiplying the second equation by 3

$\therefore 4y - 6x = 14$ (1) , $9y + 6x = 12$ (2)

Adding (1) and (2) : $\therefore 13y = 26$ $\therefore y = 2$

Substituting in the equation :

$\therefore 6 + 2x = 4$ $\therefore x = -1$

$\therefore$ The S.S. = $\{(-1, 2)\}$

13 $\because x + 2y = 1$ $\therefore x = 1 - 2y$ (1)

$\therefore 2x + 4y = -5$ (2)

Substituting from (1) in (2) :

$\therefore 2(1 - 2y) + 4y = -5$

$\therefore 2 - 4y + 4y = -5$ $\therefore 2 = -5$

This is contrary $\therefore$ The S.S. = $\varnothing$

14 $\because \frac{x}{6} + \frac{y}{3} = \frac{1}{3}$, multiplying by 6

$\therefore x + 2y = 2$ (1)

, $\because \frac{x}{2} + \frac{2y}{3} = 1$, multiplying by 6

$\therefore 3x + 4y = 6$ (2)

, multiplying equation (1) by -2

$\therefore -2x - 4y = -4$ (3)

, adding (2) , (3) : $\therefore x = 2$

, substituting in (1) :

$\therefore 2 + 2y = 2$ $\therefore y = 0$

$\therefore$ The S.S. = $\{(2, 0)\}$

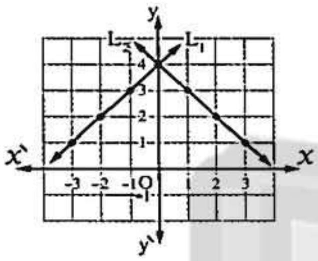
3

1 Graphically :

$y = x + 4$, $y = 4 - x$

x	−1	−2	−3
y	3	2	1

x	1	2	3
y	3	2	1



from the graph , the S.S. = $\{(0 , 4)\}$

Algebraically :

Substituting by the value of y from the first equation in the second equation

$\therefore x + 4 + x = 4$ $\quad \therefore 2x = 0$ $\quad \therefore x = 0$

Substituting in the first equation : $\therefore y = 4$

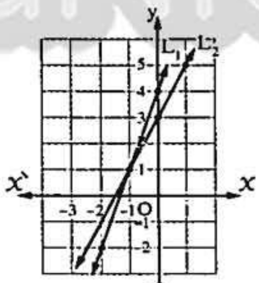
$\therefore$ The S.S. = $\{(0 , 4)\}$

2 Graphically :

$y = 3x + 4$, $y = 2x + 3$

x	−2	−1	0
y	−2	1	4

x	−1	0	1
y	1	3	5



from the graph , the S.S. = $\{(-1 , 1)\}$

Algebraically :

Substituting by $y = 2x + 3$ in the first equation

$\therefore 3x - 2x - 3 + 4 = 0$ $\quad \therefore x = -1$ $\quad \therefore y = 1$

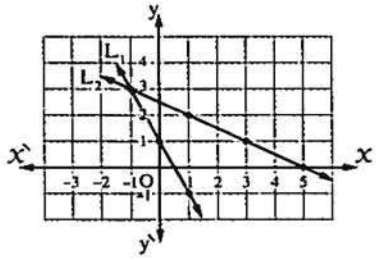
$\therefore$ The S.S. = $\{(-1 , 1)\}$

3 Graphically :

$y = 1 - 2x$, $x = 5 - 2y$

x	−1	0	1
y	3	1	−1

x	1	3	5
y	2	1	0



from the graph , the S.S. = $\{(-1 , 3)\}$

Algebraically : $\because 2x + y = 1$ $\therefore$ multiplying by -2

$\therefore -4x - 2y = -2$ (1) , $x + 2y = 5$ (2)

Adding (1) and (2) : $\therefore -3x = 3$ $\quad \therefore x = -1$

Substituting in (2) : $\therefore y = 3$

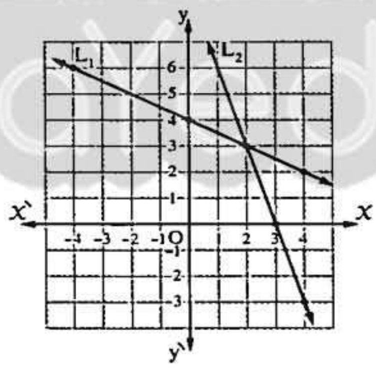
$\therefore$ The S.S. = $\{(-1 , 3)\}$

4 Graphically :

$x = 8 - 2y$, $y = 9 - 3x$

x	−4	0	4
y	6	4	2

x	2	3	4
y	3	0	−3



from the graph , the S.S. = $\{(2 , 3)\}$

Algebraically :

$\because x + 2y = 8$ $\quad \therefore$ multiplying by -3

$\therefore -3x - 6y = -24$ (1) , $\therefore 3x + y = 9$ (2)

Adding (1) and (2) :

$\therefore -5y = -15$ $\quad \therefore y = 3$

Substituting in (2) : $\therefore x = 2$

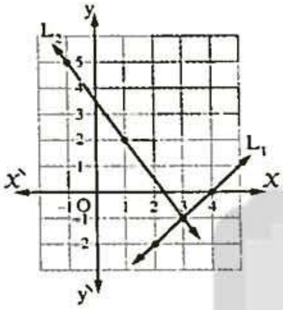
$\therefore$ The S.S. = $\{(2 , 3)\}$

5 Graphically :

$x = 4 + y$, $y = \frac{7 - 3x}{2}$

x	2	3	4
y	-2	-1	0

x	-1	1	3
y	5	2	-1



from the graph , the S.S. = $\{(3 , -1)\}$

Algebraically :

$\because x - y = 4$ $\therefore$ multiplying by 2

$\therefore 2x - 2y = 8$ (1) , $3x + 2y = 7$ (2)

Adding (1) and (2) : we find that $5x = 15$ $\therefore x = 3$

Substituting in (1) : $\therefore y = -1$

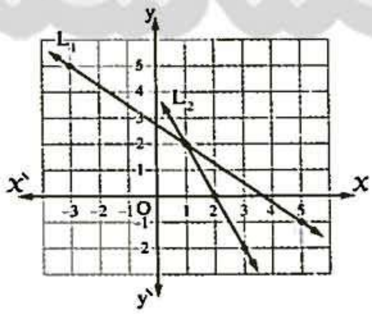
$\therefore$ The S.S. = $\{(3 , -1)\}$

6 Graphically :

$y = \frac{11 - 3x}{4}$, $y = 4 - 2x$

x	-3	1	5
y	5	2	-1

x	1	2	3
y	2	0	-2



from the graph , the S.S. = $\{(1 , 2)\}$

Algebraically : From the second equation

$\therefore y = 4 - 2x$

Substituting in the first equation :

$\therefore 3x + 16 - 8x = 11$ $\therefore -5x = -5$ $\therefore x = 1$

$\therefore y = 2$ $\therefore$ The S.S. = $\{(1 , 2)\}$

4

1 $\because m_1 = -\frac{7}{4} , m_2 = \frac{-5}{-2} = \frac{5}{2}$ $\therefore m_1 \neq m_2$

$\therefore$ The two straight lines intersect at a point

$\therefore$ The number of solutions = 1

2 $\because m_1 = \frac{-4}{2} = -2 , m_2 = -2$ $\therefore m_1 = m_2$

$\because$ The first straight line intersects y-axis at the point $(0 , 5)$

The second straight line intersects y-axis at the point $(0 , -5)$

$\therefore$ The two straight lines are parallel because of $m_1 = m_2$ and the two intersection points with y-axis are different

$\therefore$ The number of solutions = zero

3 $\because m_1 = \frac{-9}{6} = \frac{-3}{2} , m_2 = \frac{-3}{2}$ $\therefore m_1 = m_2$

$\because$ The two straight lines intersect y-axis at the same point $(0 , 4)$

$\therefore$ The two straight lines are coincident

$\therefore$ The number of solutions is an infinite

5

$\because (3 , -1)$ is a solution for the equation

$a x + b y - 5 = 0$ $\therefore 3a - b = 5$ (1)

$\because (3 , -1)$ is a solution for the equation

$3a x + b y = 17$ $\therefore 9a - b = 17$

$\therefore -9a + b = -17$ (2)

Adding (1) and (2) : $\therefore -6a = -12$ $\therefore a = 2$

Substituting in (1) : $\therefore b = 1$

6

$\because (a , 2b)$ is a solution for the equation : $3x - y = 5$

$\therefore 3a - 2b = 5$ (1)

, $\because (a , 2b)$ is a solution for the equation : $x + y = -1$

$\therefore a + 2b = -1$ (2)

Adding (1) and (2) : $\therefore 4a = 4$ $\therefore a = 1$

Substituting in (1) : $\therefore b = -1$

Another solution :

$\because 3x - y = 5$ (1) , $x + y = -1$ (2)

Adding (1) and (2) : $\therefore 4x = 4$ $\therefore x = 1$

Substituting in (2) : $\therefore y = -2$

$\therefore (1 , -2)$ is a solution for the two equations

, $\because (a , 2b)$ is a solution for the two equations

$\therefore (a , 2b) = (1 , -2)$ $\therefore a = 1$, $2b = -2$

$\therefore b = -1$

7

$\because f(x) = a x^2 + b$, $f(1) = 5$

$\therefore a + b = 5$ (1) , $\because f(2) = 11$

$\therefore 4a + b = 11$ (2)

Subtracting (1) from (2) : $\therefore 3a = 6$ $\therefore a = 2$

Substituting in (1) : $\therefore b = 3$

8

1 (3 , 1) 2 second 3 $\{(-5 , 5)\}$

4 $\varnothing$

5 $\{(x , y) : y = 6 - 4x , (x , y) \in \mathbb{R} \times \mathbb{R}\}$

6 (1 , 2) 7 $\{(-2 , -5)\}$

8 3 9 4

9

1 b 2 c 3 d 4 a 5 d

6 a 7 a 8 d 9 c 10 d

10

$\because x + y = 6$ (1)

, $y - 2x = 0$ $\therefore y = 2x$ (2)

By substituting from (2) in (1) :

$\therefore x + 2x = 6$ $\therefore 3x = 6$ $\therefore x = 2$

By substituting in (2) : $\therefore y = 4$ $\therefore B(2 , 4)$

$\therefore$ The length of the altitude drawn from B to $\overline{AO}$ is 4 length units

, $\because A \in$ straight line L_1 , $A \in \overleftrightarrow{xx'}$

at $y = 0$ in the equation $x + y = 6$

$\therefore x = 6$ $\therefore A(6 , 0)$

$\therefore AO = 6$ length units

$\therefore$ The area of $\triangle ABO = \frac{1}{2} \times 6 \times 4 = 12$ square units.

Applications on solving two equations in two unknowns of first degree

1

Let the two numbers be x and y

$\therefore x + y = 63$ (1) , $x - y = 11$ (2)

Adding (1) and (2) : $\therefore 2x = 74$ $\therefore x = 37$

Substituting in equ. (1) : $\therefore y = 26$

$\therefore$ The two numbers are 37 , 26

2

Let the two numbers be x and y

$\therefore x + y = 54$ (1) , $y = 2x$ (2)

Substituting from (2) in (1) :

$\therefore x + 2x = 54$ $\therefore 3x = 54$ $\therefore x = 18$

Substituting in (2) : $\therefore y = 36$

$\therefore$ The two numbers are 18 , 36

3

Let the first number be x , the second number be y

$\therefore 3x + 2y = 13$ (1)

, $x + 3y = 16$ (2)

From (2) : $x = 16 - 3y$ (3)

Substituting from (3) in (1) :

$\therefore 3(16 - 3y) + 2y = 13$

$\therefore 48 - 9y + 2y = 13$ $\therefore 48 - 7y = 13$

$\therefore 48 - 13 = 7y$ $\therefore 7y = 35$ $\therefore y = 5$

Substituting in (3) : $x = 1$

$\therefore$ The two numbers are 1 , 5

4

Let the length x cm. and the width be y cm.

$\therefore x - y = 4$ (1) , $2(x + y) = 28$ $\therefore x + y = 14$ (2)

Adding (1) and (2) : $\therefore 2x = 18$ $\therefore x = 9$

Substituting in (1) : $\therefore y = 5$

$\therefore$ The length = 9 cm. , the width = 5 cm.

$\therefore$ The area of the rectangle = $9 \times 5 = 45$ cm^2.

5

Let the number of arabian teams be x teams

and let the number of non-arabian teams be y teams

$\therefore x + y = 16$ (1) , $y - 3x = 4$ multiplying by -1

$\therefore 3x - y = -4$ (2)

Adding (1) and (2) : $\therefore 4x = 12$ $\therefore x = 3$

$\therefore$ The number of arabian teams = 3 teams

6

Let the father's age be x years

and let the son's age be y years

$\therefore x + y = 55$ (1) , $x - 4y = 5$ (2)

Multiplying the equ. (2) by -1 :

$\therefore -x + 4y = -5$ (3)

Adding (1) and (3) : $\therefore 5y = 50$ $\therefore y = 10$

Substituting in (1) : $\therefore x = 45$

$\therefore$ The son's age = 10 years

and the father's age = 45 years

7

Let the number of girls be x

and let the number of boys be y

$\therefore 2x - y = 50$ (1) , $2y - 3x = 50$ (2)

Multiplying equ. (1) by 2 : $\therefore 4x - 2y = 100$ (3)

Adding (2) and (3) : $\therefore x = 150$

Substituting in (1) : $\therefore y = 250$

$\therefore$ The number of boys = 250

The number of girls = 150

8

Let the measure of the greater angle be $x°$

and let the measure of the smaller angle be y°

$\therefore x + y = 180$ (1) , $2x = 7y$ (2)

$\therefore 2x - 7y = 0$

From (1) : $x = 180 - y$ (3)

Substituting in (2) : $\therefore 2(180 - y) - 7y = 0$

$\therefore 360 - 9y = 0$ $\therefore y = \frac{360}{9} = 40$

From (3) : $\therefore x = 140$

$\therefore$ The two measures of the two angles are 140° , 40°

9

Let the measure of the first angle be $x°$

and let the measure of the second angle be y°

$\therefore x + y = 90$ (1) , $x - y = 50$ (2)

Adding (1) and (2) : $\therefore 2x = 140$ $\therefore x = 70$

Substituting in (1) : $\therefore y = 20$

$\therefore$ The two measures are 70° , 20°

10

Let the price of one pen be x pounds and the price of one book be y pounds

$\therefore 4x + 2y = 22$ (1) , $5x + y = 20$ (2)

Multiplying equ. (2) by -2 :

$\therefore -10x - 2y = -40$ (3)

Adding (1) and (3) : $\therefore -6x = -18$ $\therefore x = 3$

Substituting in (2) : $\therefore y = 5$

$\therefore$ The price of the pen = L.E. 3

The price of the book = L.E. 5

11

Let the units digit be x and the tens digit be y

$\therefore x + y = 3x$ $\therefore 2x - y = 0$ (1)

$, y - x = 4$ (2)

Adding (1) and (2) : $\therefore x = 4$

Substituting in (2) : $\therefore y = 8$ $\therefore$ The number is 84

12

Let the units digit be x and the tens digit be y

$\therefore x + y = 11$ (1)

$, (y + 10x) - (x + 10y) = 27$ $\therefore 9x - 9y = 27$

$\therefore x - y = 3$ (2)

Adding (1) and (2) : $\therefore 2x = 14$ $\therefore x = 7$

Substituting in (1) : $\therefore y = 4$

$\therefore$ The number is 47

13

Let the units digit be x and the tens digit be y

$\therefore x + 10y = 5(x + y)$ $\therefore 5y - 4x = 0$ (1)

$, (y + 10x) - (x + 10y) = 9$ $\therefore 9x - 9y = 9$

$\therefore x - y = 1$ $\therefore 5x - 5y = 5$ (2)

Adding (1) and (2) : $\therefore x = 5$

Substituting in (1) : $\therefore y = 4$

$\therefore$ The number is 45

14

Let the rational number be $\frac{a}{b}$

$\therefore \frac{a-1}{b-1} = \frac{1}{2}$ $\therefore 2a - 2 = b - 1$

$\therefore 2a - b = 1$ (1)

$, \frac{a}{b+5} = \frac{1}{3}$ $\therefore 3a = b + 5$

$\therefore 3a - b = 5$ (2)

Multiplying equ. (1) by -1 :

$\therefore -2a + b = -1$ (3)

Adding (2) and (3) : $\therefore a = 4$

Substituting in (1) : $\therefore b = 7$

$\therefore$ The rational number = $\frac{4}{7}$

15

Let Ahmed's age now be x years

and Osama's age now be y years

$\therefore x + y = 43$ (1)

$, (x + 5) - (y + 5) = 3$ $\therefore x - y = 3$ (2)

Adding (1) and (2) : $\therefore 2x = 46$ $\therefore x = 23$

Substituting in (2) : $\therefore y = 20$

$\therefore$ Ahmed's age now = 23 years

and Ahmed's age after 7 years from now = 23 + 7 = 30 years

, Osama's age now = 20 years and

Osama's age after 7 years from now = 20 + 7 = 27 years

16

Let Magdi's age now be x years

and the age of his daughter Dina be y years

$\therefore (x - 5) = 5(y - 5) \quad \therefore x - 5y = -20 \qquad (1)$

$x + 4 = 3(y + 4) \quad \therefore x - 3y = 8 \qquad (2)$

Subtracting (1) from (2) : $\therefore 2y = 28 \quad \therefore y = 14$

Substituting in (1) : $\therefore x = 50$

$\therefore$ The age of Magdi now = 50 years

and the age of Dina = 14 years

17

$\because$ The triangle is equilateral.

$\therefore x + 2y = x + y + 2 \quad \therefore y = 2 \qquad (1)$

$3x - y = x + y + 2 \quad \therefore 2x - 2y = 2$

Substituting from (1) :

$\therefore 2x - 4 = 2 \quad \therefore x = 3$

$\therefore$ The side length = 7 cm.

18

$\because$ The triangle is isosceles

$\therefore$ The two base angles are equal in measure.

$\therefore 5x - 5y = 3x + 5y \quad \therefore 2x - 10y = 0 \; (1)$

$\because$ The measure of the vertex angle = $2x$

$\therefore 180 - (5x - 5y + 3x + 5y) = 2x$

$\therefore 180 - 8x = 2x \quad \therefore x = 18$

Substituting in (1) : $\therefore y = 3.6$

Excellent pupils

1

$\because (-d, 2c)$ is a solution for the equation : $x + y = 4$

$\therefore -d + 2c = 4 \qquad (1)$

$, \because (3d - 2, 3 - c)$ is a solution for the equation :

$x + y = 4 \quad \therefore 3d - 2 + 3 - c = 4 \quad \therefore 3d - c = 3 \qquad (2)$

Multiplying the equation (2) by 2 : $\therefore 6d - 2c = 6$ (3)

Adding (1) and (3) : $\therefore 5d = 10 \quad \therefore d = 2$

Substituting in (1) : $\therefore c = 3$

2

Let $\frac{1}{\ell} = x, \frac{1}{m} = y \quad \therefore x + y = 3 \qquad (1)$

$, \frac{2}{\ell} = 2x, \frac{3}{m} = 3y \quad \therefore 2x + 3y = 10 \qquad (2)$

Multiplying (1) by -2 : $\therefore -2x - 2y = -6 \qquad (3)$

Adding (2) and (3) : $\therefore y = 4$

Substituting in (1) : $\therefore x = -1$,

$\because \frac{1}{\ell} = x \quad \therefore \frac{1}{\ell} = -1 \quad \therefore \ell = -1$

$\because \frac{1}{m} = y \quad \therefore \frac{1}{m} = 4 \quad \therefore m = \frac{1}{4}$

3

Let the length of the rectangle be x cm. and the width be y cm.

$\therefore 2(x + y) = 24 \quad \therefore x + y = 12 \qquad (1)$

$, x - 4 = y + 2 \quad \therefore x - y = 6 \qquad (2)$

Adding (1) and (2) : $\therefore 2x = 18 \quad \therefore x = 9$

$\therefore$ The side length of the square = 9 − 4 = 5 cm.

$\therefore$ The area of the square = 25 cm^2.

4

Let the number of banknotes of P.T. 25 be x

and the number of banknotes of P.T. 50 be y

$\therefore x + y = 21$ (1) , $25x + 50y = 800$

dividing by 25 $\quad \therefore x + 2y = 32 \qquad (2)$

Subtracting (1) from (2) : $\therefore y = 11$

Substituting in (1) : $\therefore x = 10$

$\therefore$ The number of banknote of P.T. 25 = 10

and the number of banknote of P.T. 50 = 11

Answers of Exercise 2

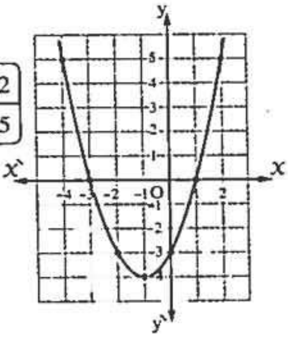
1

1 a	2 a	3 c	4 d
5 c	6 b	7 b	8 c
9 b	10 a	11 c	12 c

2

1 $f(x) = x^2 + 2x - 3$

x	-4	-3	-2	-1	0	1	2
y	5	0	-3	-4	-3	0	5



From the graph :

The S.S. = $\{-3 , 1\}$

2 $x^2 + 2x - 3 = 0$

$\therefore (x+3)(x-1) = 0$

$\therefore x + 3 = 0$,

then $x = -3$ or $x - 1 = 0$, then $x = 1$

$\therefore$ The S.S. = $\{-3 , 1\}$

3 $\because a = 1 , b = 2 , c = -3$

$\therefore x = \frac{-2 \pm \sqrt{4+12}}{2} = \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm 4}{2} = -1 \pm 2$

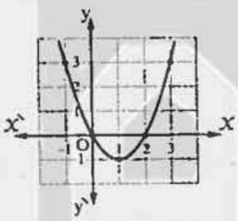
$\therefore$ The S.S. = $\{-3 , 1\}$

4 The S.S. = $\{-3 , 1\}$

3

$f(x) = x^2 - 2x$

x	-1	0	1	2	3
y	3	0	-1	0	3

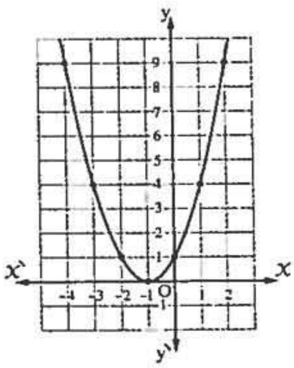


From the graph : The S.S. = $\{0 , 2\}$

4

$f(x) = x^2 + 2x + 1$

x	-4	-3	-2	-1	0	1	2
y	9	4	1	0	1	4	9

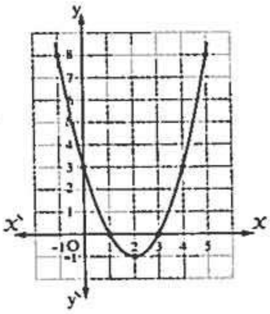


From the graph : The S.S. = $\{-1\}$

5

$f(x) = x^2 - 4x + 3$

x	-1	0	1	2	3	4	5
y	8	3	0	-1	0	3	8



From the graph :

1 The minimum value = -1

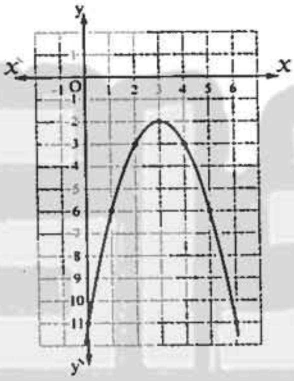
2 The equation of the axis of symmetry is $x = 2$

3 The S.S. = $\{1 , 3\}$

6

$f(x) = -x^2 + 6x - 11$

x	0	1	2	3	4	5	6
y	-11	-6	-3	-2	-3	-6	-11

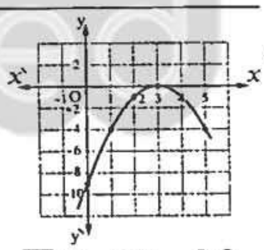


From the graph : The S.S. = $\varnothing$

7

$f(x) = 6x - x^2 - 9$

x	0	1	2	3	4	5
y	-9	-4	-1	0	-1	-4



From the graph :

1 The maximum value = 0

2 The S.S. = $\{3\}$

8

$f(x) = 4x^2 - 12x + 9$

x	0	1	2	3
y	9	1	1	9

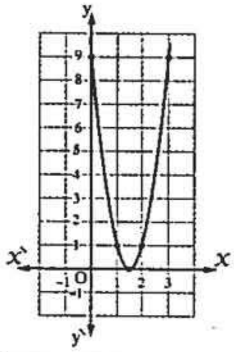
$\because$ The x-coordinate of the vertex point of the curve

$= \frac{-b}{2a} = \frac{12}{8} = \frac{3}{2} = 1\frac{1}{2}$

$f\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right)^2 - 12 \times \frac{3}{2} + 9 = 0$

$\therefore$ The vertex point of the curve is $\left(1\frac{1}{2} , 0\right)$

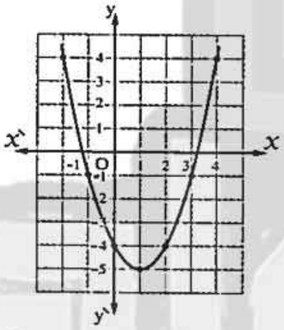
From the graph :
The S.S. $= \{1\frac{1}{2}\}$



9

1. $f(x) = x^2 - 2x - 4$

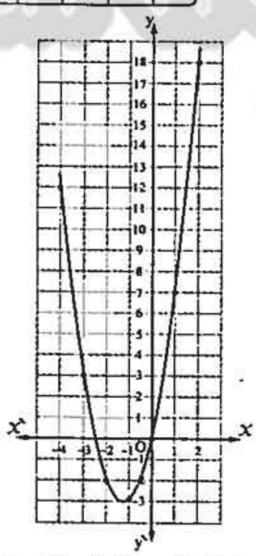
x	-2	-1	0	1	2	3	4
y	4	-1	-4	-5	-4	-1	4



From the graph : The S.S. $= \{-1.2 , 3.2\}$ approximately.

2. $f(x) = 2x^2 + 5x$

x	-4	-3	-2	-1	0	1	2
y	12	3	-2	-3	0	7	18



From the graph : The S.S. $= \{-2.5 , 0\}$

3. $f(x) = -x^2 + 3x + 2$

x	-1	0	1	2	3	4
y	-2	2	4	4	2	-2

Draw by yourself and from the graph
The S.S. $= \{-0.6 , 3.6\}$ approximately.

4. $f(x) = x^2 - 5x + 3$

x	0	1	2	3	4	5
y	3	-1	-3	-3	-1	3

Draw by yourself and from the graph
The S.S. $= \{0.7 , 4.3\}$ approximately.

5. $f(x) = 2x^2 + 3x - 6$

x	-3	-2	-1	0	1	2
y	3	-4	-7	-6	-1	8

Draw by yourself and from the graph
The S.S. $= \{-2.6 , 1.1\}$ approximately.

6. $f(x) = 2x^2 - 5x - 1$

x	-1	0	1	2	3
y	6	-1	-4	-3	2

Draw by yourself and from the graph
The S.S. $= \{-0.2 , 2.7\}$ approximately.

7. $f(x) = x^2 - 7x + 8$

x	1	2	3	4	5	6	7
y	2	-2	-4	-4	-2	2	8

Draw by yourself and from the graph
The S.S. $= \{1.4 , 5.6\}$ approximately.

10

1. $\because a = 1 , b = 7 , c = 2$

$\therefore x = \frac{-7 \pm \sqrt{49 - 8}}{2} = \frac{-7 \pm \sqrt{41}}{2}$

$\therefore x \approx -0.3$ or $x \approx -6.7$

$\therefore$ The S.S. $= \{-0.3 , -6.7\}$

2. $\because a = 1 , b = -4 , c = 1$

$\therefore x = \frac{4 \pm \sqrt{16 - 4}}{2} = \frac{4 \pm \sqrt{12}}{2}$

$\therefore x \approx 0.27$ or $x \approx 3.73$

$\therefore$ The S.S. $= \{0.27 , 3.73\}$

3 $\because a = 2, b = -4, c = 1$

$\therefore x = \frac{4 \pm \sqrt{16-8}}{4} = \frac{4 \pm \sqrt{8}}{4}$

$\therefore x \approx 0.293$ or $x \approx 1.707$

$\therefore$ The S.S. $= \{0.293, 1.707\}$

4 $\because a = 3, b = -6, c = 1$

$\therefore x = \frac{6 \pm \sqrt{36-12}}{6} = \frac{6 \pm \sqrt{24}}{6}$

$\therefore x \approx 0.184$ or $x \approx 1.816$

$\therefore$ The S.S. $= \{0.184, 1.816\}$

5 $\because a = 2, b = 5, c = 0$

$\therefore x = \frac{-5 \pm \sqrt{25-0}}{4} = \frac{-5 \pm 5}{4}$

$\therefore x = \frac{0}{4} = 0$ or $x = \frac{-10}{4} = -2.5$

$\therefore$ The S.S. $= \{0, -2.5\}$

6 $\because a = 1, b = 3, c = 5$

$\therefore x = \frac{-3 \pm \sqrt{9-20}}{2} = \frac{-3 \pm \sqrt{-11}}{2}$

$\therefore$ The S.S. $= \varnothing$

7 $\because a = 1, b = 8, c = 9$

$\therefore x = \frac{-8 \pm \sqrt{64-36}}{2} = \frac{-8 \pm 2\sqrt{7}}{2}$

$= -4 \pm \sqrt{7} = -4 \pm 2.65$

$\therefore$ The S.S. $= \{-1.35, -6.65\}$

8 $\because a = 2, b = -1, c = -2$

$\therefore x = \frac{1 \pm \sqrt{1+16}}{4} = \frac{1 \pm \sqrt{17}}{4} = \frac{1 \pm 4.12}{4}$

$\therefore$ The S.S. $= \{-0.78, 1.28\}$

11

1 $\because x^2 - 6x + 7 = 0$

$\therefore a = 1, b = -6, c = 7$

$\therefore x = \frac{6 \pm \sqrt{36-28}}{2} = \frac{6 \pm \sqrt{8}}{2}$

$\therefore x \approx 1.586$ or $x \approx 4.414$

$\therefore$ The S.S. $= \{1.586, 4.414\}$

2 $\because 2x^2 - 10x - 1 = 0$

$\therefore a = 2, b = -10, c = -1$

$\therefore x = \frac{10 \pm \sqrt{100+8}}{4} = \frac{10 \pm \sqrt{108}}{4} = \frac{10 \pm 6\sqrt{3}}{4}$

$= \frac{5 \pm 3\sqrt{3}}{2}$

$\therefore x \approx 5.098$ or $x \approx -0.098$

$\therefore$ The S.S. $= \{-0.098, 5.098\}$

3 $\because x^2 - x - 4 = 0$

$\therefore a = 1, b = -1, c = -4$

$\therefore x = \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$

$\therefore x \approx 2.562$ or $x \approx -1.562$

$\therefore$ The S.S. $= \{2.562, -1.562\}$

4 $\because 2x^2 + 3x - 6 = 0$

$\therefore a = 2, b = 3, c = -6$

$\therefore x = \frac{-3 \pm \sqrt{9+48}}{4} = \frac{-3 \pm \sqrt{57}}{4}$

$\therefore x \approx 1.137$ or $x \approx -2.637$

$\therefore$ The S.S. $= \{1.137, -2.637\}$

5 $\because x^2 - 3x + 1 = 0$

$\therefore a = 1, b = -3, c = 1$

$\therefore x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$

$\therefore x \approx 0.382$ or $x \approx 2.618$

$\therefore$ The S.S. $= \{0.382, 2.618\}$

6 $\because x^2 - 11x + 9 = 0$

$\therefore a = 1, b = -11, c = 9$

$\therefore x = \frac{11 \pm \sqrt{121-36}}{2} = \frac{11 \pm \sqrt{85}}{2}$

$\therefore x \approx 10.110$ or $x \approx 0.890$

$\therefore$ The S.S. $= \{10.110, 0.890\}$

7 Multiplying the equation by x:

$\therefore x^2 + 4 = 6x$ $\quad\therefore x^2 - 6x + 4 = 0$

$\therefore a = 1, b = -6, c = 4$

$\therefore x = \frac{6 \pm \sqrt{36-16}}{2} = \frac{6 \pm \sqrt{20}}{2}$

$\therefore x \approx 5.236$ or $x \approx 0.764$

$\therefore$ The S.S. $= \{5.236, 0.764\}$

8 Multiplying the equation by x^2:

$\therefore 8 + x = x^2$ $\quad\therefore x^2 - x - 8 = 0$

$\therefore a = 1, b = -1, c = -8$

$\therefore x = \frac{1 \pm \sqrt{1+32}}{2} = \frac{1 \pm \sqrt{33}}{2}$

$\therefore x \approx 3.372$ or $x \approx -2.372$

$\therefore$ The S.S. $= \{3.372, -2.372\}$

9 $\because x(5 - x) = 3$ $\quad\therefore 5x - x^2 = 3$

$\therefore -x^2 + 5x - 3 = 0$

$\therefore a = -1, b = 5, c = -3$

$\therefore x = \frac{-5 \pm \sqrt{25-12}}{-2} = \frac{-5 \pm \sqrt{13}}{-2}$

$\therefore x \approx 4.303$ or $x \approx 0.697$

$\therefore$ The S.S. $= \{4.303, 0.697\}$

10 Multiplying by 9

$\therefore x^2 - 12x = -18 \qquad \therefore x^2 - 12x + 18 = 0$

$\therefore a = 1 , b = -12 , c = 18$

$\therefore x = \frac{12 \pm \sqrt{144 - 72}}{2} = \frac{12 \pm \sqrt{72}}{2}$

$\therefore x \approx 10.243$ or $x \approx 1.757$

$\therefore$ The S.S. $= \{10.243 , 1.757\}$

Applications on solving an equation of the second degree in one unknown

1

$\because -0.2x^2 + 2x = 0$

$\therefore a = -0.2 , b = 2 , c = 0 \qquad \therefore x = \frac{-2 \pm \sqrt{4}}{-0.4}$

$\therefore x = 10$ or $x = 0$ (refused)

$\therefore$ The covered distance by the dolphin from the starting moment of its jumping from water till it returns again to water = 10 feet

2

$\because -0.3x^2 + 0.78x + 0.38 = 0$

$\therefore a = -0.3 , b = 0.78 , c = 0.38$

$\therefore x = \frac{-0.78 \pm \sqrt{0.6084 + 0.456}}{-0.6} = \frac{-0.78 \pm \sqrt{0.0644}}{-0.6}$

$\therefore x \approx 3$ or $x \approx -0.4$ (refused)

$\therefore$ The horizontal distance far from the cannon which the bullet covers till it strikes the land = 3 km. approximately.

3

$\because -0.06x^2 + 0.39x + 0.79 = 0$

$\therefore a = -0.06 , b = 0.39 , c = 0.79$

$\therefore x = \frac{-0.39 \pm \sqrt{0.1521 + 0.1896}}{-0.12} = \frac{-0.39 \pm \sqrt{0.3417}}{-0.12}$

$\therefore x \approx 8.12$ or $x \approx -1.62$ (refused)

$\therefore$ The maximum distance to which water reaches = 8.12 m. = 812 cm.

4

$\because -4.9t^2 + 3.5t + 10 = 0$

$\therefore a = -4.9 , b = 3.5 , c = 10$

$\therefore t = \frac{-3.5 \pm \sqrt{12.25 + 196}}{-9.8}$

$\therefore t \approx 1.83$ or $t \approx -1.12$ (refused)

$\therefore$ The diver reaches the water surface after 1.83 seconds approximately.

5

1 $\because -16t^2 + 80t + 20 = 0$

$\therefore a = -16 , b = 80 , c = 20$

$\therefore t = \frac{-80 \pm \sqrt{6400 + 1280}}{-32}$

$\therefore t \approx 5.24$ or $t \approx -0.24$ (refused)

$\therefore$ The ball reaches the floor surface after 5.24 seconds approximately.

2 Calculate the vertex of the curve to know the maximum height , the ball reaches to :

$\because$ The vertex of the curve $= \left(-\frac{b}{2a} , f\left(\frac{-b}{2a}\right)\right)$

$= (2.5 , 120)$

i.e.: The maximum height , the ball reaches to is 120 feets and that is after 2.5 seconds , and hence it can't reach the height 130 feet.

6

$\because 160 = 24t + 4.9t^2$

$\therefore 4.9t^2 + 24t - 160 = 0$

$\therefore a = 4.9 , b = 24 , c = -160$

$\therefore t = \frac{-24 \pm \sqrt{576 + 3136}}{9.8} = \frac{-24 \pm \sqrt{3712}}{9.8}$

$\therefore t \approx 3.77$ or $t \approx -8.67$ (refused)

$\therefore$ The snake will be able to escape at less than 3.77 seconds.

Excellent pupils

1 2   2 $\{-3\}$ 3 7   4 -1

Answers of Exercise 3

1

1 Substituting from equ. (1) in equ. (2) :

$\therefore x^2 + x^2 = 2 \qquad \therefore 2x^2 = 2$

$\therefore x^2 = 1 \qquad \therefore x = 1$ or $x = -1$

$\therefore y = 1$ or $y = -1$

The S.S. $= \{(1, 1) , (-1 , -1)\}$

2 $\because x-3=0$ $\therefore x=3$

Substituting in second equation :

$\therefore 9+y^2=25$ $\therefore y^2=16$

$\therefore y=4$ or $y=-4$

$\therefore$ The S.S. $=\{(3,4),(3,-4)\}$

3 $\because x-2y=0$ $\therefore x=2y$ (1)

Substituting in the other equation :

$\therefore (2y)^2-y^2=3$ $\therefore 4y^2-y^2=3$

$\therefore 3y^2=3$ $\therefore y^2=1$

$\therefore y=1$ or $y=-1$

From (1) : $\therefore x=2$ or $x=-2$

$\therefore$ The S.S. $=\{(2,1),(-2,-1)\}$

4 $\because x-y=0$ $\therefore x=y$ (1)

Substituting in the other equation :

$\therefore x^2+x\times x+x^2=27$

$\therefore 3x^2=27$ $\therefore x^2=9$

$\therefore x=3$ or $x=-3$

From (1) : $\therefore y=3$ or $y=-3$

$\therefore$ The S.S. $=\{(3,3),(-3,-3)\}$

5 $\because y-2x=0$ $\therefore y=2x$ (1)

Substituting in the other equation :

$\therefore x(2x)=18$ $\therefore 2x^2=18$ $\therefore x^2=9$

$\therefore x=3$ or $x=-3$

From (1) : $\therefore y=6$ or $y=-6$

$\therefore$ The S.S. $=\{(3,6),(-3,-6)\}$

6 $\because y+2x=0$ $\therefore y=-2x$ (1)

Substituting in equ. (2) :

$\therefore 6x^2-(-2x)^2=72$ $\therefore 6x^2-4x^2=72$

$\therefore 2x^2=72$ $\therefore x^2=36$

$\therefore x=6$ or $x=-6$

From (1) : $\therefore y=-12$ or $y=12$

$\therefore$ The S.S. $=\{(6,-12),(-6,12)\}$

7 Substituting from equ. (2) in equ. (1) :

$\therefore y^2+y=0$ $\therefore y(y+1)=0$

$\therefore y=0$ or $y=-1$

Substituting in equ. (1) : $\therefore x=0$ or $x=1$

$\therefore$ The S.S. $=\{(0,0),(1,-1)\}$

8 $\because x-y=0$ $\therefore x=y$ (1)

substituting in the second equation :

$\therefore y=\frac{4}{y}$ $\therefore y^2=4$

$\therefore y=2$ or $y=-2$

From (1) : $\therefore x=2$ or $x=-2$

$\therefore$ The S.S. $=\{(2,2),(-2,-2)\}$

2

1 Substituting from equ. (1) in equ. (2) :

$\therefore (x-1)^2+x=7$

$\therefore x^2-2x+1+x-7=0$

$\therefore x^2-x-6=0$ $\therefore (x-3)(x+2)=0$

$\therefore x=3$ or $x=-2$

Substituting in equ. (1) : $\therefore y=2$ or $y=-3$

$\therefore$ The S.S. $=\{(3,2),(-2,-3)\}$

2 Substituting from equ. (1) in equ. (2) :

$\therefore (5-y)^2-y^2=55$

$\therefore 25-10y+y^2-y^2=55$

$\therefore -10y=30$ $\therefore y=-3$

Substituting in equ. (1) : $\therefore x=8$

$\therefore$ The S.S. $=\{(8,-3)\}$

3 $\because x-y=1$ $\therefore x=1+y$ (1)

Substituting in the second equation :

$\therefore (1+y)^2+y^2=25$

$\therefore 1+2y+y^2+y^2=25$ $\therefore 2y^2+2y-24=0$

$\therefore y^2+y-12=0$ $\therefore (y+4)(y-3)=0$

$\therefore y=-4$ or $y=3$

And from (1) : $\therefore x=-3$ or $x=4$

$\therefore$ The S.S. $=\{(-3,-4),(4,3)\}$

4 $\because x+y=7$ $\therefore y=7-x$ (1)

Substituting in the other equation :

$\therefore (7-x)^2-x^2=7$

$\therefore 49-14x+x^2-x^2=7$

$\therefore -14x=-42$ $\therefore x=3$

From (1) : $\therefore y=4$

$\therefore$ The S.S. $=\{(3,4)\}$

5 $\because x-y-2=0$ $\therefore x=y+2$

Substituting in the second equation :

$\therefore (y+2)^2-y^2=0$

$\therefore y^2+4y+4-y^2=0$

$\therefore 4y=-4$ $\therefore y=-1$

From (1) : $\therefore x=1$

$\therefore$ The S.S. $=\{(1,-1)\}$

6 $\because 2x + y = 10$ $\therefore y = 10 - 2x$ (1)

Substituting in the second equation :

$\therefore x^2 + (10 - 2x)^2 = 25$

$\therefore x^2 + 100 - 40x + 4x^2 - 25 = 0$

$\therefore 5x^2 - 40x + 75 = 0$ $\therefore x^2 - 8x + 15 = 0$

$\therefore (x - 3)(x - 5) = 0$ $\therefore x = 3$ or $x = 5$

From (1) : $\therefore y = 4$ or $y = 0$

$\therefore$ The S.S. $= \{(3, 4), (5, 0)\}$

7 $\because y - x = 3$ $\therefore y = x + 3$ (1)

Substituting in the equ. (2) :

$\therefore x^2 - 2x + 3(x + 3) = 15$

$\therefore x^2 - 2x + 3x + 9 - 15 = 0$

$\therefore x^2 + x - 6 = 0$ $\therefore (x + 3)(x - 2) = 0$

$\therefore x = -3$ or $x = 2$

From (1) : $\therefore y = 0$ or $y = 5$

$\therefore$ The S.S. $= \{(-3, 0), (2, 5)\}$

8 $\because y + 2x = 7$ $\therefore y = 7 - 2x$ (1)

Substituting in the second equation :

$\therefore 2x^2 + x + 3(7 - 2x) = 19$

$\therefore 2x^2 + x + 21 - 6x - 19 = 0$

$\therefore 2x^2 - 5x + 2 = 0$

$\therefore (2x - 1)(x - 2) = 0$ $\therefore x = \frac{1}{2}$ or $x = 2$

And from (1) : $\therefore y = 6$ or $y = 3$

$\therefore$ The S.S. $= \{(\frac{1}{2}, 6), (2, 3)\}$

3

1 $\because x + y = 7$ $\therefore y = 7 - x$ (1)

Substituting in the second equation :

$\therefore x(7 - x) = 12$ $\therefore 7x - x^2 = 12$

$\therefore x^2 - 7x + 12 = 0$ $\therefore (x - 3)(x - 4) = 0$

$\therefore x = 3$ or $x = 4$

From (1) : $\therefore y = 4$ or $y = 3$

$\therefore$ The S.S. $= \{(3, 4), (4, 3)\}$

2 $\because x + y = 5$ $\therefore y = 5 - x$ (1)

$\because \frac{xy}{6} = 1$ $\therefore xy = 6$ (2)

Substituting from (1) in (2) :

$\therefore x(5 - x) = 6$ $\therefore 5x - x^2 - 6 = 0$

$\therefore x^2 - 5x + 6 = 0$ $\therefore (x - 2)(x - 3) = 0$

$\therefore x = 2$ or $x = 3$

And from (1) : $\therefore y = 3$ or $y = 2$

$\therefore$ The S.S. $= \{(2, 3), (3, 2)\}$

3 $\because y - x = 2$ $\therefore y = x + 2$ (1)

Substituting in the second equation :

$\therefore x^2 + x(x + 2) - 4 = 0$

$\therefore x^2 + x^2 + 2x - 4 = 0$

$\therefore 2x^2 + 2x - 4 = 0$ $\therefore x^2 + x - 2 = 0$

$\therefore (x + 2)(x - 1) = 0$ $\therefore x = -2$ or $x = 1$

From (1) : $\therefore y = 0$ or $y = 3$

$\therefore$ The S.S. $= \{(-2, 0), (1, 3)\}$

4 $\because x - 2y - 1 = 0$ $\therefore x = 2y + 1$

Substituting in the second equation :

$\therefore (2y + 1)^2 - y(2y + 1) = 0$

$\therefore 4y^2 + 4y + 1 - 2y^2 - y = 0$

$\therefore 2y^2 + 3y + 1 = 0$ $\therefore (2y + 1)(y + 1) = 0$

$\therefore y = -\frac{1}{2}$ or $y = -1$

From (1) : $\therefore x = 0$ or $x = -1$

$\therefore$ The S.S. $= \{(0, -\frac{1}{2}), (-1, -1)\}$

5 $\because x + y = 1$ $\therefore y = 1 - x$ (1)

Substituting in the second equation :

$\therefore x^2 + x(1 - x) + (1 - x)^2 = 3$

$\therefore x^2 + x - x^2 + 1 - 2x + x^2 - 3 = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore (x - 2)(x + 1) = 0$ $\therefore x = 2$ or $x = -1$

From (1) : $\therefore y = -1$ or $y = 2$

$\therefore$ The S.S. $= \{(2, -1), (-1, 2)\}$

6 $\because x + 2y = 4$ $\therefore x = 4 - 2y$ (1)

Substituting in the second equation :

$\therefore (4 - 2y)^2 + y(4 - 2y) + y^2 = 7$

$\therefore 16 - 16y + 4y^2 + 4y - 2y^2 + y^2 - 7 = 0$

$\therefore 3y^2 - 12y + 9 = 0$ $\therefore y^2 - 4y + 3 = 0$

$\therefore (y - 1)(y - 3) = 0$ $\therefore y = 1$ or $y = 3$

And from (1) : $\therefore x = 2$ or $x = -2$

$\therefore$ The S.S. $= \{(2, 1), (-2, 3)\}$

7 $\because y - x = 3$ $\therefore y = 3 + x$ (1)

Substituting in the second equation :

$\therefore x^2 + (3 + x)^2 - x(3 + x) = 13$

$\therefore x^2+9+6x+x^2-3x-x^2-13=0$

$\therefore x^2+3x-4=0$ $\therefore (x-1)(x+4)=0$

$\therefore x=1$ or $x=-4$

And from (1) : $\therefore y=4$ or $y=-1$

$\therefore$ The S.S. $=\{(1,4),(-4,-1)\}$

8 $\because x-y=10$ $\therefore x=y+10$ (1)

Substituting in the second equation :

$\therefore (y+10)^2-4y(y+10)+y^2=52$

$\therefore y^2+20y+100-4y^2-40y+y^2-52=0$

$\therefore -2y^2-20y+48=0$ $\therefore y^2+10y-24=0$

$\therefore (y+12)(y-2)=0$ $\therefore y=-12$ or $y=2$

From (1) : $\therefore x=-2$ or $x=12$

$\therefore$ The S.S. $=\{(-2,-12),(12,2)\}$

4

1 Substituting from equ. (1) in equ. (2) :

$\therefore y^2+3y-10=0$ $\therefore (y-2)(y+5)=0$

$\therefore y=2$ or $y=-5$

$\therefore$ The S.S. $=\{(0,2),(0,-5)\}$

2 Substituting from equ. (2) in equ. (1) :

$\therefore y^2-2y=8$ $\therefore y^2-2y-8=0$

$\therefore (y+2)(y-4)=0$ $\therefore y=-2$ or $y=4$

Substituting in equ. (1) :

$\therefore x=4$ or $x=16$

$\therefore$ The S.S. $=\{(4,-2),(16,4)\}$

3 $\because x^2+2xy=2$ $\therefore x(x+2y)=2$

$,\because x+2y=2$ $\therefore 2x=2$ $\therefore x=1$

Substituting in the first equation :

$\therefore 1+2y=2$ $\therefore y=\frac{1}{2}$

$\therefore$ The S.S. $=\{(1,\frac{1}{2})\}$

4 $\because x^2+2xy+y^2+y=6$

$\therefore (x+y)^2+y=6$ $,\because x+y=2$

$\therefore 2^2+y=6$ $\therefore y=2$

Substituting in the first equation :

$\therefore x+2=2$ $\therefore x=0$

$\therefore$ The S.S. $=\{(0,2)\}$

5 $\because x+y=2$ $\therefore x=2-y$ (1)

$\because \frac{1}{x}+\frac{1}{y}=2$ $\therefore y+x=2xy$

$\therefore y+x-2xy=0$ (2)

Substituting from (1) in (2) :

$\therefore y+2-y-2y(2-y)=0$

$\therefore 2y^2-4y+2=0$

$\therefore y^2-2y+1=0$ $\therefore (y-1)^2=0$ $\therefore y=1$

From (1) : $\therefore x=1$

$\therefore$ The S.S. $=\{(1,1)\}$

5

1 a	2 d	3 d	4 d
5 b	6 d	7 d	8 a

Applications on solving two equations in two variables one of them of the first degree and the other of the second degree

1

Let the two numbers be x and y :

$\therefore x+y=17$ (1)

$,xy=72$ (2)

From (1) : $\therefore x=17-y$ (3)

Substituting from (3) in (2) :

$\therefore (17-y)y=72$ $\therefore 17y-y^2-72=0$

$\therefore y^2-17y+72=0$ $\therefore (y-9)(y-8)=0$

$\therefore y=9$ or $y=8$

Substituting in (3) : $\therefore x=8$ or $x=9$

$\therefore$ The two numbers are 8 and 9

2

Let the two numbers be x and y :

$\therefore x+y=9$ (1)

$,x^2-y^2=45$ (2)

From (1) : $\therefore x=9-y$ (3)

Substituting from (3) in (2) : $\therefore (9-y)^2-y^2=45$

$\therefore 81-18y+y^2-y^2=45$ $\therefore 81-18y=45$

$\therefore 18y=36$ $\therefore y=2$

Substituting in (3) : $\therefore x=9-2=7$

$\therefore$ The two numbers are 7 and 2

3

Let the two numbers be x and y :

$\therefore x-3y=1$ (1)

, $x^2 + y^2 = 17$ (2)

From (1) : $\therefore x = 1 + 3y$ (3)

Substituting in (2) : $\therefore (1 + 3y)^2 + y^2 = 17$

$\therefore 1 + 6y + 9y^2 + y^2 - 17 = 0$

$\therefore 10y^2 + 6y - 16 = 0$

$\therefore 5y^2 + 3y - 8 = 0$ $\therefore (5y + 8)(y - 1) = 0$

$\therefore y = \frac{-8}{5}$ (refused) or $y = 1$

And from (3) : $\therefore x = 4$

$\therefore$ The two numbers are 1 and 4

4

Let the length of the rectangle = x cm.
and the width = y cm.

$\therefore (x + y) \times 2 = 18$ $\therefore x + y = 9$ (1)

, $xy = 18$ (2)

From (1) : $\therefore y = 9 - x$ (3)

Substituting in (2) : $\therefore x(9 - x) = 18$

$\therefore 9x - x^2 = 18$ $\therefore x^2 - 9x + 18 = 0$

$\therefore (x - 3)(x - 6) = 0$ $\therefore x = 3$ or $x = 6$

Substituting in (3) : $\therefore y = 6$ or $y = 3$

$\therefore$ The two dimensions are 6 cm. and 3 cm.

5

Let the length of the rectangle be x cm.
and its width be y cm.

$\therefore x - y = 3$ (1)

, $xy = 28$ (2)

From (1) : $\therefore x = y + 3$ (3)

Substituting from (3) in (2) :

$\therefore y(y + 3) = 28$ $\therefore y^2 + 3y - 28 = 0$

$\therefore (y + 7)(y - 4) = 0$

$\therefore y = -7$ (refused) or $y = 4$

Substituting in (3) : $\therefore x = 7$

$\therefore$ The two dimensions of the rectangle are 4 cm. and 7 cm.

$\therefore$ The perimeter of the rectangle = $(7 + 4) \times 2 = 22$ cm.

6

Let the length of the greatest diagonal be x cm.
and the smallest one be y cm.

$\therefore x - y = 4$ (1)

$\because$ The perimeter of the rhombus = 40 cm.

$\therefore$ The side length = 10 cm.

$\because$ The two diagonals of the rhombus are perpendicular and bisects each other

$\therefore \left(\frac{1}{2}x\right)^2 + \left(\frac{1}{2}y\right)^2 = (10)^2$ $\therefore \frac{1}{4}x^2 + \frac{1}{4}y^2 = 100$

$\therefore x^2 + y^2 = 400$ (2)

From (1) : $\therefore x = y + 4$ (3)

Substituting in (2) : $\therefore (y + 4)^2 + y^2 = 400$

$\therefore y^2 + 8y + 16 + y^2 - 400 = 0$

$\therefore 2y^2 + 8y - 384 = 0$ $\therefore y^2 + 4y - 192 = 0$

$\therefore (y + 16)(y - 12) = 0$

$\therefore y = -16$ (refused) or $y = 12$

Substituting in (3) : $\therefore x = 16$

$\therefore$ The lengths of the two diagonals are 16 cm. and 12 cm.

7

Let the side length of the great square be x metre
and the side length of the small square be y metre

$\therefore 4x - 4y = 8$

$\therefore x - y = 2$ (1)

, $x^2 - y^2 = 20$ (2)

From (1) : $\therefore x = 2 + y$ (3)

Substituting in (2) : $\therefore (2 + y)^2 - y^2 = 20$

$\therefore 4 + 4y + y^2 - y^2 = 20$

$\therefore 4y = 16$ $\therefore y = 4$

And from (3) : $\therefore x = 6$

$\therefore$ The side length of the great square = 6 metres
and the side length of the small square = 4 metres

8

Let the lengths of the two sides of the right angle be x cm. and y cm.

$\therefore x + y + 13 = 30$ $\therefore x + y = 17$ (1)

, $\because x^2 + y^2 = 169$ (2)

From (1) : $\therefore x = 17 - y$ (3)

Substituting in (2) : $\therefore (17 - y)^2 + y^2 = 169$

$\therefore y^2 - 34y + 289 + y^2 - 169 = 0$

$\therefore 2y^2 - 34y + 120 = 0$ $\therefore y^2 - 17y + 60 = 0$

$\therefore (y - 12)(y - 5) = 0$ $\therefore y = 12$ or $y = 5$

Substituting in (3) : $\therefore x = 5$ or $x = 12$

$\therefore$ The side lengths of the right angle are 5 cm. and 12 cm.

9

Let the length of the hypotenuse = x cm.

, the length of the other side = y cm.

$\therefore x + y + 5 = 30 \qquad \therefore x + y = 25 \qquad (1)$

$, x^2 = y^2 + 25 \qquad (2)$

From (1) : $\therefore x = 25 - y \qquad (3)$

Substituting in (2) : $\therefore (25 - y)^2 = y^2 + 25$

$\therefore 625 - 50 y + y^2 - y^2 - 25 = 0$

$\therefore 600 - 50 y = 0 \qquad \therefore 50 y = 600$

$\therefore y = 12$ cm.

$\therefore$ The area of a triangle $= \frac{1}{2} \times 12 \times 5 = 30$ cm^2.

10

Let the lengths of the two sides of the right angle be x cm. and y cm. , $x > y$

$\therefore x - y = 3 \qquad (1)$

$, x^2 + y^2 = 225 \qquad (2)$

From (1) : $\therefore x = 3 + y \qquad (3)$

Substituting in (2) : $\therefore (3 + y)^2 + y^2 = 225$

$\therefore 9 + 6 y + y^2 + y^2 - 225 = 0$

$\therefore 2 y^2 + 6 y - 216 = 0 \qquad \therefore y^2 + 3y - 108 = 0$

$\therefore (y + 12)(y - 9) = 0$

$\therefore y = -12$ (refused) or $y = 9$

Substituting in (3) : $\therefore x = 12$

$\therefore$ The perimeter of the triangle = 9 + 12 + 15 = 36 cm.

11

$\because x y = 77 \qquad (1)$

$, x - 2 = y + 2 \qquad \therefore x = y + 4 \qquad (2)$

Substituting in (1) : $\therefore (y + 4) \times y = 77$

$\therefore y^2 + 4 y - 77 = 0 \qquad \therefore (y + 11)(y - 7) = 0$

$\therefore y = -11$ (refused) or $y = 7$

Substituting in (2) : $\therefore x = 11$

$\therefore$ The side length of the square $= x - 2 = 9$ cm.

$\therefore$ The area of the square = 81 cm^2.

12

Let Ayman's age be x year and the age of his son Bassem be y year.

$\therefore x - 3 y = 1 \qquad (1)$

$, x^2 + y^2 - 3 x y = 181 \qquad (2)$

From (1) : $\therefore x = 1 + 3 y \qquad (3)$

Substituting in (2) :

$\therefore (1 + 3 y)^2 + y^2 - 3 y (1 + 3 y) = 181$

$\therefore 1 + 6 y + 9 y^2 + y^2 - 3 y - 9 y^2 - 181 = 0$

$\therefore y^2 + 3 y - 180 = 0 \qquad \therefore (y + 15)(y - 12) = 0$

$\therefore y = -15$ (refused) or $y = 12$

Substituting in (3) : $\therefore x = 37$

$\therefore$ Ayman's age = 37 years

and the age of his son Bassem = 12 years

13

Let the units digit be x and the tens digit be y

$\therefore x = 2 y \qquad (1)$

$\therefore x y = \frac{1}{2}(x + 10 y) \qquad \therefore x y = \frac{1}{2} x + 5 y \qquad (2)$

Substituting from (1) in (2) :

$\therefore 2 y^2 = y + 5 y \qquad \therefore 2 y^2 = 6 y$

$\therefore 2 y = 6$ (since $y \neq 0$) $\qquad \therefore y = 3$

From (1) : $\therefore x = 6 \qquad \therefore$ The number is 36

14

Let the units digit be x and the tens digit be y

$\therefore y - x = 1 \qquad \therefore y = x + 1 \qquad (1)$

$\therefore (x + 10 y)(y + 10 x) = 252 \qquad (2)$

Substituting from (1) in (2) :

$\therefore \left(x + 10 (x + 1)\right)(x + 1 + 10 x) = 252$

$\therefore (11 x + 10)(11 x + 1) = 252$

$\therefore 121 x^2 + 121 x - 242 = 0 \qquad \therefore x^2 + x - 2 = 0$

$\therefore (x + 2)(x - 1) = 0$

$\therefore x = -2$ (refused) or $x = 1$

Substituting in (1) : $\therefore y = 2$

$\therefore$ The original number is 21

15

Let the x coordinate be x and the y coordinate be y

$\therefore y = 2 x^2 \qquad (1)$

$\because$ The point moves on the straight line

$\therefore$ The coordinates of the point satisfy its equation

$\therefore 5 x - 2 y - 1 = 0 \qquad (2)$

Substituting from (1) in (2) :

$\therefore 5 x - 2 (2 x^2) - 1 = 0 \qquad \therefore -4 x^2 + 5 x - 1 = 0$

$\therefore 4 x^2 - 5 x + 1 = 0 \qquad \therefore (4 x - 1)(x - 1) = 0$

$\therefore x = \frac{1}{4}$ or $x = 1$

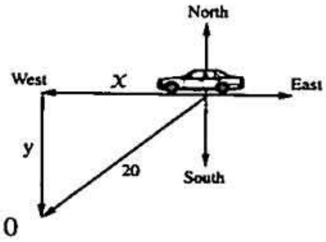
From (1) : $\therefore y = \frac{1}{8}$ or $y = 2$

$\therefore$ The point is $\left(\frac{1}{4}, \frac{1}{8}\right)$ or (1 , 2)

16

$\because x + y = 28$ (1)

And from the opposite figure :



$\therefore x^2 + y^2 = (20)^2$

$\therefore x^2 + y^2 - 400 = 0$ (2)

From (1) :

$\therefore x = 28 - y$ (3)

Substituting in (2) :

$\therefore (28 - y)^2 + y^2 - 400 = 0$

$\therefore 784 - 56\,y + y^2 + y^2 - 400 = 0$

$\therefore 2\,y^2 - 56\,y + 384 = 0$ $\therefore y^2 - 28\,y + 192 = 0$

$\therefore (y - 12)(y - 16) = 0$ $\therefore y = 12$ or $y = 16$

Substituting in (3) : $\therefore x = 16$ or $x = 12$

$\therefore$ The driver moved towards the west a distance of 16 km. , then he moved towards south a distance of 12 km. or the driver moved 12 km. towards west and 16 km. towards south.

Excellent pupils

1

[1] Substituting from the first equation in the second equation.

$\therefore y^2 - 2\,y - 2 = 0$ $\therefore y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore y = \frac{2 \pm \sqrt{4 - 4 \times 1 \times -2}}{2 \times 1} = \frac{2 \pm \sqrt{4 + 8}}{2}$

$= \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$

$\therefore y = 1 + \sqrt{3}$ or $y = 1 - \sqrt{3}$

Substituting in the first equation :

$\therefore x = 1 + \sqrt{3}$ or $x = 1 - \sqrt{3}$

$\therefore$ The S.S. $= \{(1 + \sqrt{3}, 1 + \sqrt{3}), (1 - \sqrt{3}, 1 - \sqrt{3})\}$

[2] $\because \sqrt{x} + y = 5$ $\therefore \sqrt{x} = 5 - y$ (1)

Squaring the two sides : $\therefore x = 25 - 10\,y + y^2$

Substituting in the equation : $x + y = 7$

$\therefore 25 - 10\,y + y^2 + y = 7$ $\therefore y^2 - 9\,y + 18 = 0$

$\therefore (y - 3)(y - 6) = 0$ $\therefore y = 3$

And substituting in (1) :

$\therefore x = 4$ or $y = 6$ (refused) because from the equation (1) : $\therefore 5 - 6 = -1$ and $\sqrt{x}$ should not be negative

$\therefore$ The S.S. $= \{(4, 3)\}$

2

$\because (-2, 4)$ is a solution for the equation : $a\,x + b\,y = 2$

$\therefore -2\,a + 4\,b = 2$ $\therefore a - 2\,b = -1$

$\therefore a = 2\,b - 1$ (1)

$\because (-2, 4)$ is a solution for the equation $a\,b\,x\,y + 2\,x^2 = 0$

$\therefore -8\,a\,b + 8 = 0$ $\therefore a\,b = 1$ (2)

Substituting from (1) in (2) :

$\therefore (2\,b - 1)\,b = 1$ $\therefore 2\,b^2 - b - 1 = 0$

$\therefore (2\,b + 1)(b - 1) = 0$

$\therefore b = -\frac{1}{2}$ (refused) or $b = 1$

Substituting in (1) : $\therefore a = 1$

$\therefore (a, b) = (1, 1)$

Answers of exams on unit one

Model 1

1

[1] d	[2] b	[3] d
[4] b	[5] c	[6] c

2

[a] The S.S. $= \{3.65, -1.65\}$

[b] The S.S. $= \{(2, -3), (8, 3)\}$

3

[a] Draw by yourself.

From the graph : The S.S. $= \{(1, 3)\}$

[b] The two dimensions of a rectangle are : 3 cm. , 4 cm.

4

[a] The S.S. $= \{(1, 1)\}$

[b] Draw by yourself.

From the graph we find that :

[1] The vertex of the curve is : $(1, -4)$

[2] The minimum value $= -4$

[3] The S.S. $= \{-1, 3\}$

5

[a] $a = 5$, $b = -2$

[b] The two numbers are : 5 , 7

Model 2

1

1 c 2 d 3 b

4 b 5 c 6 b

2

[a] The S.S. = $\{(1, 2)\}$

[b] The S.S. = $\{0.84, -0.24\}$

3

[a] $a = -1$, $b = 0$

[b] The S.S. = $\{(-4, -1), (1, 4)\}$

4

[a] The two numbers are : 1 , 5

[b] The two numbers are : 7 , 3

5

[a] Draw by yourself , the S.S. = $\{0, 2\}$

[b] Draw by yourself , the S.S. = $\varnothing$

Answers of unit two

Answers of Exercise 4

1

1 z (f) = {– 2}

2 $f(x) = x(x - 2)$ $\therefore z(f) = \{0, 2\}$

3 $f(x) = (x + 4)(x - 4)$ $\therefore z(f) = \{-4, 4\}$

4 $\varnothing$

5 $f(x) = (5 + 3x)(5 - 3x)$ $\therefore z(f) = \{-\frac{5}{3}, \frac{5}{3}\}$

6 $f(x) = 5x(x + 2)(x - 2)$ $\therefore z(f) = \{0, -2, 2\}$

7 $f(x) = (x - 5)(x^2 + 5x + 25)$ $\therefore z(f) = \{5\}$

8 $f(x) = 2x(x + 3)(x^2 - 3x + 9)$ $\therefore z(f) = \{0, -3\}$

9 $f(x) = (x - 4)(x + 1)$ $\therefore z(f) = \{4, -1\}$

10 $f(x) = (2x + 3)(x - 4)$ $\therefore z(f) = \{-\frac{3}{2}, 4\}$

11 $f(x) = x(x - 2)(x + 1)$ $\therefore z(f) = \{0, 2, -1\}$

12 $f(x) = -2x(x - 2)(x - 1)$ $\therefore z(f) = \{0, 2, 1\}$

13 $f(x) = x^2(2x - 3)(x + 2)$ $\therefore z(f) = \{0, \frac{3}{2}, -2\}$

14 $f(x) = (x + 2)(x - 7)$ $\therefore z(f) = \{-2, 7\}$

15 $f(x) = (x + 2)(x - 1)$ $\therefore z(f) = \{-2, 1\}$

16 Put $f(x)$ in the form $x^2 + 2x - 6 = 0$

$\therefore a = 1, b = 2, c = -6$

$x = \frac{-2 \pm \sqrt{4 + 24}}{2}$ $\therefore x = -1 + \sqrt{7}$ or $x = -1 - \sqrt{7}$

$\therefore z(f) = \{-1 + \sqrt{7}, -1 - \sqrt{7}\}$

17 Put $f(x)$ in the form $2x^2 - x + 5 = 0$

$a = 2, b = -1, c = 5$

$\therefore x = \frac{1 \pm \sqrt{1 - 40}}{4} = \frac{1 \pm \sqrt{-39}}{4}$

$\because \sqrt{-39} \notin \mathbb{R}$ $\therefore z(f) = \varnothing$

18 $f(x) = (x^3 - 4x) - (3x^2 - 12)$

$= x(x^2 - 4) - 3(x^2 - 4) = (x - 3)(x^2 - 4)$

$= (x - 3)(x - 2)(x + 2)$

$\therefore z(f) = \{3, 2, -2\}$

19 $f(x) = (x^3 - 8) + (x^2 - 2x)$

$= (x - 2)(x^2 + 2x + 4) + x(x - 2)$

$= (x - 2)(x^2 + 3x + 4)$

$\therefore z(f) = \{2\}$

20 $f(x) = (x^2 - 1)(x^2 - 9)$

$= (x - 1)(x + 1)(x - 3)(x + 3)$

$\therefore z(f) = \{1, -1, 3, -3\}$

2

1 a	2 b	3 c	4 c	5 c
6 a	7 c	8 c	9 a	10 d
11 d	12 a	13 a	14 a	15 c

3

1 {5} 2 $\varnothing$ 3 {2} 4 {3}

5 {1, –2} 6 {1, –2} 7 {0, 3}

8 {3, –3} 9 $\varnothing$ 10 – 9

4

$\because f(5) = (5)^3 - 2(5)^2 - 75 = 125 - 50 - 75 = 0$

$\therefore$ the number 5 is one of zeroes of the function f

5

$\because z(f) = \{0, 1\}$ $\therefore f(0) = 0$

$\therefore b = 0$ $\therefore f(x) = ax^2 + x$

$, \because f(1) = 0$ $\therefore a \times 1^2 + 1 = 0$

$\therefore a + 1 = 0$ $\therefore a = -1$

6

$\because f(3) = 0$ $\therefore 9a + 3b + 15 = 0$

$\therefore 3a + b = -5$ (1)

$\because f(5) = 0$ $\therefore 25a + 5b + 15 = 0$

$\therefore 5a + b = -3$ (2)

Subtracting (1) from (2) : $\therefore 2a = 2$ $\therefore a = 1$

And from (1) : $\therefore b = -8$

Excellent pupils

1

$\because z(h) = \{-2, 0\}$ $\therefore h(0) = 0$ $\therefore c = 0$

$, h(-2) = 0$ $\therefore 4a - 2b = 0$

$\therefore 2a - b = 0$ (1)

$\because h(3) = 15$ $\therefore 9a + 3b = 15$

$\therefore 3a + b = 5$ (2)

Adding (1) and (2) : $\therefore 5a = 5$ $\therefore a = 1$

And from (1) : $\therefore b = 2$

$\therefore h(x) = x^2 + 2x$

$h(2) = (2)^2 + 2 \times 2 = 8$

2

Putting $a X - 3 = 0$ $\therefore X = \frac{3}{a}$

$\therefore z(g) = \{\frac{3}{a}\}$

$\because z(g) = z(f)$

$\therefore a^2 \times \left(\frac{3}{a}\right)^2 - 12 \times \frac{3}{a} + 9 = 0$

$\therefore a^2 \times \frac{9}{a^2} - \frac{36}{a} + 9 = 0$

$\therefore \frac{36}{a} = 18$ $\therefore a = \frac{36}{18} = 2$

$\therefore z(g) = z(f) = \{\frac{3}{a}\} = \{\frac{3}{2}\}$

Answers of Exercise 5

1

1. The domain of $n = \mathbb{R} - \{2\}$
2. The domain of $n = \mathbb{R} - \{-2\}$
3. The domain of $n = \mathbb{R}$
4. The domain of $n = \mathbb{R} - \{0\}$
5. The domain of $n = \mathbb{R} - \{0\}$
6. $\because n(X) = \frac{X^2+1}{X(X-1)}$

   $\therefore$ The domain of $n = \mathbb{R} - \{0, 1\}$
7. $\because n(X) = \frac{X^2+9}{(X+4)(X-4)}$

   $\therefore$ The domain of $n = \mathbb{R} - \{-4, 4\}$
8. The domain of $n = \mathbb{R}$
9. $\because n(X) = \frac{X^2+25}{X(X^2+25)}$

   $\therefore$ The domain of $n = \mathbb{R} - \{0\}$
10. $\because n(X) = \frac{X^2-4}{(X-3)(X+2)}$

    $\therefore$ The domain of $n = \mathbb{R} - \{3, -2\}$
11. $\because n(X) = \frac{X^2-4X+3}{8(X+1)(X^2-X+1)}$

    $\therefore$ The domain of $n = \mathbb{R} - \{-1\}$
12. $\because n(X) = \frac{X^2-5X+6}{(X^2+9)(X-3)(X+3)}$

    $\therefore$ The domain of $n = \mathbb{R} - \{3, -3\}$
13. $\because n(X) = \frac{X+1}{-X(X-2)^2}$

    $\therefore$ The domain of $n = \mathbb{R} - \{0, 2\}$
14. $\because n(X) = \frac{X^2-3}{X^2-3X+5}$

    Putting $X^2 - 3X + 5 = 0$

    $\therefore a = 1$, $b = -3$, $c = 5$

    $X = \frac{3 \pm \sqrt{9-20}}{2} = \frac{3 \pm \sqrt{-11}}{2}$

    $\because \sqrt{-11} \notin \mathbb{R}$

    $\therefore$ The domain of $n = \mathbb{R}$

2

Consider the algebraic fractions in each problem be n_1 , n_2 and n_3 respectively the solution will be as follows :

1. The domain of $n_1 = \mathbb{R}$, the domain of $n_2 = \mathbb{R} - \{0\}$

   $\therefore$ The common domain $= \mathbb{R} - \{0\}$
2. The domain of $n_1 = \mathbb{R} - \{-5\}$

   , the domain of $n_2 = \mathbb{R} - \{7\}$

   $\therefore$ The common domain $= \mathbb{R} - \{-5, 7\}$
3. The domain of $n_1 = \mathbb{R} - \{2\}$

   , $\because n_2(X) = \frac{X+3}{(X+3)(X-3)}$

   $\therefore$ The domain of $n_2 = \mathbb{R} - \{-3, 3\}$

   $\therefore$ The common domain $= \mathbb{R} - \{2, -3, 3\}$
4. The domain of $n_1 = \mathbb{R} - \{0\}$

   $\because n_2(X) = \frac{X^2-1}{X(X-1)}$

   $\therefore$ The domain of $n_2 = \mathbb{R} - \{0, 1\}$

   $\therefore$ The common domain $= \mathbb{R} - \{0, 1\}$
5. $\because n_1(X) = \frac{X}{(X+2)(X-2)}$

   $\therefore$ The domain of $n_1 = \mathbb{R} - \{-2, 2\}$

   The domain of $n_2 = \mathbb{R} - \{2\}$

   $\therefore$ The common domain $= \mathbb{R} - \{-2, 2\}$
6. $\because n_1(X) = \frac{X^2+3X}{X(X+3)(X-3)}$

   $\therefore$ The domain of $n_1 = \mathbb{R} - \{0, -3, 3\}$

   , $\because n_2(X) = \frac{X^2+3X+9}{(X-3)(X^2+3X+9)}$

   $\therefore$ The domain of $n_2 = \mathbb{R} - \{3\}$

   $\therefore$ The common domain $= \mathbb{R} - \{0, -3, 3\}$

7 $\because n_1(x) = \frac{(x-4)}{(x-2)(x-3)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{2, 3\}$

, $\because n_2(x) = \frac{2x}{x(x+3)(x-3)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0, -3, 3\}$

$\therefore$ The common domain $= \mathbb{R} - \{2, 3, 0, -3\}$

8 $\because n_1(x) = \frac{x^2+4}{(x+2)(x-2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-2, 2\}$

, $\because n_2(x) = \frac{7}{(x+2)^2}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{-2\}$

$\therefore$ The common domain $= \mathbb{R} - \{-2, 2\}$

9 The domain of $n_1 = \mathbb{R} - \{-2\}$

The domain of $n_2 = \mathbb{R}$

The domain of $n_3 = \mathbb{R} - \{3\}$

$\therefore$ The common domain $= \mathbb{R} - \{-2, 3\}$

10 The domain of $n_1 = \mathbb{R} - \{-4\}$

The domain of $n_2 = \mathbb{R} - \{3\}$

The domain of $n_3 = \mathbb{R}$

$\therefore$ The common domain $= \mathbb{R} - \{-4, 3\}$

11 The domain of $n_1 = \mathbb{R}$

, $\because n_2(x) = \frac{3}{(x+3)(x-3)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{-3, 3\}$

, $\because n_3(x) = \frac{3x}{x(x-3)}$

$\therefore$ The domain of $n_3 = \mathbb{R} - \{0, 3\}$

$\therefore$ The common domain $= \mathbb{R} - \{0, -3, 3\}$

12 $\because n_1(x) = \frac{x^2-4}{(x-2)(x-3)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{2, 3\}$

, $\because n_2(x) = \frac{7}{(x+3)(x-3)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{-3, 3\}$

, $\because n_3(x) = \frac{x^2-3x-4}{(x+2)(x-1)}$

$\therefore$ The domain of $n_3 = \mathbb{R} - \{-2, 1\}$

$\therefore$ The common domain $= \mathbb{R} - \{2, 3, -3, -2, 1\}$

3

1 $\mathbb{R} - \{0\}$ 2 $\mathbb{R} - \{-2\}$

3 $\mathbb{R} - \{0, 2\}$ 4 $\{1, 6\}$ 5 $\{-2\}$

6 $\mathbb{R} - \{0\}$ 7 3 8 2

9 25 10 7 11 4

4

1 c 2 d 3 d 4 a 5 d 6 d

7 c 8 c 9 c 10 c 11 c 12 d

13 b 14 c 15 a 16 c

5

$n(x) = \frac{2x+1}{(x-3)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, 2\}$, $n(0) = \frac{1}{6}$

$n(2)$ is meaningless because $2 \notin$ the domain of n

6

$\because$ The domain of $n = \mathbb{R} - \{3\}$

$\therefore$ At $x = 3$, then $x^2 - ax + 9 = 0$

$\therefore 9 - 3a + 9 = 0$ $\therefore 3a = 18$ $\therefore a = 6$

7

$\because n(a)$ is undefined

$\therefore$ At $x = a$ $\therefore$ denominator $= 0$

$\therefore 4a^2 - 12a + 9 = 0$

$\therefore (2a-3)^2 = 0$ $\therefore 2a - 3 = 0$

$\therefore 2a = 3$ $\therefore a = \frac{3}{2}$

8

$\because$ The domain of $f = \mathbb{R} - \{2, c\}$

$\therefore$ When $x = 2$ $\therefore x^2 - 5x + m = 0$

$\therefore 4 - 5 \times 2 + m = 0$ $\therefore m = 6$

$\therefore f(x) = \frac{x}{x^2-5x+6}$ $\therefore f(x) = \frac{x}{(x-2)(x-3)}$

$\therefore$ The domain of $f = \mathbb{R} - \{2, 3\}$ $\therefore c = 3$

9

$\because$ The domain $= \mathbb{R} - \{-2\}$

$\therefore$ When $x = -2$ $\quad \therefore x + a = 0$

$\therefore -2 + a = 0$ $\quad \therefore a = 2$

$\therefore f(x) = \frac{x+b}{x+2}$ $\quad , \because f(0) = 3$ $\quad \therefore \frac{0+b}{0+2} = 3$

$\therefore \frac{b}{2} = 3$ $\quad \therefore b = 6$

10

$\because z(f) = \{4\}$ $\quad \therefore$ At $x = 4$

$\therefore a x^2 - 6x + 8 = 0$

$\therefore a \times 4^2 - 6 \times 4 + 8 = 0$

$\therefore 16a - 16 = 0$ $\quad \therefore 16a = 16$ $\quad \therefore a = 1$

$, \because$ The domain of $f = \mathbb{R} - \{2\}$

$\therefore$ At $x = 2$ $\quad \therefore bx - 4 = 0$

$\therefore 2b - 4 = 0$ $\quad \therefore 2b = 4$ $\quad \therefore b = 2$

Excellent pupils

1

$\because$ The domain of $n = \mathbb{R} - \{1, 3\}$

$\therefore$ At $x = 1$, then $x^2 + ex + a = 0$

$\therefore 1 + e + a = 0$ $\quad \therefore e + a = -1$ $\quad (1)$

and at $x = 3$, then $x^2 + ex + a = 0$

$\therefore 9 + 3e + a = 0$ $\quad \therefore 3e + a = -9$ $\quad (2)$

Subtracting (1) from (2) :

$\therefore 2e = -8$ $\quad \therefore e = -4$

Substituting in (1) : $\therefore a = 3$

2

The common domain $= \mathbb{R} - \{3\}$

$\because$ The domain of $n_1 = \mathbb{R}$ $\quad \therefore 3 \notin$ the domain of n_2

$\therefore$ At $x = 3$, then $x^2 - 6x - a = 0$

$\therefore 9 - 18 - a = 0$ $\quad \therefore a = -9$

Answers of Exercise 6

1

1. $n(x) = \frac{2(x+4)}{x+4}$

   $\therefore$ The domain of $n = \mathbb{R} - \{-4\}$

   $, n(x) = 2$

2. $n(x) = \frac{x(x-2)}{x(x+3)}$

   $\therefore$ The domain of $n = \mathbb{R} - \{0, -3\}$

   $, n(x) = \frac{x-2}{x+3}$

3. $n(x) = \frac{x(x-4)}{(x-4)(x+4)}$

   $\therefore$ The domain of $n = \mathbb{R} - \{4, -4\}$

   $, n(x) = \frac{x}{x+4}$

4. $n(x) = \frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)}$

   $\therefore$ The domain of $n = \mathbb{R} - \{2\}$

   $, n(x) = \frac{x+2}{x^2+2x+4}$

5. $n(x) = \frac{4x(3x-2)}{2x(3x-2)}$

   $\therefore$ The domain of $n = \mathbb{R} - \{0, \frac{2}{3}\}$

   $, n(x) = 2$

6. $n(x) = \frac{(x-2)(x+2)}{(x-2)(x-3)}$

   $\therefore$ The domain of $n = \mathbb{R} - \{2, 3\}$

   $, n(x) = \frac{x+2}{x-3}$

7. $n(x) = \frac{(x-3)^2}{2x(x-3)(x+3)}$

   $\therefore$ The domain of $n = \mathbb{R} - \{0, 3, -3\}$

   $, n(x) = \frac{x-3}{2x(x+3)}$

8. $n(x) = \frac{(x+3)(x-2)}{(x+3)(x-5)}$

   $\therefore$ The domain of $n = \mathbb{R} - \{-3, 5\}$

   $, n(x) = \frac{x-2}{x-5}$

9. $n(x) = \frac{(2x+3)(x+2)}{(2x-1)(2x+3)}$

   $\therefore$ The domain of $n = \mathbb{R} - \{\frac{1}{2}, \frac{-3}{2}\}$

   $, n(x) = \frac{x+2}{2x-1}$

10. $n(x) = \frac{(x+1)(x^2-x+1)}{x(x^2-x+1)}$

    $\therefore$ The domain of $n = \mathbb{R} - \{0\}$

    $, n(x) = \frac{x+1}{x}$

11. $n(x) = \frac{-(x+2)(x-3)}{(x-2)(x-3)}$

    $\therefore$ The domain of $n = \mathbb{R} - \{2, 3\}$

    $, n(x) = \frac{-(x+2)}{x-2}$

12. $n(x) = \frac{(x^2-4)(x^4+4x^2+16)}{x^4+4x^2+16}$

    $\therefore$ The domain of $n = \mathbb{R}$

    $, n(x) = x^2 - 4$

13 $n(x)=\frac{(x-3)(x-1)}{x(x-3)}$

$\therefore$ The domain of $n=\mathbb{R}-\{0,3\}$

, $n(x)=\frac{x-1}{x}$

14 $n(x)=\frac{\frac{x^2+1}{x}}{\frac{4x^2+4}{x}}$

$\therefore$ The domain of $n=\mathbb{R}-\{0\}$

, $n(x)=\frac{x^2+1}{4x^2+4}=\frac{x^2+1}{4(x^2+1)}=\frac{1}{4}$

15 $n(x)=\frac{(x-3)(x+2)}{(x^3+2x^2)-(9x+18)}$

$=\frac{(x-3)(x+2)}{x^2(x+2)-9(x+2)}$

$=\frac{(x-3)(x+2)}{(x-3)(x+3)(x+2)}$

$\therefore$ The domain of $n=\mathbb{R}-\{-3,3,-2\}$

, $n(x)=\frac{1}{x+3}$

16 $n(x)=\frac{(x^3-1)+(x^2-1)}{x-1}$

$=\frac{(x-1)(x^2+x+1)+(x-1)(x+1)}{(x-1)}$

$=\frac{(x-1)(x^2+2x+2)}{x-1}$

$\therefore$ The domain of $n=\mathbb{R}-\{1\}$

, $n(x)=x^2+2x+2$

2

1 $\because n_1(x)=\frac{(2x-3)(2x+3)}{3(2x-3)}$

$\therefore$ The domain of $n_1=\mathbb{R}-\{\frac{3}{2}\}$

, $n_1(x)=\frac{2x+3}{3}$ $\because n_2(x)=\frac{x(2x+3)}{3x}$

$\therefore$ The domain of $n_2=\mathbb{R}-\{0\}$

, $n_2(x)=\frac{2x+3}{3}$ $\therefore n_1(x)=n_2(x)$

For all the values of $x\in\mathbb{R}-\{\frac{3}{2},0\}$

2 $\because n_1(x)=\frac{x^2-3x+9}{(x+3)(x^2-3x+9)}$

$\therefore$ The domain of $n_1=\mathbb{R}-\{-3\}$

, $n_1(x)=\frac{1}{x+3}$

, $\because n_2(x)=\frac{2}{2(x+3)}$

$\therefore$ The domain of $n_2=\mathbb{R}-\{-3\}$

, $n_2(x)=\frac{1}{x+3}$

$\therefore n_1(x)=n_2(x)$ for all the values of $x\in\mathbb{R}-\{-3\}$

3 $\because n_1(x)=\frac{(x-2)(x+2)}{(x-2)(x+3)}$

$\therefore$ The domain of $n_1=\mathbb{R}-\{2,-3\}$

, $n_1(x)=\frac{x+2}{x+3}$

, $\because n_2(x)=\frac{x(x-3)(x+2)}{x(x-3)(x+3)}$

$\therefore$ The domain of $n_2=\mathbb{R}-\{0,3,-3\}$

, $n_2(x)=\frac{x+2}{x+3}$

$\therefore n_1(x)=n_2(x)$ for all the values of $x\in\mathbb{R}-\{0,2,3,-3\}$

4 $\because n_1(x)=\frac{(x+4)(x-3)}{(x+4)(x+1)}$

$\therefore$ The domain of $n_1=\mathbb{R}-\{-4,-1\}$

, $n_1(x)=\frac{x-3}{x+1}$ $\because n_2(x)=\frac{(x-3)(x+1)}{(x+1)^2}$

$\therefore$ The domain of $n_2=\mathbb{R}-\{-1\}$

, $n_2(x)=\frac{x-3}{x+1}$ $\therefore n_1(x)=n_2(x)$

For all the values of $x\in\mathbb{R}-\{-4,-1\}$

3

1 $\because n_1(x)=\frac{3x}{3(x-2)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1=\mathbb{R}-\{2\}\\ , n_1(x)=\frac{x}{x-2}\end{array}\right\}(1)$

$\because n_2(x)=\frac{2x}{2(x-2)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2=\mathbb{R}-\{2\}\\ , n_2(x)=\frac{x}{x-2}\end{array}\right\}(2)$

From (1) and (2) : $\therefore n_1=n_2$

2 $\because n_1(x) = \dfrac{x}{(x-1)(x+1)}$

$$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{1, -1\}\end{array}\right\} (1)$$

$\because n_2(x) = \dfrac{5x}{5(x-1)(x+1)}$

$$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{1, -1\} \\ , n_2(x) = \dfrac{x}{(x-1)(x+1)}\end{array}\right\} (2)$$

From (1) and (2) : $\therefore n_1 = n_2$

3 $\because n_1(x) = \dfrac{2x}{2(x+2)}$

$$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{-2\} \\ , n_1(x) = \dfrac{x}{x+2}\end{array}\right\} (1)$$

$\because n_2(x) = \dfrac{x(x+2)}{(x+2)^2}$

$$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{-2\} \\ , n_2(x) = \dfrac{x}{x+2}\end{array}\right\} (2)$$

From (1) and (2) : $\therefore n_1 = n_2$

4 $\because n_1(x) = \dfrac{(x-1)(x^2+x+1)}{x(x^2+x+1)}$

$$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{0\} \\ , n_1(x) = \dfrac{x-1}{x}\end{array}\right\} (1)$$

$\because n_2(x) = \dfrac{(x-1)(x^2+1)}{x(x^2+1)}$

$$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{0\} \\ , n_2(x) = \dfrac{x-1}{x}\end{array}\right\} (2)$$

From (1) and (2) : $\therefore n_1 = n_2$

5 $\because n_1(x) = \dfrac{x(x-1)}{x^2(x-2)}$

$$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{0, 2\} \\ , n_1(x) = \dfrac{x-1}{x(x-2)}\end{array}\right\} (1)$$

$\because n_2(x) = \dfrac{(x-2)(x-1)}{x(x-2)^2}$

$$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{0, 2\} \\ , n_2(x) = \dfrac{x-1}{x(x-2)}\end{array}\right\} (2)$$

From (1) and (2) : $\therefore n_1 = n_2$

6 $\because n_1(x) = \dfrac{x^2}{x^2(x-1)}$

$$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{0, 1\} \\ , n_1(x) = \dfrac{1}{x-1}\end{array}\right\} (1)$$

$\because n_2(x) = \dfrac{x(x^2+x+1)}{x(x-1)(x^2+x+1)}$

$$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{0, 1\} \\ , n_2(x) = \dfrac{1}{x-1}\end{array}\right\} (2)$$

From (1) and (2) : $\therefore n_1 = n_2$

7 $\because n_1(x) = \dfrac{x(x^2+1)}{(x^3+x^2)+(x+1)} = \dfrac{x(x^2+1)}{x^2(x+1)+(x+1)} = \dfrac{x(x^2+1)}{(x+1)(x^2+1)}$

$$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{-1\} \\ , n_1(x) = \dfrac{x}{x+1}\end{array}\right\} (1)$$

$$\left.\begin{array}{l}\because n_2(x) = \dfrac{x}{x+1} \\ \therefore \text{The domain of } n_2 = \mathbb{R} - \{-1\}\end{array}\right\} (2)$$

From (1) and (2) : $\therefore n_1 = n_2$

4

1 $$\left.\begin{array}{l}\because n_1(x) = \dfrac{x-1}{x} \\ \therefore \text{The domain of } n_1 = \mathbb{R} - \{0\}\end{array}\right\} (1)$$

$$\left.\begin{array}{l}\because n_2(x) = \dfrac{x-1}{x} \\ \therefore \text{The domain of } n_2 = \mathbb{R} - \{0\}\end{array}\right\} (2)$$

From (1) and (2) : $\therefore n_1 = n_2$

because $n_1(x) = n_2(x)$

and the domain of n_1 = the domain of n_2

2 $\because n_1(x) = \dfrac{2x(x^2+3)}{(x-1)(x^2+3)}$

$$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{1\} \\ , n_1(x) = \dfrac{2x}{x-1}\end{array}\right\} (1)$$

$$\left.\begin{array}{l}\because n_2(x) = \dfrac{2x}{x-1} \\ \therefore \text{The domain of } n_2 = \mathbb{R} - \{1\}\end{array}\right\} (2)$$

From (1) and (2) : $\therefore n_1 = n_2$

because $n_1(x) = n_2(x)$

and the domain of n_1 = the domain of n_2

3 $\because n_1(x)=\frac{x+5}{(x-5)(x+5)}$

$\therefore$ The domain of $n_1=\mathbb{R}-\{5,-5\}$, $n_1(x)=\frac{1}{x-5}$ (1)

$\because n_2(x)=\frac{3}{3(x-5)}$

$\therefore$ The domain of $n_2=\mathbb{R}-\{5\}$, $\therefore n_2(x)=\frac{1}{x-5}$ (2)

From (1) and (2) : $\therefore n_1\neq n_2$

because the domain of $n_1\neq$ the domain of n_2

4 $\because n_1(x)=\frac{(x-3)(x+3)}{(x+1)(x+3)}$

$\therefore$ The domain of $n_1=\mathbb{R}-\{-1,-3\}$, $n_1(x)=\frac{x-3}{x+1}$ (1)

$\because n_2(x)=\frac{x-3}{x+1}$

$\therefore$ The domain of $n_2=\mathbb{R}-\{-1\}$ (2)

From (1) and (2) : $\therefore n_1\neq n_2$

because the domain of $n_1\neq$ the domain of n_2

5 $\because n_1(x)=\frac{(x-2)(x+2)}{(x-2)(x+3)}$

$\therefore$ The domain of $n_1=\mathbb{R}-\{2,-3\}$, $n_1(x)=\frac{x+2}{x+3}$ (1)

$\because n_2(x)=\frac{(x-3)(x+2)}{(x-3)(x+3)}$

$\therefore$ The domain of $n_2=\mathbb{R}-\{3,-3\}$, $n_2(x)=\frac{x+2}{x+3}$ (2)

From (1) and (2) : $\therefore n_1\neq n_2$

because the domain of $n_1\neq$ the domain of n_2

6 $\because n_1(x)=\frac{(x+1)(x^2-x+1)}{x(x^2-x+1)}$

$\therefore$ The domain of $n_1=\mathbb{R}-\{0\}$, $n_1(x)=\frac{x+1}{x}$ (1)

$\because n_2(x)=\frac{(x^3+x^2)+(x+1)}{x(x^2+1)}=\frac{x^2(x+1)+(x+1)}{x(x^2+1)}$

$=\frac{(x^2+1)(x+1)}{x(x^2+1)}$

$\therefore$ The domain of $n_2=\mathbb{R}-\{0\}$, $n_2(x)=\frac{x+1}{x}$ (2)

From (1) and (2) : $\therefore n_1=n_2$

because $n_1(x)=n_2(x)$

and the domain of $n_1=$ the domain of n_2

7 $\because n_1(x)=\frac{x}{x}-\frac{1}{x}=\frac{x-1}{x}$

$\therefore$ The domain of $n_1=\mathbb{R}-\{0\}$, $n_1(x)=\frac{x-1}{x}$ (1)

, $\because$ The domain of $n_2=\mathbb{R}-\{0\}$, $n_2(x)=-\frac{x-1}{x}$ (2)

From (1) and (2) : $\therefore n_1\neq n_2$

because : $n_1(x)\neq n_2(x)$

5

1 -1	2 $2x-1$	3 $\mathbb{R}-\{0,2\}$	
4 $\mathbb{R}-\{0,-1\}$	5 3	6 -2	7 4

6

1 b　2 b　3 c　4 c

Excellent pupils

1

$\because n(x)=\frac{(5x+3+3x-1)(5x+3-3x+1)}{8(4x+1)}$

$=\frac{(8x+2)(2x+4)}{8(4x+1)}=\frac{2(4x+1)\times 2(x+2)}{8(4x+1)}$

$=\frac{4(4x+1)(x+2)}{8(4x+1)}$

$\therefore$ The domain of $n=\mathbb{R}-\{-\frac{1}{4}\}$

, $n(x)=\frac{x+2}{2}$　　$\therefore n(0)=\frac{0+2}{2}=1$

$\because (n(x))^2=4$　　$\therefore \left(\frac{x+2}{2}\right)^2=4$

$\therefore \frac{(x+2)^2}{4}=4$　　$\therefore (x+2)^2=16$

$\therefore x+2=\pm 4$　　$\therefore x=2$ or $x=-6$

2

$\because$ The domain of $n_1 = \mathbb{R} - \{-a\}$

$\therefore -a \notin$ the domain of n_1 , $n_1 = n_2$

$\therefore -a \notin$ the domain of n_2

$\therefore -a$ is a root of the equation $x^3 + a x^2 + x + 5 = 0$

$\therefore (-a)^3 + a(-a)^2 - a + 5 = 0$

$\therefore -a^3 + a^3 - a + 5 = 0$ $\therefore -a + 5 = 0$

$\therefore a = 5$ $\therefore n_1(x) = \frac{x}{x+5}$

$\therefore n_2(x) = \frac{x^3 + b x}{x^3 + 5x^2 + x + 5}$

$\because n_1 = n_2$ $\therefore n_1(x) = n_2(x)$

$\therefore \frac{x}{x+5} = \frac{x^3 + b x}{x^3 + 5x^2 + x + 5}$

$\therefore \frac{x}{x+5} = \frac{x(x^2+b)}{x^2(x+5) + (x+5)}$

$\therefore \frac{x}{x+5} = \frac{x(x^2+b)}{(x^2+1)(x+5)}$ $\therefore b = 1$

Answers of Exercise 7

1

1 b 2 a 3 c 4 a

5 b 6 b 7 b

2

1 $n(x) = \frac{2x}{x+2} + \frac{4}{x+2}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2\}$

$\therefore n(x) = \frac{2x+4}{x+2} = \frac{2(x+2)}{x+2} = 2$

2 $\because n(x) = \frac{3x}{x-3} - \frac{9}{x-3}$

$\therefore$ The domain of $n = \mathbb{R} - \{3\}$

$\therefore n(x) = \frac{3x-9}{x-3} = \frac{3(x-3)}{x-3} = 3$

3 $\because n(x) = \frac{2x^2}{2x+5} + \frac{2x^2-25}{2x+5}$

$\therefore$ The domain of $n = \mathbb{R} - \{-\frac{5}{2}\}$

$\therefore n(x) = \frac{4x^2-25}{2x+5} = \frac{(2x+5)(2x-5)}{2x+5} = 2x-5$

4 $\because n(x) = \frac{x^2}{(x-1)(x+1)} - \frac{x}{(x-1)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1, -1\}$

$\therefore n(x) = \frac{x^2-x}{(x-1)(x+1)} = \frac{x(x-1)}{(x-1)(x+1)} = \frac{x}{x+1}$

3

1 $\because n(x) = \frac{x}{x(x+2)} + \frac{x+1}{x+2}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, -2\}$

$\therefore n(x) = \frac{1}{x+2} + \frac{x+1}{x+2} = \frac{x+2}{x+2} = 1$

2 $\because n(x) = \frac{x}{x-4} - \frac{x+4}{(x+4)(x-4)}$

$\therefore$ The domain of $n = \mathbb{R} - \{4, -4\}$

$\therefore n(x) = \frac{x}{x-4} - \frac{1}{x-4} = \frac{x-1}{x-4}$

3 $\because n(x) = \frac{(x-2)(x+3)}{x+3} + \frac{(x-2)(x+2)}{x+2}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3, -2\}$

$\therefore n(x) = (x-2) + (x-2) = 2x - 4$

4 $\because n(x) = \frac{x(x+3)}{(x+3)(x-1)} - \frac{x-2}{(x-2)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3, 1, 2\}$

, $n(x) = \frac{x}{x-1} - \frac{1}{x-1} = \frac{x-1}{x-1} = 1$

5 $\because n(x) = \frac{x^2-2x+4}{(x+2)(x^2-2x+4)} + \frac{(x-1)(x+1)}{(x+2)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2, 1\}$

$\therefore n(x) = \frac{1}{x+2} + \frac{x+1}{x+2} = \frac{x+2}{x+2} = 1$

6 $\because n(x) = \frac{2(x+3)}{(x+3)(x-2)} - \frac{x(x-6)}{(x-6)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3, 2, 6\}$

$\therefore n(x) = \frac{2}{x-2} - \frac{x}{x-2} = \frac{2-x}{x-2}$

$= \frac{-(x-2)}{x-2} = -1$

7 $\because n(x) = \frac{x-6}{(2x-3)(x-6)} + \frac{x-5}{(2x-3)(x-5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{\frac{3}{2}, 6, 5\}$

$\therefore n(x) = \frac{1}{2x-3} + \frac{1}{2x-3} = \frac{2}{2x-3}$

8 $\because n(x) = \frac{(x+2)(x-1)}{(x+1)(x-1)} - \frac{x+5}{(x+5)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1, -1, -5\}$

$\therefore n(x) = \frac{x+2}{x+1} - \frac{1}{x+1} = \frac{x+1}{x+1} = 1$

9 $\because n(x) = \frac{3(x+5)}{(x+2)(x+5)} + \frac{(2x+1)(x-2)}{(x-2)(x+2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2, -5, 2\}$

$\therefore n(x) = \frac{3}{x+2} + \frac{2x+1}{x+2} = \frac{2x+4}{x+2}$

$= \frac{2(x+2)}{x+2} = 2$

10 $\because n(x) = \frac{3(x-2)}{(x-2)(x+2)} - \frac{x(x-3)}{x(x+2)(x-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -2, 0, 3\}$

$\therefore n(x) = \frac{3}{x+2} - \frac{1}{x+2} = \frac{2}{x+2}$

4

1 $\because n(x) = \frac{x-2}{x} + \frac{3+x}{2x}$

$\therefore$ The domain of $n = \mathbb{R} - \{0\}$

$\therefore n(x) = \frac{2(x-2)+3+x}{2x} = \frac{2x-4+3+x}{2x}$

$= \frac{3x-1}{2x}$

2 $\because n(x) = \frac{x}{x-2} - \frac{x}{x+2}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -2\}$

$\therefore n(x) = \frac{x(x+2) - x(x-2)}{(x-2)(x+2)}$

$= \frac{x^2+2x-x^2+2x}{(x-2)(x+2)} = \frac{4x}{(x-2)(x+2)}$

3 $\because n(x) = \frac{2}{x+3} + \frac{x+3}{x(x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3, 0\}$

$\therefore n(x) = \frac{2x+x+3}{x(x+3)} = \frac{3x+3}{x(x+3)}$

4 $\because n(x) = \frac{x+3}{2x} - \frac{x}{2x-1}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, \frac{1}{2}\}$

$\therefore n(x) = \frac{(x+3)(2x-1)-2x^2}{2x(2x-1)}$

$= \frac{2x^2+5x-3-2x^2}{2x(2x-1)} = \frac{5x-3}{2x(2x-1)}$

5 $\because n(x) = \frac{x}{x(x+2)} + \frac{x+2}{(x+2)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, 2, -2\}$

$\therefore n(x) = \frac{1}{x+2} + \frac{1}{x-2} = \frac{x-2+x+2}{(x+2)(x-2)}$

$= \frac{2x}{(x+2)(x-2)}$

6 $\because n(x) = \frac{2x-1}{(x-2)(x+1)} - \frac{1}{x-2}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -1\}$

$\therefore n(x) = \frac{2x-1-x-1}{(x-2)(x+1)} = \frac{x-2}{(x-2)(x+1)} = \frac{1}{x+1}$

7 $\because n(x) = \frac{3x-4}{(x-3)(x-2)} + \frac{2(x+3)}{(x+3)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, 2, -3\}$

$\therefore n(x) = \frac{3x-4}{(x-3)(x-2)} + \frac{2}{x-2}$

$= \frac{3x-4+2x-6}{(x-3)(x-2)} = \frac{5x-10}{(x-3)(x-2)}$

$= \frac{5(x-2)}{(x-3)(x-2)} = \frac{5}{x-3}$

8 $\because n(x) = \frac{x^2+2x+4}{(x-2)(x^2+2x+4)} + \frac{(x-4)(x+3)}{(x-3)(x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, 3, -3\}$

$\therefore n(x) = \frac{1}{x-2} + \frac{x-4}{x-3} = \frac{x-3+(x-2)(x-4)}{(x-2)(x-3)}$

$= \frac{x-3+x^2-6x+8}{(x-2)(x-3)} = \frac{x^2-5x+5}{(x-2)(x-3)}$

9 $\because n(x) = \frac{3x-2}{(3x-2)(x+1)} - \frac{3x-4}{(2x-5)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{\frac{2}{3}, -1, \frac{5}{2}\}$

$\therefore n(x) = \frac{1}{x+1} - \frac{3x-4}{(2x-5)(x+1)}$

$= \frac{2x-5-3x+4}{(2x-5)(x+1)} = \frac{-(x+1)}{(2x-5)(x+1)}$

$= \frac{-1}{2x-5}$

10 $\because$ The domain of $n = \mathbb{R} - \{3\}$

$\therefore n(x) = \frac{(x+3)(x-3)-x^2}{x-3} = \frac{x^2-9-x^2}{x-3}$

$= \frac{-9}{x-3}$

5

1 $\because n(x) = \frac{x^2}{x-1} - \frac{x}{x-1}$

$\therefore$ The domain of $n = \mathbb{R} - \{1\}$

$, n(x) = \frac{x^2 - x}{x-1} = \frac{x(x-1)}{x-1} = x$

2 $\because n(x) = \frac{3x(x+2)}{(x-2)(x+2)} - \frac{6}{x-2}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -2\}$

$\therefore n(x) = \frac{3x}{x-2} - \frac{6}{x-2} = \frac{3x-6}{x-2} = \frac{3(x-2)}{x-2} = 3$

3 $\because n(x) = \frac{x^2+2x+4}{(x-2)(x^2+2x+4)} + \frac{(x+3)(x-3)}{(x+3)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -3\}$

$\therefore n(x) = \frac{1}{x-2} + \frac{x-3}{x-2} = \frac{x-2}{x-2} = 1$

4 $\because n(x) = \frac{x(x+1)}{(x-1)(x+1)} + \frac{x-5}{(x-5)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-1, 1, 5\}$

$\therefore n(x) = \frac{x}{x-1} + \frac{1}{x-1} = \frac{x+1}{x-1}$

5 $\because n(x) = \frac{2x(x-4)}{(2x-3)(x-4)} - \frac{3(2x+3)}{(2x-3)(2x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \left\{\frac{3}{2}, 4, \frac{-3}{2}\right\}$

$\therefore n(x) = \frac{2x}{2x-3} - \frac{3}{2x-3} = \frac{2x-3}{2x-3} = 1$

6 $\because n(x) = \frac{x+3}{x^2-9} - \frac{2x+2}{x^2-2x-3}$

$= \frac{(x+3)}{(x-3)(x+3)} - \frac{2(x+1)}{(x-3)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, -3, -1\}$

$\therefore n(x) = \frac{1}{x-3} - \frac{2}{x-3} = \frac{-1}{x-3}$

7 $\because n(x) = \frac{3x-6}{x^2-4} + \frac{9}{x^2+x-2}$

$= \frac{3(x-2)}{(x-2)(x+2)} + \frac{9}{(x+2)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -2, 1\}$

$\therefore n(x) = \frac{3}{x+2} + \frac{9}{(x+2)(x-1)}$

$= \frac{3(x-1)+9}{(x+2)(x-1)} = \frac{3x-3+9}{(x+2)(x-1)}$

$= \frac{3x+6}{(x+2)(x-1)} = \frac{3(x+2)}{(x+2)(x-1)} = \frac{3}{x-1}$

8 $\because n(x) = \frac{x-5}{(x-5)(2x-3)} + \frac{x+3}{-(2x-3)(x-6)}$

$\therefore$ The domain of $n = \mathbb{R} - \left\{5, \frac{3}{2}, 6\right\}$

$\therefore n(x) = \frac{1}{2x-3} - \frac{x+3}{(2x-3)(x-6)}$

$= \frac{x-6-x-3}{(2x-3)(x-6)} = \frac{-9}{(2x-3)(x-6)}$

9 $\because n(x) = \frac{(x-2)(x+2)}{(x+2)(x-1)} + \frac{-5(2x-1)}{-(2x-1)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \left\{-2, 1, \frac{1}{2}\right\}$

$\therefore n(x) = \frac{x-2}{x-1} + \frac{5}{x-1} = \frac{x+3}{x-1}$

10 $\because n(x) = \frac{x-3}{(x-3)(x-4)} + \frac{x-3}{x-3}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, 4\}$

$\therefore n(x) = \frac{1}{x-4} + 1 = \frac{1}{x-4} + \frac{x-4}{x-4} = \frac{x-3}{x-4}$

6

$\because n(x) = \frac{x(x-5)}{(x-3)(x-5)} - \frac{x^2+3x+9}{(x-3)(x^2+3x+9)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, 5\}$

$\therefore n(x) = \frac{x}{x-3} - \frac{1}{x-3} = \frac{x-1}{x-3}$

$, n(1) = 0$, $n(5)$ is undefined

7

$n(x) = \frac{x+3}{(x+3)^2} + \frac{x+2}{x+3}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3\}$

$\therefore n(x) = \frac{1}{x+3} + \frac{x+2}{x+3} = \frac{x+3}{x+3} = 1$

, $n(-3)$ is undefined because $-3 \notin$ the domain of n

, $n(2016) = 1$

8

$\because n(x) = \frac{12}{3(2x-1)(2x+1)} - \frac{2}{2x(2x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \left\{0, -\frac{1}{2}, \frac{1}{2}\right\}$

$\therefore n(x) = \frac{4}{(2x-1)(2x+1)} - \frac{1}{x(2x-1)}$

$= \frac{4x-(2x+1)}{x(2x-1)(2x+1)}$

$= \frac{2x-1}{x(2x-1)(2x+1)} = \frac{1}{x(2x+1)}$

, $n(0)$ is undefined because $0 \notin$ the domain of n

$, n(-1) = \frac{1}{-1(2 \times -1 + 1)} = 1$

9

$\because n(x) = \frac{x(x-2)}{x^2(x^2-3x+2)} + \frac{x^2-4}{x^2+x-2}$

$= \frac{x(x-2)}{x^2(x-2)(x-1)} + \frac{(x-2)(x+2)}{(x-1)(x+2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, 2, 1, -2\}$

$, n(x) = \frac{1}{x(x-1)} + \frac{x-2}{x-1}$

$= \frac{1+x^2-2x}{x(x-1)} = \frac{x^2-2x+1}{x(x-1)}$

$= \frac{(x-1)^2}{x(x-1)} = \frac{x-1}{x}$

$, \because n(x) = 0 \quad \therefore \frac{x-1}{x} = 0 \quad \therefore x - 1 = 0$

$\therefore x = 1 \quad \therefore$ The S.S. $= \varnothing$

10

$\because n(x) = \frac{x^2+x+1}{x^4-x} - \frac{x+3}{x^2+2x-3}$

$= \frac{x^2+x+1}{x(x-1)(x^2+x+1)} - \frac{x+3}{(x+3)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, 1, -3\}$

$, n(x) = \frac{1}{x(x-1)} - \frac{1}{x-1} = \frac{1-x}{x(x-1)}$

$= \frac{-(x-1)}{x(x-1)} = \frac{-1}{x}$

$, \because n(a) = -2 \quad \therefore \frac{-1}{a} = -2$

$\therefore -2a = -1 \quad \therefore a = \frac{1}{2}$

11

$\because z(f_1) = \{5\} \quad \therefore$ at $x = 5$

$\therefore x - a = 0 \quad \therefore 5 - a = 0 \quad \therefore a = 5$

$, \because$ the domain of $f_1 = \mathbb{R} - \{3\}$

$\therefore$ at $x = 3 \quad \therefore x + b = 0$

$\therefore 3 + b = 0 \quad \therefore b = -3 \quad \therefore f_1(x) = \frac{x-5}{x-3}$

$\therefore f_1(x) + f_2(x) = \frac{x-5}{x-3} + \frac{x-1}{x-3}$

$\therefore$ The domain $= \mathbb{R} - \{3\}$

$, f_1(x) + f_2(x) = \frac{x-5+x-1}{x-3} = \frac{2x-6}{x-3} = \frac{2(x-3)}{x-3} = 2$

12

$\because$ The domain of $n = \mathbb{R} - \{0, 4\} \quad \therefore a = -4$

$\therefore n(x) = \frac{b}{x} + \frac{9}{x-4} \quad , \because n(5) = 2$

$\therefore \frac{b}{5} + 9 = 2 \quad \therefore \frac{b}{5} = -7 \quad \therefore b = -35$

Excellent pupils

1

$\because n(x) = \frac{5(x+2)}{(x+2)(x-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2, 3\} \quad \therefore n(x) = \frac{5}{x-3}$

$\because k(x)$ is the additive inverse of $n(x)$

$\therefore k(x) = \frac{5}{3-x}$ and the domain of $k = \mathbb{R} - \{-2, 3\}$

$\therefore k(2) = \frac{5}{3-2} = 5$

, $k(3)$ is undefined because $3 \notin$ the domain of k

2

1. $\frac{4x}{x-1} - \frac{3x}{x+1} = 1$

$\therefore \frac{4x(x+1) - 3x(x-1)}{(x-1)(x+1)} = 1$

$\therefore 4x(x+1) - 3x(x-1) = (x-1)(x+1)$

$\therefore 4x^2 + 4x - 3x^2 + 3x = x^2 - 1$

$\therefore 7x = -1 \quad \therefore x = -\frac{1}{7}$

2. $\frac{3}{\sqrt{x}-\sqrt{7}} - \frac{3}{\sqrt{x}+\sqrt{7}} = \frac{1}{2\sqrt{7}}$

$\therefore \frac{3(\sqrt{x}+\sqrt{7}) - 3(\sqrt{x}-\sqrt{7})}{(\sqrt{x}-\sqrt{7})(\sqrt{x}+\sqrt{7})} = \frac{1}{2\sqrt{7}}$

where $x \neq \pm\sqrt{7}$

$\therefore \frac{6\sqrt{7}}{x-7} = \frac{1}{2\sqrt{7}} \quad \therefore x - 7 = 84 \quad \therefore x = 91$

3. $\frac{1}{(x+1)(x-5)} - \frac{4x+5}{x^2(x+1)(x-5)} = 1$

where $x \notin \{0, -1, 5\}$

$\therefore \frac{x^2-4x-5}{x^2(x+1)(x-5)} = 1$

$\therefore \frac{(x+1)(x-5)}{x^2(x+1)(x-5)} = 1$

$\therefore x^2 = 1 \quad \therefore x = 1$ or $x = -1$ (refused)

Answers of Exercise 8

1

1 d	2 c	3 d	4 d
5 d	6 d	7 d	8 b

2

1 $\mathbb{R}-\{2\}$ 2 $X-2$

3 $\mathbb{R}-\{2\}$ 4 $\mathbb{R}-\{5, 2\}$

5 $\frac{5}{X}$, $\mathbb{R}-\{0, -3\}$ 6 -1 7 -2

3

1 $n(X)=\frac{3(X-5)}{X+3}\times\frac{4(X+3)}{5(X-5)}$

$\therefore$ The domain of $n=\mathbb{R}-\{-3, 5\}$

, $n(X)=\frac{12}{5}$

2 $n(X)=\frac{X+2}{(X-2)(X+2)}\times\frac{2(X-2)}{X-3}$

$\therefore$ The domain of $n=\mathbb{R}-\{2, -2, 3\}$

, $n(X)=\frac{2}{X-3}$

3 $n(X)=\frac{(X+1)^2}{2(X-4)}\times\frac{X-4}{X+1}$

$\therefore$ The domain of $n=\mathbb{R}-\{4, -1\}$

, $n(X)=\frac{X+1}{2}$

4 $n(X)=\frac{(X-1)(X^2+X+1)}{(X-1)^2}\times\frac{2(X-1)}{(X^2+X+1)}$

$\therefore$ The domain of $n=\mathbb{R}-\{1\}$

, $n(X)=2$

5 $n(X)=\frac{2(X-5)}{(X-5)(X+5)}\times\frac{X(X+5)}{X-3}$

$\therefore$ The domain of $n=\mathbb{R}-\{5, -5, 3\}$

, $n(X)=\frac{2X}{X-3}$

6 $n(X)=\frac{(X-4)(X+1)}{(X-1)(X+1)}\times\frac{X(X-1)}{X(X+3)}$

$\therefore$ The domain of $n=\mathbb{R}-\{0, 1, -1, -3\}$

, $n(X)=\frac{X-4}{X+3}$

7 $n(X)=\frac{3X(2X+1)}{X+2}\times\frac{(X+2)^2}{3(2X+1)}$

$\therefore$ The domain of $n=\mathbb{R}-\left\{-2, -\frac{1}{2}\right\}$

, $n(X)=X(X+2)=X^2+2X$

8 $n(X)=\frac{(X-1)(X^2+X+1)}{X(X-1)}\times\frac{X+3}{X^2+X+1}$

$\therefore$ The domain of $n=\mathbb{R}-\{0, 1\}$

, $n(X)=\frac{X+3}{X}$

9 $n(X)=\frac{5(X+1)}{X+6}\times\frac{(X+6)(X-3)}{(X-3)(X+1)}$

$\therefore$ The domain of $n=\mathbb{R}-\{-6, 3, -1\}$

, $n(X)=5$, $n(2)=5$

10 $n(X)=\frac{X(X+2)}{(X-3)(X+3X+9)}\times\frac{X^2+3X+9}{X+2}$

$\therefore$ The domain of $n=\mathbb{R}-\{3, -2\}$

, $n(X)=\frac{X}{X-3}$, $n(6)=\frac{6}{6-3}=2$

, $n(-2)$ is undefined because $-2\notin$ the domain of n

11 $n(X)=\frac{(X-2)(X^2+2X+4)}{(X-2)(X+5)}\times\frac{2(X+3)}{X^2+2X+4}$

$\therefore$ The domain of $n=\mathbb{R}-\{2, -5\}$

$\therefore n(X)=\frac{2(X+3)}{X+5}$ $\therefore n^{-1}(X)=\frac{X+5}{2(X+3)}$

, the domain of $n^{-1}=\mathbb{R}-\{2, -5, -3\}$

$\therefore n^{-1}(1)=\frac{1+5}{2(1+3)}=\frac{6}{8}=\frac{3}{4}$

12 $n(X)=\frac{2(X-2)(X^2+2X+4)}{(X-2)(X-5)}$

$\times\frac{(3X+5)(X-5)}{(X^2+2X+4)}$

$\therefore$ The domain of $n=\mathbb{R}-\{2, 5\}$

, $n(X)=2(3X+5)=6X+10$

13 $n(X)=\frac{(X-3)(X+1)}{5(X-3)(X^2+3X+9)}\times\frac{5(X^2+3X+9)}{X+1}$

$\therefore$ The domain of $n=\mathbb{R}-\{3, -1\}$

, $n(X)=1$

14 $n(X)=\frac{X-2}{X(2X-3)}\times\frac{-(2X-3)(2X+3)}{-(X-2)(2X+3)}$

$\therefore$ The domain of $n=\mathbb{R}-\left\{0, \frac{3}{2}, 2, -\frac{3}{2}\right\}$

, $n(X)=\frac{1}{X}$

15 $n(X)=\frac{(X-6)^2}{X(X-6)}\times\frac{4(X+6)}{-(X-6)(X+6)}$

$\therefore$ The domain of $n=\mathbb{R}-\{0, 6, -6\}$

, $n(X)=\frac{-4}{X}$

4

1 $n(X)=\frac{3(X-5)}{X+3}\times\frac{4(X+3)}{5(X-5)}$

$\therefore$ The domain of $n=\mathbb{R}-\{-3, 5\}$

, $n(X)=\frac{12}{5}$

2 $n(X)=\frac{X-1}{(X-1)(X+1)}\times\frac{(X-5)(X+1)}{X(X-5)}$

$\therefore$ The domain of $n=\mathbb{R}-\{1, -1, 0, 5\}$

, $n(X)=\frac{1}{X}$

3 $n(X)=\frac{(X+3)(X-1)}{X+3}\times\frac{(X+1)}{(X-1)(X+1)}$

$\therefore$ The domain of $n=\mathbb{R}-\{-3, 1, -1\}$

, $n(X)=1$

4 $n(x) = \dfrac{(x-5)(x+3)}{(x+3)(x-3)} \times \dfrac{(x-3)^2}{2(x-5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3, 3, 5\}$

, $n(x) = \frac{1}{2}(x-3)$

5 $n(x) = \dfrac{(x-2)(x^2+2x+4)}{(x+3)(x-2)} \times \dfrac{2(x+3)}{x^2+2x+4}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3, 2\}$, $n(x) = 2$

6 $n(x) = \dfrac{(x-1)^2}{(x-1)(x^2+x+1)} \times \dfrac{x^2+x+1}{x-1}$

$\therefore$ The domain of $n = \mathbb{R} - \{1\}$, $n(x) = 1$

7 $n(x) = \dfrac{(x-3)(x+3x+9)}{(x-3)(x+3)} \times \dfrac{2x}{x(x^2+3x+9)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, -3, 0\}$

, $n(x) = \dfrac{2}{x+3}$

8 $n(x) = \dfrac{x(x-3)}{(2x+3)(x-2)} \times \dfrac{(2x-3)(2x+3)}{x(2x-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-\frac{3}{2}, 2, 0, \frac{3}{2}\}$

, $n(x) = \dfrac{x-3}{x-2}$

9 $n(x) = \dfrac{(x+1)(x-2)}{(x-2)(x+3)} \times \dfrac{(x+3)(x-5)}{(x+1)(x-5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -3, -1, 5\}$

, $n(x) = 1$

10 $n(x) = \dfrac{(x-3)(x+3)}{x(2x+3)} \times \dfrac{(2x-3)(2x+3)}{3(x+5)(x-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, -\frac{3}{2}, -5, 3, \frac{3}{2}\}$

, $n(x) = \dfrac{(x+3)(2x-3)}{3x(x+5)}$

11 $n(x) = \dfrac{(x-2)(x+2)}{(3x-5)(x+2)} \times \dfrac{x(3x-5)}{(x-2)(6x+7)}$

$\therefore$ The domain of $n = \mathbb{R} - \{\frac{5}{3}, -2, 2, -\frac{7}{6}, 0\}$

, $n(x) = \dfrac{x}{6x+7}$

12 $n(x) = \dfrac{(x-1)(x-2)}{-(x-1)(x+1)} \times \dfrac{(x-1)(x-5)}{3(x-5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1, -1, 5\}$

, $n(x) = \dfrac{(x-2)(x-1)}{-3(x+1)}$

13 $n(x) = \dfrac{3(x-3)}{(x-2)(x-3)} \times \dfrac{-(x-2)(x+3)}{2(x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, 3, -3\}$

, $n(x) = -\frac{3}{2}$

14 $n(x) = \dfrac{x-2}{x(2x-3)} \times \dfrac{(2x+3)(2x-3)}{(2x+3)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, \frac{3}{2}, -\frac{3}{2}, 2\}$

, $n(x) = \dfrac{1}{x}$

15 $n(x) = \dfrac{(x+3)(x-2)}{(x+3)(x+2)} \div \dfrac{x(x^2+1) - 2(x^2+1)}{x^2(x+2) + (x+2)}$

$= \dfrac{(x+3)(x-2)}{(x+3)(x+2)} \times \dfrac{(x^2+1)(x+2)}{(x^2+1)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3, -2, 2\}$, $n(x) = 1$

5

First : $n(x) = \dfrac{x(x-2)}{(x-2)(x^2+2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2\}$

, $n(x) = \dfrac{x}{x^2+2}$ $\therefore n^{-1}(x) = \dfrac{x^2+2}{x}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{2, 0\}$

Second : $\dfrac{x^2+2}{x} = 3$ $\therefore x^2 - 3x + 2 = 0$

$\therefore (x-2)(x-1) = 0$

$\therefore x = 2$ (refused) or $x = 1$

6

$\because n(x) = \dfrac{x(x+2)(x+1)}{x(x+2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, -2\}$

, $n(x) = x + 1$, $n^{-1}(x) = \dfrac{1}{x+1}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, -2, -1\}$

$n^{-1}(-2)$ is undefined because $-2 \notin$ the domain of n^{-1}

7

$\because n(x) = \dfrac{x(x-2)+x}{x-2} = \dfrac{x^2-2x+x}{x-2}$

$= \dfrac{x^2-x}{x-2} = \dfrac{x(x-1)}{x-2}$

$\therefore n^{-1}(x) = \dfrac{x-2}{x(x-1)}$

, the domain of $n^{-1} = \mathbb{R} - \{2, 1, 0\}$

8

$n(x) = \dfrac{(x-7)(x+7)}{(x-2)(x^2+2x+4)} \times \dfrac{x-2}{x+7}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -7\}$

, $n(x) = \dfrac{x-7}{x^2+2x+4}$, $n(1) = \dfrac{1-7}{1+2+4} = -\dfrac{6}{7}$

9

$n(x) = \frac{x(x+1)(x-2)}{(x-2)(x-3)} \times \frac{(x-3)(x+5)}{x(x+1)(x+5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, 3, 0, -1, -5\}$

$n(x) = 1$, $n(7) = 1$

, $n(3)$ is undefined because $3 \notin$ the domain of n

10

$f(x) = \frac{(x+3)(x-5)}{(x-3)(x+3)} \times \frac{x(x-3)}{(x-5)(x+5)}$

$\therefore$ The domain of $f = \mathbb{R} - \{3, -3, 5, -5, 0\}$

, $f(x) = \frac{x}{x+5}$ $\quad \because f(a) = \frac{1}{3}$ $\quad \therefore \frac{a}{a+5} = \frac{1}{3}$

$\therefore 3a = a + 5$ $\quad \therefore 2a = 5$ $\quad \therefore a = \frac{5}{2}$

11

$n_1(x) = \frac{(2x+7)}{(2x-1)(2x+1)} \times \frac{(2x-1)(4x^2+2x+1)}{(2x+7)(2x-1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-\frac{7}{2}, -\frac{1}{2}, \frac{1}{2}\}$

$\therefore n_1(x) = \frac{4x^2+2x+1}{(2x-1)(2x+1)}$

, $n_2(x) = \frac{3(2x-1)(2x+1)}{3(4x^2+2x+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R}$

, $n_2(x) = \frac{(2x-1)(2x+1)}{4x^2+2x+1}$

, $n(x) = \frac{4x^2+2x+1}{(2x-1)(2x+1)} \times \frac{(2x-1)(2x+1)}{4x^2+2x+1}$

, $n(x) = 1$

Where the domain of $n = \mathbb{R} - \{-\frac{7}{2}, -\frac{1}{2}, \frac{1}{2}\}$

12

$n(x) = \left(\frac{3(x+5)}{(x+5)(x+2)} + \frac{2x+1}{x+2}\right) \times \frac{(x-3)(x^2+3x+9)}{(x^2+3x+9)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-5, -2\}$

, $n(x) = \left(\frac{3}{x+2} + \frac{2x+1}{x+2}\right) \times (x-3)$

$= \left(\frac{2(x+2)}{x+2}\right)(x-3) = 2(x-3)$

$\because n(x) = 2$ $\quad \therefore 2(x-3) = 2$

$\therefore x - 3 = 1$ $\quad \therefore x = 4$

Excellent pupils

1

$\because n_1(x) \times n_1^{-1}(x) = 1$

$\therefore \frac{x^2 - ax + 12}{(x-4)(x+1)} \times \frac{(x+1)}{x-3} = 1$

$\therefore x^2 - ax + 12 = (x-4)(x-3) = x^2 - 7x + 12$

$\therefore a = 7$

2

$n(x) = \left(x + \frac{1}{x-2}\right) \div \left(4x + \frac{4}{x-2}\right)$

$= \frac{x(x-2)+1}{x-2} \div \frac{4x(x-2)+4}{x-2}$

$= \frac{x^2-2x+1}{x-2} \div \frac{4x^2-8x+4}{x-2}$

$= \frac{(x-1)^2}{x-2} \times \frac{x-2}{4(x-1)^2}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, 1\}$, $n(x) = \frac{1}{4}$

$\therefore n(1)$ is undefined because $1 \notin$ the domain of n

, $n(8) = \frac{1}{4}$

Answers of exams on unit two

Model 1

1

1 d	2 c	3 c
4 c	5 c	6 a

2

[a] Prove by yourself.

[b] The domain of $n = \mathbb{R} - \{2, -2, 3\}$, $n(x) = 1$

3

[a] $a = 1$, $b = -5$

[b] The domain of $n = \mathbb{R} - \{2, -2, 0, -1\}$

, $n(x) = 3$

4

[a] 1 $n^{-1}(x) = \frac{x-2}{x}$

, the domain of $n^{-1} = \mathbb{R} - \{3, 2, 0\}$

2 $x = -2$

[b] 1 $n(x) = \frac{x}{(x+2)(x-3)}$

, the domain of $n = \mathbb{R} - \{-2, 2, -3, 3\}$

2 $n(-1) = \frac{1}{4}$

5

[a] The domain of $n = \mathbb{R} - \{-1, 0, 1\}$

, $n(x) = \frac{x}{x-1}$, $n(2) = 2$, $n(1)$ is undefined

[b] The domain of $n = \mathbb{R} - \left\{1, 2, 0, \frac{-3}{2}\right\}$

, $n(x) = \frac{2x+3}{x}$

Model 2

1

1 d	2 a	3 b
4 b	5 a	6 a

2

[a] Prove by yourself.

[b] The domain of $n = \mathbb{R} - \{2, -2, 3\}$, $n(x) = 1$

3

[a] The domain of $n = \mathbb{R} - \{2, -2, 3\}$

, $n(x) = \frac{1}{x-3}$

[b] $a = 6$, $b = -2$

4

[a] The domain of $n = \mathbb{R} - \{0, 1, -3\}$

, $n(x) = \frac{-1}{x}$, $a = \frac{1}{2}$

[b] The domain of $n = \mathbb{R} - \{2, -3\}$

, $n(x) = 1$

5

[a] $n_1(x) = n_2(x)$ for all values of $x \in \mathbb{R} - \{2, -2, 1\}$

[b] The domain of $n = \mathbb{R} - \{0\}$

, $n(x) = \frac{1}{4}$

Answers of unit three

Answers of Exercise 9

1

1 b 2 c 3 d 4 d
5 b 6 c 7 a 8 a
9 a 10 b 11 d

2

1 0.7 2 0.2 3 0.65

3

1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{2}{3} - \frac{1}{3} = \frac{5}{6}$

2 $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \frac{5}{8} = \frac{3}{8} + \frac{1}{2} - P(A \cap B)$

$\therefore P(A \cap B) = \frac{3}{8} + \frac{1}{2} - \frac{5}{8} = \frac{1}{4}$

3 [i] $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{8} = \frac{17}{24}$

[ii] $\because$ A and B are two mutually exclusive events

$\therefore P(A \cap B) = \text{zero}$

$\therefore P(A \cup B) = P(A) + P(B) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

4

1 $\because$ A and B are two mutually exclusive events

$\therefore P(A \cap B) = \text{zero}$ $\therefore P(A \cup B) = P(A) + P(B)$

$\therefore \frac{1}{3} = P(A) + \frac{1}{12}$

$\therefore P(A) = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$

2 $B \subset A$ $\therefore P(A) = P(A \cup B) = \frac{1}{3}$

5 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \frac{4}{9} = \frac{1}{6} + P(B) - \frac{1}{18}$

$\therefore P(B) = \frac{4}{9} - \frac{1}{6} + \frac{1}{18} = \frac{1}{3}$

6

1 $\because A \subset B$ $\therefore A \cap B = A$

$\therefore P(A) = P(A \cap B) = \frac{2}{5}$

2 $\because A \subset B$ $\therefore A \cup B = B$

$\therefore P(B) = P(A \cup B) = \frac{4}{5}$

7

1 $\because A \subset B$ $\therefore P(B) = P(A \cup B)$

$\therefore 2x = 0.8$ $\therefore x = 0.4$

2 $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore 0.8 = 0.5 + 2x - 0.1$

$\therefore 2x = 0.8 - 0.5 + 0.1 = 0.4$

$\therefore x = 0.2$

8

1 $P(A \cap B) = \frac{1}{10}$, $P(A \cup B) = \frac{6}{10} = \frac{3}{5}$

2 $P(A \cap C) = \text{zero}$, $P(A \cup C) = \frac{7}{10}$

3 $P(B \cap C) = \text{zero}$, $P(B \cup C) = \frac{6}{10} = \frac{3}{5}$

9

1 $P(A \cap B) = \frac{1}{7}$ 2 $P(A \cup B) = \frac{7}{7} = 1$

3 $P(A \cap C) = \frac{2}{7}$ 4 $P(A \cup C) = \frac{4}{7}$

5 $P(B \cap C) = \text{zero}$ 6 $P(B \cup C) = \frac{6}{7}$

7 $P(A) + P(B) - P(A \cap B) = P(A \cup B) = 1$

10

1 $P(A) = \frac{13}{24}$

2 The probability of occurrence of the two events A and B together $= P(A \cap B)$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \frac{5}{6} = \frac{13}{24} + \frac{5}{12} - P(A \cap B)$

$\therefore P(A \cap B) = \frac{13}{24} + \frac{5}{12} - \frac{5}{6} = \frac{1}{8}$

11

1 The probability that the drawn ball is blue $= \frac{5}{12}$

2 The probability that the drawn ball is not red

= the probability that the drawn ball is blue or white

$= \frac{5}{12} + \frac{3}{12} = \frac{2}{3}$

3 The probability that the drawn ball is blue or red $= \frac{5}{12} + \frac{4}{12} = \frac{3}{4}$

12

1 The number of the black balls $= 25 - (4 + 7) = 14$
The probability that the drawn ball is black $= \frac{14}{25}$

2 The probability that the drawn ball is yellow or black $= \frac{4}{25} + \frac{14}{25} = \frac{18}{25}$

3 The probability that the drawn ball is not yellow = the probability that the drawn ball is red or black $= \frac{7}{25} + \frac{14}{25} = \frac{21}{25}$

4 The probability that the drawn ball is green = zero

13

$A = \{2, 4, 6\}$, $B = \{1, 3, 5\}$, $C = \{2\}$

1 The probability of occuring the two events A and B together $= P(A \cap B) =$ zero

2 $\because$ The probability of occuring the events A or C $= P(A \cup C)$

, $\because P(A \cap C) = \frac{1}{6}$ (Where $A \cap C = \{2\}$)

$\therefore P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$= \frac{3}{6} + \frac{1}{6} - \frac{1}{6} = \frac{1}{2}$

14

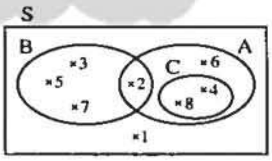
First :

1 $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$

2 $A = \{2, 4, 6, 8\}$ (3) $B = \{2, 3, 5, 7\}$

3 $C = \{4, 8\}$

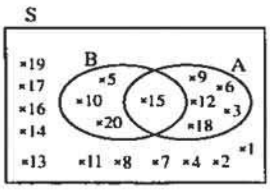
Second :



1 The probability of occuring A and B together $= P(A \cap B) = \frac{1}{8}$

2 The probability of occuring one of the two events B or C at least $= P(B \cup C) = \frac{6}{8} = \frac{3}{4}$

15



1 $P(A) = \frac{6}{20} = \frac{3}{10}$

2 $P(B) = \frac{4}{20} = \frac{1}{5}$

3 $P(A \cap B) = \frac{1}{20}$

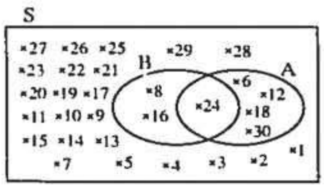
4 $P(A \cup B) = \frac{9}{20}$

16

1 The probability that the written number is odd and divisible by $5 = \frac{3}{30} = \frac{1}{10}$

2 The probability that the written number is prime or divisible by $7 = \frac{13}{30}$

17



1 $P(A) = \frac{5}{30} = \frac{1}{6}$

2 $P(B) = \frac{3}{30} = \frac{1}{10}$

3 $P(A \cap B) = \frac{1}{30}$

4 $P(A \cup B) = \frac{7}{30}$

18

1 The probability of the drawn ball is red or carrying an odd number $= \frac{11}{15}$

2 The probability of the drawn ball is green and carrying an even number $= \frac{4}{15}$

19

$\because P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

$\because P(1) = P(2) = P(3) = P(4) = P(5)$

, $\because P(6) = 3\,P(1)$ $\therefore 5\,P(1) + 3\,P(1) = 1$

$\therefore 8\,P(1) = 1$ $\therefore P(1) = \frac{1}{8}$

1 $P(6) = \frac{3}{8}$

2 $\because$ The event of appearance of an odd prime number $= \{3, 5\}$

$\therefore$ The probability of appearance of an odd prime number $= \frac{2}{8} = \frac{1}{4}$

20

$\because$ A and B are two mutually exclusive events

$\therefore P(A \cup B) = P(A) + P(B)$

$\therefore 0.64 = P(A) + P(B)$

, $\because P(B) = 3\,P(A)$ $\therefore 0.64 = P(A) + 3\,P(A)$

$\therefore 0.64 = 4\,P(A)$ $\therefore P(A) = 0.16$

$\therefore P(B) = 0.48$

Excellent pupils

1

$\because P(A) = 2\,P(B)$, $P(B) = P(C)$

, $\because P(A) + P(B) + P(C) = 1$

$\therefore 2\,P(B) + P(B) + P(B) = 1$

$\therefore 4\,P(B) = 1 \quad \therefore P(B) = \frac{1}{4} \quad \therefore P(C) = \frac{1}{4}$

$\because$ The event that the player B wins and the event that the player C wins are mutually exclusive

$\therefore$ The probability that the player B or the player C $= P(B \cup C) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

2

$\because 7\,P(A \cap B) = 2 - P(B)$

$\therefore P(B) = 2 - 7\,P(A \cap B)$ (1)

$\because P(A) + P(B) - P(A \cap B) = P(A \cup B)$ (2)

Substituting from (1) in (2) :

$\therefore P(A) + 2 - 7\,P(A \cap B) - P(A \cap B) = P(A \cup B)$

$\therefore \frac{2}{5} + 2 - 7\,P(A \cap B) - P(A \cap B) = \frac{4}{5}$

$\therefore 8\,P(A \cap B) = \frac{8}{5} \qquad \therefore P(A \cap B) = \frac{1}{5}$

Substituting in (1) : $P(B) = \frac{3}{5}$

Answers of Exercise 10

1

1 35 % 2 $\frac{1}{2}$ 3 A , B 4 $\varnothing$, zero

5 (1) S (2) $\varnothing$ (3) 1 (4) zero

6 0.2 7 $\frac{19}{26}$

2

1 a 2 b 3 c 4 b 5 a

6 a 7 c 8 d 9 d

3

event A	event A'	P(A)	P(A')	P(A) + P(A')
{2 , 4 , 6}	{1 , 3 , 5}	$\frac{1}{2}$	$\frac{1}{2}$	1
{1 , 2 , 4 , 5}	{3 , 6}	$\frac{2}{3}$	$\frac{1}{3}$	1
{5}	{1 , 2 , 3 , 4 , 6}	$\frac{1}{6}$	$\frac{5}{6}$	1
{1 , 2 , 3 , 4 , 5 , 6}	$\varnothing$	1	zero	1

4

1 $P(A \cap B) = \frac{2}{6} = \frac{1}{3}$

2 $P(A - B) = \frac{1}{6}$

3 The probability of non occurrence of the event A $= P(A') = \frac{3}{6} = \frac{1}{2}$

5

1 $P(A') = 1 - P(A) = 1 - \frac{1}{5} = \frac{4}{5}$

2 $P(B') = 1 - P(B) = 1 - \frac{3}{5} = \frac{2}{5}$

3 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= \frac{1}{5} + \frac{3}{5} - \frac{1}{10} = \frac{7}{10}$

4 $P(A - B) = P(A) - P(A \cap B) = \frac{1}{5} - \frac{1}{10} = \frac{1}{10}$

5 $P(B - A) = P(B) - P(A \cap B) = \frac{3}{5} - \frac{1}{10} = \frac{1}{2}$

6

1 $P(X') = 1 - P(X) = 1 - 0.35 = 0.65$
$P(Y') = 1 - P(Y) = 1 - 0.48 = 0.52$

2 $\because P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$
$\therefore P(X \cap Y) = P(X) + P(Y) - P(X \cup Y)$
$= 0.35 + 0.48 - 0.6 = 0.23$

3 $P(X - Y) = P(X) - P(X \cap Y) = 0.35 - 0.23 = 0.12$

4 $P(X \cap Y)' = 1 - P(X \cap Y) = 1 - 0.23 = 0.77$

7

1 $P(A) = P(A - B) + P(A \cap B) = \frac{1}{4} + \frac{1}{12} = \frac{1}{3}$

2 $P(A) = P(A - B) = \frac{1}{4}$

3 $\because B \subset A \qquad \therefore P(A \cap B) = P(B) = \frac{1}{3}$
$\therefore P(A) = P(A - B) + P(A \cap B) = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$

8

1 $\because P(X) = P(X') \qquad , P(X) + P(X') = 1$
$\therefore P(X) = \frac{1}{2}$

2 $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$
$= \frac{1}{2} + \frac{2}{5} - \frac{1}{5} = \frac{7}{10}$

9

$\because P(A) = P(A') , P(A) + P(A') = 1$

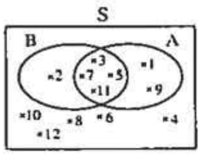
$\therefore P(A) = \frac{1}{2}$

1 $P(B) = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$

2 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= \frac{1}{2} + \frac{5}{16} - \frac{1}{16} = \frac{3}{4}$

3 $P(A - B) = P(A) - P(A \cap B) = \frac{1}{2} - \frac{1}{16} = \frac{7}{16}$

10



$P(A) = \frac{6}{12} = \frac{1}{2} , P(B) = \frac{5}{12}$

$, P(A') = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{5}{12} - \frac{1}{3} = \frac{7}{12}$

, $P(A - B) = P(A) - P(A \cap B) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

11

1 The probability that the drawn ball is red $= \frac{8}{20} = \frac{2}{5}$

2 The probability that the drawn ball is white or green $= \frac{7}{20} + \frac{5}{20} = \frac{12}{20} = \frac{3}{5}$

3 The probability that the drawn ball is not white $= 1 - \frac{7}{20} = \frac{13}{20}$

12

1 The probability of non occurrence the two events A and B together $= P(\overline{A \cap B}) = 1 - P(A \cap B)$

$= 1 - 0.6 = 0.4$

2 The probability of occurrence of one of the two events at least $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.8 + 0.7 - 0.6 = 0.9$

13

1 The probability of occurrence of one of the two events at least $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.6 - 0.4 = 0.7$

2 The probability of occurrence of the event B and non occurrence the event A

$= P(B - A) = P(B) - P(A \cap B) = 0.6 - 0.4 = 0.2$

3 The probability of non occurrence of the event A

$= P(\acute{A}) = 1 - P(A) = 1 - 0.5 = 0.5$

4 The probability of non occurrence of any one of the two events $= P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.7 = 0.3$

5 The probability of occurrence of one of the two events but not the other $= P(A - B) + P(B - A)$

$= P(A) + P(B) - 2\,P(A \cap B)$

$= 0.5 + 0.6 - 2 \times 0.4 = 0.3$

6 The probability of occurrence of the event A only

$= P(A - B) = P(A) - P(A \cap B) = 0.5 - 0.4 = 0.1$

14

1 $\because$ The probability of non occurrence of the event $A = P(\acute{A}) = \frac{1}{4}$

$\therefore P(A) = 1 - P(\acute{A}) = 1 - \frac{1}{4} = \frac{3}{4}$

2 $\because$ The probability of occurrence of one of the two events at most $= P(\overline{A \cap B}) = \frac{3}{5}$

$\therefore$ The probability of occurrence the two events together $= P(A \cap B)$

$= 1 - P(\overline{A \cap B}) = 1 - \frac{3}{5} = \frac{2}{5}$

3 $\because$ The probability of non occurrence of the event $B = P(\acute{B}) = \frac{1}{2}$

$\therefore P(B) = 1 - P(\acute{B}) = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore$ The probability of occurrence of any of the two events $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{4} + \frac{1}{2} - \frac{2}{5} = \frac{17}{20}$

4 $\because$ The probability of occurrence of the event A only

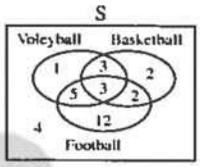
$= P(A - B) = P(A) - P(A \cap B) = \frac{3}{4} - \frac{2}{5} = \frac{7}{20}$

5 $\because$ The probability of occurrence of one of the two events only $= P(A - B) + P(B - A)$

$= P(A) + P(B) - 2\,P(A \cap B)$

$= \frac{3}{4} + \frac{1}{2} - 2 \times \frac{2}{5} = \frac{9}{20}$

15



The number of members in each set.

1 Two students

2 4 students

3 The probability that the student is one of football team only $= \frac{12}{32} = \frac{3}{8}$

16

Assuming that

A is the event that the student reads Al Akhbar Newspaper and B is the event that the student reads Al Ahram Newspaper

1 The probability that the student reads Al Akhbar Newspaper $= P(A) = \frac{18}{40} = \frac{9}{20}$

2 The probability that the student does not read Al Akhbar Newspaper $= P(\acute{A})$

$= 1 - P(A) = 1 - \frac{9}{20} = \frac{11}{20}$

3 The probability that the student reads Al Ahram Newspaper $= P(B) = \frac{15}{40} = \frac{3}{8}$

4 The probability that the student reads the two Newspaper together $= P(A \cap B) = \frac{8}{40} = \frac{1}{5}$

5 The probability that the student reads Al Akhbar Newspaper only $= P(A - B) = P(A) - P(A \cap B)$

$= \frac{9}{20} - \frac{1}{5} = \frac{1}{4}$

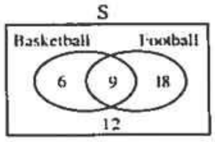
6 The probability that the student reads Al Ahram Newspaper only $= P(B - A) = P(B) - P(A \cap B)$

$= \frac{3}{8} - \frac{1}{5} = \frac{7}{40}$

7 The probability that the student reads Al-Akhbar only or Al Ahram only $= P(A - B) + P(B - A)$

$= \frac{1}{4} + \frac{7}{40} = \frac{17}{40}$

17



The number of participants in each set

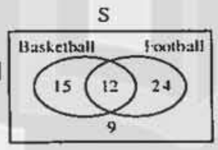
1 The probability that the selected student is participant in football team $= \frac{27}{45} = \frac{3}{5}$

2 The probability that the selected student is participant in basketball team $= \frac{15}{45} = \frac{1}{3}$

3 The probability that the selected student is participant in football team and basketball team $= \frac{9}{45} = \frac{1}{5}$

4 The probability that the selected student is not participant in any team $= \frac{12}{45} = \frac{4}{15}$

18



1 The probability that the chosen student is participant in football team and not participant in basketball team $= \frac{24}{60} = \frac{2}{5}$

2 The probability that the chosen student is participant in one team at least $= \frac{51}{60} = \frac{17}{20}$

3 The probability that the chosen student is not a participant in any team of the previous teams $= \frac{9}{60} = \frac{3}{20}$

19

1 The probability that the two events occur together $= P(A \cap B)$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.6 + \frac{12}{30} - \frac{13}{15} = \frac{2}{15}$

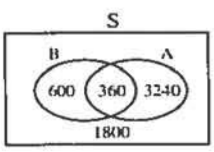
2 The probability of occuring one of the two events but not the other $= P(A - B) + P(B - A)$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= P(A) + P(B) - 2\,P(A \cap B)$

$= 0.6 + \frac{12}{30} - 2 \times \frac{2}{15} = \frac{11}{15}$

20



1 (1) $P(A) = \frac{3\,600}{6\,000} = \frac{3}{5}$

(2) $P(B) = \frac{960}{6\,000} = \frac{4}{25}$

2 (1) $P(A \cap B) = \frac{360}{6\,000} = \frac{3}{50}$

(2) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{5} + \frac{4}{25} - \frac{3}{50} = \frac{7}{10}$

(3) $P(A - B) = P(A) - P(A \cap B) = \frac{3}{5} - \frac{3}{50} = \frac{27}{50}$

(4) $P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - \frac{7}{10} = \frac{3}{10}$

3 $\because$ The probability that the mother live in urban and of age 30 years and more $= \frac{1\,500}{6\,000} = \frac{1}{4}$

$\therefore$ The number of births in urban if the number of births is 9 000 cases $= \frac{1}{4} \times 9\,000 = 2\,250$ cases.

Excellent pupils

1

Assuming that the whit cows is A and the brown kind is B

$\because$ The farm contains cows of the two colours

$\therefore P(A \cup B) = 1$, $P(A) = \frac{5}{7}$, $P(B) = \frac{11}{28}$

1 The probability that the cow has the two colours $= P(A \cap B)$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{5}{7} + \frac{11}{28} - 1 = \frac{3}{28}$

2 The probability that the cow is white only

$= P(A - B) = P(A) - P(A \cap B) = \frac{5}{7} - \frac{3}{28} = \frac{17}{28}$

2

$S = \{(H , H) , (H , T) , (T , H) , (T , T)\}$

1 The probability of non occurrence of a head in the second toss $= \frac{2}{4} = \frac{1}{2}$

2 The probability of non occurrence of a head in the two tosses together $= \frac{3}{4}$

Answers of exam on unit three

1

1 d 2 b 3 b
4 b 5 a 6 d

2

[a] 1 $\frac{3}{4}$ 2 $\frac{1}{8}$ 3 Prove by yourself
[b] 1 $\frac{3}{5}$ 2 1

3

1 $\frac{1}{5}$ 2 $\frac{1}{4}$ 3 $\frac{1}{20}$ 4 $\frac{2}{5}$

4

1 0.6 2 0.9 3 0.4 4 0.2

5

[a] 1 $\frac{8}{21}$ 2 $\frac{2}{3}$ 3 $\frac{13}{21}$
[b] $\frac{3}{4}$

Answers of accumulative basic skills

1 a	2 d	3 b	4 d
5 c	6 a	7 b	8 a
9 c	10 c	11 c	12 b
13 b	14 a	15 b	16 d
17 d	18 d	19 c	20 b
21 a	22 d	23 a	24 a
25 d	26 c	27 c	28 c

Guide Answers

of Geometry Exercises

Answers of unit four

Answers of Exercise 1

1

1 the radius 2 a chord 3 the diameter

4 an axis of symmetry

5 an infinite number , 1

6 perpendicular to this chord

7 bisects 8 the centre of the circle

9 the circumference of the circle

10 3 11 3

2

1 100° 2 20° 3 54° , 72°

4 10 , 35° 5 80° , 50° 6 20° , 90°

3

1 40° 2 120° 3 5

4 16 5 20 6 45°

7 24 , 8 8 45° , $5\sqrt{2}$ 9 616

4 1 120° 2 6 3 60

5 $\because$ MA = MD = r

$\therefore \Delta$ AMD is an isosceles triangle.

$\therefore m(\angle DAM) = m(\angle ADM) = 25°$

$\therefore m(\angle DAC) = 25° + 40° = 65°$

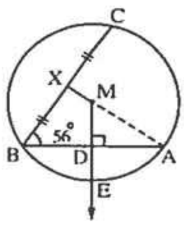
$\therefore$ In Δ ADC :

$m(\angle ACD) = 180° - (25° + 65°) = 90°$

$\therefore \overline{DC} \perp \overline{AB}$ $\because M \in \overline{DC}$

$\therefore$ C is the midpoint of $\overline{AB}$ (Q.E.D)

6 $\because$ X is the midpoint of $\overline{CB}$



$\therefore \overline{MX} \perp \overline{BC}$

$\therefore m(\angle DMX)$

$= 360° - (90° + 90° + 56°)$

$= 124°$ (First req.)

$, \because \overline{MD} \perp \overline{AB}$

$\therefore$ D is the midpoint of $\overline{AB}$

$\therefore$ AD = 4 cm.

In Δ ADM :

$\because m(\angle ADM) = 90°$, AM = r = 5 cm.

$\therefore MD = \sqrt{(AM)^2 - (AD)^2} = \sqrt{25 - 16}$

$= \sqrt{9} = 3$ cm.

$\therefore$ DE = 5 – 3 = 2 cm. (Second req.)

7 $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$

$\therefore m(\angle BDM) = 90°$ similarly $m(\angle MEA) = 90°$

$\therefore$ From Δ AFE : $m(\angle DFM) = 45°$

and from Δ DFM : $m(\angle DMF) = 45°$

$\therefore \Delta$ DFM is an isosceles triangle. (Q.E.D)

8 $\because$ C is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$

In Δ ACM : $\because m(\angle ACM) = 90°$

$\therefore (MC)^2 = (AM)^2 - (AC)^2$ (Pythagoras' theorem)

$\therefore (MC)^2 = (13)^2 - (12)^2 = 25$ $\therefore$ MC = 5 cm.

$\therefore$ CD = MD – MC = 13 – 5 = 8 cm.

$\therefore$ The area of Δ ADB $= \frac{1}{2} \times 24 \times 8 = 96$ cm.^{2}

(The req.)

9 $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle AXY) = 90°$

$, \because \overline{AB} // \overline{CD}$, $\overleftrightarrow{XY}$ is a transversal

$\therefore m(\angle XYD) = m(\angle AXY)$

$= 90°$ (alternate angles)

$\therefore \overline{MY} \perp \overline{CD}$

$\therefore$ Y is the midpoint of $\overline{CD}$ (Q.E.D)

10 $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$

$\because$ E is the midpoint of $\overline{AC}$ $\therefore \overline{ME} \perp \overline{AC}$

$\therefore m(\angle DME) = 360° - (120° + 90° + 90°) = 60°$

$\therefore m(\angle XMY) = m(\angle DME) = 60°$ (V.O.A)

$\because$ MX = MY = r

$\therefore \Delta$ XYM is an equilateral triangle. (Q.E.D)

11 $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

$\therefore m(\angle AXY) = 90° - 30° = 60°$

$, \because$ AB = AC $\therefore \frac{1}{2}$ AB $= \frac{1}{2}$ AC

$\therefore$ AX = AY $, \because m(\angle AXY) = 60°$

$\therefore \Delta$ AXY is an equilateral triangle. (Q.E.D.)

12 In the great circle :

$\because \overline{ME} \perp \overline{AB}$ $\therefore$ E is the midpoint of $\overline{AB}$

$\therefore AE = EB$ (1)

In the small circle :

$\because \overline{ME} \perp \overline{CD}$ $\therefore$ E is the midpoint of $\overline{CD}$

$\therefore CE = ED$ (2)

Subtracting (2) from (1) : $\therefore AE - CE = EB - ED$

$\therefore AC = BD$ (Q.E.D)

13 $\because \overline{MD} \perp \overline{BC}$ $\therefore$ D is the midpoint of $\overline{BC}$

$\because \overline{ME} \perp \overline{AC}$ $\therefore$ E is the midpoint of $\overline{AC}$

$\therefore$ In Δ ABC :

$\because$ D and E are the two midpoints of $\overline{BC}$ and $\overline{AC}$ respectively.

$\therefore \overline{ED} \parallel \overline{AB}$ (Q.E.D 1)

$\because$ D is the midpoint of $\overline{BC}$

$\therefore DC = \frac{1}{2} BC$ (1)

$\because$ E is the midpoint of $\overline{AC}$

$\therefore EC = \frac{1}{2} AC$ (2)

$\because$ D and E are the two midpoints of $\overline{BC}$ and $\overline{AC}$ respectively.

$\therefore DE = \frac{1}{2} AB$ (3)

Adding (1) , (2) and (3) :

$\therefore$ The perimeter of Δ CDE

$= \frac{1}{2}$ the perimeter of Δ ABC (Q.E.D. 2)

14 $\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

$\therefore m(\angle XMY) = 360° - (60° + 90° + 90°) = 120°$

$\because \overline{MX} \perp \overline{AB}$ $\therefore$ X is the midpoint of $\overline{AB}$

$\because \overline{MY} \perp \overline{AC}$ $\therefore$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{XY} \parallel \overline{BC}$

$\therefore m(\angle AXY) = m(\angle ABC) = 70°$

(Corresponding angles)

$\therefore m(\angle MXY) = m(\angle AXM) - m(\angle AXY)$

$= 90° - 70° = 20°$

$\therefore$ In Δ MXY :

$m(\angle XYM) = 180° - (120° + 20°) = 40°$ (The req.)

15 In Δ AMC :

$\because AM = MC = r$ $\therefore m(\angle MAC) = m(\angle ACM)$

$\because m(\angle BAC) = m(\angle MAC)$

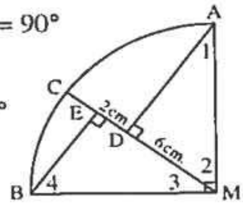
$\therefore m(\angle BAC) = m(\angle ACM)$ and they are alternate angles

$\therefore \overline{AB} \parallel \overline{CM}$

$\because$ D is the midpoint of $\overline{AB}$ $\therefore \overline{MD} \perp \overline{AB}$

$\because \overline{AB} \parallel \overline{CM}$ $\therefore \overline{DM} \perp \overline{CM}$ (Q.E.D)

16 In Δ AMD : $\because m(\angle ADM) = 90°$



$\therefore m(\angle 1) + m(\angle 2) = 90°$

$, \because m(\angle 2) + m(\angle 3) = 90°$

$\therefore m(\angle 1) = m(\angle 3)$

In $\Delta\Delta$ ADM , MEB

$\because m(\angle ADM) = m(\angle MEB) = 90°$

$, m(\angle 1) = m(\angle 3)$ $\therefore m(\angle 2) = m(\angle 4)$

$\therefore$ In $\Delta\Delta$ ADM , MEB

$\begin{cases} MA = MB = r \\ m(\angle 1) = m(\angle 3) \\ m(\angle 2) = m(\angle 4) \end{cases}$

$\therefore \Delta ADM \equiv \Delta MEB$

$\therefore MA = \sqrt{8^2 + 6^2} = 10$ cm.

$, \because MA = MC = r$ $\therefore MC = 10$ cm.

$\therefore EC = 10 - (6 + 2) = 2$ cm. (The req.)

17 $\because AB = AC$, $m(\angle A) = 60°$

$\therefore \Delta$ ABC is an equilateral triangle

$\therefore m(\angle B) = 60°$

$\because m(\angle BXM) = 90°$ $\therefore m(\angle BMX) = 30°$

$\therefore BM = 2\,BX = 5$ cm. $\therefore BC = 2\,BM = 10$ cm.

$\therefore AB = 10$ cm. (1)

$\because \overline{MX} \perp \overline{BE}$ $\therefore$ X is the midpoint of $\overline{BE}$

$\therefore BE = 2\,BX = 2 \times 2.5 = 5$ cm. (2)

$, \because AE = AB - BE$

$\therefore AE = 10 - 5 = 5$ cm. (The req.)

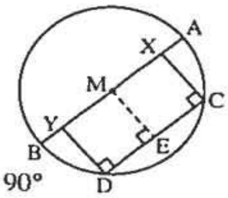
18 In Δ MNC : $\because NC + MC > NM$ (triangle inequality)

$, \because MA = MC = r$, $NM = AN + MA$

$\therefore NC + MC > AN + MA$

$\therefore NC > AN$ (Q.E.D.)

19 **Construction :**



Draw $\overline{ME} \perp \overline{CD}$ to cut it at E

Proof : $\because \overline{ME} \perp \overline{CD}$

$\therefore$ E is the midpoint of $\overline{CD}$

, m ($\angle$ XCE) = m ($\angle$ MED) = 90°

but they are corresponding angles

$\therefore \overleftrightarrow{XC} \,//\, \overleftrightarrow{ME}$ similarly $\overleftrightarrow{ME} \,//\, \overleftrightarrow{YD}$

$\therefore \overleftrightarrow{XC} \,//\, \overleftrightarrow{ME} \,//\, \overleftrightarrow{YD}$

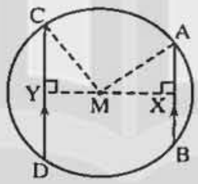
$\because \overleftrightarrow{XY}$ and $\overleftrightarrow{CD}$ are two transversals to them

, CE = ED $\qquad \therefore$ XM = MY

$\because$ AM = BM = r $\quad \therefore$ AM − XM = BM − MY

$\therefore$ AX = BY (Q.E.D.)

20 **Construction :**



Draw $\overline{MA}$, $\overline{MC}$,

$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

Proof : $\because \overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$

$\therefore$ AX = 6 cm.

$\therefore$ In Δ AXM : XM = $\sqrt{(10)^2 - (6)^2}$ = 8 cm. (1)

$\because \overline{MY} \perp \overline{CD}$

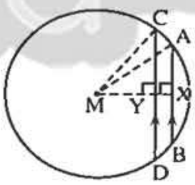
$\therefore$ Y is the midpoint of $\overline{CD}$ $\qquad \therefore$ CY = 8 cm.

$\therefore$ In Δ CYM : YM = $\sqrt{(10)^2 - (8)^2}$ = 6 cm. (2)

Adding (1) and (2) :

$\therefore$ The distance between $\overline{AB}$, $\overline{CD}$ = 14 cm. (The req.)

Yes there is another solution :



In the same way of the previous proof we find that :

XM = 8 cm. , YM = 6 cm.

$\therefore$ XY = 8 − 6 = 2 cm. (The req.)

21 $\because \overline{AB}$ is a diameter of the circle

$\therefore$ M is the midpoint of $\overline{AB}$

$\therefore$ M = $\left(\frac{3+3}{2}, \frac{4-3}{2}\right) = \left(3, \frac{1}{2}\right)$ (First req.)

$\because \overline{AM}$ is a radius in the circle M

$\therefore$ AM = $\sqrt{(3-3)^2 + \left(4 - \frac{1}{2}\right)^2}$

$= \sqrt{\left(3\frac{1}{2}\right)^2} = 3\frac{1}{2}$ length units

$\therefore$ The circumference of the circle = 2 π r

$= 2 \times \frac{22}{7} \times \frac{7}{2}$ = 22 length units (Second req.)

22 $\because$ MA = $\sqrt{(-1-2)^2 + (2-6)^2} = \sqrt{9+16} = \sqrt{25}$

= 5 length units

, MB = $\sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16} = \sqrt{25}$

= 5 length units

$\therefore$ MA = MB

$\therefore$ M is the centre of a circle passes through the two points A and B (Q.E.D. 1)

Let D be the midpoint of $\overline{AB}$

$\therefore$ D = $\left(\frac{2+2}{2}, \frac{6-2}{2}\right)$ = (2 , 2)

$\therefore \overline{MD} \perp \overline{AB}$

$\therefore$ MD = $\sqrt{(-1-2)^2 + (2-2)^2} = \sqrt{9}$ = 3 length units (Q.E.D. 2)

23 Let the equation of $\overleftrightarrow{MD}$ be y = a x + b

$\because$ The slope of $\overleftrightarrow{AB} = \frac{5-1}{-4-4} = \frac{4}{-8} = -\frac{1}{2}$

$\because$ D is the midpoint of $\overline{AB}$

$\therefore \overleftrightarrow{MD} \perp \overleftrightarrow{AB}$

$\therefore$ The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{MD}$ = − 1

$\therefore -\frac{1}{2}$ × the slope of $\overleftrightarrow{MD}$ = − 1

$\therefore$ The slope of $\overleftrightarrow{MD}$ = − 1 × − 2 = 2

$\therefore$ The equation of $\overleftrightarrow{MD}$: y = 2 x + b

$\because$ D is the midpoint of $\overline{AB}$

$\therefore$ D = $\left(\frac{4-4}{2}, \frac{1+5}{2}\right)$ = (0 , 3)

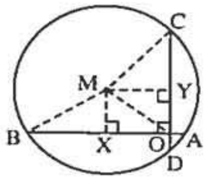
$\because$ D $\in \overleftrightarrow{MD}$ $\qquad \therefore$ It satisfies its equation

$\therefore$ 3 = 0 + b $\qquad \therefore$ b = 3

$\therefore$ The equation of $\overleftrightarrow{MD}$: y = 2 x + 3 (Q.E.D.)

Excellent pupils

1 **Construction :**



Draw $\overline{MC}$, $\overline{MB}$, $\overline{MO}$

, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

Proof : $\because \overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$

$\therefore$ XB = 6 cm.

$\therefore$ In Δ MXB : $(MX)^2 = (7)^2 - (6)^2 = 13$

$\because \overline{MY} \perp \overline{CD}$

$\therefore$ Y is the midpoint of $\overline{CD}$ $\qquad \therefore$ YC = 5 cm.

$\therefore$ In Δ MYC : $\therefore (MY)^2 = (7)^2 - (5)^2 = 24$

In the quadrilateral MXOY :

m (∠ MXO) = m (∠ XOY) = m (∠ OYM) = 90°

∴ The figure MXOY is a rectangle.

∴ XO = MY ∴ $(XO)^2 = (MY)^2 = 24$

In Δ MXO :

∵ $(MO)^2 = (MX)^2 + (XO)^2 = 13 + 24 = 37$

∴ MO = $\sqrt{37}$ cm. (The req.)

2 ∵ $\overline{MA} \perp \overline{BC}$

∴ D is the midpoint of $\overline{BC}$

∴ BD = $7\sqrt{3}$ cm.

∵ AB = BM , BM = MA (The lengths of two radii)

∴ Δ ABM is an equilateral triangle.

∵ $\overline{BD} \perp \overline{AM}$

∴ D is the midpoint of $\overline{AM}$

∴ DM = $\frac{1}{2}$ MB let DM = ℓ , MB = 2ℓ

In Δ BDM which is right-angled at D

$(7\sqrt{3})^2 = (2\ell)^2 - \ell^2$

∴ $147 = 3\ell^2$ ∴ $\ell^2 = 49$

∴ $\ell = 7$ cm. ∴ MB = 14 cm.

∴ The radius length of the circle = 14 cm. (The req.)

Answers of Exercise 2

1

1 outside 2 on 3 inside

4 inside , the centre of the circle

2

1 outside the circle M 2 $\overline{AB}$

3 the radius 4 a tangent to it

5 parallel 6 4

3

1 is a secant to the circle M

2 lies outside the circle

3 is a tangent to the circle M 4 $[\frac{5}{3}, 4[$

5 {– 3 , 3}

4

1 c 2 b 3 c 4 b 5 c 6 c

7 c 8 a 9 b 10 a 11 a

5

1 b 2 d 3 a 4 a 5 c

6

1 35° 2 30° 3 130° 4 4 5 60 6 32

7 ∵ $\overrightarrow{BC}$ is a tangent to the circle M at B

∴ $\overline{BC} \perp \overline{MB}$

In Δ ABC : m (∠ A) = 180° – (45° + 90°) = 45°

, ∵ D is the midpoint of $\overline{AH}$ ∴ $\overline{MD} \perp \overrightarrow{AH}$

In Δ ADM :

m (∠ DMA) = 180° – (45° + 90°) = 45°

∴ m (∠ DAM) = m (∠ DMA)

∴ DA = DM (Q.E.D.)

8 In Δ MDB :

∵ MD = MB = r

∴ m (∠ MBD) = m (∠ MDB) = $\frac{180° - 100°}{2}$ = 40°

, ∵ $\overleftrightarrow{AC}$ is a tangent to the circle M at A

∴ $\overline{MA} \perp \overline{AC}$

In Δ ABC :

m (∠ C) = 180° – (90° + 40°) = 50° (First req.)

, m (∠ CDM) = 180° – 40° = 140° (Second req.)

9 ∵ MZ = r = 5 cm. ∴ MY = 13 cm.

, ∵ $(MY)^2 = 169$, $(MX)^2 = 25$

, $(XY)^2 = 144$

∴ $(MX)^2 + (XY)^2 = (MY)^2$

∴ m (∠ MXY) = 90° ∴ $\overline{XY} \perp \overline{MX}$

∴ $\overleftrightarrow{XY}$ is a tangent to the circle M at X (Q.E.D.)

10

1 In Δ MAB :

∵ The sum of measures of the interior angles of the triangle = 180°

∴ m (∠ MAB) = 180° – (54° + 36°) = 90°

∴ $\overline{MA} \perp \overline{AB}$

∴ $\overleftrightarrow{AB}$ is a tangent to the circle M (Q.E.D.)

2 ∵ MA = AC , MA = MC = r ∴ AC = MC = CB

∴ $\overline{AC}$ is a median of Δ AMB , AC = $\frac{1}{2}$ MB

∴ m (∠ BAM) = 90° ∴ $\overline{MA} \perp \overline{AB}$

∴ $\overleftrightarrow{AB}$ is a tangent to the circle M (Q.E.D.)

3 In Δ MAD : $\because$ MA = MD = r

m ($\angle$ MDA) = m ($\angle$ MAD)

, $\because$ m ($\angle$ MDA) = m ($\angle$ ADB)

$\therefore$ m ($\angle$ MAD) = m ($\angle$ ADB) but they are alternate angles

$\therefore$ $\overleftrightarrow{AM} \parallel \overleftrightarrow{BD}$

$\therefore$ m ($\angle$ MAB) = m ($\angle$ DBE) = 90°

(Corresponding angles)

$\therefore$ $\overline{MA} \perp \overrightarrow{AB}$

$\therefore$ $\overrightarrow{AB}$ is a tangent to the circle M (Q.E.D.)

11 $\because$ $\overrightarrow{AB}$ is a tangent to the circle M at A

$\therefore$ $\overline{MA} \perp \overline{AB}$

$\therefore$ m ($\angle$ MAB) = 90°

In Δ MAB : $\because$ m ($\angle$ ABM) = 30°

$\therefore$ MB = 2 MA = 16 cm.

$\therefore$ AB $= \sqrt{(MB)^2 - (MA)^2} = \sqrt{256 - 64}$

$= \sqrt{192}$

$= 8\sqrt{3}$ cm. (First req.)

In Δ ABC which is right-angled at C

$\because$ m ($\angle$ ABC) = 30°

$\therefore$ AC $= \frac{1}{2}$ AB $= \frac{1}{2} \times 8\sqrt{3}$

$= 4\sqrt{3}$ cm. (Second req.)

Another solution for the first requirement.

$\because$ $\overrightarrow{AB}$ is a tangent to the circle M at A

$\therefore$ m ($\angle$ MAB) = 90°

$\therefore$ tan ($\angle$ B) $= \frac{MA}{AB}$ $\therefore$ tan 30° $= \frac{8}{AB}$

$\therefore \frac{1}{\sqrt{3}} = \frac{8}{AB}$ $\therefore$ AB $= 8\sqrt{3}$ cm.

12 $\because$ $\overleftrightarrow{XY}$ is a tangent to the circle at X

$\therefore$ $\overline{MX} \perp \overline{XY}$ $\therefore$ m ($\angle$ MXY) = 90°

$\therefore$ In Δ MXY : $(MY)^2 = (MX)^2 + (XY)^2$

$\therefore$ $(MZ + 8)^2 = (MX)^2 + 144$

, $\because$ MZ = MX = r $\therefore$ $(r + 8)^2 = r^2 + 144$

$\therefore$ $r^2 + 16\,r + 64 = r^2 + 144$ $\therefore$ 16 r = 80

$\therefore$ r $= \frac{80}{16} = 5$ cm. (The req.)

13 $\because$ $\overrightarrow{AC}$ is a tangent to the circle M at A

$\therefore$ $\overline{MA} \perp \overline{AC}$ $\therefore$ m ($\angle$ MAC) = 90°

In $\Delta\Delta$ MAC , MBD :

MA = MB (lengths of two radii)

MC = MD (given)

m ($\angle$ AMC) = m ($\angle$ BMD) (V.O.A.)

$\therefore$ The two triangles are congruent and we deduce that m ($\angle$ MAC) = m ($\angle$ MBD) = 90°

$\therefore$ $\overline{BD} \perp \overline{MB}$

$\therefore$ $\overrightarrow{BD}$ is a tangent to the circle M at B (Q.E.D.)

14 $\because$ $\overline{AB}$ is a tangent to the circle M at B

$\therefore$ $\overline{MB} \perp \overline{AB}$ $\therefore$ m ($\angle$ ABM) = 90°

, $\because$ $\overline{AC}$ is a tangent to the circle M at C

$\therefore$ $\overline{MC} \perp \overline{AC}$ $\therefore$ m ($\angle$ ACM) = 90°

$\therefore$ In $\Delta\Delta$ ABM , ACM which are right-angled

MB = MC = r

$\overline{AM}$ is a common hypotenuse

$\therefore$ Δ ABM $\equiv$ Δ ACM

$\therefore$ m ($\angle$ AMB) = m ($\angle$ AMC)

$\therefore$ $\overrightarrow{MA}$ bisects $\angle$ BMC (First req.)

From Δ ABM : m ($\angle$ AMB) = 180° − (90° + 25°)

= 65°

$\therefore$ m ($\angle$ BMC) = 2 × 65° = 130° (Second req.)

15 In the small circle :

$\because$ $\overline{AB}$ is a tangent atc $\therefore$ $\overline{MC} \perp \overline{AB}$

In the great circle : $\because$ $\overline{MC} \perp \overline{AB}$

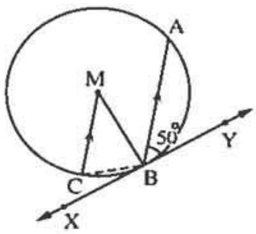
$\therefore$ C is the midpoint of $\overline{AB}$ $\therefore$ AC = 4 cm.

, $\because$ AM = 5 cm.

$\therefore$ In Δ ACM which is right-angled at C

MC $= \sqrt{(AM)^2 - (AC)^2} = \sqrt{25 - 16} = 3$ cm. (The req.)

16 $\because$ $\overleftrightarrow{YX}$ is a tangent to the circle M at B



$\therefore$ $\overline{MB} \perp \overleftrightarrow{YX}$

$\therefore$ m ($\angle$ MBY) = 90°

$\therefore$ m ($\angle$ MBA) = 90° − 50°

= 40°

$\because$ $\overline{BA} \parallel \overline{MC}$, $\overline{BM}$ is a transversal to them

$\therefore$ m ($\angle$ BMC) = m ($\angle$ ABM) = 40°

(alternate angles)

$\because MB = MC = r$

$\therefore \Delta\ MBC$ is an isosceles triangle

$\therefore m(\angle MBC) = m(\angle MCB) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$

$\therefore m(\angle CBX) = 90^\circ - 70^\circ = 20^\circ$ (The req.)

17 $\because \overleftrightarrow{DC}$ is a tangent to the circle M at C

$\therefore \overline{MC} \perp \overline{DC}$ $\therefore m(\angle MCD) = 90^\circ$

$\therefore$ In $\Delta\ DMC$: $m(\angle DMC) = 180^\circ - (90^\circ + 20^\circ) = 70^\circ$

, $\because \overleftrightarrow{AB} \parallel \overline{MD}$, $\overleftrightarrow{AE}$ is a transversal to them

$\therefore m(\angle MEC) = m(\angle BAE) = 80^\circ$

(corresponding angles)

$\therefore$ In $\Delta\ MEC$:

$m(\angle ECM) = 180^\circ - (70^\circ + 80^\circ) = 30^\circ$ (The req.)

18 $\because MX = MY$ (lengths of two radii)

, $BX = CY$ (Given) , by adding $\therefore MB = MC$

, $\because \overleftrightarrow{BC}$ is a tangent to the circle M at A

$\therefore \overline{MA} \perp \overline{BC}$

, $\because \Delta\ MBC$ is an isosceles triangle in which :

$MB = MC$, $\overline{MA} \perp \overline{BC}$

$\therefore \overleftrightarrow{MA}$ bisects $\angle BMC$

$\therefore m(\angle BMA) = m(\angle CMA)$ (Q.E.D.)

19 $\because BC = BM$, $MB = MC$ (lengths of two radii)

$\therefore \Delta\ BCM$ is an equilateral triangle.

$\therefore m(\angle CBM) = 60^\circ$

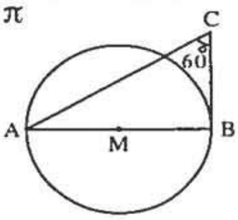
, $\because \angle MCB$ is an exterior angle of $\Delta\ ABC$

$\therefore m(\angle A) = m(\angle ABC) = \frac{60^\circ}{2} = 30^\circ$

$\therefore m(\angle ABM) = 90^\circ$ $\therefore \overline{MB} \perp \overline{AB}$

$\therefore \overleftrightarrow{AB}$ is a tangent to the circle M at B (Q.E.D.)

20 $\because$ The area of the circle $= 36\pi$



$\therefore r^2\pi = 36\pi$

$\therefore r^2 = 36$

$\therefore r = 6$ cm.

$\therefore AB = 12$ cm.

, $\because \overleftrightarrow{BC}$ is a tangent to the circle M at B

$\therefore \overline{BC} \perp \overline{AB}$

In $\Delta\ ABC$: $\tan(\angle C) = \frac{AB}{BC}$ $\therefore \tan 60^\circ = \frac{12}{BC}$

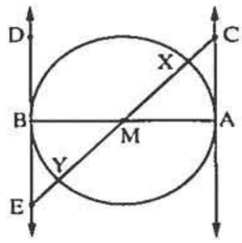
$\therefore BC = \frac{12}{\tan(60^\circ)} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ cm.

$\therefore$ The area of $\Delta\ ABC$

$= \frac{1}{2} AB \times BC = \frac{1}{2} \times 12 \times 4\sqrt{3}$

$= 24\sqrt{3}$ cm^2 (The req.)

21 $\because \overleftrightarrow{AC}$ is a tangent to the circle M at A



$\therefore \overline{MA} \perp \overleftrightarrow{AC}$

$\therefore m(\angle CAM) = 90^\circ$

$\because \overleftrightarrow{BD}$ is a tangent to the circle M at B

$\therefore \overline{MB} \perp \overleftrightarrow{BD}$

$\therefore m(\angle EBM) = 90^\circ$

In $\Delta\Delta$ CAM , EBM :

$m(\angle CAM) = m(\angle EBM) = 90^\circ$

$m(\angle AMC) = m(\angle BME)$ (V.O.A)

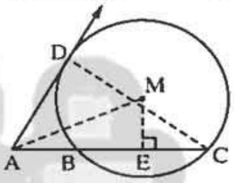
$MA = MB$ (lengths of two radii)

$\therefore$ The two triangles are congruent and we deduce that $CM = EM$

, $\because XM = YM$ (lengths of two radii) , by subtracting

$\therefore CX = YE$ (Q.E.D.)

22 **Construction :**



Draw $\overline{ME} \perp \overline{BC}$,

draw $\overline{MC}$, $\overline{MD}$, $\overline{MA}$

Proof :

$\because AC = 12$ cm. , $AB = 4$ cm. $\therefore BC = 8$ cm.

, $\because \overline{ME} \perp \overline{BC}$ $\therefore CE = EB = 4$ cm.

, $\because r = 5$ cm. $\therefore MC = 5$ cm.

$\because$ In $\Delta\ MEC$: $m(\angle MEC) = 90^\circ$

$\therefore (ME)^2 = (MC)^2 - (CE)^2 = 25 - 16 = 9$

$\therefore ME = 3$ cm.

$\therefore$ The distance between the chord $\overline{BC}$ and the centre $= 3$ cm. (First req.)

In $\Delta\ MEA$: $m(\angle MEA) = 90^\circ$

$\therefore (MA)^2 = (ME)^2 + (AE)^2 = 9 + 64 = 73$

, $\because \overleftrightarrow{AD}$ is a tangent to the circle $\therefore \overline{MD} \perp \overleftrightarrow{AD}$

$\therefore$ In $\Delta\ AMD$: $m(\angle ADM) = 90^\circ$

$\therefore (AD)^2 = (AM)^2 - (MD)^2 = 73 - 25 = 48$

$\therefore AD = \sqrt{48} = 4\sqrt{3}$ cm. (Second req.)

23

1 $\because MA = \sqrt{(0+3)^2 + (0-4)^2} = \sqrt{9+16} = \sqrt{25}$

$= 5$ length units

$\therefore MA = r$ $\therefore$ A lies on the circle.

2 $\because MB = \sqrt{(0-2)^2 + (0-3)^2} = \sqrt{4+9}$

$= \sqrt{13}$ length units

$\therefore MB < r$ $\therefore$ B lies inside the circle.

3 $\because MC = \sqrt{(0-6)^2 + (0-8)^2} = \sqrt{36+64} = \sqrt{100}$
$= 10$ length units

$\therefore MC > r \qquad \therefore$ C lies outside the circle.

24

$\because MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9} = \sqrt{25}$
$= 5$ length units

$, MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16} = \sqrt{25}$
$= 5$ length units

$, MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16} = \sqrt{25}$
$= 5$ length units

$\therefore MA = MB = MC$

$\therefore$ The points A , B and C lie on the circle M (Q.E.D.1)

, its circumference $= 10\,\pi$ length units. (Q.E.D.2)

25 $\because \overline{CD}$ is a diameter in the circle M

$\therefore$ M is the midpoint of $\overline{CD}$

Let C (x , y) $\qquad \therefore (1 , 1) = \left(\frac{x+3}{2} , \frac{y-2}{2}\right)$

$\therefore \frac{x+3}{2} = 1 \qquad \therefore x + 3 = 2 \qquad \therefore x = -1$

$, \frac{y-2}{2} = 1 \qquad \therefore y - 2 = 2 \qquad \therefore y = 4$

$\therefore C = (-1 , 4)$

$, \because$ the slope of $\overleftrightarrow{CD} = \frac{-2-4}{3+1} = \frac{-6}{4} = -\frac{3}{2}$

$\therefore$ The slope of the perpendicular straight line to $\overleftrightarrow{CD} = \frac{2}{3}$

$, \because$ the tangent to the circle M at C is perpendicular to $\overline{CD}$

$\therefore$ The slope of the tangent to the circle at C $= \frac{2}{3}$

$\therefore$ The equation of the tangent is : $y = \frac{2}{3} x + c$

$, \because$ the tangent passes through the point C $(-1 , 4)$

$\therefore 4 = \frac{2}{3} \times -1 + c \qquad \therefore c = 4\frac{2}{3}$

$\therefore$ The equation is : $y = \frac{2}{3} x + 4\frac{2}{3}$ (The req.)

Excellent pupils

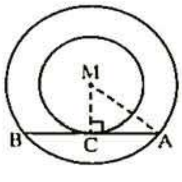
1 $\because \overline{AB}$ touches the small circle at C

$\therefore \overline{MC} \perp \overline{AB}$

$, \because \overline{AB}$ is a chord of the great circle $, \overline{MC} \perp \overline{AB}$

$\therefore$ C is the midpoint of $\overline{AB}$

$\therefore AC = \frac{14}{2} = 7$ cm.



$\because \Delta AMC$ is right angled at C

$\therefore (AC)^2 = (MA)^2 - (MC)^2$

$\therefore (7)^2 = (MA)^2 - (MC)^2$

$\therefore (MA)^2 - (MC)^2 = 49$

$\therefore$ The area of the included part between the two circles = the area of the greater circle − the area of the smaller circle $= \pi\,(MA)^2 - \pi\,(MC)^2$
$= \pi\,[(MA)^2 - (MC)^2] = 49\,\pi\ \text{cm}^2$ (The req.)

2 $\because \overleftrightarrow{AB}$ is a tangent to the circle M at B

$\therefore \overline{MB} \perp \overleftrightarrow{AB} \qquad \therefore m(\angle ABM) = 90^\circ$

$\because MB = MD$ (lengths of two radii)

$\therefore m(\angle MBD) = m(\angle MDB) = 2x^\circ$

In ΔABD : $m(\angle A) + m(\angle ABD) + m(\angle D) = 180^\circ$

$\therefore x^\circ + 90^\circ + 2x^\circ + 2x^\circ = 180^\circ$

$\therefore 5x^\circ = 90^\circ \qquad \therefore x = 18^\circ$ (The req.)

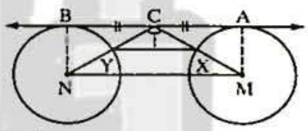
3 **Construction :** We draw $\overline{MA}$ and $\overline{NB}$

Proof : $\because \overleftrightarrow{AB}$ is a tangent to the circle M at A

$\therefore \overline{MA} \perp \overleftrightarrow{AB}$

similarly $\overline{NB} \perp \overleftrightarrow{AB}$

$\therefore \overline{MA} \parallel \overline{BN}$



$, \because MA = NB$ (two radii of two congruent circles)

$\therefore AMNB$ is a parallelogram

$\therefore \overleftrightarrow{AB} \parallel \overline{MN}$ (Q.E.D. 1)

In $\Delta\Delta$ AMC , BNC :

$\left\{\begin{array}{ll} AC = BC & \text{(given)} \\ MA = BN & \text{(given)} \\ m(\angle MAC) = m(\angle NBC) = 90^\circ & \text{(proved)} \end{array}\right.$

$\therefore \Delta AMC \equiv \Delta BNC \qquad \therefore MC = NC$ (1)

$\therefore \Delta CMN$ is an isosceles triangle (Q.E.D. 2)

$\because MX = NY$ (2)

$\therefore$ Subtracting (2) from (1) :

$\therefore MC - MX = NC - NY \qquad \therefore CX = CY$

$\therefore$ From the isosceles triangle XCY

$\therefore m(\angle CXY) = \frac{180^\circ - m(\angle 1)}{2}$ (3)

and from the isosceles triangle MNC

$\therefore m(\angle CMN) = \frac{180^\circ - m(\angle 1)}{2}$ (4)

From (3) and (4) : $m(\angle CXY) = m(\angle CMN)$

and they are corresponding angles

$\therefore \overline{XY} \parallel \overline{MN}$ (Q.E.D. 3)

Answers of Exercise 3

1

1 distant
2 touching externally
3 one is inside the other , touching internally
4 one is inside the other , distant.
5 touching internally , touching externally
6 the common chord , bisects it
7 the common tangent at the point of tangency
8 $\overleftrightarrow{MN}$
9 distant
10 intersecting
11 touching externally

2

1 d, 2 c, 3 b, 4 d, 5 a, 6 b
7 c, 8 b, 9 d, 10 c, 11 d, 12 b
13 d, 14 b, 15 b, 16 d

3

1 50°, 2 110°, 3 90°
4 6, 5 $12 , \sqrt{105}$, 6 $4\sqrt{10}$

4

1 10, 2 $45° \frac{1}{3}$

5

$\because MN = MA + NA$

$, \because NA = NB = 7$ cm. (lengths of two radii)

$\therefore 12 = MA + 7 \quad \therefore MA = 5$ cm. (The req.)

6

$\because$ The two circles are touching internally at A

$\therefore MN = 10 - 6 = 4$ cm. $, \overleftrightarrow{MN} \perp \overline{AB}$

$\therefore$ The area of Δ BMN $= \frac{1}{2} \times MN \times AB$

$\therefore 24 = \frac{1}{2} \times 4 \times AB \quad \therefore AB = 12$ cm. (The req.)

7

$\because \overleftrightarrow{MN}$ is the line of centres $, \overline{AB}$ is the common chord

$\therefore \overline{AB} \perp \overline{MN} \qquad \therefore m(\angle AEN) = 90°$

$\because$ The sum of the measures of the interior angles of the quadrilateral CDNE = 360°

$\therefore m(\angle CDN) = 360° - (55° + 125° + 90°) = 90°$

$\therefore \overleftrightarrow{ND} \perp \overleftrightarrow{CD}$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

8

$\because \overline{NM}$ is the line of centres $, \overline{AB}$ is the common chord

$\therefore \overline{MN} \perp \overline{AB} \qquad \therefore m(\angle AEM) = 90°$

$, \because$ C is the midpoint of $\overline{XY}$

$\therefore \overline{MC} \perp \overline{XY} \qquad \therefore (\angle MCX) = 90°$

In the quadrilateral DCME :

$m(\angle CME) = 360° - (90° + 90° + 40°) = 140°$ (First req.)

$, \because \overleftrightarrow{FZ}$ is a tangent to the circle N at F

$\therefore \overline{NF} \perp \overleftrightarrow{FZ} \qquad \therefore m(\angle NFZ) = 90°$

$\therefore m(\angle MEA) = m(\angle NFZ)$ and they are corresponding angles

$\therefore \overleftrightarrow{FZ} \,//\, \overline{AB}$ (Second req.)

9 $\because \overline{MN}$ is the line of centres $, \overline{AB}$ is the common cho rd of the two circles

$\therefore \overline{MN} \perp \overline{AB}$

$\because \overline{AB}$ // the straight line L

$\therefore$ The straight line L $\perp \overline{MN}$

$\therefore \overline{EF} \perp \overline{MX} \qquad \therefore XE = XF \quad (1)$

Similarly $\overline{NX} \perp \overline{CD} \qquad \therefore CX = XD \quad (2)$

Subtracting (1) from (2) :

$\therefore CX - XE = XD - XF$

$\therefore CE = FD$ (Q.E.D.)

10 $\because \overleftrightarrow{MN}$ is the line of centres $, \overline{AB}$ is the common chord of the two circles

$\therefore \overline{MN} \perp \overline{AB}$, C is the midpoint of $\overline{AB}$

$\therefore AC = \frac{1}{2} \times 12 = 6$ cm.

$\therefore MC = \sqrt{(AM)^2 - (AC)^2} = \sqrt{100 - 36} = 8$ cm.

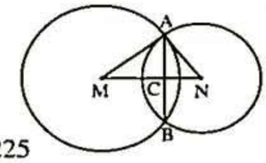
In Δ AMN :

$\because AM = AN = r \qquad , \overline{AC} \perp \overline{MN}$

$\therefore$ C is the midpoint of $\overline{MN}$

$\therefore MN = 2\, MC = 2 \times 8 = 16$ cm. (The req.)

11 $\because$ $\overleftrightarrow{MN}$ is the line of centres ,
$\overline{AB}$ is the common chord of the two circles



$\therefore \overleftrightarrow{MN} \perp \overline{AB}$, $AC = CB$

In Δ AMN : $(AN)^2 = 81$

, $(AM)^2 = 144$, $(MN)^2 = 225$

$\therefore (MN)^2 = (AM)^2 + (AN)^2$

$\therefore \Delta$ AMN is right-angled at A , $\because \overline{AC} \perp \overline{MN}$

$\therefore AC = \frac{AM \times AN}{MN} = \frac{12 \times 9}{15} = 7.2$ cm.

$\therefore AB = 2\,AC = 14.4$ cm. (The req.)

12 $\because$ $\overleftrightarrow{MN}$ is the line of centres

, $\overline{AB}$ is the comnon chord

$\therefore$ $\overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$

$\therefore CA = CB$

$\therefore$ In Δ ABC : m ($\angle$ CAB) = m ($\angle$ CBA) (1)

, $\because$ DA = DB

$\therefore$ In Δ ABD : m ($\angle$ DAB) = m ($\angle$ DBA) (2)

By adding (1) , (2) :

$\therefore$ m ($\angle$ CAD) = m ($\angle$ CBD) (Q.E.D.)

13 $\because$ $\overleftrightarrow{MN}$ is the line of centres , $\overline{AB}$ is the common chord.

$\therefore \overleftrightarrow{MN} \perp \overline{AB}$ i.e. m ($\angle$ AFM) = 90°

, $\because$ $\overline{CD}$ is a diameter of the circle M

, $\overleftrightarrow{CX}$ is a tangent of it at C

$\therefore \overleftrightarrow{CX} \perp \overline{CD}$ i.e. m ($\angle$ ECD) = 90°

$\therefore$ m ($\angle$ CEF) + m ($\angle$ CMF)

= 360° – (90° + 90°) = 180°

$\because$ m ($\angle$ DMF) + m ($\angle$ CMF) = 180°

$\therefore$ m ($\angle$ DMN) = m ($\angle$ CEB) (Q.E.D.)

14 In Δ ANB : $\because$ NA = NB , m ($\angle$ N) = 60°

$\therefore \Delta$ ANB is an equilateral triangle.

$\therefore$ AB = AN = r but MA = NA = r

because the two circles are congruent

$\therefore$ AB = MA = AN = r $\therefore AB = \frac{1}{2}$ MN

$\therefore$ m ($\angle$ MBN) = 90°

since $\overline{BN}$ is a radius of the circle N

$\therefore$ $\overleftrightarrow{MB}$ touches the circle N at B (Q.E.D.)

15 $\because$ The area of the shaded part = 550 cm^2

$\therefore$ The area of the great circle – The area of the small circle = 550 cm^2

$\therefore \pi r_1^2 - \pi r_2^2 = 550$ $\therefore \pi (r_1^2 - r_2^2) = 550$

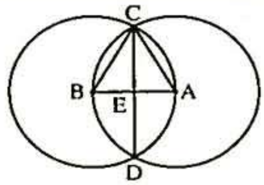
$\therefore r_1^2 - r_2^2 = 550 \times \frac{7}{22}$ $\therefore (r_1 - r_2)(r_1 + r_2) = 175$

$\because M_1 M_2 = r_1 - r_2$ because the two circles are touching internally

$\therefore 7 (r_1 + r_2) = 175$ $\therefore r_1 + r_2 = \frac{175}{7} = 25$ cm.

(The req.)

16 $\because$ $\overline{AB}$ is a radius of the circle A and a radius of the circle B



$\therefore$ The two circles are congruent

$\therefore$ CA = CB = AB = 3 cm.

$\therefore \Delta$ ABC is an equilateral triangle

$\therefore$ m ($\angle$ ACB) = 60° (First req.)

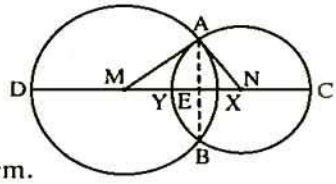
$\therefore \sin A = \frac{CE}{AC}$ $\therefore \sin 60° = \frac{CE}{3}$

$\therefore CE = 3 \sin 60° = \frac{3\sqrt{3}}{2}$

$\therefore CD = 2\,CE = 2 \times \frac{3\sqrt{3}}{2} = 3\sqrt{3}$ cm. (Second req.)

17

1. 4 , 8 , 24
2. Yes , because
   AN = CN (lengths of two radii in the circle N)
   , AM = MD (lengths of two radii in the circle M)
   i.e. AN + AM + NM = CN + MD + NM
   $\therefore$ The perimeter of Δ ANM = CD
3. m ($\angle$ NAM) = 90° because $(NM)^2 = (NA)^2 + (AM)^2$
4. The area of Δ NAM = $\frac{1}{2} \times NA \times MA$
   $= \frac{1}{2} \times 6 \times 8 = 24$ cm^2
5. **Construction :** Draw $\overline{AB}$ to cut $\overline{MN}$ at E
   Proof : $\because$ $\overline{AB}$ intersects $\overline{MN}$ at E



   $\therefore \overline{AB} \perp \overline{MN}$
   $\because AE = \frac{AM \times AN}{MN}$
   $= \frac{8 \times 6}{10} = 4.8$ cm.

$\because \overline{MN}$ bisects $\overline{AB}$ $\quad \therefore AE = EB = 4.8$

$\therefore AB = 4.8 \times 2 = 9.6$ cm. (The req.)

18 $\because r_1 = 6$ length unit , $r_2 = 4$ length unit

$\therefore r_1 + r_2 = 10$ length unit , $r_1 - r_2 = 2$ length unit

1 $\because MN = \sqrt{(-4-5)^2 + (8+4)^2} = \sqrt{81 + 144}$

$= \sqrt{225} = 15$ length unit

$\therefore MN > r_1 + r_2$

$\therefore$ The two circles are distant.

2 $\because MN = \sqrt{(2-6)^2 + (1+2)^2} = \sqrt{16 + 9}$

$= \sqrt{25} = 5$ length unit

$\therefore r_1 - r_2 < MN < r_1 + r_2$

$\therefore$ The two circles are intersecting.

19 Let the equation of $\overleftrightarrow{MN}$ be $y = m\,x + c$

$\because$ The two circles M and N are intersecting at A and B

$\therefore \overline{MN} \perp \overline{AB}$ and bisects it

$\because$ The slope of $\overleftrightarrow{AB} = \frac{-1-3}{-4-0} = 1$

$\because$ The slope of $\overleftrightarrow{MN} \times$ The slope of $\overleftrightarrow{AB} = -1$

$\therefore$ The slope of $\overleftrightarrow{MN} \times 1 = -1$

$\therefore$ The slope of $\overleftrightarrow{MN} = -1$

$\therefore$ The equation of $\overleftrightarrow{MN}$ is $y = -x + c$

$\because$ The midpoint of $\overline{AB}$ is $\left(\frac{0-4}{2}, \frac{3-1}{2}\right)$

$= (-2, 1)$

The midpoint of $\overline{AB}$ belongs to $\overleftrightarrow{MN}$

$\therefore$ It satisfies its equation

$\therefore 1 = 2 + c \quad \therefore c = -1$

$\therefore y = -x - 1$ (The req.)

20 $\because MA = \sqrt{(3+1)^2 + (5+3)^2} = \sqrt{16 + 64}$

$= 4\sqrt{5}$ length unit

$\therefore A \in$ the circle M

, $\because NA = \sqrt{(-3+1)^2 + (-7+3)^2} = \sqrt{4 + 16}$

$= 2\sqrt{5}$ length units

$\therefore A \in$ the circle N

$\because MN = \sqrt{(3+3)^2 + (5+7)^2} = \sqrt{36 + 144}$

$= \sqrt{180} = 6\sqrt{5}$ length units

$\therefore MN = MA + NA$

$\therefore$ The two circles are touching externally. (Q.E.D.)

Excellent pupils

1 $\because \overleftrightarrow{AC}$ is a tangent to the circle M at A

$\therefore \overline{MA} \perp \overline{AC}$

In $\triangle ACM$: $\because m(\angle A) = 90°$, $m(\angle C) = 30°$

$\therefore AM = \frac{1}{2} CM \quad \therefore CM = 12$ cm.

$\because DM = AM = 6$ cm. $\quad \therefore CD = 12 - 6 = 6$ cm.

$\because \overleftrightarrow{CB}$ is a tangent of the circle N at B

$\therefore \overline{NB} \perp \overline{CB}$

In $\triangle CBN$: $\because m(\angle CBN) = 90°$, $m(\angle C) = 30°$

$\therefore BN = \frac{1}{2} CN \quad , \because BN = NE$

$\therefore CE = EN = ND = \frac{6}{3} = 2$ cm. (The req.)

2 Assuming that the radii lengths of the circles L , M and N are r_1 , r_2 , r_3 respectively.

$\because LM = 5$ cm. $\quad \therefore r_1 + r_2 = 5$ (1)

$\because MN = 8$ cm. $\quad \therefore r_2 + r_3 = 8$ (2)

$\because LN = 7$ cm. $\quad \therefore r_3 + r_1 = 7$ (3)

Adding (1) , (2) and (3) :

$\therefore 2(r_1 + r_2 + r_3) = 20 \quad \therefore r_1 + r_2 + r_3 = 10$

$\because r_2 + r_3 = 8 \quad \therefore r_1 + 8 = 10$

$\therefore r_1 = 2$ cm.

and from (1) : $r_2 = 5 - 2 = 3$ cm.

and from (2) : $r_3 = 8 - 3 = 5$ cm. (The req.)

Answers of Exercise 4

«Notice that : Lengths are not real»

1

1 d	2 d	3 a	4 a	5 c	6 a
7 d	8 b	9 d	10 b	11 a	12 d

2

1 its radius

2 the circumcircle of this triangle

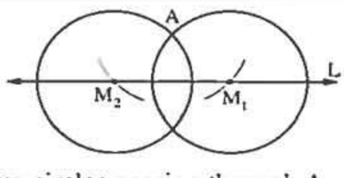
3 one circle

4 two circles

5 one circle

6 14 cm.

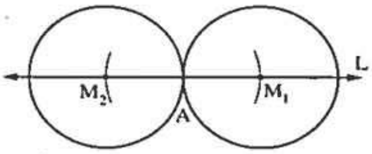
3



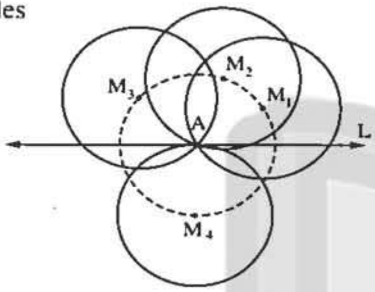
There are two circles passing through A

4

1 When $M \in L$ we can draw two circles



2 When $M \notin L$ we can draw an infinite number of circles



5

1 We can draw two circles

2 In $\Delta\ AM_1D$:

$\because \overline{M_1D} \perp \overline{AB}$

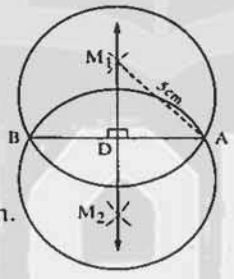
$\therefore$ D is the midpoint of $\overline{AB}$

$\therefore AD = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3$ cm.

$, \because m(\angle ADM_1) = 90°$

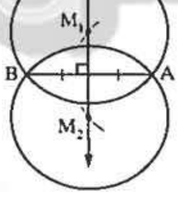
$\therefore M_1D = \sqrt{(AM_1)^2 - (AD)^2}$

$= \sqrt{25 - 9} = 4$ cm. (The req.)

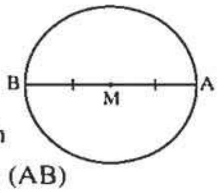


6

1 We can draw two circles.



2 We can draw one circle only.

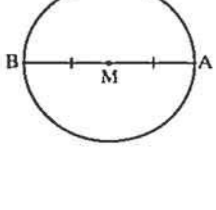


3 It is impossible to draw the circle because the radius length $= 1.5$ cm. which is less than $\frac{1}{2}$ (AB)

7

$\because$ The radius length is the smallest

$\therefore r = 3$ cm.



8

In $\Delta\ ADM$: $\because \overline{AD} \perp \overline{MD}$

$\therefore$ D is the midpoint of $\overline{AB}$

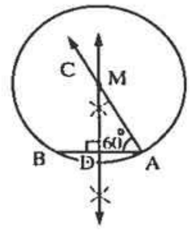
$\therefore AD = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3$ cm.

$, \because m(\angle ADM) = 90°$

$, m(\angle A) = 60°$

$\therefore m(\angle AMD) = 180° - (90° + 60°) = 30°$

$\therefore AM = 2\,AD = 2 \times 3 = 6$ cm. (The req.)



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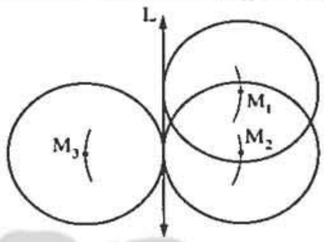
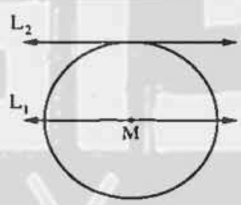


There are an infinite number of circles whose centres lie on a straight line parallel to the straight line L at a distance 3 cm. from it.

10



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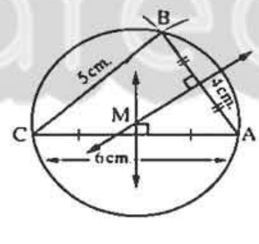
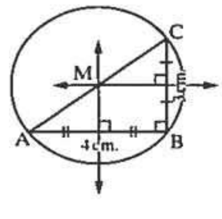


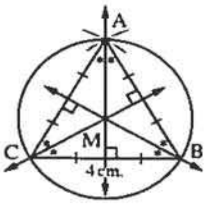
* Δ ABC is an acute-angled triangle
* The centre of the circle lies inside the triangle ABC

12



The centre of the circle lies at the midpoint of the hypotenuse $\overline{AC}$

13



1 The centre of the circle is the point of intersection of :

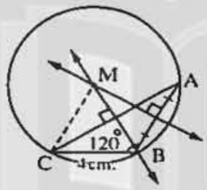
– The heights of the triangle

– The medians of the triangle

– The bisectors of the interior angles of the triangle

2 Three axes of symmetry.

14



In Δ ABC :

$\because$ AB = BC

$\therefore$ Δ ABC is an isosceles triangle

, $\because$ $\overline{BM} \perp \overline{AC}$

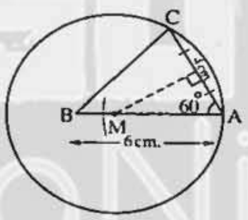
$\therefore$ $\overline{BM}$ bisects $\angle$ ABC $\quad\therefore$ m ($\angle$ MBC) = 60°

, $\because$ MB = MC = r

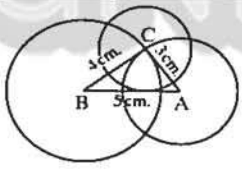
$\therefore$ Δ MBC is an equilateral triangle

$\therefore$ MB = MC = BC = r = 4 cm. (The req.)

15

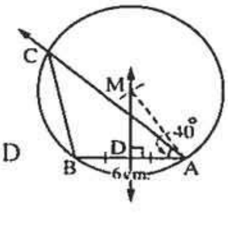


16



The type of this triangle according to the measures of its angle is right-angled triangle at C

17



$\because$ D is the midpoint of $\overline{AB}$

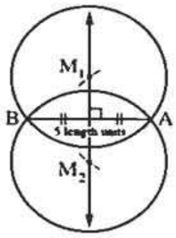
$\therefore$ AD = 3 cm.

In Δ AMD which is right-angled at D

$\therefore$ MD = $\sqrt{(AM)^2-(AD)^2}$

$= \sqrt{25-9} = \sqrt{16} = 4$ cm. (The req.)

18



AB $=\sqrt{(2+2)^2+(0-3)^2}$

$= \sqrt{16+9} = \sqrt{25}$

= 5 length units

$\therefore$ There are two solutions

19

$\because$ AB $=\sqrt{(1-1)^2+(3+1)^2} = \sqrt{16} = 4$ length units

, BC $=\sqrt{(1+3)^2+(-1+1)^2} = \sqrt{16} = 4$ length units

, AC $=\sqrt{(1+3)^2+(3+1)^2} = \sqrt{32} = 4\sqrt{2}$ length units

$\because$ $(AB)^2 = 16$, $(BC)^2 = 16$, $(AC)^2 = 32$

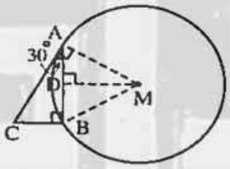
$\therefore$ $(AC)^2 = (AB)^2 + (BC)^2$

$\therefore$ Δ ABC is right-angled at B

$\therefore$ The centre of the circumcircle of Δ ABC is the midpoint of the hypotenuse $\overline{AC}$

$\therefore$ M = $\left(\frac{1-3}{2} , \frac{3-1}{2}\right) = (-1 , 1)$ (The req.)

Excellent pupils



$\because$ $\overline{AC}$ touches the circle M at A

$\therefore$ $\overline{MA} \perp \overline{AC}$

$\therefore$ m ($\angle$ MAB) = 90° – 30° = 60°

$\because$ MA = MB = r

$\therefore$ Δ ABM is an equilateral triangle

$\because$ $\overline{MD} \perp \overline{AB}$ $\qquad\therefore$ D is the midpoint of $\overline{BA}$

$\therefore$ AD = 2 cm. $\qquad$, $\because$ AM = AB = 4 cm.

In Δ AMD which is right-angled at D

$\therefore$ MD = $\sqrt{(AM)^2-(AD)^2} = \sqrt{16-4} = 2\sqrt{3}$

$\therefore$ The area of Δ ABM = $\frac{1}{2} \times 4 \times 2\sqrt{3} = 4\sqrt{3}$ cm^2.

(First req.)

The area of the circle M = π r^2 = 16 π cm^2. (Second req.)

Answers of Exercise 5

1

1 equidistant , centre

2 equal in length

3 equidistant

4 $\overline{AB}$

5 54°

6 $\because$ MF < ME $\therefore$ CD > AB $\therefore x + 1 > 7$ $\therefore x > 6$

$\because \overline{CD}$ is a chord doesn't pass through the centre of the circle M

$\therefore$ CD < 10 $\therefore x < 9$

$\therefore 6 < x < 9$ i.e. $x \in]6 , 9[$

2

1 MY , MF , FY　　2 MF , 4 , 16

3 MY , 40°　　4 NY , rectangle

3

1 14　　2 AB , 3 , 6 , 10

3 50°　　4 NY , congruent , AC

4

In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

, $\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$

, $\because \overline{MY} \perp \overline{AC}$, AB = AC $\therefore$ MX = MY (Q.E.D.)

5

$\because$ MF = ME (lengths of two radii)

, FX = EY

By subtracting : $\therefore$ MX = MY

, $\because$ X is the midpoint of $\overline{AC}$ $\therefore \overline{MX} \perp \overline{AC}$

, $\because$ Y is the midpoint of $\overline{BC}$ $\therefore \overline{MY} \perp \overline{BC}$

$\therefore$ AC = BC , $\because$ m ($\angle$ A) = 60°

$\therefore \Delta$ ABC is an equilateral triangle. (Q.E.D.)

6

$\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$

$\because$ Y is the midpoint of $\overline{AC}$ $\therefore \overline{MY} \perp \overline{AC}$

$\because$ The sum of measures of the interior angles of the quadrilateral AXMY = 360°

$\therefore$ m ($\angle$ XMY) = 360° − (70° + 90° + 90°) = 110° (First req.)

$\because$ AB = AC $\therefore$ MX = MY

$\because$ MD = ME (lengths of two radii)

by subtracting $\therefore$ XD = YE (Second req.)

7

$\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$

$\because$ AB = AC $\therefore$ MX = MY ,

$\because$ MD = ME (lengths of two radii) by subtracting

$\therefore$ XD = YE (Q.E.D. 1)

In Δ XMY : $\because$ MX = MY

$\therefore$ m ($\angle$ MXY) = m ($\angle$ MYX)

$\because$ m ($\angle$ MXB) = m ($\angle$ MYC) = 90°

by adding $\therefore$ m ($\angle$ YXB) = m ($\angle$ XYC) (Q.E.D. 2)

8

$\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

$\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\because$ AB = AC

$\therefore$ MX = MY

$\therefore \Delta$ MXY is an isosceles triangle (Q.E.D. 1)

$\because$ m ($\angle$ AXM) = 90° , m ($\angle$ MXY) = 30°

$\therefore$ m ($\angle$ AXY) = 90° − 30° = 60°

$\because$ X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$, AB = AC

$\therefore$ AX = AY

$\therefore \Delta$ AXY is an equilateral triangle (Q.E.D. 2)

9

$\because$ AB = CD ,

MB = MC (lengths of two radii)

$\because$ AM = DM

$\because \overline{MA} \perp \overline{XE}$, $\overline{MD} \perp \overline{EY}$ $\therefore$ XE = EY

$\because \overline{MA} \perp \overline{XE}$ $\therefore$ A is the midpoint of $\overline{XE}$

$\therefore$ XE = 6 cm. $\therefore$ EY = 6 cm. (The req.)

10

$\because \overline{AB}$ is the common chord of the two circles M , N

, $\overline{MN}$ is the line of centres

$\therefore \overleftrightarrow{MN} \perp \overline{AB}$ $\therefore \overline{MD} \perp \overline{AB}$

, $\because \overline{MX} \perp \overline{AC}$, AC = AB

$\therefore$ MX = MD (1)

, $\because$ MY = ME (lengths of two radii) (2)

Subtracting (1) from (2) :

$\therefore$ XY = DE (Q.E.D.)

11

In the circle M : $\because$ E is the midpoint of $\overline{CD}$

$\therefore \overline{ME} \perp \overline{CD}$

, $\because$ $\overline{AB}$ is the common chord , $\overleftrightarrow{MN}$ is the line of centres

$\therefore \overleftrightarrow{MN} \perp \overline{AB}$, $\because$ ME = ML

$\therefore$ AB = CD (1)

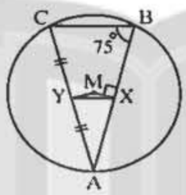
In the circle N : $\because \overleftrightarrow{MN} \perp \overline{AB}$, $\overline{NZ} \perp \overline{XY}$

, NL = NZ $\therefore$ AB = XY (2)

From (1) and (2) :

$\therefore$ CD = XY (Q.E.D.)

12



$\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$

, $\because \overline{MX} \perp \overline{AB}$, MX = MY

$\therefore$ AB = AC

$\therefore$ m ($\angle$ C) = 75°

$\therefore$ m ($\angle$ A) = 180° – (75° + 75°) = 30° (First req.)

$\because \overline{MX} \perp \overline{AB}$ $\therefore$ X is the midpoint of $\overline{AB}$

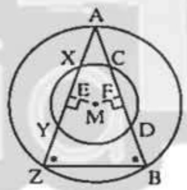
$\therefore$ In Δ ABC :

XY = $\frac{1}{2}$ BC , AX = $\frac{1}{2}$ AB , AY = $\frac{1}{2}$ AC

$\therefore$ The perimeter of Δ AXY

= $\frac{1}{2}$ The perimeter of Δ ABC (Second req.)

13



Constr. :

Draw : $\overline{MF} \perp \overline{AB}$, $\overline{ME} \perp \overline{AZ}$

Proof : In the great circle :

$\because$ m ($\angle$ ABZ) = m ($\angle$ AZB)

$\therefore$ AB = AZ

$\because \overline{MF} \perp \overline{AB}$, $\overline{ME} \perp \overline{AZ}$ $\therefore$ MF = ME

In the small circle :

$\because \overline{MF} \perp \overline{CD}$, $\overline{ME} \perp \overline{XY}$, MF = ME

$\therefore$ CD = XY (Q.E.D.)

14

$\because$ MF = ME (lengths of two radii)

, XF = YE $\therefore$ MX = MY

$\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$ $\therefore$ AB = CD (Q.E.D. 1)

$\because \overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$ $\therefore$ AX = $\frac{1}{2}$ AB

$\because \overline{MY} \perp \overline{CD}$

$\therefore$ Y is the midpoint of $\overline{CD}$ $\therefore$ CY = $\frac{1}{2}$ CD

$\because$ AB = CD $\therefore$ AX = CY

$\therefore \Delta$ AXF , Δ CYE

In them $\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90° \end{cases}$

$\therefore \Delta AXF \equiv \Delta CYE$ then we deduce that AF = CE

(Q.E.D. 2)

15

$\because$ Y is the midpoint of $\overline{AC}$ $\therefore \overline{MY} \perp \overline{AC}$ (1)

Similarly $\overline{MX} \perp \overline{AB}$

$\because$ AC = AB $\therefore$ MY = MX

and from Δ YMX : $\because$ m ($\angle$ M) = 120°

$\therefore$ m ($\angle$ MYX) = m ($\angle$ YXM) = $\frac{180° - 120°}{2}$ = 30° (2)

from (1) and (2) : $\therefore$ m ($\angle$ AYX) = 90° – 30° = 60°

$\because \overrightarrow{YZ}$ bisects $\angle$ AYX

$\therefore$ m ($\angle$ ZYX) = $\frac{60°}{2}$ = 30°

$\therefore$ m ($\angle$ ZYX) = m ($\angle$ YXM)

but they are alternate angles

$\therefore \overrightarrow{YZ} // \overrightarrow{MX}$ (Q.E.D.)

16

$\because$ The circle M $\cap$ the circle N = {A , B}

$\therefore \overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$

$\therefore$ In Δ ABD : $\overleftrightarrow{DC}$ is the axis of symmetry of $\overline{AB}$

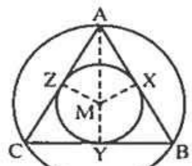
$\therefore$ AD = BD

$\because \overline{MX} \perp \overline{AD}$, $\overline{MY} \perp \overline{BD}$ $\therefore$ MX = MY (Q.E.D.)

17

Constr. : Draw $\overline{MX}$, $\overline{MY}$, $\overline{MZ}$, $\overline{MA}$

Proof :



$\because \overleftrightarrow{AB}$ is a tangent to the smaller circle M

$\therefore \overline{MX} \perp \overline{AB}$

, similary : $\overline{MY} \perp \overline{BC}$, $\overline{MZ} \perp \overline{AC}$

, $\because$ MX = MY = MZ = r in the smaller circle

$\therefore$ AB = BC = AC

$\therefore \Delta$ ABC is an equilateral triangle (First req.)

$\therefore$ m ($\angle$ B) = 60°

, ∵ the greater circle M is the circumcircle of Δ ABC

∴ M is the point of intersection of the altitudes of Δ ABC

∴ $\overline{AY}$ is an altitude in Δ ABC

∴ In Δ ABY which is right at Y : $\sin B = \frac{AY}{AB}$

, ∵ AY = AM + MY = 4 + 2 = 6 cm.

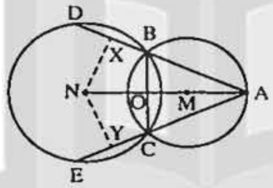
∴ $\sin 60° = \frac{6}{AB}$ ∴ $\frac{\sqrt{3}}{2} = \frac{6}{AB}$

∴ $AB = \frac{2 \times 6}{\sqrt{3}} = 4\sqrt{3}$ cm. ∴ $BC = AB = 4\sqrt{3}$ cm.

∴ The area of Δ ABC = $\frac{1}{2} \times BC \times AY$

$= \frac{1}{2} \times 4\sqrt{3} \times 6 = 12\sqrt{3}$ cm². (Second req.)

18



Constr. : Draw $\overline{NX} \perp \overline{BD}$, $\overline{NY} \perp \overline{EC}$

Proof :

∵ $\overleftrightarrow{MN}$ is the line of centres

, $\overline{BC}$ is the common chord of the two circles

∴ $\overleftrightarrow{MN} \perp \overline{BC}$, O is the midpoint of $\overline{BC}$

∴ OB = OC

∴ In ΔΔ AOB , AOC

$\begin{cases} OB = OC \\ \overline{AO} \text{ is common side} \\ m(\angle AOB) = m(\angle AOC) = 90° \end{cases}$

∴ Δ AOB ≡ Δ AOC

∴ m (∠ BAO) = m (∠ CAO)

In ΔΔ AXN , AYN

∵ m (∠ AXN) = m (∠ AYN) = 90°

, m (∠ XAN) = m (∠ YAN)

∴ m (∠ ANX) = m (∠ ANY)

∴ In ΔΔ AXN , AYN

$\begin{cases} m(\angle ANX) = m(\angle ANY) \\ m(\angle XAN) = m(\angle YAN) \\ \overline{AN} \text{ is a common side} \end{cases}$

∴ Δ AXN ≡ Δ AYN ∴ NX = NY

, ∵ $\overline{NX} \perp \overline{BD}$, $\overline{NY} \perp \overline{CE}$

∴ BD = CE (Q.E.D.)

19

∵ Z is the midpoint of $\overline{AB}$

∴ $\overline{MZ} \perp \overline{AB}$ similarly $\overline{MX} \perp \overline{CD}$ (1)

∵ AB = CD ∴ MZ = MX

From Δ MZX : ∵ m (∠ M) = 120°

∴ m (∠ MZX) = m (∠ MXZ) = $\frac{180° - 120°}{2}$ = 30° (2)

From (1) and (2) :

∴ m (∠ YZX) = m (∠ YXZ) = 90° – 30° = 60°

∴ Δ ZYX is an equilateral triangle. (Q.E.D.)

20

∵ Δ MXA and Δ MYB which are right-angled triangles

In them $\begin{cases} MA = MB \text{ (lengths of two radii)} \\ MX = MY \end{cases}$

∴ The two triangles are congruent , then we deduce that :

m (∠ MAX) = m (∠ MBY)

∴ Δ HAB is an isosceles triangle. (Q.E.D. 1)

, ∵ $\overline{MX} \perp \overline{AC}$, $\overline{MY} \perp \overline{BD}$

, MX = MY

∴ AC = BD , ∵ AH = BH

∴ AH – AC = BH – BD ∴ HC = HD (Q.E.D. 2)

21

∵ X is the midpoint of $\overline{AB}$ ∴ $\overline{MX} \perp \overline{AB}$

∵ Y is the midpoint of $\overline{AC}$ ∴ $\overline{MY} \perp \overline{AC}$

∵ MX = MY ∴ AB = AC

, ∵ m (∠ BAC) = 60°

∴ Δ ABC is an equilateral triangle (Q.E.D. 1)

∵ BM = CM = r ∴ M ∈ the axis of symmetry of $\overline{BC}$

∵ AB = AC ∴ A ∈ the axis of symmetry of $\overline{BC}$

∴ $\overleftrightarrow{AM}$ is the axis of symmetry of $\overline{BC}$

∴ $\overleftrightarrow{AM} \perp \overline{BC}$ (Q.E.D. 2)

22

∵ X is the midpoint of $\overline{AB}$

∴ $\overline{MX} \perp \overline{AB}$ similarly $\overline{MY} \perp \overline{CD}$,

∵ AB = CD ∴ MX = MY

∴ Δ MYX is an isosceles triangle

∵ $\overline{ML} \perp \overline{XY}$ ∴ XL = LY (1)

∵ $\overline{ML} \perp$ the chord $\overline{EF}$ ∴ EL = LF (2)

subtracting (1) from (2) : ∴ XE = YF (Q.E.D.)

23

∵ $\overline{MA} \perp \overline{ZC}$ ∴ A is the midpoint of $\overline{ZC}$

similarly B is the midpoint of $\overline{ZD}$

∵ MA = MB ∴ ZC = ZD

∴ $\frac{1}{2}$ ZC = $\frac{1}{2}$ ZD ∴ AZ = BZ

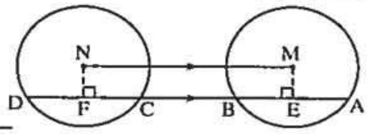
$\therefore \triangle XAZ$ and $\triangle YBZ$

In them $\begin{cases} AZ = BZ \text{ (Proved)} \\ m(\angle ZAX) = m(\angle ZBY) = 90^\circ \\ \angle Z \text{ is a common angle} \end{cases}$

$\therefore \triangle XAZ \equiv \triangle YBZ$ and we deduce that $XZ = YZ$

$\because ZD = ZC$ $\therefore CY = DX$ (Q.E.D.)

24



Constr. :

Draw :

$\overline{ME} \perp \overline{AB}$, $\overline{NF} \perp \overline{CD}$

Proof : $\because \overline{EF} // \overline{MN}$, $\overline{ME} \perp \overline{AB}$, $\overline{NF} \perp \overline{CD}$

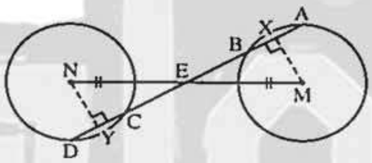
$\therefore \overline{ME} // \overline{NF}$

$\therefore$ The figure MNFE is a rectangle $\therefore ME = NF$

$\because$ M and N are two congruent circles $\therefore AB = CD$

Adding BC to both sides $\therefore AC = BD$ (Q.E.D.)

25



Constr. :

Draw $\overline{MX} \perp \overline{AB}$

, $\overline{NY} \perp \overline{CD}$

Proof :

In $\triangle\triangle$ MXE , NYE

In them $\begin{cases} m(\angle MXE) = m(\angle NYE) = 90^\circ \\ m(\angle MEX) = m(\angle NEY) \text{ (V.O.A)} \\ ME = NE \text{ (given)} \end{cases}$

$\therefore \triangle MXE \equiv \triangle NYE$ $\therefore MX = NY$

$\therefore AB = CD$ (First req.)

, $\because \overline{MX} \perp \overline{AB}$, $\overline{NY} \perp \overline{CD}$

$\therefore$ X is midopint of $\overline{AB}$

, Y is the midpoint of $\overline{CD}$

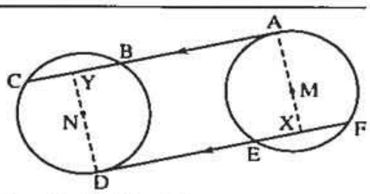
, $\because AB = CD$ $\therefore XA = YD$

, $\because XE = YE$ ($\triangle MXE \equiv \triangle NYE$)

By adding : $\therefore AE = DE$

$\therefore$ E is the midpoint of $\overline{AD}$ (Second req.)

26



Constr. :

Draw $\overline{AX}$ and $\overline{DY}$

Proof :

$\because \overleftrightarrow{AB}$ is a tangent to the circle M at A

$\therefore \overline{MA} \perp \overleftrightarrow{AB}$

, $\because \overline{AC} // \overline{FD}$ $\therefore m(\angle AXE) = 90^\circ$

Similarly ,

$\because \overleftrightarrow{DE}$ is a tangent of the circle N at D

$\therefore \overline{ND} \perp \overleftrightarrow{DE}$, $\because \overline{AC} // \overline{FD}$

$\therefore m(\angle DYB) = 90^\circ$

$\therefore$ AXDY is rectangle. $\therefore AX = DY$

, $\because$ M and N are two congruent circles.

$\therefore MA = ND$ $\therefore MX = NY$

, $\because \overline{MX} \perp \overline{EF}$, $\overline{NY} \perp \overline{BC}$

$\therefore BC = EF$ (Q.E.D. 1)

, $\because AY = XD$ (1)

, $\because \overleftrightarrow{MX} \perp \overline{EF}$ $\therefore$ X is the midpoint of $\overline{EF}$

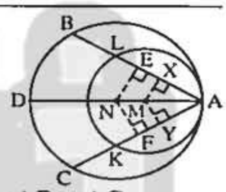
Similarly , Y is the midpoint of $\overline{BC}$

, $\because EF = BC$ $\therefore BY = XE$ (2)

Subtracting (2) from (1) :

$\therefore AB = DE$ (Q.E.D. 2)

27



Constr. :

Draw : $\overline{NE} \perp \overline{AB}$, $\overline{NF} \perp \overline{AC}$

$\overline{MX} \perp \overline{AL}$, $\overline{MY} \perp \overline{AK}$

Proof : $\because \overline{NE} \perp \overline{AB}$, $\overline{NF} \perp \overline{AC}$, $AB = AC$

$\therefore NE = NF$

$\therefore \triangle ANE$ and $\triangle ANF$ which are right-angled

In them $\begin{cases} NE = NF \\ \overline{AN} \text{ is a common side} \end{cases}$

$\therefore \triangle ANE \equiv \triangle ANF$, then we deduce that

$m(\angle NAE) = m(\angle NAF)$

$\therefore \triangle AMX$, $\triangle AMY$

In them $\begin{cases} \overline{AM} \text{ is common side} \\ m(\angle AXM) = m(\angle AYM) = 90^\circ \\ m(\angle XAM) = m(\angle YAM) \text{ (proved)} \end{cases}$

$\therefore \triangle AMX \equiv \triangle AMY$, then we deduce that $MX = MY$

$\overline{MX} \perp \overline{AL}$, $\overline{MY} \perp \overline{AK}$

$\therefore AL = AK$ (Q.E.D.)

28

$\because \overline{MD} \perp \overline{AB}$ $\therefore$ D is the midpoint of $\overline{AB}$ (1)

, $\because \overline{ME} \perp \overline{AC}$ $\therefore$ E is the midpoint of $\overline{AC}$ (2)

, $\because AD = \sqrt{(2-1)^2 + (2-0)^2} = \sqrt{5}$ length units

, $AE = \sqrt{(2-3)^2 + (2-4)^2} = \sqrt{5}$ length units

$\therefore AD = AE$ $\therefore AB = AC$

$\therefore ME = MD$ (Q.E.D.)

29

Let D and E be the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

$\because$ D is the midpoint of $\overline{AB}$

$\therefore D = \left(\frac{4+0}{2}, \frac{3+3}{2}\right) = (2, 3)$

$\therefore MD = \sqrt{(2-2)^2 + (1-3)^2} = \sqrt{4} = 2$ length unit

$\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$

$\because$ E is the midpoint of $\overline{AC}$ $\therefore \overline{ME} \perp \overline{AC}$

$\because AB = AC$ $\therefore MD = ME$

$\therefore ME = 2$ length units

$\therefore$ The chord $\overline{AC}$ is at a distance $= 2$ length units from the centre of the circle M (The req.)

30

$\because$ F is the midpoint of $\overline{AB}$

$\therefore F = \left(\frac{4+0}{2}, \frac{-1-3}{2}\right) = (2, -2)$

$\therefore MF = \sqrt{(1-2)^2 + (0+2)^2} = \sqrt{5}$ length unit

$\because ME = \sqrt{(1+1)^2 + (0-1)^2} = \sqrt{5}$ length units

$\therefore MF = ME$

$\because$ F is the midpoint of $\overline{AB}$ $\therefore \overline{MF} \perp \overline{AB}$

$\because$ E is the midpoint of $\overline{DC}$ $\therefore \overline{ME} \perp \overline{DC}$

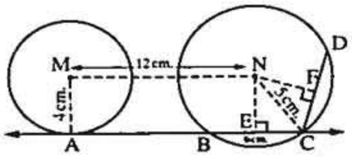
$\therefore AB = CD$ (Q.E.D.)

Excellent pupils

Constr. :

Draw :

$\overline{NE} \perp \overline{CB}$, $\overline{NF} \perp \overline{CD}$



Proof : $\because \overline{NE} \perp \overline{CB}$

$\therefore$ E is the midpoint of $\overline{CB}$

In Δ NEC which is right-angled at E

$NE = \sqrt{(NC)^2 - (CE)^2} = \sqrt{25-9} = 4$ cm.

$\therefore NE = AM$

$\because \overline{AC}$ is a tangent to the circle M , $\overline{MA}$ is a radius

$\therefore \overline{MA} \perp \overline{AC}$

, $\because m(\angle CEN) = m(\angle CAM) = 90°$

and they are alternate angles

$\therefore \overline{NE} \parallel \overline{AM}$

$\therefore$ The figure NEAM is a rectangle $\therefore \overline{NM} \parallel \overline{CA}$

$\therefore$ The figure MACN is a trapezium

Its area $= \frac{1}{2}(MN + AC) \times AM$

$= \frac{1}{2}(12 + 15) \times 4 = 54 \text{ cm}^2$ (First req.)

$\because \overline{NF} \perp \overline{CD}$, $\overline{NE} \perp \overline{CB}$, $CD = CB$

$\therefore NF = NE = 4$ cm.

$\therefore$ The distance between the point N and $\overline{CD}$ is 4 cm. (Second req.)

Answers of exams on unit Four

Model 1

1

1 b	2 a	3 c
4 c	5 a	6 d

2

[a] 1 $m(\angle DMX) = 126°$

2 $DE = 4$ cm.

[b] Prove by yourself.

3

[a] $AB = 6$ cm.

[b] Draw by yourself , you can draw two circles.

4

[a] Prove by yourself.

[b] 1 $m(\angle CME) = 130°$ 2 Prove by yourself.

5

[a] Draw by yourself ,

the centre of the circle lies at the midpoint of the hypotenuse $\overline{AC}$

[b] Prove by yourself.

Model 2

1

1 a
2 b
3 c
4 a
5 b
6 d

2

[a] $BC = \frac{14\sqrt{3}}{3}$ cm.

[b] Prove by yourself.

3

[a] Prove by yourself.

[b] Prove by yourself.

4

[a] Draw by yourself , 3 cm.

[b] 1 $m(\angle BAC) = 30°$

2 $\frac{\text{The perimeter of } \Delta ABC}{\text{The perimeter of } \Delta AXY} = 2$

5

[a] Prvoe by yourself.

[b] Prove by yourself.

Answers of unit five

Answers of Exercise 6

1

1 length
2 equal in length
3 equal
4 equal
5 360°
6 180° , π r
7 90°

2

The measure of the arc $= \frac{1}{3}$ the measure of the circle

$= \frac{1}{3} \times 360° = 120°$

The length of the arc $= \frac{1}{3}$ the circumference of the circle

$= \frac{1}{3} \times 2\pi r$

$= \frac{1}{3} \times 2 \times \frac{22}{7} \times 21 = 44$ cm.

3

$\because m(\angle AMB) = 120°$ $\therefore m(\widehat{AB}) = 120°$

$\because r = 7$ cm.

$\therefore$ the length of $\widehat{AB} = \frac{120°}{360°} \times 2\pi r$

$= \frac{120°}{360°} \times 2 \times \frac{22}{7} \times 7 = 14\frac{2}{3}$ cm.

4

1 a, 2 a, 3 c, 4 c, 5 a
6 c, 7 b, 8 c, 9 c, 10 b
11 c, 12 a

5

1 75°, 2 140°, 3 255°, 4 220°

6

1 25°, 2 50°, 3 100°, 4 130°
5 150°, 6 210°, 7 260°, 8 310°

7

$\because m(\widehat{AC}) = m(\widehat{BC})$ $\therefore AC = BC$

$\therefore$ In ΔABC : $\because AC = BC$, $m(\angle C) = 70°$

$\therefore m(\angle ABC) = \frac{180° - 70°}{2} = 55°$ (The req.)

8

$\because MA = MB = r$ $\therefore m(\angle A) = m(\angle B) = 45°$

$\therefore m(\angle M) = 180° - (45° + 45°) = 90°$

$\therefore m(\widehat{AB}) = 90°$

$\therefore$ The length of $\widehat{AB} = \frac{90}{360} \times 2\pi r$

$= \frac{90}{360} \times 2 \times \frac{22}{7} \times 7 = 11$ cm.

(The req.)

9

$\because \overline{AB} \cap \overline{CD} = \{M\}$

$\therefore m(\angle AMC) = m(\angle DMB) = 35°$

$\therefore m(\widehat{AC}) = 35°$

$\because \overline{AB} \parallel \overline{CE}$

$\therefore m(\widehat{BE}) = m(\widehat{AC}) = 35°$ (The req.)

10

$\because \overline{XB} \parallel \overline{CY}$ $\therefore m(\widehat{XC}) = m(\widehat{BY})$

$\therefore XC = BY$, $\because \overline{MA} \perp \overline{XC}$, $\overline{MD} \perp \overline{BY}$

$\therefore MA = MD$ (Q.E.D.)

11

$\because AD = BC$ $\therefore m(\widehat{AD}) = m(\widehat{BC})$

$\therefore m(\angle AND) = m(\angle CNB)$

Adding $m(\angle DNB)$ to both sides

$\therefore m(\angle ANB) = m(\angle CND)$ (Q.E.D.)

12

$\because \overline{AC}$ is a diameter in the circle M

$\therefore m(\widehat{ABC}) = m(\widehat{ADC})$ (semicircle) (1)

$\because BC = CD$ $\therefore m(\widehat{BC}) = m(\widehat{CD})$ (2)

Subtracting (2) from (1) :

$\therefore m(\widehat{AB}) = m(\widehat{AD})$ (Q.E.D.)

13

$\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\widehat{AB}) = 180°$

$\because m(\widehat{AC}) = m(\widehat{CD}) = m(\widehat{DB})$

$\therefore m(\widehat{CD}) = \frac{180}{3} = 60°$

$\therefore m(\angle CMD) = 60°$ $\because MC = MD$

$\therefore \Delta MCD$ is equilateral. (Q.E.D.)

14

$\because m(\angle CMD) = 70°$ $\therefore m(\widehat{CD}) = 70°$

$\because m(\widehat{AC}) + m(\widehat{CD}) + m(\widehat{DB}) = 180°$

$\therefore m(\widehat{AC}) + m(\widehat{DB}) = 180° - 70° = 110°$

Let $m(\overset{\frown}{AC})$ be $5x$, $m(\overset{\frown}{DB}) = 6x$

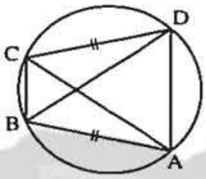
$\therefore 5x + 6x = 110$ $\therefore 11x = 110$

$\therefore x = 10°$

$\therefore m(\overset{\frown}{AC}) = 5 \times 10° = 50°$

$\therefore m(\overset{\frown}{ACD}) = 50° + 70° = 120°$ (The req.)

15



$\because AB = DC$

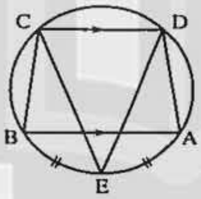
$\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{DC})$

Adding $m(\overset{\frown}{BC})$ to both sides

$\therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{BD})$

$\therefore AC = BD$ (Q.E.D.)

16



$\because \overline{AB} \parallel \overline{DC}$

$\therefore m(\overset{\frown}{BC}) = m(\overset{\frown}{AD})$

$\because$ E is the midpoint of $\overset{\frown}{AB}$

$\therefore m(\overset{\frown}{EB}) = m(\overset{\frown}{AE})$ adding

$\therefore m(\overset{\frown}{CE}) = m(\overset{\frown}{DE})$

$\therefore CE = DE$ (Q.E.D.)

17

$\because \overline{AB} \parallel \overline{EF}$ $\therefore m(\overset{\frown}{AE}) = m(\overset{\frown}{BF})$ (1)

$\because \overline{AB} \parallel \overline{CD}$ $\therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{BC})$ (2)

Adding (1) and (2) : $\therefore m(\overset{\frown}{CE}) = m(\overset{\frown}{CF})$

$\therefore CE = CF$ (Q.E.D.)

Another solution

$\because \overline{CD} \parallel \overline{EF}$ $\therefore m(\overset{\frown}{EC}) = m(\overset{\frown}{CF})$

$\therefore CE = CF$ (Q.E.D.)

18

$\because AB = CD$ (properties of the rectangle)

, $CE = CD$ $\therefore AB = CE$ $\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{CE})$

and adding $m(\overset{\frown}{BE})$ to both sides

$\therefore m(\overset{\frown}{AE}) = m(\overset{\frown}{BC})$ $\therefore AE = BC$

(Q.E.D.)

19

$\because \overleftrightarrow{BE} \parallel \overline{AC}$ $\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{CB})$ (1)

$\because \overline{BC} \parallel \overline{AD}$ $\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$ (2)

From (1) and (2) : $\therefore m(\overset{\frown}{CB}) = m(\overset{\frown}{CD})$

$\therefore CB = CD$ $\therefore \Delta BCD$ is isosceles (Q.E.D.)

20

$\because$ The length of $\overset{\frown}{AC}$ = the length of $\overset{\frown}{AB}$

$\therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{AB}) = 80°$ $\therefore m(\angle AMB) = 80°$

In ΔAMB : $\because MA = MB = r$

$\therefore m(\angle MAB) = \frac{180° - 80°}{2} = 50°$ (First req.)

$\because \overline{AB} \parallel \overline{CD}$ $\therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{BD}) = 80°$

$\because m(\overset{\frown}{CD}) + m(\overset{\frown}{AC}) + m(\overset{\frown}{AB}) + m(\overset{\frown}{BD}) = 360°$

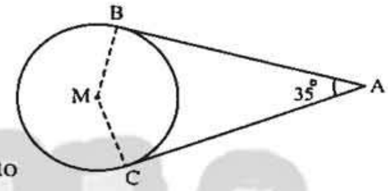
$\therefore m(\overset{\frown}{CD}) = 360° - (80° + 80° + 80°) = 120°$

(Second req.)

$\therefore$ The length of $\overset{\frown}{CD} = \frac{120}{360} \times 2 \times 3.14 \times 15 = 31.4$ cm.

(Third req.)

21



Construction :

Draw $\overline{MB}$, $\overline{MC}$

Proof :

$\because \overline{AB}$ is a tangent to the circle M at B

$\therefore \overline{MB} \perp \overline{AB}$ similarly $\overline{MC} \perp \overline{AC}$

$\therefore m(\angle BMC) = 360° - (35° + 90° + 90°) = 145°$

$\therefore m(\overset{\frown}{BC}) = 145°$

$\therefore m(\overset{\frown}{BC}$ the major$) = 360° - 145° = 215°$ (The req.)

22

$\because \overleftrightarrow{AM} \parallel \overline{CD}$, $\overleftrightarrow{MD}$ is transversal

$\therefore m(\angle CDM) + m(\angle AMD) = 180°$

(two interior angles in the same side of the transversal)

$\therefore m(\angle CDM) = 180° - 90° = 90°$

, $\because MD = \frac{1}{2} MB$, $MC = MB = r$

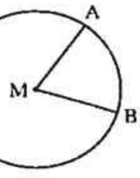
$\therefore MD = \frac{1}{2} MC$ $\therefore m(\angle MCD) = 30°$

, $\because \overleftrightarrow{AM} \parallel \overline{CD}$, $\overleftrightarrow{CM}$ is a transversal

$\therefore m(\angle AMC) = m(\angle MCD) = 30°$ (alternate angles)

$\therefore m(\overset{\frown}{AC}) = m(\angle AMC) = 30°$ (The req.)

23



Let $m(\angle AMB$ the reflex$)$ be $x°$

$\therefore m(\angle AMB) = \frac{1}{4} x$

$\therefore \frac{1}{4} x + x = 360°$

$\therefore \frac{5}{4} x = 360°$ $\therefore x = 288°$

$\therefore m(\overset{\frown}{AB}) = m(\angle AMB) = \frac{1}{4} \times 288 = 72°$ (The req.)

24

$\because AC = BD$ $\qquad \therefore m(\overset{\frown}{ABC}) = m(\overset{\frown}{BCD})$

subtracting $m(\overset{\frown}{BC})$ from both sides

$\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$ $\qquad \therefore AB = CD$

$\therefore 3x - 5 = x + 3$ $\qquad \therefore 2x = 8$ $\qquad \therefore x = 4$

$\therefore AB = 7$ cm (The req.)

25

$\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\overset{\frown}{AB}) = 180°$ $\quad , \because \overline{AB} \parallel \overline{DE}$

$\therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{CB}) = \frac{180°}{2} = 90°$

$, \because$ X is the midpoint of $\overset{\frown}{AC}$

$\therefore m(\overset{\frown}{AX}) = 45°$ $\qquad \therefore m(\angle AMX) = m(\overset{\frown}{AX}) = 45°$

$, \because \overline{DE} \parallel \overline{AB}$, $\overleftrightarrow{DM}$ is a transversal

$\therefore m(\angle EDM) = m(\angle AMD) = 45°$ (alternate angles)

$, \because m(\overset{\frown}{BY}) = 2\, m(\overset{\frown}{CY})$ $\qquad \therefore m(\overset{\frown}{BY}) = 60°$

$\therefore m(\angle YMB) = m(\overset{\frown}{BY}) = 60°$

$, \because \overline{DE} \parallel \overline{AB}$, $\overleftrightarrow{ME}$ is a transversal

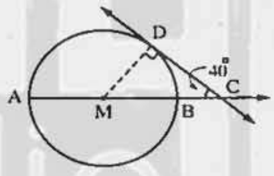
$\therefore m(\angle DEM) = m(\angle EMB) = 60°$ (alternate angles)

In $\triangle$ MDE : $m(\angle DME) = 180° - (45° + 60°) = 75°$

(The req.)

26

1



Construction : Draw $\overline{MD}$

Proof :

$\because \overleftrightarrow{CD}$ is a tangent to the circle M at D

$, \overline{MD}$ is a radius $\qquad \therefore m(\angle MDC) = 90°$

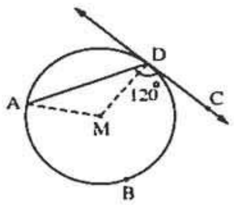
In $\triangle$ MDC : $\therefore m(\angle CMD) = 50°$

$\therefore m(\overset{\frown}{BD}) = 50°$ (First req.)

$\therefore m(\overset{\frown}{AD}) = 180° - 50° = 130°$ (Second req.)

2 **Construction :**

Draw $\overline{MD}$ and $\overline{MA}$



Proof : $\because \overleftrightarrow{CD}$ is a tangent to the circle M at D

$, \overline{MD}$ is a radius

$\therefore m(\angle MDC) = 90°$

$\therefore m(\angle MDA) = 120° - 90° = 30°$

In $\triangle$ MDA : $\because MD = MA = r$

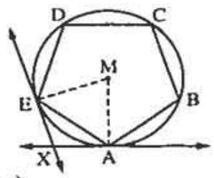
$\therefore m(\angle DMA) = 180° - (30° + 30°) = 120°$

$\therefore m(\angle DMA$ the reflex$) = 240°$

$\therefore m(\overset{\frown}{ABD}) = 240°$ (The req.)

Excellent pupils

1



Construction : Draw $\overline{AM}$, $\overline{ME}$

Proof :

$\because AB = BC = CD = DE = AE$

(the properties of the regular pentagon)

$\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{BC}) = m(\overset{\frown}{CD})$

$= m(\overset{\frown}{DE}) = m(\overset{\frown}{AE})$

$\because$ measure of the circle $= 360°$

$\therefore m(\overset{\frown}{AE}) = \frac{360}{5} = 72°$ (First req.)

$\therefore m(\angle AME) = m(\overset{\frown}{AE}) = 72°$

$\because \overleftrightarrow{AX}$ is a tangent to the circle at A ,

$\overline{MA}$ is a radius $\qquad \therefore m(\angle MAX) = 90°$

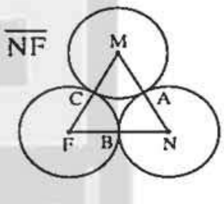
similarly $m(\angle MEX) = 90°$

In the quadrilateral MAXE :

$\therefore m(\angle AXE) = 360° - (72° + 90° + 90°) = 108°$

(Second req.)

2



Construction : Draw $\overline{MN}$, $\overline{MF}$, $\overline{NF}$

Proof : $\because A \in \overline{MN}$, $B \in \overline{NF}$

$C \in \overline{MF}$ and M , N and F

are three congruent circles

$\therefore$ The radii lengths of them are equal

$\therefore \triangle$ MNF is equilateral.

$\therefore m(\angle ANB) = m(\angle BFC) = m(\angle AMC) = 60°$

$\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{BC}) = m(\overset{\frown}{AC})$ (First req.)

$\therefore$ The perimeter of the figure ABC

$= 3 \times \frac{60}{360} \times 2 \times 3.14 \times 10 = 31.4$ cm. (Second req.)

Answers of Exercise 7

First : Problems on theorem (1) and its corollaries

1

1 55°	2 70°	3 30°	4 45°	5 40°
6 114°	7 80°	8 110°	9 20°	

10 $m(\angle A) = 70°$, $m(\overset{\frown}{AC}) = 100°$

11 $m(\angle C) = 90°$, $m(\angle B) = 26°$

12 30°　13 40°　14 75°　15 25°　16 55°

17 m (∠ M) = 80° , m (∠ C) = 140°

18 115°　19 117° 30̀　20 110°

2

1 y = 70°　2 z = 40°　3 ℓ = 40°

4 z = 62° 30̀　5 x = 135°　6 y = 40° , z = 10°

3

1 a　2 b　3 c　4 c　5 c

6 b　7 d　8 d　9 a　10 b

11 c　12 a　13 c　14 a　15 b

16 c　17 c　18 b

4

$\because$ m (∠ BAC) = $\frac{1}{2}$ m (∠ BMC)

(inscribed and central angles subtended the same arc $\widehat{BC}$)

$\therefore$ m (∠ BAC) = $\frac{1}{2}$ × 130° = 65°

$\because$ The sum of the measures of the interior angles of the triangle ABC = 180°

$\therefore$ m (∠ ABC) = 180° − (65° + 50°) = 65° (The req.)

5

$\because$ MA = MB (lengths of two radii)

$\therefore$ m (∠ MBA) = m (∠ MAB) = 26°

$\therefore$ m (∠ AMB) = 180° − (26° + 26°) = 128° (First req.)

$\because$ m (∠ ACB) = $\frac{1}{2}$ m (∠ AMB)

(inscribed and central angles subtended the same arc $\widehat{AB}$)

$\therefore$ m (∠ ACB) = $\frac{1}{2}$ × 128 = 64° (Second req.)

$\because$ m (∠ AMB the reflex) = 360° − 128° = 232°

$\therefore$ m (∠ AXB) = $\frac{1}{2}$ m (∠ AMB the reflex)

= 116° (Third req.)

(inscribed and central angles subtended the same major arc $\widehat{AB}$)

m $(\widehat{AXB})$ = m (∠ AMB) = 128° (Fourth req.)

6

$\because$ m (∠ BCD) = $\frac{1}{2}$ m (∠ BMD)

(inscribed and central angles subtended by $\widehat{BD}$)

$\therefore$ m (∠ BCD) = $\frac{1}{2}$ × 50° = 25°

, $\because$ $\overline{AB}$ is a diameter in the circle M

$\therefore$ m (∠ ACB) = 90°

$\therefore$ m (∠ ACD) = m (∠ ACB) + m (∠ BCD)

= 90° + 25° = 115° (The req.)

7

$\because$ $\overline{AB}$ is a diameter in the circle M

$\therefore$ m (∠ ACB) = 90°

$\because$ m (∠ ABC) = 180° − (90° + 35°) = 55°

similarly m (∠ ADB) = 90°

, $\because$ the length of $\widehat{AD}$ = the length of $\widehat{DB}$

$\therefore$ AD = DB , from Δ ABD :

$\therefore$ m (∠ DBA) = m (∠ DAB) = $\frac{180° - 90°}{2}$ = 45°

$\therefore$ m (∠ CBD) = 55° + 45° = 100° (The req.)

8

$\because$ m $(\widehat{AC})$ = 2 m (∠ ABC) = 140°

$\therefore$ D is the midpoint of $\widehat{AC}$

$\therefore$ m $(\widehat{AD})$ = $\frac{140°}{2}$ = 70°

$\therefore$ m (∠ DCA) = 35° (First req.)

$\because$ $\overline{AB}$ is a diameter in the circle M

$\therefore$ m (∠ ACB) = 90°

$\therefore$ m (∠ CAB) = 180° − (90° + 70°) = 20° (Second req.)

9

$\because$ $\overleftrightarrow{AC}$ touches the circle at A　$\therefore$ $\overline{MA} \perp \overleftrightarrow{AC}$

In Δ ABC : $(CB)^2 = (AB)^2 + (AC)^2 = (12)^2 + (9)^2$

= 225

$\therefore$ CB = 15 cm. (First req.)

$\because$ $\overline{AB}$ is a diameter　$\therefore$ m (∠ ADB) = 90°

$\therefore$ AD = $\frac{AC \times AB}{BC}$ = $\frac{9 \times 12}{15}$ = 7.2 cm. (Second req.)

10

$\because$ m (∠ BMC) = 2 m (∠ A)

(central and inscribed angles subtended the same arc $\widehat{BC}$)

$\therefore$ m (∠ BMC) = 2 × 30° = 60°

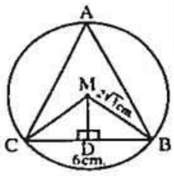
$\because MB = MC = r \quad \therefore \Delta MBC$ is equilateral

$\therefore MB = MC = BC = 7$ cm. $= r$

$\therefore$ The area of the circle M

$= \pi r^2 = \frac{22}{7} \times 49 = 154 \text{ cm}^2$ (The req.)

11



$\because \overline{MD} \perp \overline{BC}$

$\therefore$ D is the midpoint of $\overline{BC}$

$\therefore BD = 3$ cm.

$\therefore \cos(\angle MBD) = \frac{BD}{BM} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\therefore m(\angle MBD) = 30°$

In ΔMBC : $\because MB = MC = r$

$\therefore m(\angle MCB) = 30°$

$\therefore m(\angle BMC) = 180° - 2 \times 30° = 120°$

$\therefore m(\angle BAC) = \frac{1}{2} m(\angle BMC)$

(inscribed and central angles subtended by the same arc $\overset{\frown}{BC}$)

$\therefore m(\angle BAC) = \frac{1}{2} \times 120° = 60°$ (The req.)

12

$\because m(\angle C) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended the same arc $\overset{\frown}{AB}$)

$\therefore m(\angle C) = \frac{1}{2} \times 120° = 60°$

In ΔAMB : $\because MA = MB = r$

$\therefore m(\angle MAB) = m(\angle MBA) = \frac{180° - 120°}{2} = 30°$

, $m(\angle CAB) = 180° - 130° = 50°$

$\therefore$ In ΔABC :

$m(\angle ABC) = 180° - (60° + 50°) = 70°$

$\therefore m(\angle MBC) = 70° - 30° = 40°$ (The req.)

13

$\because m(\angle EBC) = \frac{1}{2} m(\angle M)$

(inscribed and central angles subtended the same arc $\overset{\frown}{EC}$)

$\therefore m(\angle EBC) = \frac{1}{2} \times 120° = 60°$

, $\because \angle EBC$ is an exterior angle of ΔABE

$\therefore m(\angle BEA) + m(\angle A) = 60°$

, $\because BE = BA \quad \therefore m(\angle A) = \frac{60°}{2} = 30°$ (The req.)

14

$\because m(\angle BAC) = \frac{1}{2} m(\angle BNC)$

(inscribed and central angles subtended the same arc $\overset{\frown}{BC}$)

$\therefore m(\angle BAC) = \frac{1}{2} \times 80° = 40° \quad \because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

$= \frac{180° - 40°}{2} = 70°$ (First req.)

$\because m(\overset{\frown}{BC}) = m(\angle N) = 80°$

$\therefore m(\overset{\frown}{BC}$ the major$) = 360° - 80° = 280°$ (Second req.)

15

$\because m(\angle ACD) = \frac{1}{2} m(\overset{\frown}{AD}) = \frac{1}{2} \times \frac{1}{2} m(\overset{\frown}{AB})$

$\therefore m(\angle ACD) = \frac{1}{4} m(\overset{\frown}{AB}) = \frac{1}{4} m(\angle ANB)$ (Q.E.D.)

16

$\because \overline{AB} \parallel \overline{CD}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore m(\angle BAC) + m(\angle ACD) = 180°$

$\therefore 100° + m(\angle ACD) = 180° \quad \therefore m(\angle ACD) = 80°$

$\therefore m(\angle AMD) = 2\, m(\angle ACD) = 160°$

(central and inscribed angles subtended the same arc $\overset{\frown}{AD}$)

(The req.)

17

$\because m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended the same arc $\overset{\frown}{AB}$)

$\therefore m(\angle ACB) = \frac{1}{2} \times 120° = 60°$ (1)

$\because \overleftrightarrow{CD} \parallel \overline{AB} \quad \therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{BC})$

$\therefore AC = BC$ (2)

From (1) and (2) : $\therefore \Delta CAB$ is equilateral (Q.E.D.)

18

$\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\overset{\frown}{AB}) = 180°$

, $\because m(\overset{\frown}{AC}) = 2\, m(\angle ABC) = 2 \times 40° = 80°$

$\therefore m(\overset{\frown}{BDC}) = 180° - 80° = 100°$

, $\because \overleftrightarrow{DH} \parallel \overline{BC}$

$\therefore m(\overset{\frown}{CD}) = m(\overset{\frown}{BD}) = \frac{100°}{2} = 50°$ (The req.)

19

$\because m(\angle ACD) = \frac{1}{2} m(\angle AMD)$

(inscribed and central angles subtended by $\overset{\frown}{AD}$)

$\therefore m(\angle ACD) = \frac{1}{2} \times 70° = 35°$ (First req.)

, $\because \overline{DC} \parallel \overline{AB}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle A) = m(\angle ACD) = 35°$ (alternate angles)

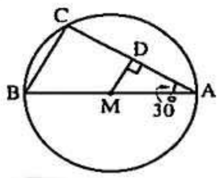
, $\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\angle ACB) = 90°$

$\therefore$ From ΔABC : $m(\angle ABC) = 180° - (90° + 35°) = 55°$

(Second req.)

20



$\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\angle C) = 90°$

$, \because \overline{MD} \perp \overline{AC}$

$\therefore m(\angle ADM) = 90°$

$\therefore m(\angle C) = m(\angle ADM) = 90°$

and they are corresponding angles

$\therefore \overline{DM} \parallel \overline{BC}$ (Q.E.D.1)

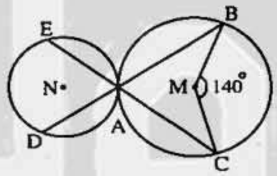
$, \because \Delta ABC$ is right-angled at C

$, m(\angle A) = 30°$

$\therefore BC = \frac{1}{2} AB$ $, \because \overline{AB}$ is a diameter in the circle M

$\therefore BC$ = the radius length of the circle (Q.E.D.2)

21



$\because m(\angle BAC) = \frac{1}{2} m(\angle M)$

(inscribed and central angles subtended by the same arc)

$\therefore m(\angle BAC) = 70°$

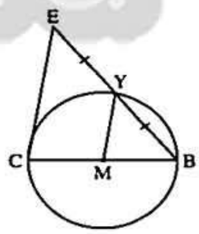
$, \because \overline{BD} \cap \overline{CE} = \{A\}$

$\therefore m(\angle EAD) = m(\angle BAC) = 70°$ (V.O.A.)

$\therefore m(\widehat{ED}) = 2\, m(\angle EAD) = 2 \times 70° = 140°$

(The req.)

22



$\because m(\angle YMC) = 2\, m(\angle YBC)$ (1)

(central and inscribed angles subtended the same arc $\widehat{CY}$)

In ΔBMY :

$\because MB = MY = r$

$\therefore m(\angle YBM) = m(\angle BYM)$ (2)

$\because$ Y and M are the midpoints of $\overline{BE}$, $\overline{BC}$ respectively

$\therefore \overline{MY} \parallel \overline{EC}$

$\because \overleftrightarrow{BE}$ is a transversal to them

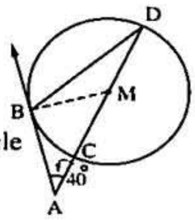
$\therefore m(\angle BYM) = m(\angle BEC)$ (3)

(corresponding angles)

From (1) , (2) and (3) :

$\therefore m(\angle YMC) = 2\, m(\angle BEC)$ (Q.E.D.)

23



Construction :

Draw $\overline{MB}$

Proof : $\because \overleftrightarrow{AB}$ is a tangent to the circle at B , $\overline{MB}$ is a radius

$\therefore m(\angle MBA) = 90°$

In ΔMBA :

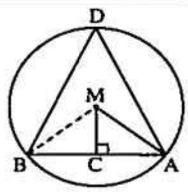
$\therefore m(\angle BMA) = 180° - (90° + 40°) = 50°$

$\because m(\angle BDC) = \frac{1}{2} m(\angle BMC)$

(inscribed and central angles subtended the same arc $\widehat{BC}$)

$\therefore m(\angle BDC) = \frac{1}{2} \times 50° = 25°$ (The req.)

24



Construction :

Draw $\overline{MB}$

Proof :

$\because m(\angle ADB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended the same arc $\widehat{AB}$) (1)

In ΔAMB : $\because AM = BM$, $\overline{MC} \perp \overline{AB}$

$\therefore \overrightarrow{MC}$ bisects $\angle AMB$

$\therefore m(\angle AMC) = \frac{1}{2} m(\angle AMB)$ (2)

From (1) and (2) :

$\therefore m(\angle AMC) = m(\angle ADB)$ (Q.E.D.)

25

$\because m(\angle AMC) = 2\, m(\angle ABC)$

(inscribed and central angles subtended the same arc $\widehat{AC}$)

$\because \overline{CM} \parallel \overline{AB}$, $\overleftrightarrow{MA}$ is a transversal to them

$\therefore m(\angle MAB) = m(\angle AMC)$ (alternate angles)

In ΔAEB : $\therefore m(\angle EAB) = 2\, m(\angle EBA)$

$\therefore m(\angle EAB) > m(\angle EBA)$

$\therefore BE > AE$ (Q.E.D.)

26

$\because m(\widehat{AB}) : m(\widehat{BC}) : m(\widehat{AC}) = 4 : 5 : 3$

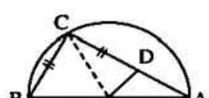
$\therefore m(\widehat{AB}) = 4x$, $m(\widehat{BC}) = 5x$, $m(\widehat{AC}) = 3x$

$\therefore 4x + 5x + 3x = 360°$ $\therefore 12x = 360°$

$\therefore x = \frac{360°}{12} = 30°$ $\therefore m(\widehat{AB}) = 4 \times 30° = 120°$

$\therefore m(\angle ACB) = \frac{1}{2} \times 120° = 60°$ (The req.)

27



Construction : Draw $\overline{MC}$

Proof : $\because CD = CB = r$

$, \because MB = MC = r$

$\therefore MC = MB = BC$ $\therefore \Delta MBC$ is equilateral

$\therefore m(\angle MCB) = 60°$

, $\because \overline{AB}$ is a diameter of the semicircle M

$\therefore m(\angle ACB) = 90°$

$\therefore m(\angle MCD) = 90° - 60° = 30°$

In ΔMCD : $\because CD = r$ $\therefore CD = CM$

$\therefore m(\angle CDM) = \frac{180° - 30°}{2} = 75°$

$\therefore m(\angle ADM) = 180° - 75° = 105°$ (The req.)

Second : Problems on wellknown problems

1

1 80°	2 20°	3 100°
4 86°	5 40°	6 60°

2

1 c	2 a	3 c
4 b	5 b	

3

$\because m(\widehat{CB}) + m(\widehat{BD}) + m(\widehat{AD}) + m(\widehat{AC}) = 360°$

$\therefore m(\widehat{CB}) = 360° - (60° + 100° + 120°) = 80°$ (First req.)

$\therefore m(\angle CEB) = \frac{1}{2}(80° + 100°) = 90°$ (Second req.)

4

$\because \frac{1}{2}[m(\widehat{EC}) - 60°] = 40°$ $\therefore m(\widehat{EC}) - 60° = 80°$

$\therefore m(\widehat{EC}) = 140°$ (First req.)

$\because m(\widehat{BD}) + m(\widehat{BC}) + m(\widehat{CE}) + m(\widehat{DE}) = 360°$

, $\because m(\widehat{BC}) = m(\widehat{DE})$

$\therefore 60° + 2\,m(\widehat{BC}) + 140° = 360°$

$\therefore m(\widehat{BC}) = 80°$ (Second req.)

5

$\because m(\angle E) = \frac{1}{2}[m(\widehat{AC}) - m(\widehat{BE})]$

$\therefore 30° = \frac{1}{2}[80° - m(\widehat{BD})]$

$\therefore 60° = 80° - m(\widehat{BD})$ $\therefore m(\widehat{BD}) = 20°$

, $\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\widehat{AB}) = 180°$

$\therefore m(\widehat{AC}) + m(\widehat{CD}) + (\widehat{BD}) = 180°$

$\therefore 80° + m(\widehat{CD}) + 20° = 180°$

$\therefore m(\widehat{CD}) = 180° - 100° = 80°$ (The req.)

6

$\because m(\angle A) = \frac{1}{2}[m(\widehat{BC}) - m(\widehat{DE})]$

$\therefore 70° = \frac{1}{2}[180° - m(\widehat{DE})]$

$\therefore 140° = 180° - m(\widehat{DE})$

$\therefore m(\widehat{DE}) = 180° - 140° = 40°$

, $\because \overline{DE} \parallel \overline{BC}$ $\therefore m(\widehat{BD}) = m(\widehat{CE})$

, $\because m(\widehat{BD}) + m(\widehat{DE}) + m(\widehat{CE}) = 180°$

$\therefore m(\widehat{BD}) + 40° + m(\widehat{BD}) = 180°$

$\therefore 2\,m(\widehat{BD}) = 180° - 40° = 140°$

$\therefore m(\widehat{BD}) = \frac{140°}{2} = 70°$ (The req.)

7

$\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\widehat{AB}) = 180°$, $\because m(\widehat{CD}) = 80°$

$\therefore m(\widehat{AC}) + m(\widehat{BD}) = 180° - 80° = 100°$

, $\because \overline{AB} \parallel \overline{CD}$

$\therefore m(\widehat{AC}) = m(\widehat{BD}) = \frac{100°}{2} = 50°$

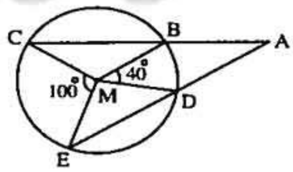
, $\because m(\angle DHB) = \frac{1}{2} m(\widehat{BD})$

$\therefore m(\angle DHB) = \frac{1}{2} \times 50° = 25°$ (First req.)

, $\because m(\angle AOH) = \frac{1}{2}[m(\widehat{AH}) + m(\widehat{BD})]$

$\therefore m(\angle AOH) = \frac{1}{2}[100° + 50°] = 75°$ (Second req.)

8



$\because m(\angle BMD) = 40°$

$\therefore m(\widehat{BD}) = 40°$

$\because m(\angle CME) = 100°$

$\therefore m(\widehat{CE}) = 100°$

$\therefore m(\angle A) = \frac{1}{2}(100° - 40°) = 30°$ (The req.)

9

$\because m(\widehat{BD}$ the major$) = 2\,m(\angle BCD) = 2 \times 100° = 200°$

$\therefore m(\widehat{BCD}) = 360° - 200° = 160°$

, $\because m(\widehat{EO}) = m(\angle EMO) = 50°$

, $\because m(\angle A) = \frac{1}{2}[m(\widehat{BCD}) - m(\widehat{EO})]$

$\therefore m(\angle A) = \frac{1}{2}[160° - 50°] = 55°$ (The req.)

10

$\because \frac{1}{2}[m(\widehat{CE}) - 44°] = 30°$ $\therefore m(\widehat{CE}) - 44° = 60°$

$\therefore m(\widehat{CE}) = 104°$ (First req.)

$\because m(\widehat{DE}) = 2\, m(\angle DCE)$ $\therefore m(\widehat{DE}) = 2 \times 48° = 96°$

$\because m(\widehat{DB}) + m(\widehat{DE}) + m(\widehat{CE}) + m(\widehat{BC}) = 360°$

$\therefore 44° + 96° + 104° + m(\widehat{BC}) = 360°$

$\therefore m(\widehat{BC}) = 116°$ (Second req.)

11

$\because m(\widehat{BD}) = 2\, m(\angle BCD)$

$\therefore m(\widehat{BD}) = 2 \times 26° = 52°$

$, \because m(\angle A) = \frac{1}{2}[m(\widehat{CH}) - m(\widehat{BD})]$

$\therefore 40° = \frac{1}{2}[m(\widehat{CH}) - 52°]$

$\therefore m(\widehat{CH}) - 52° = 80°$

$\therefore m(\widehat{CH}) = 132°$ (First req.)

$, \because m(\angle HXC) = \frac{1}{2}[m(\widehat{CH}) + m(\widehat{BD})]$

$\therefore m(\angle HXC) = \frac{1}{2}[132° + 52°] = 92°$ (Second req.)

12

$\because m(\angle BOC) = \frac{1}{2}[m(\widehat{BC}) + m(\widehat{DE})]$

$\therefore 92° = \frac{1}{2}[m(\widehat{BC}) + m(\widehat{DE})]$

$\therefore 184° = m(\widehat{BC}) + m(\widehat{DE})$ (1)

$, \because m(\angle A) = \frac{1}{2}[m(\widehat{BC}) - m(\widehat{DE})]$

$\therefore 34° = \frac{1}{2}[m(\widehat{BC}) - m(\widehat{DE})]$

$\therefore 68° = m(\widehat{BC}) - m(\widehat{DE})$ (2)

Adding (1) and (2) : $\therefore 252° = 2\, m(\widehat{BC})$

$\therefore m(\widehat{BC}) = 126°$

$, \because m(\angle CDB) = \frac{1}{2} m(\widehat{BC})$

$\therefore m(\angle CDB) = \frac{1}{2} \times 126° = 63°$ (The req.)

13

$m(\angle AEC) = \frac{1}{2} m(\widehat{AC}) = 40°$ (First req.)

$\because \overline{AB} \parallel \overline{CD}$

$\therefore m(\widehat{BD}) = m(\widehat{AC}) = 80°$ (Second req.)

$\because m(\angle AXC) = \frac{1}{2}[m(\widehat{AC}) + m(\widehat{EB})]$

$\therefore 60° = \frac{1}{2}[80° + m(\widehat{EB})]$

$\therefore m(\widehat{EB}) = 120° - 80° = 40°$ (Third req.)

14

$\because AC = DB$ $\therefore m(\widehat{AC}) = m(\widehat{DB})$

$\therefore m(\angle AEC) = \frac{1}{2}[m(\widehat{AC}) + m(\widehat{DB})]$

$= \frac{1}{2}[m(\widehat{AC}) + m(\widehat{AC})] = m(\widehat{AC})$

$\because m(\widehat{AC}) = m(\angle AMC)$

$\therefore m(\angle AMC) = m(\angle AEC)$ (Q.E.D.)

Excellent pupils

1

Let $m(\angle ABC) = m(\angle AMC) = x$,

$\because m(\angle ABC) = \frac{1}{2} m(\angle AMC$ the reflex$)$

(inscribed and central angles subtended the same arc $\widehat{AC}$ the major)

$\therefore m(\angle AMC$ the reflex$) = 2x$

$\because m(\angle AMC) + m(\angle AMC$ the reflex$) = 360°$

$\therefore x + 2x = 360°$ $\therefore 3x = 360°$

$\therefore x = 120°$ $\therefore m(\angle B) = 120°$ (The req.)

2

$\because AB = AD = AC$ $\therefore$ A is the centre of the circle which passes through the points B , D and C

$\therefore \angle BAD$ is a central angle

$, \angle BCD$ is an inscribed angle

$\therefore m(\angle BCD) = \frac{1}{2} m(\angle BAD)$

(inscribed and central angles subtended the same arc $\widehat{BD}$)

$\therefore m(\angle BCD) = \frac{1}{2} \times 50° = 25°$ (The req.)

Answers of Exercise 8

1

1 equal in measure 2 equal in measure
3 50° , 25° 4 40° , 90° 5 20° , 117°

2

1 **First** : b **Second** : c 2 d
3 **First** : b **Second** : a 4 b 5 b

3

Fig. (1) : $x = 65°$

Fig. (2) : $x = 25°$

Fig. (3) : $x = 40°$, $y = 50°$, $z = 90°$

Fig. (4) : $x = 50°$ **Fig. (5)** : $x = 60°$

Fig. (6) : $x = 40°$, $y = 40°$, $z = 30°$

Fig. (7) : $x = 53°$, $y = 53°$, $z = 53°$

Fig. (8) : $y = 10°$ **Fig.** (9) : $x = 70°$

Fig. (10) : $x = 10°$ **Fig.** (11) : $x = 15°$

Fig. (12) : $z = 25°$

5 Theoretical.

5

$\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\angle ADB) = 90°$

$\because$ in ΔABD : $m(\angle ABD) = 25°$, $m(\angle ADB) = 90°$

$\therefore m(\angle DAB) = 180° - (25° + 90°) = 65°$

$\therefore m(\angle DEB) = m(\angle DAB) = 65°$

(two inscribed angles subtended by $\overset{\frown}{BD}$) (The req.)

6

$\because \overline{AC}$ is a diameter in the circle M

$\therefore m(\angle ABC) = 90°$

$\therefore m(\angle CBD) = 90° - 60° = 30°$ (First req.)

, $\because m(\angle ADB) = m(\angle ACB)$

(two inscribed angles subtended by $\overset{\frown}{AB}$)

$\therefore m(\angle ADB) = 50°$

In ΔABD : $\therefore m(\angle BAD) = 180° - (50° + 60°) = 70°$

(Second req.)

7

$\because AB = AC$ $\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{AC})$

$\therefore m(\angle AEB) = m(\angle AEC)$ (Q.E.D.)

8

$\because m(\angle A) = m(\angle C)$

(two inscribed angles subtended by $\overset{\frown}{BD}$)

$\because \overline{AB} \parallel \overline{CD}$

$\therefore m(\angle B) = m(\angle C)$ (alternate angles)

$\therefore m(\angle A) = m(\angle B)$ $\therefore AF = FB$ (Q.E.D.)

9

$\because \overline{DE} \parallel \overline{BC}$ $\therefore m(\overset{\frown}{DB}) = m(\overset{\frown}{EC})$

$\therefore m(\angle DAB) = m(\angle EAC)$

Adding $m(\angle BAC)$ to both sides

$\therefore m(\angle DAC) = m(\angle BAE)$ (Q.E.D.)

10

$\because m(\angle A) = m(\angle C)$

(two inscribed angles subtended by $\overset{\frown}{BD}$)

, $m(\angle B) = m(\angle D)$

(two inscribed angles subtended by $\overset{\frown}{AC}$)

$\because EA = ED$ $\therefore m(\angle A) = m(\angle D)$

$\therefore m(\angle B) = m(\angle C)$ $\therefore EB = EC$ (Q.E.D.)

11

$\because AB = CD$ $\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$

Subtracting $m(\overset{\frown}{BD})$ from both sides

$\therefore m(\overset{\frown}{AD}) = m(\overset{\frown}{BC})$ $\therefore m(\angle C) = m(\angle A)$

$\therefore \Delta ACE$ is isosceles. (Q.E.D.)

12

$\because \overline{AB}$ is a diameter of the circle M

$\therefore m(\angle ACB) = 90°$

$\because \overline{DC} \parallel \overline{AB}$, $\overleftrightarrow{CB}$ is a transversal to them

$\therefore m(\angle ABC) + m(\angle DCB) = 180°$

(two interior angles in the same side of the transversal)

$\therefore m(\angle DCB) = 180° - 55° = 125°$

$\therefore m(\angle ACD) = 125° - 90° = 35°$

$\therefore m(\angle AED) = m(\angle ACD) = 35°$

(two inscribed angles subtended by $\overset{\frown}{AD}$) (The req.)

13

$\because m(\angle BDC) = m(\angle BAC)$

(two inscribed angles subtended by $\overset{\frown}{BC}$)

$\therefore m(\angle BDC) = 30°$

, $\because m(\overset{\frown}{BC}) = 2\, m(\angle BDC) = 2 \times 30° = 60°$

, $\because \overline{AB}$ is a diameter of the circle M

$\therefore m(\overset{\frown}{AB}) = 180°$

$\therefore m(\overset{\frown}{AC}) = 180° - 60° = 120°$

, $\because$ D is the midpoint of $\overset{\frown}{AC}$

$\therefore m(\overset{\frown}{AD}) = \frac{120°}{2} = 60°$ (First req.)

, $\because m(\angle ACD) = \frac{1}{2}\, m(\overset{\frown}{AD})$

$\therefore m(\angle ACD) = \frac{1}{2} \times 60° = 30°$

$\therefore m(\angle ACD) = m(\angle BAC) = 30°$

and they are alternate angles

$\therefore \overline{AB} \parallel \overline{CD}$ (Second req.)

14

$\because$ AD = BE $\quad\therefore m(\widehat{AD}) = m(\widehat{BE})$

Adding $m(\widehat{DE})$ to both sides

$\therefore m(\widehat{AE}) = m(\widehat{BD}) \quad \therefore m(\angle B) = m(\angle A)$

$\therefore$ In Δ ABC : AC = BC

$\because$ AD = BE subtracting $\quad\therefore$ CD = CE (Q.E.D.)

15

In Δ AEC : $\because$ CE = AE

$\therefore m(\angle ACE) = m(\angle CAE) \quad \therefore m(\widehat{AD}) = m(\widehat{BC})$

Adding $m(\widehat{DB})$ to both sides

$\therefore m(\widehat{ADB}) = m(\widehat{CBD})$

$\therefore m(\angle ACB) = m(\angle CAD)$ (Q.E.D.)

16

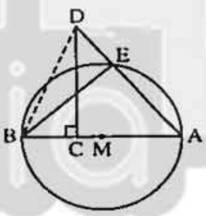
In Δ ABD : $\because$ AD = AB

$\therefore m(\angle ADB) = m(\angle ABD) = \frac{180° - 80°}{2} = 50°$

$\therefore m(\angle ADB) = m(\angle ACB) = 50°$ but they are drawn on $\overline{AB}$ and on one side of it

$\therefore$ The points A , B , C and D have one circle passing through them. (Q.E.D.)

17



$\because \overline{CD} \perp \overline{AB}$

$\therefore m(\angle BCD) = 90°$

$\because \overline{AB}$ is a diameter

$\therefore m(\angle AEB) = 90°$

$\therefore m(\angle BED) = 90°$

$\therefore m(\angle BCD) = m(\angle BED)$ but they are drawn on $\overline{BD}$ and on one side of it

$\therefore$ The points D , E , C and B have one circle passing through them. (Q.E.D.)

18

1 $\because \overline{AD} \,/\!/\, \overline{BC}$, $\overline{BD}$ is a transversal to them

$\therefore m(\angle CBD) = m(\angle ADB) = 38°$ (alternate angles)

$\because \angle$ CED is an exterior angle of Δ AED

$\therefore m(\angle DAE) = 76° - 38° = 38°$

$\therefore m(\angle CBD) = m(\angle DAC)$

but they are drawn on $\overline{CD}$ and on one side of it

$\therefore$ The points A , B , C and D have one circle passing through them. (Q.E.D.)

2 $\because \angle$ BEA is an exterior angle of Δ AED

$\therefore m(\angle EAD) = 85° - 60° = 25°$

$\because$ AD = CD $\quad\therefore m(\angle ACD) = m(\angle CAD) = 25°$

$\therefore m(\angle ABD) = m(\angle ACD)$

but they are drawn on $\overline{AD}$ and on the same side of it

$\therefore$ The points A , B , C and D have one circle passing through them. (Q.E.D.)

3 $\because$ BE = CE $\quad\therefore m(\angle EBC) = m(\angle ECB)$

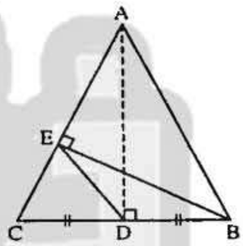
$\because \overline{AD} \,/\!/\, \overline{BC}$ and $\overleftrightarrow{AC}$ is a transversal to them

$\therefore m(\angle DAC) = m(\angle BCA)$ (alternate angles)

$\therefore m(\angle DAC) = m(\angle DBC)$, but they are drawn on $\overline{DC}$ and on one side of it

$\therefore$ The points A , B , C and D have one circle passing through them. (Q.E.D.)

19



Construction :

Draw $\overline{AD}$

Proof : $\because$ AB = AC ,

D is the midpoint of $\overline{BC}$

$\therefore \overline{AD} \perp \overline{BC}$

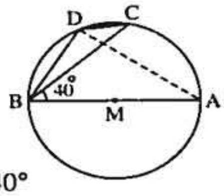
$\because \overline{BE} \perp \overline{AC}$

$\therefore m(\angle ADB) = m(\angle AEB) = 90°$

and they are drawn on $\overline{AB}$ and on one side of it

$\therefore$ The points A , B , D , and E have a circle passing through them. (Q.E.D.)

20



Construction : Draw $\overline{AD}$

Proof :

$\because \overline{AB}$ is a diameter in the circle

$\therefore m(\angle ADB) = 90°$

, $\because m(\angle ADC) = m(\angle ABC) = 40°$

(two inscribed angles subtended by $\widehat{AC}$)

$\therefore m(\angle CDB) = 90° + 40° = 130°$ (The req.)

21

$\because \overline{CD} \cap \overline{EF} = \{A\}$

$\therefore m(\angle EAC) = m(\angle FAD)$ (V.O.A.)

, $m(\angle EBC) = m(\angle EAC)$

(two inscribed angles subtended by $\widehat{EC}$)

, m (∠ FBD) = m (∠ FAD)
(two inscribed angles subtended by $\overset{\frown}{FD}$)
∴ m (∠ EBC) = m (∠ FBD) (Q.E.D.)

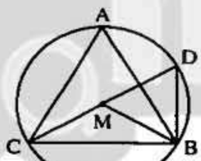
22

m (∠ BNC) = 2 m (∠ BEF) = 2 × 35° = 70°
(central and inscribed angles subtended by $\overset{\frown}{FB}$)
(First req.)
∵ $\overrightarrow{BC}$ is a tangent to the circle N at B
∴ $\overline{NB} \perp \overline{CB}$ ∴ In Δ NCB :
m (∠ BCN) = 180° – (90° + 70°) = 20° (Second req.)
∵ m (∠ FAB) = m (∠ FEB) = 35°
(two inscribed angles subtended by $\overset{\frown}{FB}$)
∴ In Δ ABD : m (∠ BDA) = 180° – (90° + 35°) = 55°
(Third req.)

23

∵ $\overleftrightarrow{BL}$ is a tangent to the circle , $\overleftrightarrow{BL}$ // $\overline{AC}$
∴ m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{BC}$) (1)
∴ m (∠ ADB) = m (∠ CDB)
∴ $\overrightarrow{DB}$ bisects ∠ ADC (Q.E.D. 1)
∵ $\overline{AD}$ // $\overline{BC}$ ∴ m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{DC}$) (2)
From (1) and (2) : ∴ m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{DC}$)
∴ m (∠ CDB) = m (∠ CBD) (Q.E.D.2)

24

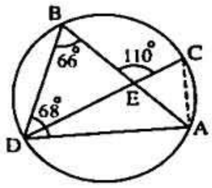


∵ Δ ABC is an equilateral triangle
∴ m (∠ A) = 60°
∴ m (∠ D) = m (∠ A)
= m (∠ ABC) = 60° (1)
∵ $\overline{DC}$ is a diameter in the circle M
∴ m (∠ DBC) = 90° ∴ m (∠ BCD) = 30°
∵ MB = MC = r
∴ m (∠ MBC) = m (∠ MCB) = 30°
From (1) : m (∠ ABM) = 60° – 30° = 30°
∴ m (∠ ABD) = m (∠ CBM) = 30°
m (∠ ABD) = m (∠ ACD)
(two inscribed angles subtended by $\overset{\frown}{AD}$)
∴ m (∠ ABD) = m (∠ CBM) = m (∠ ACD)(Q.E.D.)

25

∵ m ($\overset{\frown}{AD}$) = m ($\overset{\frown}{EB}$) and adding ($\overset{\frown}{DE}$) to both sides
∴ m ($\overset{\frown}{AE}$) = m ($\overset{\frown}{DB}$) ∴ m (∠ EBA) = m (∠ DAB)
∴ In Δ ACB : CA = CB (First req.)
∵ $\overline{AB}$ is a diameter of the circle M
∴ m ($\overset{\frown}{AEB}$) = 180°
∴ m ($\overset{\frown}{AD}$) = m ($\overset{\frown}{DE}$) = m ($\overset{\frown}{EB}$) = $\frac{180°}{3}$ = 60°
∴ m ($\overset{\frown}{DEB}$) = 60° + 60° = 120° (Second req.)

26



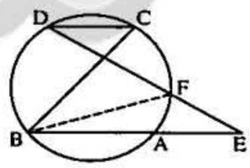
Construction : Draw $\overline{CA}$
Proof :
∵ m (∠ ACD) = m (∠ B)
= 66°
(two inscribed angles subtended by $\overset{\frown}{AD}$)
But : ∠ CEB is an exterior angle of Δ AEC
∴ m (∠ CAE) = 110° – 66° = 44° (1)
But : m (∠ BAD) = 180° – (66° + 68°) = 46° (2)
Adding (1) and (2) : ∴ m (∠ CAD) = 90°
∴ $\overline{CD}$ is a diameter in the circle. (Q.E.D.)

27

∵ Δ ABC is equilateral ∴ m (∠ B) = 60°
∴ m (∠ D) = m (∠ B) = 60°
(two inscribed angles subtended by $\overset{\frown}{AC}$)
, ∵ AD = DE
∴ Δ ADE is an equilateral triangle. (Q.E.D.)

Excellent pupils

1



Construction : Draw $\overline{BF}$
Proof :
m (∠ BCD) = m (∠ BFD)
(two inscribed angles subtended by $\overset{\frown}{BD}$) (1)
∵ ∠ BFD is an exterior angle of Δ BEF
∴ m (∠ E) < m (∠ BFD) (2)
From (1) and (2) :
∴ m (∠ E) < m (∠ BCD) (Q.E.D.)
Another solution :
∵ m (∠ E) = $\frac{1}{2}$ [m ($\overset{\frown}{BD}$) – m ($\overset{\frown}{AF}$)]
, m (∠ BCD) = $\frac{1}{2}$ m ($\overset{\frown}{BD}$)
∴ m (∠ E) < m (∠ BCD) (Q.E.D.)

2

$\because m(\angle D) = m(\angle A)$

(two inscribed angles subtended by $\overset{\frown}{CB}$)

$\therefore y + 2 = x + 3$ $\quad \therefore y = x + 1$

$\because y^2 - x^2 = 53$ $\quad \therefore (x + 1)^2 - x^2 = 53$

$\therefore x^2 + 2x + 1 - x^2 = 53$

$\therefore 2x + 1 = 53$ $\quad \therefore 2x = 52$ $\quad \therefore x = 26$

$\therefore m(\angle CAB) = 26 + 3 = 29°$

$\because m(\angle CMB) = 2\, m(\angle CAB)$

(central and inscribed angles subtended by $\overset{\frown}{CB}$)

$\therefore m(\angle CMB) = 58°$ (The req.)

Answers of exams on first part of unit five

Model 1

1

1 c 2 b 3 c 4 b 5 c 6 b

2

[a] $m(\angle BDC) = 25°$ [b] $m(\angle A) = 27°\ 30'$

3

[a] AC = 6 cm. [b] Prove by yourself.

4

[a] $m(\angle ACB) = 50°$ [b] Prove by yourself.

5

[a] Prove by yourself. [b] Prove by yourself

Model 2

1

1 a 2 d 3 c 4 a 5 d 6 b

2

[a] $m(\angle BED) = 35°$, $m(\angle ADB) = 110°$

[b] 1 $m(\overset{\frown}{CE}) = 130°$ 2 $m(\overset{\frown}{BC}) = 76°$

3

[a] $m(\overset{\frown}{AC}) = 30°$, $m(\overset{\frown}{CY}) = 60°$

[b] Prove by yourself.

4

[a] $m(\overset{\frown}{EX}) = 70°$ [b] Prove by yourself.

5

[a] AB = 7 cm.

[b] 1 $m(\angle CDB) = 70°$ 2 Prove by yourself.

Answers of Exercise 9

1

Fig. (1) : $m(\angle ACB) = 32°$

Fig. (2) : $m(\angle ADB) = 30°$, $m(\angle BDC) = 50°$

Fig. (3) : $m(\angle ACB) = 25°$, $m(\angle ABD) = 65°$, $m(\angle DAC) = 44°$, $m(\angle BDC) = 46°$, $m(\angle BAC) = 46°$

Fig. (4) : $m(\angle BDC) = 40°$

2

Fig. (1) : $m(\angle B) = 94°$

Fig. (2) : $m(\angle A) = 110°$

Fig. (3) : $m(\angle C) = 104°$, $m(\angle ADE) = 80°$

Fig. (4) : $m(\angle ADF) = 65°$, $m(\angle A) = 75°$

Fig. (5) : $m(\angle B) = 116°$ **Fig. (6) :** $m(\angle ACD) = 32°$

Fig. (7) : $m(\angle DBC) = 30°$ **Fig. (8) :** $m(\angle D) = 122°$

3

Fig. (1) : $x = 75°$, $y = 100°$

Fig. (2) : $x = 33°$, $y = 20°$

Fig. (3) : $x = 50°$, $y = 40°$

Fig. (4) : $x = 100°$, $y = 110°$

Fig. (5) : $x = 78°$, $y = 39°$

Fig. (6) : $x = 60°$, $y = 125°$

Fig. (7) : $x = 70°$, $y = 70°$, $z = 100°$

Fig. (8) : $x = 55°$, $y = 55°$

4

1 Supplementary

2 interior angle at the opposite vertex.

3 65° 4 60° 5 132° , 264°

6 First : 30° Second : 120°

7 36° 8 72° , 36° 9 105°

5

1 c 2 b 3 b

4 c 5 d

6

$\because$ m $(\overset{\frown}{AB})$ = 110°

$\therefore$ m (∠ BDA) = $\frac{1}{2}$ m $(\overset{\frown}{AB})$ = $\frac{110°}{2}$ = 55°

$\because$ ∠ CBE is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m (∠ CBE) = m (∠ ADC) = 85°

$\therefore$ m (∠ BDC) = m (∠ ADC) – m (∠ BDA)

= 85° – 55° = 30° (The req.)

7

$\because$ ∠ ABE is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m (∠ D) = m (∠ ABE) = 100°

In Δ ACD : m (∠ ACD) = 180° – (100° + 40°) = 40°

$\therefore$ m (∠ ACD) = m (∠ CAD)

$\therefore$ m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{AD})$ (Q.E.D.)

8

$\because$ ∠ CDE is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m (∠ CDE) = m (∠ ABC) = 120° (First req.)

m (∠ ADC) = 180° – 120° = 60°

$\because$ $\overline{AD}$ is a diameter in the circle M

$\therefore$ m (∠ DCA) = 90°

$\therefore$ m (∠ CAD) = 180° – (90° + 60°) = 30° (Second req.)

$\therefore$ In Δ ADC which is right-angled at C

$\because$ m (∠ DAC) = 30° $\therefore$ CD = $\frac{1}{2}$ AD

$\therefore$ AD = 14 cm. $\therefore$ r = 7 cm.

$\therefore$ The length of $\overset{\frown}{AD}$ = $\frac{1}{2}$ (2 π r)

= $\frac{22}{7}$ × 7 = 22 cm. (Third req.)

9

$\because$ ABCD is a cyclic quadrilateral

$\therefore$ m (∠ ADC) = 180° – 60° = 120°

the length of $\overset{\frown}{AD}$ = the length of $\overset{\frown}{CD}$

$\therefore$ AD = CD

$\therefore$ m (∠ DCA) = m (∠ DAC) = $\frac{180° - 120°}{2}$ = 30° (1)

$\because$ $\overline{BC}$ is a diameter in the circle M

$\therefore$ m (∠ CAB) = 90°

$\therefore$ m (∠ ACB) = 180° – (60° + 90°) = 30° (2)

From (1) and (2) :

$\therefore$ $\overrightarrow{CA}$ bisects ∠ DCB (Q.E.D.)

10

$\because$ ∠ ECD is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m (∠ A) = m (∠ ECD) = 84° (First req.)

$\because$ m (∠ B) = $\frac{1}{2}$ m (∠ D)

$\therefore$ m (∠ B) + m (∠ D) = 180°

$\therefore$ m (∠ B) + 2 m (∠ B) = 180°

$\therefore$ m (∠ B) = 60° (Second req.)

11

$\because$ ∠ DCE is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m (∠ DCE) = m (∠ A) = 130°

In Δ ABD : $\because$ AB = AD

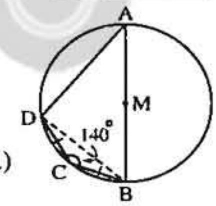
$\therefore$ m (∠ ADB) = m (∠ ABD) = $\frac{180° - 130°}{2}$ = 25°

$\because$ m (∠ ABD) = m (∠ DBC) = 25°

$\therefore$ m $(\overset{\frown}{AD})$ = m $(\overset{\frown}{DC})$

$\therefore$ AD = DC (Q.E.D.)

12



$\because$ ABCD is a cyclic quadrilateral

$\therefore$ m (∠ A) = 180° – 140° = 40° (First req.)

$\because$ CB = CD

$\therefore$ m (∠ CBD) = m (∠ CDB) = $\frac{180° - 140°}{2}$ = 20°

$\because$ $\overline{AB}$ is a diameter in the circle M

$\therefore$ m (∠ ADB) = 90°

$\therefore$ m (∠ ADC) = 90° + 20° = 110° (Second req.)

13

Fig. (1) : $\because$ $\overline{AB}$ is a diameter in the circle M

$\therefore$ m $(\overset{\frown}{AB})$ = 180°

$\therefore$ m $(\overset{\frown}{CB})$ = 180° – (80° + 60°) = 40°

$\therefore m(\angle A) = \frac{1}{2} m(\widehat{DB}) = \frac{1}{2} \times 100° = 50°$

, $m(\angle B) = \frac{1}{2} m(\widehat{AC}) = \frac{1}{2} \times 140° = 70°$

$\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle C) = 180° - m(\angle A) = 180° - 50° = 130°$

, $m(\angle D) = 180° - m(\angle B) = 180° - 70° = 110°$

(The req.)

Fig. (2) : $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle C) = 180° - m(\angle A) = 180° - 95° = 85°$

$\because \overline{AD} \parallel \overline{BE}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore m(\angle A) + m(\angle ABE) = 180°$

(two interior angles on one side of the transversal)

$\therefore m(\angle ABE) = 180° - 95° = 85°$

, $m(\angle CBE) = m(\angle CDE) = 28°$

(two inscribed angles of the same arc $\widehat{CE}$)

$\therefore m(\angle ABC) = m(\angle ABE) + m(\angle CBE)$

$= 85° + 28° = 113°$

$\therefore m(\angle ADC) = 180° - m(\angle ABC) = 180° - 113° = 67°$

(The req.)

14

$\because m(\angle BMD) = 2\, m(\angle A)$

(central and inscribed angles subtended by $\widehat{BD}$)

, $m(\angle BMD) = m(\angle BCD)$

$\therefore m(\angle BCD) = 2\, m(\angle A)$

, $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle A) + m(\angle BCD) = 180°$

$\therefore m(\angle A) + 2\, m(\angle A) = 180°$

$\therefore 3\, m(\angle A) = 180°$

$\therefore m(\angle A) = 60°$ (The req.)

15

$\because$ ABCD is a cyclic quadrilateral.

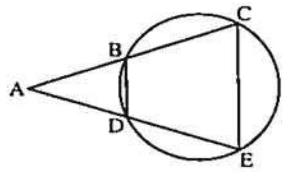
$\therefore m(\angle A) = 180° - 90° = 90°$

$\therefore \Delta ABD$ is a right-angled triangle.

$\therefore \tan(ABD) = \frac{8}{6}$

$\therefore m(\angle ABD) \simeq 53° \, 7' \, 48''$ (The req.)

16



In Δ ACE :

$\because AC = AE$

$\therefore m(\angle C) = m(\angle E)$ (1)

$\because \angle ABD$ is an exterior angle of the cyclic quadrilateral ECBD

$\therefore m(\angle ABD) = m(\angle E)$ (2)

From (1) and (2) : $\therefore m(\angle ABD) = m(\angle C)$

but they are corresponding angles

$\therefore \overline{DB} \parallel \overline{CE}$ (Q.E.D. 1)

$\therefore m(\widehat{BC}) = m(\widehat{ED})$ (Q.E.D. 2)

17

$\because \angle ECX$ is an exterior angle of the cyclic quadrilateral AECB

$\therefore m(\angle ECX) = m(\angle EAB)$ (1)

$\because m(\angle DAE) = m(\angle DCE)$ (2)

(two inscribed angles of the same arc $\widehat{DE}$)

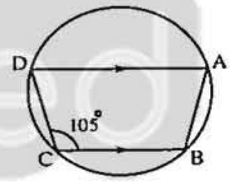
$\therefore m(\angle DAE) = m(\angle EAB)$ (3)

From (1) , (2) and (3) :

$\therefore m(\angle DCE) = m(\angle ECX)$

$\therefore \overrightarrow{CE}$ bisects $\angle XCD$ (Q.E.D.)

18



$\because$ The figure ABCD is a cyclic quadrilateral

$\therefore m(\angle A) + m(\angle C) = 180°$

$\therefore m(\angle A) = 180° - 105°$

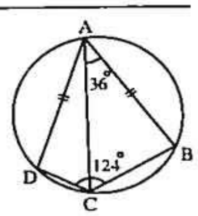
$= 75°$ (First req.)

$\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore m(\angle B) + m(\angle A) = 180°$

$\therefore m(\angle B) = 180° - 75° = 105°$ (Second req.)

19



$\because AB = AD$

$\therefore m(\widehat{AB}) = m(\widehat{AD})$

$\therefore m(\angle ACD) = m(\angle ACB)$

$= \frac{124°}{2} = 62°$

(First req.)

$\because$ The figure ABCD is a cyclic quadrilateral.

$\therefore m(\angle BAD) = 180° - 124° = 56°$

$\therefore$ m ($\angle$ CAD) = 56° − 36° = 20°

In Δ ACD :

m ($\angle$ ADC) = 180° − (62° + 20°) = 98° (Second req.)

20

$\because$ $\angle$ BCE is an exterior angle of the cyclic quadrilateral ABCD = 60°

$\therefore$ m ($\angle$ A) = m ($\angle$ BCE) = 60°

, $\because$ m ($\angle$ M) = 2 m ($\angle$ A)

(central and inscribed angles of the same arc $\overset{\frown}{BD}$)

$\therefore$ m ($\angle$ M) = 2 × 60° = 120°

$\because$ $\overline{MD}$ // $\overline{BC}$

$\therefore$ m ($\angle$ MDC) = m ($\angle$ BCE) (corresponding angles)

$\therefore$ m ($\angle$ DMB) + m ($\angle$ MDC) = 180°

(two interior angles in the same side of the transversal)

$\therefore$ $\overline{MB}$ // $\overline{DC}$

, $\because$ $\overline{MD}$ // $\overline{BC}$ $\therefore$ MDCB is a parallelogram

, $\because$ MD = MB = r

$\therefore$ The figure MDCB is a rhombus (Q.E.D. 1)

$\therefore$ CD = CB $\therefore$ m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{CB}$)

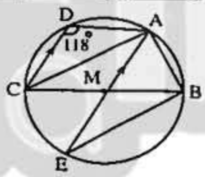
$\because$ A is the midpoint of $\overset{\frown}{BD}$ (the major)

$\therefore$ m ($\overset{\frown}{AD}$) = m ($\overset{\frown}{AB}$)

$\therefore$ m ($\overset{\frown}{CD}$) + m ($\overset{\frown}{AD}$) = m ($\overset{\frown}{CB}$) + m ($\overset{\frown}{AB}$)

$\therefore$ $\overline{AC}$ is a diameter of the circle. (Q.E.D. 2)

21



$\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ ABC) = 180° − 118°

= 62° (First req.)

$\because$ $\overline{AE}$ // $\overline{DC}$

, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ ACD) = m ($\angle$ EAC) (alternative angles)

$\because$ m ($\angle$ CBE) = m ($\angle$ EAC)

(two inscribed angles of the same arc $\overset{\frown}{CE}$)

$\therefore$ m ($\angle$ ACD) = m ($\angle$ CBE) (Second req.)

22

$\because$ $\angle$ YZN is an exterior angle of the cyclic quadrilateral YZLX

$\therefore$ m ($\angle$ YXL) = 80°

$\because$ m ($\angle$ YXZ) = m ($\angle$ YLZ) = 20°

(two inscribed angles of the same arc $\overset{\frown}{YZ}$)

$\therefore$ m($\angle$ ZXL) = 80° − 20° = 60° (First req.)

$\therefore$ m ($\overset{\frown}{ZL}$) = 2 m ($\angle$ ZXL) = 120°

$\therefore$ m ($\overset{\frown}{ZY}$) = 2 m ($\angle$ YLZ) = 40°

$\because$ m ($\overset{\frown}{XY}$) = m ($\overset{\frown}{XL}$)

$\therefore$ m ($\overset{\frown}{XY}$) = $\frac{360° - (40° + 120°)}{2}$ = 100

$\therefore$ m ($\overset{\frown}{XYZ}$) = m ($\overset{\frown}{XY}$) + m ($\overset{\frown}{YZ}$)

= 100° + 40° = 140° (Second req.)

23

$\because$ ABCD is a parallelogram

$\therefore$ m ($\angle$ A) = m ($\angle$ C) (1)

$\because$ DEBC is a cyclic quadrilateral and $\angle$ DEA is an exterior angle of it

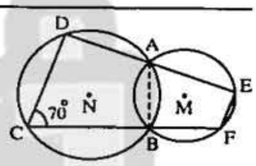
$\therefore$ m ($\angle$ DEA) = m ($\angle$ C) (2)

From (1) and (2) :

$\therefore$ m ($\angle$ A) = m ($\angle$ DEA)

$\therefore$ In Δ ADE : AD = ED (Q.E.D.)

24



Construction : Draw $\overline{AB}$

Proof : $\because$ the figure ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ BAD) = 180° − 70° = 110°

$\because$ The figure ABFE is a cyclic quadrilateral and $\angle$ BAD is an exterior angle of it

$\therefore$ m ($\angle$ F) = m ($\angle$ BAD) = 110° (First req.)

$\therefore$ m ($\angle$ F) + m ($\angle$ C) = 110° + 70° = 180°

but they are two interior angles on the same side of the transversal $\overline{FC}$

$\therefore$ $\overline{CD}$ // $\overline{EF}$ (Second req.)

25

$\because$ ABFE is a cyclic quadrilateral.

$\therefore$ m ($\angle$ E) + m ($\angle$ ABF) = 180° (1)

, $\because$ $\angle$ ABF is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m ($\angle$ ABF) = m ($\angle$ D) (2)

By substitution from (2) in (1) :

$\therefore$ m ($\angle$ E) + m ($\angle$ D) = 180°

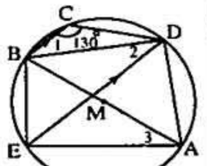
$\therefore$ 4 x° + 5 x° = 180°

$\therefore$ 9 x° = 180° $\therefore$ x = 20°

$\therefore$ m ($\angle$ D) = 5 × 20° = 100°

$\therefore$ m ($\angle$ ABF) = m ($\angle$ D) = 100° (The req.)

26



$\because \overline{CB} \parallel \overline{DE}$

$\therefore m(\angle 1) = m(\angle 2)$

(alternative angles)

but $m(\angle 3) = m(\angle 2)$

(two inscribed angles of the same arc $\overset{\frown}{BE}$)

$\therefore m(\angle 1) = m(\angle 3)$

i.e. $m(\angle DBC) = m(\angle BAE)$ (First req.)

$\therefore$ The figure CDEB is a cyclic quadrilateral

$\therefore m(\angle DEB) = 180° - 130° = 50°$

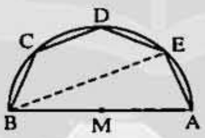
$\because \overline{AB}$ is a diameter

$\therefore m(\angle AEB) = 90°$

$\therefore m(\angle AED) = 90° - 50° = 40°$ (Second req.)

Excellent pupils

1



Construction :

Draw $\overline{BE}$

Proof : $\because \overline{AB}$ is a diameter in the circle

$\therefore m(\angle AEB) = 90°$ (1)

$\because$ The figure DEBC is a cyclic quadrilateral

$\therefore m(\angle DEB) + m(\angle DCB) = 180°$ (2)

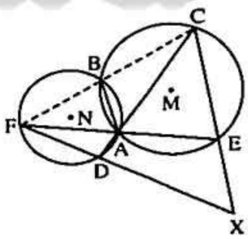
From (1) and (2) :

$\therefore m(\angle AEB) + m(\angle DEB) + m(\angle DCB)$

$= 90° + 180° = 270°$

$\therefore m(\angle AED) + m(\angle BCD) = 270°$ (Q.E.D)

2



Construction :

Draw $\overline{BC}$ and $\overline{BF}$

Proof :

$\because$ The figure AEXD is a cyclic quadrilateral

$\therefore m(\angle AEX) + m(\angle ADX) = 180°$ (1)

$\because$ The figure ABCE is a cyclic quadrilateral

$\therefore \angle AEX$ is an exterior angle of it

$m(\angle AEX) = m(\angle ABC)$ (2)

$\because$ The figure ABFD is a cyclic quadrilateral

$\therefore \angle ADX$ is an exterior angle of it

$\therefore m(\angle ADX) = m(\angle ABF)$ (3)

Substituting from (2) and (3) in (1) :

$\therefore m(\angle ABC) + m(\angle ABF) = 180°$

$\therefore$ The points C , B and F are collinear. (Q.E.D.)

Answers of Exercise 10

1

1. $\because \angle BEA$ is an exterior angle of $\triangle AED$

$\therefore m(\angle EAD) = 90° - 40° = 50°$

$\therefore m(\angle CBD) = m(\angle CAD)$

but they are drawn on $\overline{CD}$ and on one side of it

$\therefore$ The figure ABCD is a cyclic quadrilateral. (Q.E.D.)

2. In $\triangle BDC$: $\because m(\angle BDC) = 180° - (50° + 40°) = 90°$

$\therefore m(\angle BAC) = m(\angle BDC)$ but they are drawn on $\overline{BC}$ and on one side of it

$\therefore$ The figure ABCD is a cyclic quadrilateral. (Q.E.D.)

3. $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{BD}$ is a transversal to them

$\therefore m(\angle ADB) = m(\angle DBC) = 41°$ (alternate angles)

$\therefore m(\angle ACB) = 180° - (99° + 41°) = 40°$

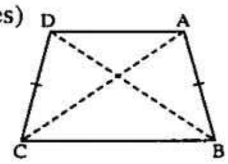
$\therefore m(\angle ACB) \neq m(\angle ADB)$

but they are drawn on $\overline{AB}$ and on one side of it

$\therefore$ The figure ABCD isn't a cyclic quadrilateral. (Q.E.D.)

4. It is impossible to draw a circle passing through the vertices of the figure ABCD

5. $\because \triangle ABC \equiv \triangle DCB$ (three sides)



$\therefore m(\angle BAC) = m(\angle CDB)$

but they are drawn on $\overline{BC}$ and on one side of it

$\therefore$ The figure ABCD is a cyclic quadrilateral. (Q.E.D.)

6. In $\triangle ABC$:

$\because m(\angle BAC) = 180° - (110° + 34°) = 36°$

$\therefore m(\angle BAC) = m(\angle BDC)$

but they are on $\overline{BC}$ and on one side of it

$\therefore$ The figure ABCD is a cyclic quadrilateral. (Q.E.D.)

2

1 In Δ ABC :

$\because$ m ($\angle$ B) = 180° – (50° + 35°) = 95°

$\therefore$ m ($\angle$ B) + m ($\angle$ D) = 95° + 85° = 180°

$\therefore$ The figure ABCD is a cyclic quadrilateral.

(The req.)

2 $\because$ m ($\angle$ EAD) = 86°

$\therefore$ m ($\angle$ DAB) = 180° – 86° = 94°

$\because$ m ($\angle$ DCF) = m ($\angle$ DAB) = 94°

$\therefore$ The figure ABCD is a cyclic quadrilateral.

(The req.)

3 $\because$ AB = AD $\therefore$ m ($\angle$ ADB) = m ($\angle$ ABD) = 30°

$\therefore$ m ($\angle$ A) = 180° – (30° + 30°) = 120°

$\because$ $\angle$ DCE is an exterior angle of the figure ABCD ,

m ($\angle$ A) = m ($\angle$ DCE) = 120°

$\therefore$ The figure ABCD is a cyclic quadrilateral.

(The req.)

4 $\because$ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ B) + m ($\angle$ A) = 180°

Two interior angles on the same side of the transversal

$\therefore$ m ($\angle$ A) = 180° – 74° = 106°

$\because$ m ($\angle$ DCF) = m ($\angle$ FCE) = 53°

$\therefore$ m ($\angle$ DCE) = 2 × 53 = 106°

$\because$ $\angle$ DCE is an exterior angle of the figure ABCD , m ($\angle$ DCE) = m ($\angle$ A) = 106°

$\therefore$ The figure ABCD is a cyclic quadrilateral.

(The req.)

5 $\because$ $\overline{AB}$ // $\overline{DE}$, $\overline{AD}$ // $\overline{BE}$

$\therefore$ The figure ABED is a parallelogram

$\therefore$ m ($\angle$ A) = m ($\angle$ E)

$\because$ BC = BE $\therefore$ m ($\angle$ BCE) = m ($\angle$ E)

$\therefore$ m ($\angle$ A) = m ($\angle$ BCE)

$\because$ $\angle$ BCE is an exterior angle of the figure ABCD

$\therefore$ The figure ABCD is a cyclic quadrilateral.

(The req.)

6 In Δ ABD : $\because$ AB = AD

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ADB)

$\therefore$ m ($\angle$ A) = 180° – 2 x

, in Δ DBC : $\because$ DB = DC

$\therefore$ m ($\angle$ C) = m ($\angle$ DBC) = 2 x

$\therefore$ m ($\angle$ A) + m ($\angle$ C) = 180° – 2 x + 2 x = 180°

$\therefore$ The figure ABCD is cyclic quadrilateral.

(The req.)

3

Theoretical.

4

$\because$ Y is the midpoint of $\overline{DC}$

$\therefore$ m ($\angle$ MYC) = 90° (1)

$\because$ X is the midpoint of $\overline{BC}$

$\therefore$ m ($\angle$ MXC) = 90° (2)

From (1) and (2) and in the figure MXCY

$\because$ m ($\angle$ MYC) + m ($\angle$ MXC) = 180°

$\therefore$ The figure MXCY is a cyclic quadrilateral

(Q.E.D. 1)

$\therefore$ m ($\angle$ XMY) = 180° – m ($\angle$ C) (3)

$\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ BAD) = 180° – m ($\angle$ C) (4)

From (3) and (4) :

$\therefore$ m ($\angle$ XMY) = m ($\angle$ BAD) (Q.E.D. 2)

5

$\because$ $\overline{BC}$ is a diameter in the circle M

$\therefore$ m ($\angle$ BAC) = 90° , $\because$ $\overline{ED}$ $\perp$ $\overline{BC}$

$\therefore$ m ($\angle$ BAC) + m ($\angle$ EDB) = 90° + 90° = 180°

$\therefore$ The figure ABDE is a cyclic quadrilateral

(Q.E.D. 1)

$\therefore$ m ($\angle$ CED) = m ($\angle$ B) = $\frac{1}{2}$ m ($\widehat{AC}$) (Q.E.D. 2)

6

$\because$ $\overline{AB}$ touches the circle M at B $\therefore$ $\overline{MB}$ $\perp$ $\overline{AB}$

$\because$ $\overline{AC}$ touches the circle M at C

$\therefore$ $\overline{MC}$ $\perp$ $\overline{AC}$ $\therefore$ m ($\angle$ ABM) + m ($\angle$ ACM) = 180°

$\therefore$ The figure ABMC is a cyclic quadrilateral (Q.E.D. 1)

$\therefore$ m ($\angle$ CMD) = m ($\angle$ A) = 45°

, $\because$ m ($\angle$ MCD) = 90°

$\therefore$ In Δ MCD : m ($\angle$ D) = 180 – (90° + 45°) = 45°

$\therefore$ m ($\angle$ CMD) = m ($\angle$ D)

$\therefore$ Δ MCD is an isosceles triangle. (Q.E.D. 2)

7

$\because$ $\overline{AB}$ is a diameter in the circle M

, $\overline{AC}$ is a tangent to the circle M at A

$\therefore \overline{AC} \perp \overline{AB}$ $\therefore m(\angle CAM) = 90°$

, $\because$ E is the midpoint of $\overline{BD}$ $\therefore \overline{ME} \perp \overline{DB}$

$\therefore m(\angle MEC) = 90°$

$\therefore m(\angle CAM) + m(\angle CEM) = 90° + 90° = 180°$

$\therefore$ The figure AMEC is a cyclic quadrilateral.

(First req.)

In ΔABC : $m(\angle C) = 180° - (40° + 90°) = 50°$

(Second req.)

8

$\because \overline{DE} \perp \overline{AD}$ $\therefore m(\angle ADE) = 90°$ (1)

$\because \overline{AB}$ is a diameter in the circle M

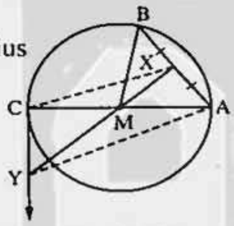
$\therefore m(\angle ACB) = 90°$ (2)

From (1) and (2) : $\therefore m(\angle ADE) = m(\angle ACE)$

but they are drawn on $\overline{AE}$ and on one side of it

$\therefore$ The figure ACDE is a cyclic quadrilateral (Q.E.D.)

9



$\because \overline{CY}$ is a tangent , $\overline{MC}$ is a radius

$\therefore \overline{MC} \perp \overline{CY}$ (1)

$\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ (2)

From (1) and (2) :

$\therefore m(\angle AXY) = m(\angle ACY) = 90°$

but they are drawn on $\overline{AY}$ and on one side of it

$\therefore$ The figure AXCY is a cyclic quadrilateral.

(Q.E.D.1)

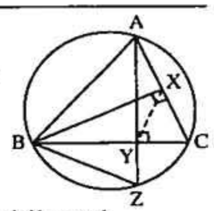
$\therefore m(\angle XAC) = m(\angle XYC)$ (two angles drawn on $\overline{XC}$ and on one side of it)

$\because m(\angle BMC) = 2\, m(\angle XAC)$

(central and inscribed angles of the same arc $\widehat{BC}$)

$\therefore m(\angle BMC) = 2\, m(\angle MYC)$ (Q.E.D.2)

10



$\because m(\angle AXB) = m(\angle AYB) = 90°$

and they are drawn on $\overline{AB}$ and on one side of it

$\therefore$ The figure ABYX is a cyclic quadrilateral.

(Q.E.D.1)

$\therefore m(\angle XAY) = m(\angle XBY)$

(they are drawn on $\overline{XY}$ and on one side of it)

, $\because m(\angle CAZ) = m(\angle CBZ)$

(two inscribed angles of the same arc $\widehat{CZ}$)

$\therefore m(\angle XBC) = m(\angle CBZ)$

$\therefore \overline{BC}$ bisects $\angle XBZ$ (Q.E.D.2)

11

$\because m(\angle BAC) = m(\angle BDC)$

(two inscribed angles of the same arc $\widehat{BC}$)

$\therefore \frac{1}{2} m(\angle BAC) = \frac{1}{2} m(\angle BDC)$

$\therefore m(\angle EAF) = m(\angle EDF)$ but they are drawn on $\overline{EF}$ and on one side of it

$\therefore$ The figure AEFD is a cyclic quadrilateral. (Q.E.D.1)

$\therefore m(\angle DEF) = m(\angle DAC)$

(two inscribed angles on $\overline{DF}$ and on one side of it)

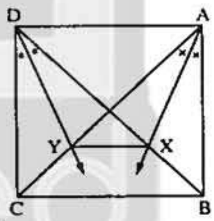
$\because m(\angle DBC) = m(\angle DAC)$

(two inscribed angles of the same arc $\widehat{DC}$)

$\therefore m(\angle DEF) = m(\angle DBC)$ but they are corresponding angles

$\therefore \overline{EF} \,//\, \overline{BC}$ (Q.E.D.2)

12



$\because$ ABCD is a square , $\overline{AC}$ and $\overline{BD}$ are two diagonals of the square

$\therefore m(\angle BAC) = m(\angle BDC)$

$\therefore \frac{1}{2} m(\angle BAC) = \frac{1}{2} m(\angle BDC)$

$\therefore m(\angle XAY) = m(\angle XDY)$ but they are drawn on $\overline{XY}$ and on one side of it

$\therefore$ The figure AXYD is a cyclic quadrilateral.

(Q.E.D.1)

$\therefore m(\angle AYX) = m(\angle ADX) = 45°$

(they are drawn on $\overline{AX}$ and on one side of it)

(Q.E.D.2)

13

$\because$ D is the midpoint of the chord $\overline{EC}$

$\therefore \overline{MD} \perp \overline{EC}$ $\therefore m(\angle MDC) = 90°$

$\because \overleftrightarrow{BC}$ is a tangent to the circle at C

$\therefore \overline{MC} \perp \overleftrightarrow{BC}$ $\therefore m(\angle MCB) = 90°$

$\because \overline{AB} \,//\, \overline{MC}$, $\overleftrightarrow{BC}$ is a transversal to them

$\therefore$ m ($\angle$ MCB) + m ($\angle$ ABC) = 180°

(two interior angles in the same side of the transversal)

$\therefore$ m ($\angle$ ABC) = 180° − 90° = 90°

$\therefore$ m ($\angle$ ADC) + m ($\angle$ ABC) = 90° + 90° = 180°

$\therefore$ The figure ABCD is a cyclic quadrilateral. (Q.E.D.)

14

In Δ AXY : $\because$ m ($\angle$ A) = 60°

$\therefore$ m ($\angle$ X) + m ($\angle$ Y) = 180° − 60° = 120°

$\therefore \frac{1}{2}$ m ($\angle$ X) + $\frac{1}{2}$ m ($\angle$ Y) = $\frac{1}{2}$ × 120° = 60°

$\therefore$ m ($\angle$ CXY) + m ($\angle$ CYX) = 60°

$\therefore$ m ($\angle$ XCY) = 180° − 60° = 120°

, $\because \overrightarrow{YB} \cap \overrightarrow{XD}$ = {C}

$\therefore$ m ($\angle$ BCD) = m ($\angle$ XCY) = 120° (V.O.A.)

$\therefore$ m ($\angle$ BCD) + m ($\angle$ A) = 120° + 60° = 180°

$\therefore$ The figure ABCD is a cyclic quadrilateral. (Q.E.D.)

15

In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$\therefore \frac{1}{2}$ m ($\angle$ ABC) = $\frac{1}{2}$ m ($\angle$ ACB)

$\therefore$ m ($\angle$ YBX) = m ($\angle$ YCX) and they are drawn on $\overline{YX}$ and on one side of it

$\therefore$ The figure BCXY is a cyclic quadrilateral. (Q.E.D.1)

$\therefore$ m ($\angle$ BXY) = m ($\angle$ BCY)

(they are drawn on $\overline{BY}$ and on one side of it)

$\because$ m ($\angle$ CBX) = m ($\angle$ BCY)

$\therefore$ m ($\angle$ CBX) = m ($\angle$ BXY)

and they are alternate angles

$\therefore \overleftrightarrow{XY}$ // $\overleftrightarrow{BC}$ (Q.E.D.2)

16

$\because \overleftrightarrow{AB}$ // $\overleftrightarrow{DE}$, $\overleftrightarrow{AD}$ is a transversal to them

$\therefore$ m ($\angle$ A) = m ($\angle$ ADE) (alternative angles)

$\because \overleftrightarrow{BC}$ // $\overleftrightarrow{DF}$, $\overleftrightarrow{CD}$ is a transversal to them

$\therefore$ m ($\angle$ C) = m ($\angle$ CDF) (alternate angles)

$\because$ m ($\angle$ ADE) + m ($\angle$ CDF) = 180°

$\therefore$ m ($\angle$ A) + m ($\angle$ C) = 180°

$\therefore$ The figure ABCD is a cyclic quadrilateral. (Q.E.D.)

17

$\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ (1)

, $\because \overleftrightarrow{MN}$ is the line of centres

, $\overline{ED}$ is the common chord

$\therefore \overleftrightarrow{MN} \perp \overline{ED}$ (2)

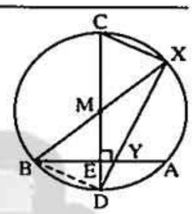
From (1) and (2) : $\therefore$ m ($\angle$ MXC) + m ($\angle$ MYC) = 180°

$\therefore$ The figure CXMY is a cyclic quadrilateral (Q.E.D.1)

, $\because$ m ($\angle$ MXC) = 90°

$\therefore$ The centre of the circle which passes through the vertices of the figure CXMY is the midpoint of $\overline{MC}$ (Q.E.D.2)

18



$\because \overline{CD} \perp \overline{AB}$

$\therefore$ m ($\angle$ AEC) = 90°

$\because \overline{CD}$ is a diameter in the circle

$\therefore$ m ($\angle$ DXC) = 90°

$\therefore$ m ($\angle$ YXC) + m ($\angle$ YEC) = 90° + 90° = 180°

$\therefore$ The figure XYEC is cyclic quadrilateral. (Q.E.D. 1)

$\therefore$ m ($\angle$ DYB) = m ($\angle$ ECX)

, $\because$ m ($\angle$ DBX) = m ($\angle$ DCX)

(two inscribed angles of the same arc $\overset{\frown}{DX}$)

$\therefore$ m ($\angle$ DYB) = m ($\angle$ DBX) (Q.E.D. 2)

19

$\because \overleftrightarrow{AB}$ // $\overleftrightarrow{DC}$, $\overleftrightarrow{AD}$ is a transversal to them

$\therefore$ m ($\angle$ A) + m ($\angle$ D) = 180° (1)

but $\angle$ CFE is an exterior angle of the cyclic quadrilateral ABFE

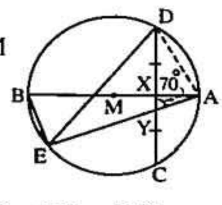
$\therefore$ m ($\angle$ CFE) = m ($\angle$ A) (2)

From (1) and (2) :

$\therefore$ m ($\angle$ CFE) + m ($\angle$ D) = 180°

$\therefore$ The figure CDEF is a cyclic quadrilateral. (Q.E.D.)

20



$\because \overline{AB}$ is a diameter in the circle M

$\therefore$ m ($\angle$ AEB) = 90°

$\because$ X is the midpoint of $\overline{DC}$

$\therefore \overline{MX} \perp \overline{DC}$

$\therefore$ m ($\angle$ YXB) + m ($\angle$ YEB) = 90° + 90° = 180°

$\therefore$ The figure XYEB is a cyclic quadrilateral (First req.)

$\therefore m(\angle B) = m(\angle AYD) = 70°$

$\because m(\angle ADE) = m(\angle B)$

(two inscribed angles of the same arc $\overset{\frown}{AE}$)

$\therefore m(\angle ADE) = 70°$ (Second req.)

21

$\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$

$\because$ Y is the midpoint of $\overline{AC}$ $\therefore \overline{MY} \perp \overline{AC}$

$\therefore m(\angle AXM) = m(\angle AYM) = 90°$ but they are drawn on $\overline{AM}$ and on one side of it

$\therefore$ The figure AXYM is a cyclic quadrilateral. (Q.E.D.1)

$\therefore m(\angle MXY) = m(\angle MAY)$

(they are drawn on $\overline{MY}$ and on one side of it) (1)

In Δ AMC : $\because$ AM = CM (two radii)

$\therefore m(\angle MCA) = m(\angle MAC)$ (2)

From (1) and (2) :

$\therefore m(\angle MXY) = m(\angle MCY)$ (Q.E.D.2)

$\because m(\angle AYM) = 90°$

$\therefore \overline{AM}$ is a diameter in the circle which passes through the points A , X , Y and M (Q.E.D.3)

22

$\because m(\angle DAB) = m(\angle DCB)$

(two inscribed angles of the same arc $\overset{\frown}{DB}$)

, $m(\angle DAB) = m(\angle EMB)$ (given)

$\therefore m(\angle ECB) = m(\angle EMB)$

but they are drawn on $\overline{EB}$ and on one side of it

$\therefore$ The figure MCBE is a cyclic quadrilateral. (Q.E.D.1)

$\therefore m(\angle CEB) = m(\angle CMB)$

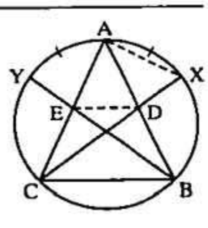
(drawn on $\overline{BC}$ and on one side of it)

$\because m(\angle CMB) = 2\, m(\angle CDB)$

(central and inscribed angles of the same arc $\overset{\frown}{CB}$)

$\therefore m(\angle CEB) = 2\, m(\angle CDB)$ (Q.E.D.2)

23



$\because m(\overset{\frown}{AX}) = m(\overset{\frown}{AY})$

$\therefore m(\angle ACX) = m(\angle ABY)$

and they are drawn on $\overline{DE}$ and on the same side of it

$\therefore$ The figure BCED is a cyclic quadrilateral (Q.E.D.1)

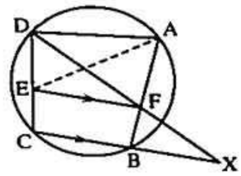
$\therefore m(\angle DEB) = m(\angle DCB)$ (they are drawn on $\overline{DB}$ and on one side of it)

$\because m(\angle XAB) = m(\angle XCB)$

(two inscribed angles of the same arc $\overset{\frown}{XB}$)

$\therefore m(\angle DEB) = m(\angle XAB)$ (Q.E.D.2)

24



$\because$ The figure ABCD is a cyclic quadrilateral

$\therefore m(\angle A) + m(\angle C) = 180°$

$\because \overline{FE} \parallel \overline{BC}$

and $\overline{DC}$ is a transversal to them

$\therefore m(\angle FED) = m(\angle C)$ (corresponding angles)

$\therefore m(\angle A) + m(\angle FED) = 180°$

$\therefore$ The figure AFED is a cyclic quadrilateral (Q.E.D. 1)

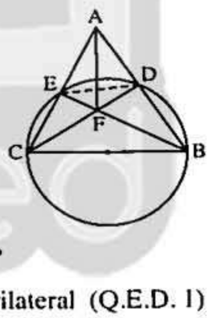
$\therefore m(\angle EAD) = m(\angle EFD)$

(drawn on $\overline{ED}$ and on one side of it. $\overline{XC} \parallel \overline{FE}$ and $\overleftrightarrow{XF}$ is a transversal to them)

$\therefore m(\angle BXF) = m(\angle EFD)$ (corresponding angles)

$\therefore m(\angle BXF) = m(\angle EAD)$ (Q.E.D. 2)

25



Construction : Draw $\overline{DE}$

Proof :

$\because \overline{BC}$ is a diameter in the circle

$\therefore m(\angle BDC) = 90°$

, $m(\angle BEC) = 90°$

$\therefore m(\angle ADF) + m(\angle AEF) = 180°$

$\therefore$ The figure ADFE is a cyclic quadrilateral (Q.E.D. 1)

$\therefore m(\angle FAD) = m(\angle DEF)$

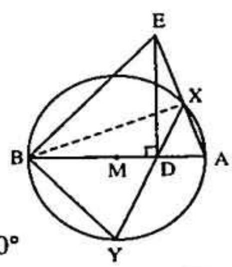
drawn on $\overline{DF}$ and on one side of it

$\because m(\angle DEF) = m(\angle DCB)$

(two inscribed angles of the same arc $\overset{\frown}{DB}$)

$\therefore m(\angle DAF) = m(\angle DCB)$ (Q.E.D. 2)

26



$\because \overline{AB}$ is a diameter in the circle

$\therefore m(\angle AXB) = 90°$

$\therefore m(\angle EXB) = 90°$

$\because \overline{DE} \perp \overline{AB}$

$\therefore m(\angle EDB) = m(\angle EXB) = 90°$

but they are drawn on $\overline{BE}$ and on one side of it

$\therefore$ The figure EBDX is a cyclic quadrilateral (Q.E.D. 1)

$\because \angle AXY$ is an exterior angle of the cyclic quadrilateral EBDX

$\therefore m(\angle AXY) = m(\angle ABE)$

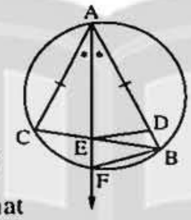
$\because m(\angle AXY) = m(\angle ABY)$

(two inscribed angles of the same arc $\overset{\frown}{AY}$)

$\therefore m(\angle ABE) = m(\angle ABY)$

$\therefore \overrightarrow{BA}$ bisects $\angle EBY$ (Q.E.D. 2)

27



$\Delta\Delta$ ADE , ACE

in them : $\begin{cases} AD = AC \\ \overline{AE} \text{ is a common side} \\ m(\angle DAE) = m(\angle CAE) \end{cases}$

$\therefore \Delta ADE \equiv \Delta ACE$ and we deduce that

$m(\angle ADE) = m(\angle ACE)$

$\because m(\angle AFB) = m(\angle ACB)$

(two inscribed angles of the same arc $\overset{\frown}{AB}$)

$\therefore m(\angle ADE)$ (the exterior) $= m(\angle BFE)$

$\therefore$ The figure BDEF is a cyclic quadrilateral (Q.E.D.)

28

$\because m(\angle ADB) = m(\angle AEB)$

but they are drawn on $\overline{AB}$ and on one side of it

$\therefore$ The figure AEDB is a cyclic quadrilateral (1)

$\because m(\angle FDC) + m(\angle FEC) = 180°$

$\therefore$ The figure FECD is a cyclic quadrilateral (2)

From (1) : $\therefore m(\angle EDA) = m(\angle EBA)$

drawn on $\overline{AE}$ and on one side of it

From (2) : $\therefore m(\angle ECF) = m(\angle EDF)$

drawn on $\overline{EF}$ and on one side of it

$\therefore m(\angle EBA) = m(\angle ECF)$ (Q.E.D.)

29

$\because \overline{BE} \perp \overline{AC}$

, $\overline{CF} \perp \overline{AB}$, $\because \overline{BE} \cap \overline{CF} = \{M\}$

$\therefore$ M is the point of intersection of the altitudes of Δ ABC

, $\because \overline{AD}$ passes through the point M

$\therefore \overline{AD} \perp \overline{BC}$

$\therefore m(\angle MDC) + m(\angle MEC) = 180°$

$\therefore$ The figure MDCE is a cyclic quadrilateral (Q.E.D.)

30

$\because$ ABCD is a parallelogram $\therefore AD = BC$

$\because BE = AD$ $\therefore BC = BE$

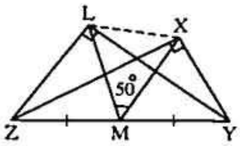
In Δ BCE : $\therefore m(\angle C) = m(\angle BEC)$

, $\because m(\angle BAD) = m(\angle C)$

$\therefore m(\angle BAD) = m(\angle BED)$ and they are drawn on $\overline{BD}$ and on one side of it

$\therefore$ The figure ABDE is a cyclic quadrilateral. (Q.E.D.)

31



Construction : Draw $\overline{XL}$

Proof :

$\because m(\angle YXZ) = m(\angle YLZ)$

$= 90°$

$\therefore$ The figure XYZL is a cyclic quadrilateral and $\overline{YZ}$ is a diameter in the circumcircle of it

$\because$ M is the midpoint of $\overline{YZ}$

$\therefore$ M is the centre of the circle which passes through the points X , Y , Z and L

$\therefore m(\angle XYL) = \frac{1}{2} m(\angle XML) = 25°$

inscribed and central angles of the same arc $(\overset{\frown}{XL})$

(First req.)

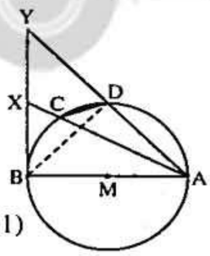
$\because$ The figure XYZL is a cyclic quadrilateral

$\therefore m(\angle XYL) = m(\angle XZL)$ (drawn on $\overline{XL}$ and on one side of it) (Second req.)

$\because m(\overset{\frown}{XZ}) = m(\overset{\frown}{XL}) + m(\overset{\frown}{LZ})$

$\therefore m(\angle XMZ) = m(\overset{\frown}{XL}) + m(\overset{\frown}{LZ})$ (Third req.)

32



Construction : Draw $\overline{DB}$

Proof :

$\because \overline{AB}$ is a diameter in the circle

$\therefore m(\angle ADB) = 90°$

$\therefore m(\angle DAB) + m(\angle DBA) = 90°$ (1)

$\because \overline{BY}$ touches the circle at B

$\therefore \overline{AB} \perp \overline{BY}$ $\therefore m(\angle ABY) = 90°$

$\therefore m(\angle Y) + m(\angle YAB) = 90°$ (2)

from (1) and (2) : $\therefore m(\angle Y) = m(\angle DBA)$

, $\because m(\angle DCA) = m(\angle DBA)$

(two inscribed angles of the same arc $\overset{\frown}{AD}$)

$\therefore m(\angle DCA)$ (the exterior) $= m(\angle Y)$

$\therefore$ The figure XYDC is a cyclic quadrilateral.

(Q.E.D.)

33

$\because m(\angle A) + m(\angle C) = 7x° + 2x° = 9x°$

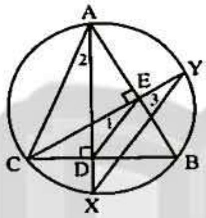
$, m(\angle B) + m(\angle D) = 4x° - 30° + 5x° - 30° = 9x°$

$\therefore m(\angle A) + m(\angle C) = m(\angle B) + m(\angle D)$

$= \frac{360°}{2} = 180°$

$\therefore$ The figure ABCD is a cyclic quadrilateral. (Q.E.D.)

34



$\because m(\angle AEC) = m(\angle CDA) = 90°$

they are drawn on $\overline{AC}$ and on one side of it

$\therefore$ the figure AEDC is a cyclic quadrilateral

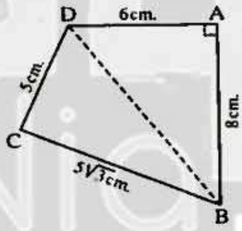
$\therefore m(\angle 1) = m(\angle 2)$ drawn on $\overline{DC}$ and on one side of it but $m(\angle 3) = m(\angle 2)$

(two inscribed angles of the same arc $\widehat{XC}$)

$\therefore m(\angle 1) = m(\angle 3)$ but they are corresponding angles.

$\therefore \overline{XY} \parallel \overline{DE}$ (Q.E.D.)

35



Construction : Draw $\overline{BD}$

Proof :

In Δ DAB which is right-angled at A

$\because (BD)^2 = (DA)^2 + (AB)^2$

$= 36 + 64 = 100$ (1)

$\therefore BD = 10$ cm.

In Δ DCB :

$\because (DC)^2 + (CB)^2 = 25 + 75 = 100$ (2)

from (1) and (2) : $\therefore (BD)^2 = (DC)^2 + (CB)^2$

$\therefore m(\angle DCB) = 90°$

$\therefore m(\angle A) + m(\angle C) = 90° + 90° = 180°$

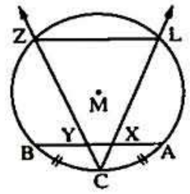
$\therefore$ The figure ABCD is a cyclic quadrilateral. (Q.E.D. 1)

$\because m(\angle BAD) = 90°$

$\therefore \overline{BD}$ is a diameter in the circumcircle of the figure ABCD and its centre is the midpoint of $\overline{BD}$

$\therefore$ The its radius length $= \frac{1}{2}$ BD = 5 cm. (Q.E.D. 2)

36



$\because \overline{AB}, \overline{CL}$ are two chords intersecting at X

$\therefore m(\angle AXL)$

$= \frac{1}{2}\left(m(\widehat{BC}) + m(\widehat{AL})\right)$

$\because m(\widehat{AC}) = m(\widehat{BC})$

$\therefore m(\angle AXL) = \frac{1}{2}\left(m(\widehat{AC}) + m(\widehat{AL})\right)$

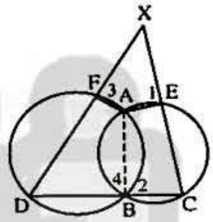
$= \frac{1}{2} m(\widehat{LAC})$

$\therefore m(\angle AXL) = m(\angle Z)$

$\because \angle AXL$ is an exterior angle of the figure XYZL

$\therefore$ The figure XYZL is a cyclic quadrilateral. (Q.E.D.)

37



Construction : Draw $\overline{AB}$

Proof :

$\because \angle 1$ is an exterior angle of the cyclic quadrilateral ECBA

$\therefore m(\angle 1) = m(\angle 2)$ similarly

$m(\angle 3) = m(\angle 4)$ but $m(\angle 2) + m(\angle 4) = 180°$

$\therefore m(\angle 1) + m(\angle 3) = 180°$, but they are opposite angles.

$\therefore$ The figure AFXE is a cyclic quadrilateral. (Q.E.D.)

Excellent pupils

1

$\because \Delta DCE \sim \Delta BAD$

$\therefore m(\angle DCE) = m(\angle BAD)$

$\because \angle DCE$ is an exterior angle of the figure ABCD

$\therefore$ The figure ABCD is a cyclic quadrilateral (Q.E.D. 1)

$\because \Delta DCE \sim \Delta BAD$

$\therefore m(\angle E) = m(\angle ADB)$ (1)

$\therefore$ The figure ABCD is a cyclic quadrilateral

$\therefore m(\angle ADB) = m(\angle ACB)$ (drawn on $\overline{AB}$ and on one side of it) (2)

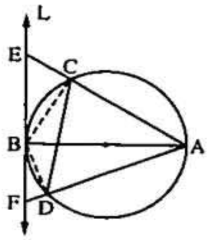
From (1) and (2) : $\therefore m(\angle E) = m(\angle ACB)$

but they are corresponding angles.

$\therefore \overline{ED} \parallel \overline{CA}$ (Q.E.D. 2)

2

Construction : Draw $\overline{BC}$ and $\overline{BD}$



Proof :

$\because \overleftrightarrow{FE}$ is a tangent to the circle at B , $\overline{AB}$ is a diameter

$\therefore \overleftrightarrow{EF} \perp \overline{AB}$

$\therefore m(\angle CBA) + m(\angle CBE) = 90°$

$\because \overline{AB}$ is a diameter

$\therefore \overline{CB} \perp \overline{AE}$

$\therefore m(\angle E) + m(\angle CBE) = 90°$

$\therefore m(\angle E) = m(\angle CBA)$

$\because m(\angle CBA) = m(\angle CDA)$

(two inscribed angles of the same arc $\overset{\frown}{AC}$)

$\therefore m(\angle E) = m(\angle CDA)$

$\because m(\angle CDA)$ is an exterior angle of the figure CDFE

$\therefore$ The figure CDFE is a cyclic quadrilateral. (Q.E.D.)

Answers of Exercise 11

1

1. parallel
2. equal in length
3. the bisectors of its interior angles
4. 4
5. zero
6. the chord of tangency of these two tangents
7. the angle between these two tangents , the angle between two radii passing through the two tangency points

2

1 a	2 b	3 c	4 a
5 b	6 a	7 b	8 b

3

1. $x = 35°$, $y = 55°$, $z = 55°$
2. $x = 65°$, $y = 25°$, $z = 130°$
3. $x = 30°$, $y = 60°$, $z = 60°$

4

1. $x = 12$ cm. , $y = 13$ cm.
2. $x = 9$ cm. , $y = 17$ cm.
3. $x = 3$ cm. , $y = 4$ cm.
4. $x = 3$ cm. , $y = 3$ cm.
5. $x = 4$ cm. , $y = 7$ cm.
6. $x = 4$ cm. , $y = 3$ cm.

5

Theoretical

6

$\because \overline{AB}$ touches the circle at B $\therefore \overline{MB} \perp \overline{AB}$

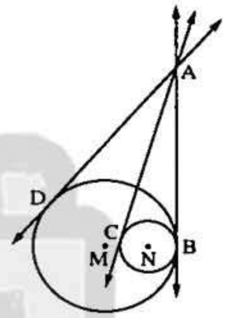
$\therefore m(\angle ABC) = 90° - 30° = 60°$

$\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M

$\therefore AB = AC$

$\therefore \triangle ABC$ is an equilateral triangle (Q.E.D.)

7



$\because \overline{AB}$, $\overline{AD}$ are two tangent segments to the circle M

$\therefore AB = AD$ (1)

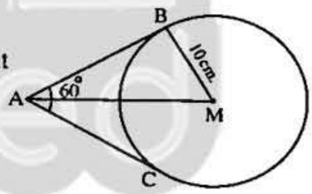
$\because \overline{AB}$, $\overline{AC}$ are two tangent segments to the circle N

$\therefore AB = AC$ (2)

From (1) and (2) :

$\therefore AD = AC$ (Q.E.D.)

8



$\because \overline{AB}$ is a tangent-segment to the circle at B

, $\overline{MB}$ is a radius

$\therefore m(\angle MBA) = 90°$

$\because \overrightarrow{AM}$ bisects $\angle BAC$ $\therefore m(\angle BAM) = 30°$

$\therefore MA = 2\,MB = 2 \times 10 = 20$ cm. (First req.)

$\because (AB)^2 = (MA)^2 - (MB)^2 = (20)^2 - (10)^2 = 300$

$\therefore AB = 10\sqrt{3}$ cm. (Second req.)

9

$\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M

$\therefore \overrightarrow{AM}$ bisects $\angle BAC$

$\therefore m(\angle BAC) = 2 \times 25 = 50°$, $AB = AC$

$\therefore$ In $\triangle ABC$:

$m(\angle ACB) = \frac{180° - 50°}{2} = 65°$ (First req.)

$\because \overline{MB}$ is a radius , $\overline{AB}$ is a tangent-segment to the circle at B

$\therefore \overline{MB} \perp \overline{AB}$ $\therefore m(\angle ABM) = 90^\circ$

similarly $m(\angle ACM) = 90^\circ$

from the quadrilateral ABMC

$m(\angle BMC) = 360^\circ - (90^\circ + 90^\circ + 50^\circ) = 130^\circ$

$\therefore m(\angle BEC) = \frac{1}{2} m(\angle BMC) = \frac{1}{2} \times 130^\circ = 65^\circ$

(inscribed and central angles of the same arc $\overset{\frown}{BC}$)

(Second req.)

10

$\because \overline{AB}, \overline{AC}$ are two tangents to the circle M

$\therefore \overrightarrow{AM}$ bisects $\angle BAC$

$\therefore m(\angle BAC) = 2 \times 30^\circ = 60^\circ$, AB = AC

$\therefore \Delta ABC$ is an equilateral triangle

$\therefore m(\angle ACB) = 60^\circ$

, $\because \overline{BD}$ is a diameter $\therefore m(\angle DCB) = 90^\circ$

$\therefore m(\angle ACD) = 90^\circ + 60^\circ = 150^\circ$ (The req.)

11

$\because \overleftrightarrow{XA}, \overleftrightarrow{XB}$ are two tangents to the circle from the point X

$\therefore XA = XB$

$\therefore m(\angle XAB) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$ (1)

$\because$ The figure ABCD is a cyclic quadrilateral

$\therefore m(\angle BAD) = 180^\circ - 125^\circ = 55^\circ$ (2)

From (1) and (2) : $\therefore m(\angle BAD) = m(\angle XAB)$

$\therefore \overrightarrow{AB}$ bisects $\angle DAX$ (Q.E.D. 1)

$\therefore m(\angle DAX) = 2 \times 55^\circ = 110^\circ$

$\therefore m(\angle DAX) + m(\angle X) = 110^\circ + 70^\circ = 180^\circ$

but they are two interior angles on the same side of The transversal $\overleftrightarrow{AX}$

$\therefore \overline{AD} // \overleftrightarrow{XB}$ (Q.E.D. 2)

12

$\because m(\angle BCD) = \frac{1}{2} m(\angle M)$

(inscribed and central angles of the same arc $\overset{\frown}{BD}$)

$\therefore m(\angle BCD) = \frac{1}{2} \times 130^\circ = 65^\circ$ (1)

$\because \overline{AB} // \overline{CD}$

$\therefore m(\angle ABC) = m(\angle BCD) = 65^\circ$ (alternate angles)

$\because \overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

$\therefore AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = 65^\circ$ (2)

From (1) , (2) : $\therefore m(\angle ACB) = m(\angle BCD)$

$\therefore \overrightarrow{CB}$ bisects $\angle ACD$ (First req.)

, $m(\angle A) = 180^\circ - 2 \times 65^\circ = 50^\circ$ (Second req.)

13

$\because \overline{AX}, \overline{AZ}$ are two tangent-segments to the circle

$\therefore AX = AZ = 2$ cm.

$\because \overline{CZ}, \overline{CY}$ are two tangent-segments to the circle

$\therefore CZ = CY = 3$ cm.

, $\because$ the perimeter of $\Delta ABC = 18$ cm.

$\therefore AX + XB + BY + YC + ZC + ZA = 18$ cm.

$\therefore 2 + XB + BY + 3 + 3 + 2 = 18$ cm.

$\therefore XB + BY = 8$ cm.

$\because \overline{XB}, \overline{BY}$ are two tangent-segments to the circle.

$\therefore XB = BY$ $\therefore 2\,BY = 8$ cm.

$\therefore BY = 4$ cm. (The req.)

14

$\because$ M is the centre of the circle which touches the sides of ΔABC

$\therefore \overrightarrow{CM}$ bisects $\angle C$

$\therefore m(\angle C) = 2 \times 20^\circ = 40^\circ$

$\because \overline{MD} \perp \overline{AD}, \overline{MF} \perp \overline{AF}$

$\therefore$ The figure MDAF is a cyclic quadrilateral.

$\therefore m(\angle A) = 180^\circ - 120^\circ = 60^\circ$

$\because$ The sum of the measures of the angles of $\Delta ABC = 180^\circ$

$\therefore m(\angle B) = 180^\circ - (40^\circ + 60^\circ) = 80^\circ$ (The req.)

15

$\because$ The length of $(\overset{\frown}{AB})$ = the length of $\overset{\frown}{BC}$

= the length of $\overset{\frown}{AC}$

$\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{BC}) = m(\overset{\frown}{AC}) = \frac{360}{3} = 120^\circ$

$\therefore m(\angle AMB) = 120^\circ$ (First req.)

$\because \overrightarrow{DA}$ touches the circle at A $\therefore \overline{MA} \perp \overrightarrow{DA}$

$\because \overrightarrow{DC}$ touches the circle at C $\therefore \overline{MC} \perp \overrightarrow{DC}$

$\because m(\angle MAD) + m(\angle MCD) = 180^\circ$

$\therefore$ The figure AMCD is a cyclic quadrilateral

(Second req.)

$\because m(\angle AMC) = m(\overset{\frown}{AC}) = 120^\circ$

$\therefore m(\angle D) = 180^\circ - 120^\circ = 60^\circ$ (1)

$\because \overline{DA}, \overline{DC}$ are two tangent-segments to the circle

$\therefore AD = CD$ (2)

From (1) and (2) :

$\therefore \Delta ACD$ is an equilateral triangle. (Third req.)

16

In Fig. (1) : $\because \overline{EA}$, $\overline{EC}$ are two tangent-segments to the circle M from the point E

$\therefore EA = EC$ (1)

, $\because \overline{EB}$, $\overline{ED}$ are two tangent-segments to the circle N from the point E

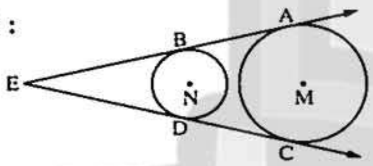
$\therefore EB = ED$ (2)

Adding (1) and (2) :

$\therefore EA + EB = EC + ED$

$\therefore AB = CD$ (Q.E.D.)

In Fig. (2) :



Assuming that $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at E

, $\because \overline{EA}$ and $\overline{EC}$ are two tangent-segments to the circle M from the point E

$\therefore EA = EC$ (1)

$\because \overline{EB}$ and $\overline{ED}$ are two tangent-segments to the circle N from the point E

$\therefore EB = ED$ (2)

Subtracting (2) from (1) :

$\therefore EA - EB = EC - ED$

$\therefore AB = CD$ (Q.E.D.)

17

$\because \overline{AB}$ is a tangent-segment to the circle M at B

, $\overline{MB}$ is a radius $\therefore m(\angle AMB) = 90°$

From ΔABM : $m(\angle MAB) = 180° - (90° + 70°)$

$= 20°$

, $\because \overrightarrow{AM}$ bisects $\angle BAC$

$\therefore m(\angle BAC) = 2 \times 20° = 40°$

, $\because \overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180° - 40°}{2} = 70°$

(First req.)

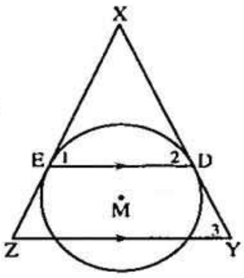
, $\because m(\angle BCD) = \frac{1}{2} m(\angle BMD)$

(inscribed and central angles of the same arc $\widehat{BD}$)

$\therefore m(\angle BCD) = \frac{1}{2} \times 70° = 35°$

$\therefore m(\angle ACD) = 70° - 35° = 35°$ (Second req.)

18



$\because \overline{XD}$ and $\overline{XE}$ are two tangent-segments to the circle

$\therefore XD = XE$

$\therefore m(\angle 1) = m(\angle 2)$ (1)

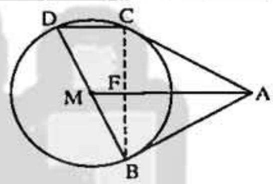
$\because \overline{DE} \parallel \overline{YZ}$ and $\overleftrightarrow{DY}$ is a transversal to them

$\therefore m(\angle 2) = m(\angle 3)$ (corresponding angles) (2)

From (1) and (2) : $\therefore m(\angle 1) = m(\angle 3)$

$\therefore$ The figure DYZE is a cyclic quadrilateral. (Q.E.D.)

19



Construction :

Draw $\overline{BC}$ to cut $\overline{AM}$ at F

Proof : $\because \overline{AC}$ and $\overline{AB}$ are two tangent-segments to the circle M

$\therefore \overleftrightarrow{AF} \perp \overline{BC}$ $\therefore m(\angle CFM) = 90°$

, $\because \overline{BD}$ is a diameter in the circle M

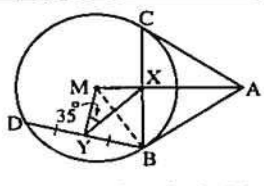
$\therefore m(\angle FCD) = 90°$

$\therefore m(\angle CFM) + m(\angle FCD) = 180°$

, but they are two interior angles in the same side of the transversal $\overleftrightarrow{BC}$

$\therefore \overline{AM} \parallel \overline{CD}$ (Q.E.D.)

20



Construction :

Draw $\overline{BM}$

Proof :

$\because \overline{AC}$, $\overline{AB}$ are two tangent-segments to the circle M

$\therefore \overleftrightarrow{AX} \perp \overline{BC}$ $\because$ Y is the midpoint of $\overline{BD}$

$\therefore \overline{MY} \perp \overline{BD}$

$\therefore m(\angle BXM) + m(\angle MYB) = 180°$

$\therefore$ The figure XBYM is a cyclic quadrilateral (First req.)

$\therefore m(\angle XBM) = m(\angle XYM) = 35°$

, $\because \overline{MB}$ is a radius , $\overline{AB}$ is a tangent-segment to the circle M at B

$\therefore \overline{MB} \perp \overline{AB}$

$\therefore m(\angle ABC) = 90° - 35° = 55°$

$\because \overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

$\therefore AB = AC$ $\therefore m(\angle ABC) = m(\angle ACB) = 55°$

$\therefore m(\angle BAC) = 180° - 2 \times 55° = 70°$ (Second req.)

21

$\because \overline{AB}$ touches the circle at B $\therefore \overline{MB} \perp \overline{AB}$

$\because \overline{AC}$ touches the circle at C $\therefore \overline{MC} \perp \overline{AC}$

$\therefore m(\angle ABM) + m(\angle ACM) = 90° + 90° = 180°$

$\therefore$ The figure ABMC is a cyclic quadrilateral (Q.E.D. 1)

$\because \angle CMD$ is an exterior angle of it

$\therefore m(\angle CMD) = m(\angle A) = 45°$

$\therefore$ In Δ MCD : $m(\angle D) = 180° - (90° + 45°) = 45°$

$\therefore CD = MC$ (1)

$\because \overline{AC}$, $\overline{AB}$ are two tangent-segments to the circle

$\therefore AC = AB$ (2)

Adding (1) and (2) :

$\therefore CD + AC = MC + AB$ $\therefore AD = AB + MC$

, $\because MC = MB$ (the lengths of two radii)

$\therefore AD = AB + MB$ (Q.E.D 2)

22

$\because \overline{CA}$, $\overline{CD}$ are two tangent-segments to the circle M

$\therefore CA = CD$ (1)

$\because \overline{CD}$ and $\overline{CB}$ are two tangent-segments to the circle N

$\therefore CD = CB$ (2)

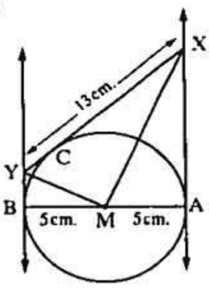
From (1) and (2) : $\therefore CA = CD = CB$

$\therefore$ C is the midpoint of $\overline{AB}$ (Q.E.D. 1)

$\therefore$ In Δ ABD : $\overline{DC}$ is a median , $DC = \frac{1}{2}$ AB

$\therefore \overline{AD} \perp \overline{BD}$ (Q.E.D. 2)

23



$\because \overline{XA}$, $\overline{XC}$ are two tangent-segments to the circle M

$\therefore \overrightarrow{XM}$ bisects $\angle AXY$

$\therefore m(\angle AXM) = m(\angle MXY)$

$\because \overline{YC}$, $\overline{YB}$ are two tangent-segments to the circle M

$\therefore \overrightarrow{YM}$ bisects $\angle BYX$

$\therefore m(\angle BYM) = m(\angle MYX)$

$\because \overline{MA} \perp \overline{AX}$, $\overline{MB} \perp \overline{BY}$ $\therefore \overline{AX} // \overline{BY}$

$\therefore m(\angle AXY) + m(\angle BYX) = 180°$

$\frac{1}{2} m(\angle AXY) + \frac{1}{2} m(\angle BYX) = 90°$

$\therefore m(\angle MXY) + m(\angle MYX) = 90°$

In Δ XMY :

$\therefore m(\angle XMY) = 90°$

$\therefore \overline{XM} \perp \overline{YM}$ (First req.)

$\because \overline{XA}$, $\overline{XC}$ are two tangent-segments to the circle M

$\therefore XA = XC$ similarly $YB = YC$

$\therefore XA + YB = XC + YC$

$\therefore XA + YB = XY = 13$ cm.

$\because \overleftrightarrow{AX} // \overleftrightarrow{BY}$, $\because AB \neq XY$ $\therefore AX \neq BY$

$\therefore$ The figure AXYB is a trapezium

$\therefore$ The area of the figure AXYB $= \frac{1}{2}(AX + BY) \times AB$

$= \frac{1}{2} \times 13 \times 10 = 65$ cm^2 (Second req.)

24

$\because BE = BD$, $CE = CF$, $AD = AF$ and adding

$\therefore BE + CE + AD = BD + CF + AF$

$\therefore BC + AD = AC + BD$ (First req.)

$\therefore 10 + AD = 8 + (7 - AD)$

$\therefore 2\,AD = 8 + 7 - 10 = 5$

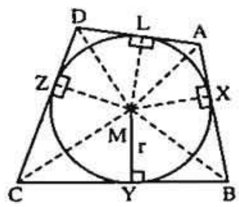
$\therefore AD = 2.5$ cm. (Second req.)

$\therefore BD = BE = 7 - 2.5 = 4.5$ cm.

$\therefore CE = 10 - 4.5 = 5.5$ cm. (Third req.)

Excellent pupils

1



Construction :

Draw $\overline{MX} \perp \overline{AB}$, $\overline{ML} \perp \overline{AD}$

$\overline{MZ} \perp \overline{DC}$

Proof : $\because$ The circle M is inscribed in the quadrilateral ABCD

$\therefore$ The circle touches the sides of the figure ABCD at X , Y , Z and L

$\therefore \overline{AX}$, $\overline{AL}$ are two tangent-segments

$\therefore AX = AL$ similarly $BX = BY$, $CZ = CY$, $DZ = DL$

Adding we find that $AX + BX + CZ + ZD$

$= AL + BY + CY + DL$

$\therefore AB + DC = AD + BC = \frac{1}{2}$ the perimeter of the figure ABCD

$\therefore$ The perimeter of the figure ABCD $= 2(AB + DC)$

$= 2(9 + 12) = 42$ cm. (First req.)

the area of the figure ABCD

= the area of Δ AMB + the area of Δ MBC + the area of Δ MCD + the area of Δ MAD

$= \frac{1}{2} AB \times r + \frac{1}{2} BC \times r + \frac{1}{2} CD \times r + \frac{1}{2} DA \times r$

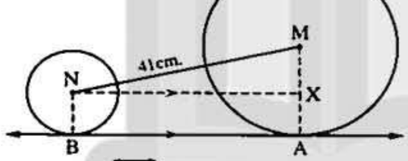
$= \frac{1}{2} r (AB + BC + CD + DA)$

$= \frac{1}{2} \times 5 \times 42 = 105 \text{ cm}^2$. (Second req.)

2

Construction :

Draw $\overline{MA}$, $\overline{NB}$, $\overline{NX} \parallel \overline{AB}$ to cut $\overline{MA}$ at X



Proof : $\because \overline{MA}$ is a radius , $\overleftrightarrow{AB}$ is a tangent to the circle M at A

$\therefore m(\angle MAB) = 90°$ (1)

similarly : $m(\angle NBA) = 90°$ (2)

$\because \overline{NX} \parallel \overline{AB}$ $\therefore m(\angle AXN) = 90°$ (3)

From (1) , (2) and (3) :

$\therefore$ The figure AXNB is a rectangle.

$\because BN = 8$ cm. $\therefore AX = 8$ cm.

, $\because MX = 9$ cm. $\because m(\angle MXN) = 90°$

$\therefore (XN)^2 = (MN)^2 - (MX)^2 = 1681 - 81 = 1600$

$\therefore XN = 40$ cm. $\therefore AB = XN = 40$ cm. (The req.)

Answers of Exercise 12

First : Problems on theorem (5) and its corollary

1

Fig. (1) : $m(\angle BAC) = 70°$

Fig. (2) : $m(\angle AMB) = 112°$

Fig. (3) : $m(\angle ADB) = 80°$

Fig. (4) : $m(\angle CAB) = 70°$

Fig. (5) : $m(\angle CAB) = 90°$

Fig. (6) : $m(\angle CAB) = 60°$

Fig. (7) : $m(\angle CAB) = 50°$

Fig. (8) : $m(\angle CAB) = 65°$, $m(\angle ADB) = 115°$

Fig. (9) : $m(\angle CAD) = 40°$

Fig. (10) : $m(\angle ABD) = 50°$

Fig. (11) : $m(\angle ADB) = 40°$

Fig. (12) : $m(\angle BAD) = 30°$, $m(\angle DAE) = 100°$

Fig. (13) : $m(\angle ABD) = 38°$

Fig. (14) : $m(\angle CAB) = 80°$, $m(\angle BDC) = 50°$

Fig. (15) : $m(\angle CAB) = 60°$, $m(\angle AEB) = 60°$

Fig. (16) : $m(\angle CAB) = 65°$

2

1 a tangent to the circle , a chord in the circle passing through the point of tangency.

2 the inscribed angle

3 central angle

4 **First :** 100° **Second :** 80° **Third :** 40°

5 60° 6 70°

3

1 c 2 d 3 a

4 d 5 b

4 Theoretical.

5

$\because \overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C

$\therefore m(\angle ABC) = m(\angle ACB)$

, $\because m(\angle ABC)$ (the tangency angle)

$= m(\angle BDC)$ (the inscribed angle) $= 65°$

$\therefore m(\angle BAC) = 180° - (65° + 65°) = 50°$ (The req.)

6

$\because m(\angle ABD)$ (the tangency angle)

$= m(\angle C)$ (the inscribed angle) (1)

$\because \overleftrightarrow{XY} \parallel \overleftrightarrow{BD}$, $\overleftrightarrow{XB}$ is a transversal to them

$\therefore m(\angle YXB) = m(\angle XBD)$ (alternate angles) (2)

From (1) and (2) : $\therefore m(\angle C) = m(\angle YXB)$

$\therefore$ The figure AXYC is a cyclic quadrilateral. (Q.E.D.)

7

$\because m(\angle ACB)$ (the inscribed angle)

$= m(\angle XAB)$ (the tangency angle) $= 40°$

$\therefore m(\angle BAC) = 180° - (40° + 110°) = 30°$

$\therefore$ m ($\angle$ CDB) = m ($\angle$ BAC) = 30° (two inscribed angles subtended by the same arc $\overset{\frown}{BC}$) (The req.)

8

In Δ ACD : $\because$ $x° + 6x° + 2x° = 180°$

$\therefore 9x° = 180°$ $\therefore x° = 20°$

$\therefore$ m ($\angle$ ADC) = 2 × 20° = 40°

$\therefore$ m ($\angle$ BAC) (the tangency angle)

= m ($\angle$ ADC) (the inscribed angle)

$\therefore$ m ($\angle$ BAC) = 40° (The req.)

9

$\because$ $\overline{AB}$ is a diameter in the circle

$\therefore$ m ($\overset{\frown}{ACB}$) = 180°

$\because$ $\overline{CD}$ // $\overline{AB}$

$\therefore$ m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{CB}$) = $\frac{180°}{2}$ = 90°

$\therefore$ m ($\angle$ DCA) (the tangency angle) = $\frac{1}{2}$ m ($\overset{\frown}{AC}$) = 45° (First req.)

The length of $\overset{\frown}{AC}$ = $\frac{90°}{360°}$ × 44 = 11 cm. (Second req.)

10

$\because$ $\overrightarrow{XZ}$ and $\overrightarrow{XY}$ are two tangents to the circle

$\therefore$ XZ = XY

$\therefore$ m ($\angle$ XZY) = $\frac{180° - 40°}{2}$ = 70°

, $\because$ m ($\angle$ ZEY) (the inscribed angle)

= m ($\angle$ XZY) (the tangency angle)

$\therefore$ m ($\angle$ ZEY) = 70° (1)

, $\because$ the figure YZDE is a cyclic quadrilateral

$\therefore$ m ($\angle$ ZYE) + m ($\angle$ D) = 180°

$\therefore$ m ($\angle$ ZYE) = 180° – 110° = 70° (2)

From (1) and (2) :

m ($\angle$ ZEY) = m ($\angle$ ZYE)

$\therefore$ ZE = ZY

$\therefore$ m ($\overset{\frown}{ZDE}$) = m ($\overset{\frown}{ZY}$) (Q.E.D.)

11

$\because$ m ($\angle$ ABC) (the tangency angle)

= m ($\angle$ BDC) (the inscribed angle) (1)

$\because$ BC = CD

$\therefore$ m ($\angle$ CBD) = m ($\angle$ CDB) (2)

From (1) and (2) :

$\therefore$ m ($\angle$ ABC) = m ($\angle$ CBD) (First req.)

$\because$ BDEC is a cyclic quadrilateral

$\therefore$ m ($\angle$ DBC) = 180° – 110° = 70°

$\therefore$ m ($\angle$ ABC) = 70° $\because$ AB = AC

$\therefore$ m ($\angle$ A) = 180° – (70° + 70°) = 40° (Second req.)

12

$\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ ABC) = 180° – 100° = 80° (First req.)

$\because$ $\overline{AB}$ // $\overline{DC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ BAC) = m ($\angle$ ACD) = 50° (alternate angles)

$\therefore$ m ($\angle$ CBE) (the tangency angle)

= m ($\angle$ BAC) the inscribed angle = 50° (Second req.)

$\therefore$ m ($\angle$ ABF) = 180° – [m ($\angle$ ABC) + m ($\angle$ CBE)]

= 180° – (80° + 50°) = 50°

$\because$ FA = FB

$\therefore$ m ($\angle$ F) = 180° – (50° + 50°) = 80° (Third req.)

13

$\because$ AC = AB , m ($\angle$ A) = 40°

$\therefore$ m ($\angle$ ACB) = m ($\angle$ ABC) = $\frac{180° - 40°}{2}$ = 70° (First req.)

$\because$ $\overline{AC}$ // $\overline{BD}$, $\overleftrightarrow{CB}$ is a transversal to them

$\therefore$ m ($\angle$ ACB) = m ($\angle$ CBD) = 70° (alternate angles)

$\therefore$ m ($\angle$ ECD) (the tangency angle)

= m ($\angle$ CBD) (the inscribed angle) = 70° (Second req.)

$\because$ m ($\angle$ CDB) (the inscribed angle)

= m ($\angle$ ACB) (the tangency angle) = 70°

$\therefore$ m ($\angle$ CBD) = m ($\angle$ CDB) = 70°

$\therefore$ In Δ CBD : CB = CD (Third req.)

14

$\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ ABC) = 180° – 120° = 60°

$\because$ $\overline{BC}$ is a diameter in the circle

$\therefore$ m ($\angle$ BAC) = 90°

$\therefore$ m ($\angle$ ACB) = 180° – (90° + 60°) = 30°

$\therefore$ m ($\angle$ EAB) (the tangency angle)

= m ($\angle$ ACB) (the inscribed angle) = 30° (1)

$\because$ $\angle$ ABC is an exterior angle of Δ ABE

$\therefore$ m ($\angle$ E) = 60° – 30° = 30° (2)

From (1) and (2) : $\therefore$ m ($\angle$ EAB) = m ($\angle$ E)

$\therefore$ BA = BE (Q.E.D. 1)

$\because \angle ABE$ is an exterior angle of ΔABC

$\therefore m(\angle ABE) = m(\angle BAC) + m(\angle ACB)$

$\because m(\angle EAB)$ (the tangency angle)

$= m(\angle ACB)$ (the inscribed angle)

$\therefore m(\angle ABE) = m(\angle BAC) + m(\angle EAB)$

$\therefore m(\angle ABE) = m(\angle EAC)$ (Q.E.D. 2)

15

$\because$ D is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

$\therefore \overline{AB} \parallel \overline{DE}$ (Q.E.D. 1)

$\therefore m(\angle BDE) = m(\angle ABD)$ (alternate angles)

$\because m(\angle ABN)$ (the tangency angle)

$= m(\angle NCE)$ (the inscribed angle)

$\therefore m(\angle NDE) = m(\angle NCE)$

but they are drawn on $\overline{NE}$ and on one side of it

$\therefore$ The figure NDCE is a cyclic quadrilateral

$\therefore$ The points N , D , C and E has one circle passing through them. (Q.E.D. 2)

16

$\because AB = AC$

$\therefore m(\widehat{AB}) = m(\widehat{AC})$

$\because m(\angle CYD) = \frac{1}{2}\left(m(\widehat{AB}) + m(\widehat{DC})\right)$

$\therefore m(\angle CYD) = \frac{1}{2}\left(m(\widehat{AC}) + m(\widehat{DC})\right)$

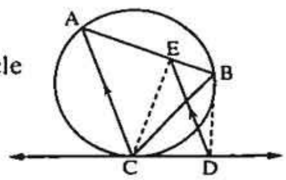
$\therefore m(\angle CYD) = \frac{1}{2} m(\widehat{AD})$

$\because m(\angle XDY) = \frac{1}{2} m(\widehat{AD})$

$\therefore m(\angle XYD) = m(\angle XDY)$

$\therefore XY = XD$ (Q.E.D.)

17



$\because \overleftrightarrow{DC}$ is a tangent to the circle

$\therefore m(\angle BCD)$ (the tangency angle)

$= m(\angle BAC)$ (the inscribed angle)

$\because \overline{DE} \parallel \overline{AC}$, $\overleftrightarrow{AE}$ is a transversal to them

$\therefore m(\angle BED) = m(\angle BAC)$ (corresponding angles)

$\therefore m(\angle BCD) = m(\angle BED)$

but they are drawn on $\overline{BD}$ and on the same side of it

$\therefore$ The figure BECD is a cyclic quadrilateral. (Q.E.D.)

18

$\because \overline{AB}$ is a diameter $\therefore m(\angle ACB) = 90°$

$\because \overline{FD} \perp \overline{AB}$

$\therefore m(\angle ACB) + m(\angle ADE) = 90° + 90° = 180°$

$\therefore$ The figure ADEC is a cyclic quadrilateral. (Q.E.D. 1)

$\therefore m(\angle FEC)$ (the exterior angle) $= m(\angle A)$

, $\because m(\angle FCE)$ (the tangency angle)

$= m(\angle A)$ (the inscribed)

$\therefore m(\angle FCE) = m(\angle FEC)$

$\therefore \Delta FCE$ is an isosceles triangle. (Q.E.D. 2)

$\because m(\angle ACE) = 90°$

$\therefore \overline{AE}$ is a diameter of the circle passing through the vertices of the figure ADEC

$\therefore$ The centre of the circle is the midpoint of $\overline{AE}$ (Q.E.D. 3)

19

$\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$

$\because \overleftrightarrow{EC}$ is a tangent , $\overline{MC}$ is a radius

$\therefore \overline{MC} \perp \overline{EC}$

$\therefore m(\angle EXM) + m(\angle ECM) = 90° + 90° = 180°$

$\therefore$ The figure ECMX is a cyclic quadrilateral (Q.E.D. 1)

$\therefore m(\angle EMX) = m(\angle ECX)$

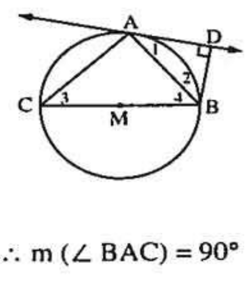
drawn on $\overline{EX}$ and on one side of it but

, $m(\angle ECD)$ (the tangency angle)

$= m(\angle DBC)$ (the inscribed angle)

$\therefore m(\angle EMX) = m(\angle DBC)$ (Q.E.D. 2)

20



$\because \overline{AD}$ is a tangent-segment to the circle

$\therefore m(\angle 1)$ (the tangency angle)

$= m(\angle 3)$ (the inscribed angle)

$\because \overline{BC}$ is a diameter $\therefore m(\angle BAC) = 90°$

$\therefore$ In ΔADB , ΔCAB

$m(\angle 1) = m(\angle 3)$, $m(\angle D) = m(\angle BAC) = 90°$

$\therefore m(\angle 2) = m(\angle 4)$ (Q.E.D.)

21

$\because$ ABCDE is a regular pentagon

$\therefore AB = BC = CD = DE = AE$

$\therefore m(\widehat{AB}) = m(\widehat{BC}) = m(\widehat{CD}) = m(\widehat{DE}) = m(\widehat{AE})$

$\therefore m(\widehat{AE}) = \frac{360^\circ}{5} = 72^\circ$ (First req.)

$\therefore m(\angle AEX)$ (The tangency angle)

$= \frac{1}{2} m(\widehat{AE}) = \frac{1}{2} \times 72^\circ = 36^\circ$

$, \because AX = EX \quad \therefore m(\angle EAX) = m(\angle AEX) = 36^\circ$

$\therefore$ In Δ AEX : $m(\angle AXE) = 180^\circ - (36^\circ + 36^\circ)$

$= 108^\circ$ (Second req.)

22

In the small circle

$\because m(\angle XAB)$ (the tangency angle)

$= m(\angle ADB)$ (the inscribed angle) (1)

In the great circle

$\because m(\angle XAC)$ (the tangency angle)

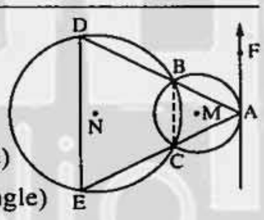
$= m(\angle AEC)$ (the inscribed angle) (2)

From (1) and (2) :

$\therefore m(\angle ADB) = m(\angle AEC)$ but they are corresponding

$\therefore \overline{DB} \parallel \overline{EC}$ (Q.E.D.)

23



Construction : Draw $\overline{BC}$

Proof :

$m(\angle FAB)$ (the tangency angle)

$= m(\angle ACB)$ (the inscribed angle)

$\because \angle ACB$ is an exterior angle of the cyclic quadrilateral BCED

$\therefore m(\angle ACB) = m(\angle D)$

$\therefore m(\angle FAB) = m(\angle D)$ but they are alternate angles

$\therefore \overleftrightarrow{AF} \parallel \overline{DE}$ (Q.E.D.)

24

$\because m(\angle ECD)$ (the tangency angle)

$= m(\angle CAB)$ (the inscribed angle)

$, m(\angle EDC)$ (the tangency angle)

$= m(\angle DAB)$ (the inscribed angle)

By adding :

$\therefore m(\angle ECD) + m(\angle EDC)$

$= m(\angle CAB) + m(\angle DAB)$

$\therefore m(\angle ECD) + m(\angle EDC) = m(\angle CAD)$ (1)

(Q.E.D. 1)

In Δ CED : $m(\angle ECD) + m(\angle EDC) + m(\angle E) = 180^\circ$

From (1) : $\therefore m(\angle CAD) + m(\angle E) = 180^\circ$

$\therefore$ The figure ACED is a cyclic quadrilateral.

(Q.E.D.2)

Second : Problems on the converse of theorem (5)

1

1 $\because AD = BD$

$\therefore m(\angle B) = m(\angle BAD) = 70^\circ$

$\therefore m(\angle D) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$

$\therefore m(\angle CAB) = m(\angle D)$

$\therefore \overrightarrow{AC}$ touches the circle M at A (Q.E.D.)

2 $\because m(\angle BAD) = 90^\circ$ (drawn in a semicircle)

$\therefore m(\angle ADB) = 90^\circ - 50^\circ = 40^\circ$

$\therefore m(\angle BAC) = m(\angle D)$

$\therefore \overrightarrow{AC}$ touches the circle M at A (Q.E.D.)

3 $\because m(\angle AMB)$ (central) $= 110^\circ$

$\therefore m(\angle CAB) = 55^\circ$

$\therefore m(\angle CAB) = \frac{1}{2} m(\angle BMA)$ (central)

$\therefore \overrightarrow{AC}$ touches circle M at A (Q.E.D.)

4 $\because AB = BD$

$\therefore m(\angle BAD) = m(\angle BDA)$

$= \frac{1}{2}(180^\circ - 50^\circ) = 65^\circ$

$\therefore m(\angle CAB) = m(\angle D)$

$\therefore \overrightarrow{AC}$ touches the circle M at A (Q.E.D.)

2

1 $\because BA = BC$

$\therefore m(\angle BAC) = m(\angle C) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$

$\therefore m(\angle BAD) = 140^\circ - 70^\circ = 70^\circ$

$\therefore m(\angle BAD) = m(\angle C)$

$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC (Q.E.D.)

2 $\because \Delta$ ABC is equilateral

$\therefore m(\angle C) = m(\angle B) = 60^\circ$

$\because \overrightarrow{AD} \parallel \overline{CB}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle C) = m(\angle DAC) = 60^\circ$ (alternate angles)

$\therefore m(\angle B) = m(\angle DAC)$

$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC (Q.E.D.)

3 $\because$ The sum of measures of the interior angles of the triangle = 180°

$\therefore 60° + 3x + 5x = 180°$

$\therefore 8x = 120°$ $\qquad \therefore x = 15°$

$\therefore m(\angle C) = 5 \times 15° = 75°$

$\therefore m(\angle C) = m(\angle DAB)$

$\therefore \overline{AD}$ is a tangent to the circle passing through the vertices of ΔABC (Q.E.D.)

4 $\because \Delta ABC$ is right-angled at A , $AC = \frac{1}{2}BC$

$\therefore m(\angle B) = 30°$

$\therefore m(\angle C) = 180° - (90° + 30°) = 60°$

$\therefore m(\angle C) = m(\angle DAB)$

$\therefore \overline{AD}$ is a tangent to the circle passing through the vertices of ΔABC (Q.E.D.)

3

$\because \overrightarrow{AE} \parallel \overline{BD}$, $\overline{AB}$ is a transversal to them

$\therefore m(\angle ABD) = m(\angle BAE) = 55°$ (alternate angles)

$\because AB = AD$

$\therefore m(\angle ADB) = m(\angle ABD) = 55°$

$\therefore m(\angle BAD) = 180° - (55° + 55°) = 70°$

$\therefore m(\angle BAD) + m(\angle BCD) = 70° + 110° = 180°$

$\therefore$ The figure ABCD is a cyclic quadrilateral (Q.E.D.1)

$\because m(\angle ADB) = m(\angle BAE) = 55°$

$\therefore \overrightarrow{AE}$ is a tangent to the circumcircle of ΔABD

, $\because$ the circumcircle of ΔABD and the circumcircle of the cyclic quadrilateral ABCD are the same circle because they have 3 common point.

$\therefore \overrightarrow{AE}$ is a tangent to the circle passing through the vertices of the figure ABCD (Q.E.D. 2)

4

$\because$ The figure ABCD is a cyclic quadrilateral

$\therefore m(\angle A) = 180° - 140° = 40°$

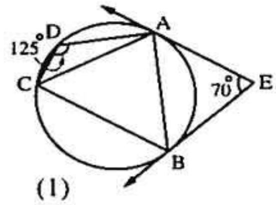
$\because AB = AD$

$\therefore m(\angle ABD) = \frac{180° - 40°}{2} = 70°$

$\therefore m(\angle ABD) = m(\angle ADE)$

$\therefore \overrightarrow{DE}$ is a tangent to the circle at D (Q.E.D.)

5



$\because$ The figure ABCD is a cyclic quadrilateral

$\therefore m(\angle ABC) = 180° - 125° = 55°$ (1)

$\because \overrightarrow{EA}$, $\overrightarrow{EB}$ are two tangents to the circle at A and B

$\therefore EA = EB$ $\qquad \because m(\angle E) = 70°$

$\therefore m(\angle EAB) = \frac{180° - 70°}{2} = 55°$

$\because \overrightarrow{EA}$ is a tangent to the circle at A

$\therefore m(\angle EAB)$ (tangency) $= m(\angle ACB)$ (inscribed)

$\therefore m(\angle ACB) = 55°$ (2)

From (1) and (2) : $\therefore m(\angle ACB) = m(\angle ABC) = 55°$

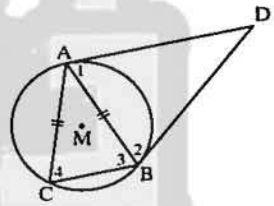
$\therefore AB = AC$ (Q.E.D. 1)

$\therefore m(\angle BAC) = 180° - 2 \times 55° = 70°$

$\therefore m(\angle BAC) = m(\angle E) = 70°$

$\therefore \overleftrightarrow{AC}$ is a tangent to the circle passing through the points A , B and E (Q.E.D. 2)

6



$\because \overline{DA}$ and $\overline{DB}$ are two tangent-segments to the circle M at A and B

$\therefore DA = DB$

$\therefore m(\angle 1) = m(\angle 2)$

$\therefore m(\angle D) = 180° - 2\,m(\angle 1)$ (1)

In ΔABC : $\therefore AB = AC$

$\therefore m(\angle 3) = m(\angle 4)$

$\therefore m(\angle BAC) = 180° - 2\,m(\angle 4)$ (2)

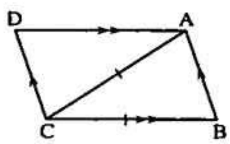
$\because \overline{AD}$ is a tangent-segment to the circle

$\therefore m(\angle 4)$ (inscribed) $= m(\angle 1)$ (tangency) (3)

From (1) , (2) and (3) : $\therefore m(\angle D) = m(\angle BAC)$

$\therefore \overleftrightarrow{AC}$ is a tangent to the circle passing through the vertices of ΔABD (Q.E.D.)

7



In ΔABC :

$\because AC = BC$

$\therefore m(\angle B) = m(\angle BAC)$ (1)

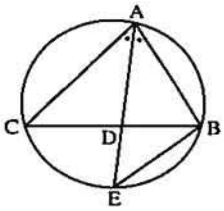
$\because \overline{AB} \parallel \overline{CD}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore m(\angle DCA) = m(\angle BAC)$ (alternate angles) (2)

From (1) and (2) : $\therefore$ m ($\angle$ B) = m ($\angle$ DCA)

$\therefore$ $\overline{CD}$ is a tangent to the circumcircle of Δ ABC (Q.E.D.)

8



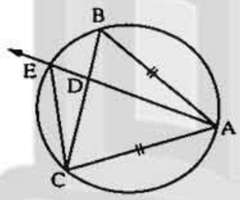
$\because$ m ($\angle$ CBE) = m ($\angle$ CAE) (two inscribed angles subtended by the same arc $\widehat{CE}$)

, m ($\angle$ CAE) = m ($\angle$ DAB)

$\therefore$ m ($\angle$ CBE) = m ($\angle$ DAB)

$\therefore$ $\overleftrightarrow{BE}$ is a tangent to the circle passing through the points A , B and D (Q.E.D.)

9



$\because$ m ($\angle$ AEC) = m ($\angle$ B) (1) (two inscribed angles subtended by the same arc $\widehat{AC}$)

In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ACB) = m ($\angle$ B) (2)

From (1) and (2) : $\therefore$ m ($\angle$ ACB) = m ($\angle$ AEC)

$\therefore$ $\overline{AC}$ is a tangent-segment to the circumcircle of Δ CDE (Q.E.D.)

10

$\because$ $\overline{NC} \perp \overline{AB}$, NA = NC = r

$\therefore$ m ($\angle$ NCA) = m ($\angle$ NAC) = $\frac{180° - 90°}{2}$ = 45°

$\therefore$ m ($\angle$ AEC) = $\frac{1}{2}$m ($\angle$ ANC) = 45° (inscribed and central angles subtended by the same arc $\widehat{AC}$)

$\therefore$ m ($\angle$ DCA) = m ($\angle$ DEC) = 45°

$\therefore$ $\overline{AC}$ is a tangent to the circumcircle of Δ CDE (Q.E.D.)

11

In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$\because$ $\overleftrightarrow{LE}$ is a tangent to the circle

$\therefore$ m ($\angle$ LAB) (tangency) = m ($\angle$ ACB) (inscribed)

$\therefore$ m ($\angle$ LAB) = m ($\angle$ ABC) (Q.E.D. 1)

$\because$ m ($\angle$ LAB) = m ($\angle$ ABC) and they are alternate angles

$\therefore$ $\overline{BC}$ // $\overleftrightarrow{LE}$

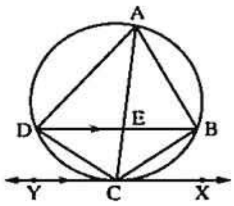
$\therefore$ m ($\angle$ BEA) = m ($\angle$ CBE) (alternate angles)

, $\because$ m ($\angle$ CAD) = m ($\angle$ CBD) (two inscribed angles subtended by the same arc $\widehat{CD}$)

$\therefore$ m ($\angle$ DEA) = m ($\angle$ CAD)

$\therefore$ $\overline{AC}$ is tangent to the circumcircle of Δ ADE (Q.E.D. 2)

12



$\because$ $\overleftrightarrow{XY}$ is a tangent to the circle

$\therefore$ m ($\angle$ XCB) (tangency) = m ($\angle$ BAC) (inscribed) (1)

$\because$ $\overleftrightarrow{XY}$ // $\overline{BD}$

$\overleftrightarrow{BC}$ is a transversal to them

$\therefore$ m ($\angle$ XCB) = m ($\angle$ CBD) (alternate angles) (2)

$\because$ m ($\angle$ CBD) = m ($\angle$ CAD) (3) (two inscribed angles subtended by the same arc $\widehat{CD}$)

from (1) , (2) and (3) : $\therefore$ m ($\angle$ BAC) = m ($\angle$ CAD)

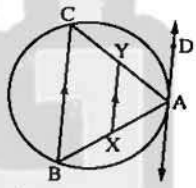
$\therefore$ $\overrightarrow{AC}$ bisects $\angle$ BAD (Q.E.D. 1)

and from (1) and (2) we deduce that

m ($\angle$ BAE) = m ($\angle$ CBE)

$\therefore$ $\overleftrightarrow{BC}$ touches the circumcircle of Δ ABE (Q.E.D. 2)

13



$\because$ $\overleftrightarrow{AD}$ is a tangent to the circle at A

$\therefore$ m ($\angle$ DAC) (tangency) = m ($\angle$ B) (inscribed)

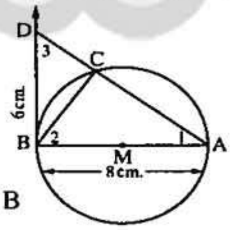
$\because$ $\overleftrightarrow{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ AXY) = m ($\angle$ B) (corresponding angles)

$\therefore$ m ($\angle$ DAC) = m ($\angle$ AXY)

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

14



$\because$ $\overline{AB}$ is a diameter in the cicle

$\therefore$ m ($\angle$ ACB) = 90°

$\therefore$ m ($\angle$ 1) + m ($\angle$ 2) = 90° (1)

$\because$ $\overline{BD}$ is a tangent to the circle at B

$\therefore$ $\overline{AB} \perp \overline{BD}$

$\therefore$ m ($\angle$ 1) + m ($\angle$ 3) = 90° (2)

From (1) and (2) : $\therefore$ m ($\angle$ 2) = m ($\angle$ 3)

$\therefore$ $\overline{AB}$ is a tangent to the circumcircle of Δ CBD (First req.)

In Δ ABD : $\because$ m ($\angle$ ABD) = 90°

$\therefore$ AD = $\sqrt{(AB)^2 + (BD)^2} = \sqrt{64 + 36} = \sqrt{100}$ = 10 cm.

$\because$ $\overline{BC} \perp \overline{AD}$

$\therefore$ BC = $\frac{AB \times BD}{AD} = \frac{8 \times 6}{10}$ = 4.8 cm. (Second req.)

15

$\because \overrightarrow{AL}$, $\overrightarrow{AC}$ are two tangents to the circle M

$\therefore$ AL = AC = 7 cm. (First req.)

$\therefore$ In Δ ALC : m ($\angle$ ALC) = m ($\angle$ ACL)

, $\because$ m ($\angle$ LAC) = 90°

$\therefore$ m ($\angle$ ACL) = $\frac{180° - 90°}{2}$ = 45°

, $\because \overrightarrow{AM}$ bisects $\angle$ LAC $\therefore$ m ($\angle$ LAM) = 45°

$\therefore$ m ($\angle$ LAN) = m ($\angle$ ACL) = 45°

$\because \overrightarrow{AL}$ is a tangent to the circle passing through the vertices of Δ ANC (Second req.)

16

$\because$ The figure DBCE is cyclic quadrilateral

$\therefore$ The exterior angle m ($\angle$ AEC) = m ($\angle$ B) (1)

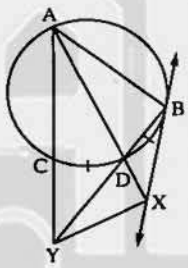
$\because \overleftrightarrow{AX}$ // $\overline{CE}$, $\overrightarrow{AE}$ is a transversal to them

$\therefore$ m ($\angle$ XAE) = m ($\angle$ AEC) (alternate angles) (2)

From (1) and (2) : $\therefore$ m ($\angle$ B) = m ($\angle$ XAD)

$\therefore \overrightarrow{AX}$ is a tangent to the circumcircle of Δ ABD (Q.E.D.)

17



$\because$ m ($\angle$ XBD) (the tangency)
= m ($\angle$ BAD) (the inscribed)

$\because$ m ($\widehat{BD}$) = m ($\widehat{CD}$)

$\therefore$ m ($\angle$ BAD) = m ($\angle$ DAC)

$\therefore$ m ($\angle$ XBY) = m ($\angle$ XAY)

but they are drawn on $\overline{XY}$ and on one side of it

$\therefore$ The figure ABXY is a cyclic quadrilateral (Q.E.D. 1)

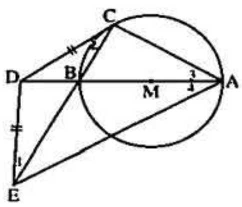
$\therefore$ m ($\angle$ BYX) = m ($\angle$ BAX) (drawn on $\overline{BX}$ and on one side of it)

$\because$ m ($\angle$ BAX) = m ($\angle$ XAY)

$\therefore$ m ($\angle$ BYX) = m ($\angle$ XAY)

$\therefore \overline{XY}$ is a tangent of the circumcircle of Δ ADY (Q.E.D. 2)

18



In Δ CDE :

$\because$ DC = DE

$\therefore$ m ($\angle$ 1) = m ($\angle$ 2)

, $\because$ m ($\angle$ 3) (the inscribed angle)
= m ($\angle$ 2) (the tangency angle)

$\therefore$ m ($\angle$ 1) = m ($\angle$ 3)

but they are drawn on $\overline{CD}$ and on one side of it.

$\therefore$ The figure ACDE is a cyclic quadrilateral. (Q.E.D.1)

, $\because \overline{AB}$ is a diameter

$\therefore$ m ($\angle$ ACB) = 90° $\therefore$ m ($\angle$ ACE) = 90°

$\therefore \overline{AE}$ is a diameter of the circumcircle of the figure ACDE (Q.E.D.2)

, $\because$ ACDE is a cyclic quadrilateral

$\therefore$ m ($\angle$ 4) = m ($\angle$ 2)

(they are drawn on $\overline{DE}$ and on one side of it).

, $\because$ m ($\angle$ 1) = m ($\angle$ 2) $\therefore$ m ($\angle$ 4) = m ($\angle$ 1)

$\therefore \overleftrightarrow{ED}$ is a tangent to the circumcircle of Δ ABE (Q.E.D. 3)

19

$\because \Delta$ ABC is an equilateral triangle

$\therefore$ m ($\angle$ ACB) = 60°

, $\because \Delta$ DCE is an equilateral triangle

$\therefore$ m ($\angle$ CDE) = 60°

$\therefore$ m ($\angle$ ACB) = m ($\angle$ CDE)

$\therefore \overline{AC}$ is a tangent-segment to the circle which passes through the vertices of Δ CED (First req.)

, $\because \Delta$ ABC is an equilateral triangle

, E is the midpoint of $\overline{BC}$

$\therefore \overline{AE} \perp \overline{BC}$ $\therefore$ m ($\angle$ CEW) = 90° (1)

, $\because$ E is the midpoint of $\overline{BC}$

$\therefore \overline{ED}$ is a median in Δ BCD

, $\because$ ED = $\frac{1}{2}$ BC $\therefore$ m ($\angle$ BDC) = 90° (2)

From (1) and (2) : $\therefore$ m ($\angle$ CEW) + m ($\angle$ BDC) = 180°

$\therefore$ The figure CDWE is a cyclic quadrilateral (Second req.)

, $\because$ m ($\angle$ CEW) = 90°

$\therefore \overline{WC}$ is the hypotenuse of Δ WEC

$\therefore$ The midpoint of the hypotenuse $\overline{WC}$ is the centre of the circle which passes through the vertices of the quadrilateral CDWE (Third req.)

20

$\because \overline{AD}$, $\overline{AC}$ are two tangent-segments to the circle M

$\therefore$ AD = AC

, $\because \overline{AC}$, $\overline{AB}$ are two tangent-segments to the circle N

$\therefore$ AC = AB $\therefore$ AD = AC = AB (First req.)

$\therefore$ AB = AD = 5 cm.

, $\because$ MD + NB = MN = 6 cm.

$\therefore$ The perimeter of the figure ABNMD

= 5 + 5 + 6 + 6 = 22 cm. (Second req.)

, $\because$ $\overline{AC}$, $\overline{AB}$ are two tangent-segments to the circle N

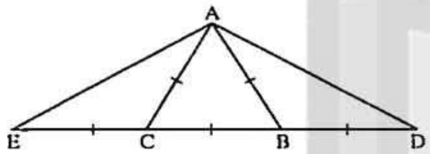
$\therefore$ $\overrightarrow{NA}$ bisects $\angle$ CNB (Third req.)

$\therefore$ m ($\angle$ ANC) = $\frac{110°}{2}$ = 55°

$\therefore$ m ($\angle$ ANC) = m ($\angle$ DAC) = 55°

$\therefore$ $\overline{AD}$ is a tangent-segment to the circle passing through the vertices of Δ ACN (Fourth req.)

21



$\because$ Δ ABC is equilateral

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = 60°

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ACE) = 120°

$\because$ AB = BD

$\therefore$ m ($\angle$ BAD) = m ($\angle$ D) = $\frac{180° - 120°}{2}$ = 30°

similarly m ($\angle$ CAE) = m ($\angle$ E) = 30°

$\therefore$ m ($\angle$ BAD) = m ($\angle$ AEB) = 30°

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circumcircle of Δ ABE (Q.E.D.)

22

$\because$ AB = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ BCA)

$\because$ $\overrightarrow{BX}$ is a tangent to the circle M

$\therefore$ m ($\angle$ XBC) (the tangency)

= m ($\angle$ BAC) (the inscribed)

$\therefore$ m ($\angle$ XBC) = m ($\angle$ BCA)

, they are alternate angles

$\therefore$ $\overrightarrow{BX}$ // $\overrightarrow{CE}$ (Q.E.D. 1)

$\because$ m ($\angle$ YBD) (the tangency)

= m ($\angle$ DCB) (the inscribed)

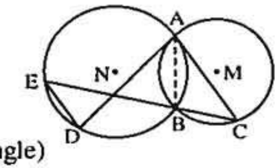
but m ($\angle$ YBE) = m ($\angle$ E) (alternate angles)

$\therefore$ m ($\angle$ DCB) = m ($\angle$ E)

$\therefore$ $\overleftrightarrow{BC}$ is a tangent to the circle passing through the points C , D and E (Q.E.D. 2)

Excellent pupils

1



Construction : Draw $\overline{AB}$

Proof :

$\because$ m ($\angle$ DAB) (the tangency angle)

= m ($\angle$ C) (the inscribed angle)

, m ($\angle$ DAB) = m ($\angle$ DEB)

(two inscribed angles subtended by the same arc $\overset{\frown}{BD}$)

$\therefore$ m ($\angle$ DEB) = m ($\angle$ C) but they are alternate angles

$\therefore$ $\overline{AC}$ // $\overline{DE}$ (Q.E.D.)

2

$\because$ BC = BE

$\therefore$ m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{BE}$)

$\because$ m ($\angle$ A)

= $\frac{1}{2}$ [m ($\overset{\frown}{BC}$) − m ($\overset{\frown}{DE}$)]

= $\frac{1}{2}$ [m ($\overset{\frown}{BE}$) − m ($\overset{\frown}{DE}$)] = $\frac{1}{2}$ m ($\overset{\frown}{BD}$)

$\because$ m ($\angle$ DEB) the inscribed = $\frac{1}{2}$ m ($\overset{\frown}{BD}$)

$\therefore$ m ($\angle$ A) = m ($\angle$ DEB)

$\therefore$ $\overline{BE}$ is a tangent-segment to the circle passing through the vertices of Δ ADE (Q.E.D.)

Answers of exams on second part of unit five

Model 1

1

1 d	2 a	3 b
4 d	5 c	6 a

2

[a] 1 m ($\angle$ A) = 40° 2 Prove by yourself.

[b] Prove by yourself.

3

[a] Prove by yourself.

[b] Prove by yourself.

4

[a] m ($\angle$ D) = 65°

[b] Prove by yourself.

5

[a] $x = 9$, $y = 17$

[b] Prove by yourself.

Model 2

1

1 c	2 a	3 d
4 d	5 b	6 c

2

[a] 1 m (∠ ABC) = 50° 2 m $(\overset{\frown}{ABD})$ = 220°

[b] 1 AB = 8 cm. 2 right-angled at B

3

[a] Prove by yourself.

[b] 1 m (∠ ABC) = 70° 2 Prove by yourself.

4

[a] Prove by yourself.

[b] 1 m (∠ ABC) = 50° , m (∠ BEC) = 50°

2 Prove by yourself

5

[a] 1 m (∠ ABC) = 70° 2 m (∠ ACD) = 35°

[b] Prove by yourselft.

Answers of accumulative basic skills

1 a	2 c	3 b	4 a
5 b	6 c	7 a	8 b
9 d	10 c	11 c	12 d
13 b	14 c	15 b	16 b
17 b	18 c	19 d	20 a
21 b	22 a	23 b	24 a
25 a	26 d	27 c	28 b
29 b	30 c		

Guide Answers

of The Notebook

- Quizzes.
- Final Examinations.

Answers of quizzes of algebra and probability

Quiz 1

1

1 c 2 c 3 d

2 [a] The S.S. = $\{(2 , 1)\}$

[b] 63 cm^2.

Quiz 2

1

1 c 2 b 3 d

2 [a] The S.S. = $\{-6.6 , -1.4\}$

[b] Represent by yourself , the roots are : -2 , 2

Quiz 3

1

1 d 2 b 3 b

2 [a] 24 cm^2.

[b] The S.S. = $\left\{\frac{3-\sqrt{17}}{2} , \frac{3+\sqrt{17}}{2}\right\}$

Quiz 4

1

1 c 2 d 3 c

2 [a] $a = 3$, $b = 6$

[b] Represent by yourself , then check algebraically by yourself , the S.S. = $\{(3 , -1)\}$

Quiz 5

1

1 b 2 c 3 b

2 [a] $\mathbb{R} - \{2 , -2\}$

[b] $a = 6$

Quiz 6

1

1 b 2 d 3 b

2 [a] Prove by yourself , the common domain = $\mathbb{R} - \{-3 , 0 , 2 , 3\}$

[b] The S.S. = $\{2.73 , -0.73\}$

Quiz 7

1

1 b 2 b 3 c

2 [a] The domain of n = $\mathbb{R} - \{-2 , 1 , 2\}$, $n(x) = \frac{3}{x-1}$

[b] Check by yourself.

Quiz 8

1

1 b 2 d 3 c

2 [a] The domain of n = $\mathbb{R} - \{2 , 3\}$, $n(x) = 1$

[b] $a = -4$, $b = -35$

Quiz 9

1

1 a 2 a 3 a

2 [a] 1 The probability of occurring the event A = 0.44

2 The probability of occurring the event B = 0.22

[b] The domain of n = $\mathbb{R} - \{0 , 6 , -6\}$, $n(x) = \frac{-4}{x}$

Quiz (10)

1

1 a   2 b   3 a

2 [a] 1 $\frac{1}{2}$ 2 $\frac{2}{5}$ 3 $\frac{4}{5}$ 4 $\frac{7}{10}$

[b] The domain of $n = \mathbb{R} - \{2, -1, 3\}$

$, n(X) = \frac{2}{X+1}$

Answers of school book examinations in algebra and probability

Model 1

1

1 b 2 a 3 d 4 c 5 b 6 a

2

[a] $\because 2x^2-5x+1=0$

$\therefore a=2 \quad , \quad b=-5 \quad , \quad c=1$

$\therefore x=\frac{5\pm\sqrt{(-5)^2-4\times2\times1}}{2\times2}=\frac{5\pm\sqrt{17}}{4}$

$\therefore x\simeq2.3$ or $x\simeq0.2$

$\therefore$ The S.S. $=\{2.3 , 0.2\}$

[b] $\because n(x)=\frac{x-3}{(x-3)(x-4)}-\frac{4}{x(x-4)}$

$\therefore$ The domain of $n=\mathbb{R}-\{3 , 4 , 0\}$

$, n(x)=\frac{1}{x-4}-\frac{4}{x(x-4)}=\frac{x-4}{x(x-4)}=\frac{1}{x}$

3

[a] $\because x-y=0 \qquad \therefore x=y$ (1)

$, x^2+xy+y^2=27$ (2)

Substituting from (1) in (2) :

$\therefore y^2+y^2+y^2=27 \qquad \therefore 3y^2=27$

$\therefore y^2=9 \qquad \therefore y=3$ or $y=-3$

Substituting in (1) : $\therefore x=3$ or $x=-3$

$\therefore$ The S.S. $=\{(3 , 3) , (-3 , -3)\}$

[b] $\because n(x)=\frac{(x+3)(x+1)}{(x-3)(x^2+3x+9)}\div\frac{x+3}{x^2+3x+9}$

$\therefore$ The domain of $n=\mathbb{R}-\{3 , -3\}$

$, n(x)=\frac{(x+3)(x+1)}{(x-3)(x^2+3x+9)}\times\frac{x^2+3x+9}{x+3}$

$=\frac{x+1}{x-3}$

$, n(2)=\frac{2+1}{2-3}=\frac{3}{-1}=-3$

$, n(-3)$ undefined because $-3\notin$ the domain of n

4

[a] Let the length be x cm. and the width be y cm.

$\therefore x=y+4$ (1)

$, \because 28=2(x+y) \quad \therefore x+y=14$ (2)

Substituting from (1) in (2) :

$\therefore y+4+y=14 \quad \therefore 2y=10 \quad \therefore y=5$

Substituting in (1) : $\therefore x=9$

$\therefore$ The length = 9 cm. , the width = 5 cm.

$\therefore$ The area $=9\times5=45$ cm^2.

[b] 1 $\because n(x)=\frac{x(x-2)}{(x-2)(x-1)}$

$\therefore n^{-1}(x)=\frac{(x-2)(x-1)}{x(x-2)}$

$\therefore$ The domain of $n^{-1}=\mathbb{R}-\{0 , 1 , 2\}$

$, n^{-1}(x)=\frac{x-1}{x}$

2 $\because n^{-1}(x)=3 \qquad \therefore \frac{x-1}{x}=3$

$\therefore 3x=x-1 \qquad \therefore 3x-x=-1$

$\therefore 2x=-1 \qquad \therefore x=\frac{-1}{2}$

5

[a] $\because n_1(x)=\frac{x^2}{x^2(x-1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1=\mathbb{R}-\{0 , 1\}\\ , \because n_1(x)=\frac{1}{x-1}\end{array}\right\}$ (1)

$\because n_2(x)=\frac{x(x^2+x+1)}{x(x-1)(x^2+x+1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2=\mathbb{R}-\{0 , 1\}\\ , n_2(x)=\frac{1}{x-1}\end{array}\right\}$ (2)

From (1) and (2) : $\therefore n_1=n_2$

[b] 1 $P(A\cap B)=\frac{2}{6}=\frac{1}{3}$

2 $P(A-B)=\frac{1}{6}$

3 The probability of non-occurrence of the event $A=\frac{3}{6}=\frac{1}{2}$

Model 2

1

1 a 2 d 3 a 4 b 5 c 6 a

2

[a] $\because 3x^2 - 5x + 1 = 0$

$\therefore a = 3 \quad , \quad b = -5 \quad , \quad c = 1$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 3 \times 1}}{2 \times 3} = \frac{5 \pm \sqrt{25 - 12}}{6}$

$= \frac{5 \pm \sqrt{13}}{6}$

$\therefore x \approx 1.43 \quad \text{or} \quad x \approx 0.23$

$\therefore$ The S.S. $= \{1.43 , 0.23\}$

[b] $\because n(x) = \frac{(x-2)(x^2+2x+4)}{(x-2)(x+3)} \times \frac{x+3}{x^2+2x+4}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -3\}$

$, n(x) = 1$

3

[a] $\because x - y = 1 \qquad \therefore x = y + 1 \qquad (1)$

$, x^2 + y^2 = 25 \qquad (2)$

Substituting from (1) in (2) :

$\therefore (y+1)^2 + y^2 = 25$

$\therefore y^2 + 2y + 1 + y^2 - 25 = 0$

$\therefore 2y^2 + 2y - 24 = 0 \qquad \therefore y^2 + y - 12 = 0$

$\therefore (y-3)(y+4) = 0$

$\therefore y = 3 \quad \text{or} \quad y = -4$

Substituting in (1) : $\therefore x = 4 \quad \text{or} \quad x = -3$

$\therefore$ The S.S. $= \{(4 , 3) , (-3 , -4)\}$

[b] 1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.6 - 0.2 = 0.7$

2 $P(A - B) = P(A) - P(A \cap B)$

$= 0.3 - 0.2 = 0.1$

4

[a] $\because 2x - y = 3 \qquad \therefore y = 2x - 3 \qquad (1)$

$, x + 2y = 4 \qquad (2)$

Substituting from (1) in (2) :

$\therefore x + 2(2x - 3) = 4$

$\therefore x + 4x - 6 = 4 \qquad \therefore 5x = 10 \qquad \therefore x = 2$

Substituting in (1) : $\therefore y = 1$

[b] $\because n(x) = \frac{x(x+3)}{(x+3)(x-3)} \div \frac{2x}{x+3}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3 , 3 , 0\}$

$, n(x) = \frac{x}{x-3} \times \frac{x+3}{2x} = \frac{x+3}{2(x-3)}$

5

[a] $\because n(x) = \frac{x(x+2)}{(x-2)(x+2)} + \frac{x+3}{(x-2)(x-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -2 , 3\}$

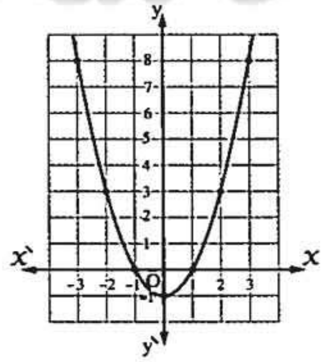
$, n(x) = \frac{x}{(x-2)} + \frac{x+3}{(x-2)(x-3)}$

$= \frac{x(x-3) + x + 3}{(x-2)(x-3)} = \frac{x^2 - 3x + x + 3}{(x-2)(x-3)}$

$= \frac{x^2 - 2x + 3}{(x-2)(x-3)}$

[b] $f(x) = x^2 - 1$

x	-3	-2	-1	0	1	2	3
y	8	3	0	-1	0	3	8



From the graph :

$\therefore$ The S.S. $= \{-1 , 1\}$

Model examination for the merge students

1

1 0 | 2 $\frac{1}{x-2}$ | 3 $\frac{2}{3}$

4 second | 5 second | 6 $\{5\}$

2

1 a | 2 b | 3 c

4 b | 5 c | 6 a

3

1 ✗ | 2 ✗ | 3 ✓

4 ✓ | 5 ✗ | 6 ✓

4

1 $\{(2, 1)\}$ | 2 $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3 $\mathbb{R} - \{1, -1\}$ | 4 $\frac{x}{x^2 + 4}$

5 $\{5\}$ | 6 $\frac{1}{3}$

Answers of governorates' examinations of algebra & probability

1 Cairo

1

1 d 2 c 3 d 4 b 5 d 6 c

2

[a] Let x and y be two real numbers

$\therefore x + y = 40$ (1)

$, x - y = 10$ (2)

Adding (1) and (2) : $\therefore 2x = 50$ $\therefore x = 25$

Substituting in (1) : $\therefore y = 15$

$\therefore$ The two real numbers are 25 , 15

[b] $\because n(x) = \frac{x}{x-2} - \frac{2(x+2)}{(x+2)(x-2)}$

$\therefore$ The domain of n $= \mathbb{R} - \{2, -2\}$

$, n(x) = \frac{x}{x-2} - \frac{x}{x-2} = \frac{x-2}{x-2} = 1$

3

[a] $\because x - 3 = 0$ $\therefore x = 3$ (1)

$, x^2 + y^2 = 25$ (2)

Substituting from (1) in (2) : $\therefore 9 + y^2 = 25$

$\therefore y^2 = 16$ $\therefore y = 4$ or $y = -4$

$\therefore$ The S.S. $= \{(3, 4), (3, -4)\}$

[b] $\because n_1(x) = \frac{x^2}{x^2(x-1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{0, 1\}$

$, n_1(x) = \frac{1}{x-1}$, $\because n_2(x) = \frac{x^2+x+1}{(x-1)(x^2+x+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{1\}$

$, n_2(x) = \frac{1}{x-1}$

$\therefore n_1(x) = n_2(x)$ for all the values of $x \in \mathbb{R} - \{0, 1\}$

4

[a] $\because n(x) = \frac{(x-2)(x^2+2x+4)}{(x+3)(x-2)} \times \frac{x+3}{x^2+2x+4}$

$\therefore$ The domain of n $= \mathbb{R} - \{2, -3\}$, $n(x) = 1$

[b] $\because 2x^2 + 5x - 6 = 0$ $\therefore a = 2, b = 5, c = -6$

$\therefore x = \frac{-5 \pm \sqrt{(5)^2 - 4 \times 2 \times (-6)}}{2 \times 2} = \frac{-5 \pm \sqrt{73}}{4}$

$\therefore x \approx 0.9$ or $x \approx -3.4$

$\therefore$ The S.S. $= \{0.9, -3.4\}$

5

[a] 1 $P(A \cup B) = P(A) + P(B) - P(P \cap B)$

$= 0.7 + 0.5 - 0.3 = 0.9$

2 $P(A - B) = P(A) - P(A \cap B) = 0.7 - 0.3 = 0.4$

[b] 1 $\because n(x) = \frac{x}{x+3}$ $\therefore n^{-1}(x) = \frac{x+3}{x}$

, the domain of $n^{-1} = \mathbb{R} - \{0, -3\}$

2 $\because n^{-1}(x) = 4$ $\therefore \frac{x+3}{x} = 4$

$\therefore 4x = x + 3$ $\therefore 3x = 3$ $\therefore x = 1$

2 Giza

1

1 c 2 d 3 b 4 a 5 c 6 b

2

[a] $\because 2x^2 - 5x + 1 = 0$ $\therefore a = 2, b = -5, c = 1$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 1}}{2 \times 2} = \frac{5 \pm \sqrt{17}}{4}$

$\therefore x \approx 2.28$ or $x \approx 0.22$

$\therefore$ The S.S. $= \{2.28, 0.22\}$

[b] $\because n(x) = \frac{(x+2)(x-2)}{(x-2)(x^2+2x+4)} \div \frac{(x+2)(x-3)}{x^2+2x+4}$

$\therefore$ The domain of n $= \mathbb{R} - \{2, -2, 3\}$

$, n(x) = \frac{(x+2)(x-2)}{(x-2)(x^2+2x+4)} \times \frac{x^2+2x+4}{(x+2)(x-3)}$

$= \frac{1}{x-3}$

3

[a] Let the lengths of the two sides of the right angle be x cm. and y cm.

$\therefore x + y + 10 = 24$ $\therefore x + y = 14$

$\therefore x = 14 - y$ (1)

$, x^2 + y^2 = 100$ (2)

Substituting from (1) in (2) : $\therefore (14 - y)^2 + y^2 = 100$

$\therefore 196 - 28y + y^2 + y^2 - 100 = 0$

$\therefore 2y^2 - 28y + 96 = 0$ (Dividing by 2)

$\therefore y^2 - 14y + 48 = 0$ $\therefore (y-6)(y-8) = 0$

$\therefore y = 6$ or $y = 8$

Substituting in (1) : $\therefore x = 8$ or $x = 6$

$\therefore$ The side lengths of the right angle are 6 cm. and 8 cm.

[b] $\because$ A , B are two mutually exclusive events

$\therefore P(A \cup B) = P(A) + P(B) = 0.2 + 0.5 = 0.7$

, $P(A - B) = P(A) = 0.2$

4

[a] 1 $\therefore n(x) = \frac{x(x-3)}{(x-2)(x-3)}$

$\therefore n^{-1}(x) = \frac{(x-2)(x-3)}{x(x-3)}$

, the domain of $n^{-1} = \mathbb{R} - \{0 , 3 , 2\}$

$\therefore n^{-1}(x) = \frac{x-2}{x}$

2 $\because n^{-1}(x) = 2$ $\therefore \frac{x-2}{x} = 2$

$\therefore x - 2 = 2x$ $\therefore x = -2$

[b] $\because x + 2y = 4$ (1)

, $3x - y = 5$ (multiplying by 2)

$\therefore 6x - 2y = 10$ (2)

Adding (1) and (2) : $\therefore 7x = 14$ $\therefore x = 2$

Substituting in (1) : $\therefore y = 1$

$\therefore$ The S.S. $= \{(2 , 1)\}$

5

[a] $\because n(x) = \frac{x^2}{x-1} - \frac{x}{x-1}$

$\therefore$ The domain of $n = \mathbb{R} - \{1\}$, $n(x) = \frac{x(x-1)}{x-1}$

$\therefore n(x) = x$

[b] $\because n_1(x) = \frac{(x+3)(x-2)}{(x+2)(x-2)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{-2 , 2\} \\ , n_1(x) = \frac{x+3}{x+2}\end{array}\right\}$ (1)

, $\because n_2(x) = \frac{(x+3)(x-3)}{(x-3)(x+2)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{3 , -2\} \\ , n_2(x) = \frac{x+3}{x+2}\end{array}\right\}$ (2)

From (1) and (2) : $\therefore n_1 \neq n_2$

Because the domain of $n_1 \neq$ the domain of n_2

3 Alexandria

1

1 b 2 d 3 b 4 d 5 a 6 a

2

[a] $\because x - y = 0$ $\therefore x = y$ (1)

, $x^2 + xy + y^2 = 27$ (2)

Substituting from (1) in (2) : $\therefore y^2 + y^2 + y^2 = 27$

$\therefore 3y^2 = 27$ $\therefore y^2 = 9$

$\therefore y = 3$ or $y = -3$

Substituting in (1) : $\therefore x = 3$ or $x = -3$

$\therefore$ The S.S. $= \{(3 , 3) , (-3 , -3)\}$

[b] $\because n_1(x) = \frac{(x-3)(x+4)}{(x+1)(x+4)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-1 , -4\}$

, $n_1(x) = \frac{x-3}{x+1}$

, $\because n_2(x) = \frac{(x-3)(x+1)}{(x+1)(x+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{-1\}$, $n_2(x) = \frac{x-3}{x+1}$

$\therefore n_1(x) = n_2(x)$ for all values of $x \in \mathbb{R} - \{-1 , -4\}$

3

[a] $\because 2x^2 + 5x = 0$ $\therefore a = 2 , b = 5 , c = 0$

$\therefore x = \frac{-5 \pm \sqrt{(5)^2 - 4 \times 2 \times 0}}{2 \times 2} = \frac{-5 \pm 5}{4}$

$\therefore x = 0$ or $x = -2.5$

$\therefore$ The S.S. $= \{0 , -2.5\}$

[b] $\because n(x) = \frac{(x-1)(x^2+x+1)}{x(x-1)} \times \frac{x+3}{x^2+x+1}$

$\therefore$ The domain of $n = \mathbb{R} - \{0 , 1\}$, $n(x) = \frac{x+3}{x}$

4

[a] $\because 2x + y = 1$ $\therefore y = 1 - 2x$ (1)

, $x + 2y = 5$ (2)

Substituting from (1) in (2) :

$\therefore x + 2(1 - 2x) = 5$ $\therefore x + 2 - 4x = 5$

$\therefore -3x = 3$ $\therefore x = -1$

Substituting in (1) : $\therefore y = 3$

$\therefore$ The S.S. $= \{(-1 , 3)\}$

[b] $\because n(x) = \frac{x(x-1)}{(x-1)(x+1)} + \frac{x+5}{(x+1)(x+5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1 , -1 , -5\}$

, $n(x) = \frac{x}{x+1} + \frac{1}{x+1} = \frac{x+1}{x+1} = 1$

5

[a] 1 $\because n(x) = \frac{x(x-2)}{(x-2)(x^2+2)}$

$\therefore n^{-1}(x) = \frac{(x-2)(x^2+2)}{x(x-2)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0 , 2\}$

, $n^{-1}(x) = \frac{x^2+2}{x}$

2 $\because n^{-1}(x)=3 \quad \therefore \frac{x^2+2}{x}=3$

$\therefore x^2+2=3x \quad \therefore x^2-3x+2=0$

$\therefore (x-2)(x-1)=0$

$\therefore x=2$ (refused) or $x=1$

[b] A and B are mutually exclusive events

$\therefore P(A\cup B)=P(A)+P(B)$

$\therefore P(B)=P(A\cup B)-P(A)=\frac{7}{12}-\frac{1}{3}=\frac{1}{4}$

4 El-Kalyoubia

1

1 b 2 d 3 c 4 a 5 c 6 c

2

[a] 1 $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=0.8+0.7-0.6=0.9$

2 $P(\acute{A})=1-P(A)=1-0.8=0.2$

[b] Let the length be x cm. and the width be y cm.

$\therefore x-y=4$ (1)

$, 2(x+y)=28$ (Dividing by 2)

$\therefore x+y=14$ (2)

Adding (1) and (2) : $\therefore 2x=18 \quad \therefore x=9$

Substituting in (1) : $\therefore y=5$

$\therefore$ The length = 9 cm. , the width = 5 cm.

$\therefore$ The area of the rectangle $=9\times 5=45$ cm^2.

3

[a] $\because x-y=0 \quad \therefore x=y$ (1)

$, x^2+xy+y^2=27$ (2)

Substituting from (1) in (2) : $\therefore y^2+y^2+y^2=27$

$\therefore 3y^2=27 \quad \therefore y^2=9$

$\therefore y=3$ or $y=-3$

Substituting in (1) : $\therefore x=3$ or $x=-3$

$\therefore$ The S.S. $=\{(3,3),(-3,-3)\}$

[b] $\because n(x)=\frac{x(x+2)}{(x-3)(x^2+3x+9)}\div\frac{x+2}{x^2+3x+9}$

$\therefore$ The domain of $n=\mathbb{R}-\{3,-2\}$

$, n(x)=\frac{x(x+2)}{(x-3)(x^2+3x+9)}\times\frac{x^2+3x+9}{x+2}$

$=\frac{x}{x-3}$

4

[a] $\because 2x^2-4x+1=0$

$\therefore a=2, b=-4, c=1$

$\therefore x=\frac{4\pm\sqrt{(-4)^2-4\times2\times1}}{2\times2}=\frac{4\pm2\sqrt{2}}{4}=\frac{2\pm\sqrt{2}}{2}$

$\therefore x\approx1.7$ or $x\approx0.3 \quad \therefore$ The S.S. $=\{1.7, 0.3\}$

[b] $\because n_1(x)=\frac{2x}{2(x+2)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1=\mathbb{R}-\{-2\}\\ , n_1(x)=\frac{x}{x+2}\end{array}\right\}$ (1)

$, \because n_2(x)=\frac{x(x+2)}{(x+2)(x+2)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2=\mathbb{R}-\{-2\}\\ , n_2(x)=\frac{x}{x+2}\end{array}\right\}$ (2)

From (1) and (2) : $\therefore n_1=n_2$

5

[a] $\because n(x)=\frac{x^2+2x+4}{(x-2)(x^2+2x+4)}+\frac{(x-3)(x+3)}{(x+3)(x-2)}$

$\therefore$ The domain of $n=\mathbb{R}-\{2,-3\}$

$, n(x)=\frac{1}{x-2}+\frac{x-3}{x-2}=\frac{x-2}{x-2}=1$

[b] $\because$ The domain of $f=\mathbb{R}-\{2,k\}$

$\therefore$ where $x=2 \quad \therefore x^2-5x+m=0$

$\therefore 4-5\times2+m=0 \quad \therefore m=6$

$\therefore f(x)=\frac{x}{x^2-5x+6}$

$\therefore f(x)=\frac{x}{(x-2)(x-3)}$

$\therefore$ The domain of $f=\mathbb{R}-\{2,3\} \quad \therefore k=3$

5 El-Sharkia

1

1 d 2 b 3 d 4 a 5 d 6 d

2

[a] $\because x(x-2)=1 \quad \therefore x^2-2x-1=0$

$\therefore a=1, b=-2, c=-1$

$\therefore x=\frac{2\pm\sqrt{(-2)^2-4\times1\times(-1)}}{2\times1}$

$=\frac{2\pm\sqrt{8}}{2}=\frac{2\pm2\sqrt{2}}{2}=1\pm\sqrt{2}$

$\therefore x=1+\sqrt{2}$ or $x=1-\sqrt{2}$

$\therefore$ The S.S. $=\{1+\sqrt{2}, 1-\sqrt{2}\}$

[b] $\because n(x) = \frac{x(x^2+1)}{x^2+1} + \frac{x^2+2x+4}{(x-2)(x^2+2x+4)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2\}$

, $n(x) = x + \frac{1}{x-2} = \frac{x(x-2)+1}{x-2}$

$= \frac{x^2-2x+1}{x-2} = \frac{(x-1)(x-1)}{x-2}$

3

[a] $\because 2x - y = 3$ (1)

, $x + 2y = 4$ $\quad \therefore x = 4 - 2y$ (2)

Substituting from (2) in (1) : $\therefore 2(4-2y) - y = 3$

$\therefore 8 - 4y - y = 3$ $\quad \therefore 8 - 5y = 3$

$\therefore -5y = -5$ $\quad \therefore y = 1$

Substituting in (2) : $\therefore x = 2$

$\therefore$ The S.S. $= \{(2, 1)\}$

[b] $\because n(x) = \frac{(x-5)(x+3)}{(x-3)(x+3)} \div \frac{-2(x-5)}{(x-3)(x-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, -3, 5\}$

, $n(x) = \frac{x-5}{x-3} \times \frac{(x-3)(x-3)}{-2(x-5)} = \frac{x-3}{-2}$

4

[a] $\because x + 2y = 2$ $\quad \therefore 2y = 2 - x$ (1)

, $x^2 + 2xy = 2$ (2)

Substituting from (1) in (2) :

$\therefore x^2 + x(2-x) = 2$ $\quad \therefore x^2 + 2x - x^2 = 2$

$\therefore 2x = 2$ $\quad \therefore x = 1$

Substituting in (1) : $\therefore y = \frac{1}{2}$

$\therefore$ The S.S. $= \left\{\left(1, \frac{1}{2}\right)\right\}$

[b] $\because n_1(x) = 1 - \frac{1}{x}$

$\left.\begin{array}{l} \therefore \text{The domain of } n_1 = \mathbb{R} - \{0\} \\ , n_1(x) = \frac{x-1}{x} \end{array}\right\}$ (1)

$\left.\begin{array}{l} , \because n_2(x) = \frac{1-x}{x} \\ \therefore \text{The domain of } n_2 = \mathbb{R} - \{0\} \end{array}\right\}$ (2)

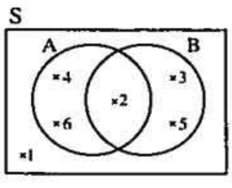
From (1) and (2) : $\therefore n_1 \neq n_2$

5

[a] $P(A) = \frac{3}{6} = \frac{1}{2}$

$P(B) = \frac{3}{6} = \frac{1}{2}$

$P(A \cup B) = \frac{5}{6}$



[b] $\because$ The domain of $n = \mathbb{R} - \{0, 4\}$

$\therefore 4 + m = 0$ $\quad \therefore m = -4$

, $\because n(5) = 2$ $\quad \therefore \frac{k}{5} + \frac{9}{5-4} = 2$ $\therefore \frac{k}{5} + 9 = 2$

$\therefore \frac{k}{5} = -7$ $\quad \therefore k = -35$

6 El-Monofia

1

1 d   2 b   3 c   4 d   5 c   6 a

2

[a] $\because 2x - y = 3$ (1)

, $x + 2y = 4$ $\quad \therefore x = 4 - 2y$ (2)

Substituting from (2) in (1) : $\therefore 2(4-2y) - y = 3$

$\therefore 8 - 4y - y = 3$ $\quad \therefore 8 - 5y = 3$

$\therefore -5y = -5$ $\quad \therefore y = 1$

Substituting in (2) : $\therefore x = 2$

$\therefore$ The S.S. $= \{(2, 1)\}$

[b] $\because 3x^2 = 5x - 1$ $\quad \therefore 3x^2 - 5x + 1 = 0$

$\therefore a = 3, b = -5, c = 1$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 3 \times 1}}{2 \times 3} = \frac{5 \pm \sqrt{13}}{6}$

$\therefore x \simeq 1.43$ or $x \simeq 0.23$

$\therefore$ The S.S. $= \{1.43, 0.23\}$

3

[a] $\because z(f) = \{3\}$ $\quad \therefore$ At $x = 3$

$\therefore x^2 - ax + 9 = 0$ $\quad \therefore 3^2 - a \times 3 + 9 = 0$

$\therefore 9 - 3a + 9 = 0$ $\quad \therefore -3a = -18$ $\quad \therefore a = 6$

, $\because$ The domain of $f = \mathbb{R} - \{2\}$

$\therefore$ At $x = 2$ $\quad \therefore bx + 4 = 0$

$\therefore 2b + 4 = 0$ $\quad \therefore 2b = -4$ $\quad \therefore b = -2$

[b] $\because n(x) = \frac{(x-2)(x^2+2x+4)}{(x-2)(x-1)} \div \frac{x(x^2+2x+4)}{(2x+3)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, 1, 0, -\frac{3}{2}\}$

, $n(x) = \frac{x^2+2x+4}{x-1} \times \frac{(2x+3)(x-1)}{x(x^2+2x+4)} = \frac{2x+3}{x}$

4

[a] $\because n(x) = \frac{x-3}{(x-3)(x-4)} - \frac{4}{x(x-4)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, 4, 0\}$

, $n(x) = \frac{1}{x-4} - \frac{4}{x(x-4)} = \frac{x-4}{x(x-4)} = \frac{1}{x}$

, n (4) in undefined because $4 \notin$ the domain of n

[b] $\because x + y = 4 \quad \therefore y = 4 - x \quad (1)$

$, \frac{1}{x} + \frac{1}{y} = 1 \quad \therefore y + x = xy \quad (2)$

Substituting from (1) in (2) :

$\therefore 4 - x + x = x(4 - x) \quad \therefore 4 = 4x - x^2$

$\therefore x^2 - 4x + 4 = 0 \quad \therefore (x-2)(x-2) = 0$

$\therefore x = 2$

Substituting in (1) : $\therefore y = 2$

$\therefore$ The S.S. $= \{(2, 2)\}$

5

[a] $\because n_1 = \frac{(x+3)(x+2)}{(x+2)(x-1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-2, 1\}$

$, n_1(x) = \frac{x+3}{x-1}$ (1)

$, \because n_2(x) = \frac{(x-5)(x+3)}{(x-5)(x-1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{5, 1\}$

$, n_2(x) = \frac{x+3}{x-1}$ (2)

From (1) and (2) : $\therefore n_1 \neq n_2$

because the domain of $n_2 \neq$ the domain of n_2

[b] 1 $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{1}{4} + \frac{1}{2} - \frac{5}{8} = \frac{1}{8}$

2 $P(B - A) = P(B) - P(A \cap B) = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

3 $P(A \cup B)' = 1 - P(A \cup B) = 1 - \frac{5}{8} = \frac{3}{8}$

7 El-Gharbia

1

1 c 2 d 3 b 4 d 5 c 6 d

2

[a] $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore 0.8 = 0.5 + x - 0.1 \quad \therefore x = 0.4$

$, \because P(A - B) = P(A) - P(A \cap B) = 0.5 - 0.1 = 0.4$

[b] $\because n(x) = \frac{x(x-2) + x}{x-2} = \frac{x^2 - 2x + x}{x-2}$

$= \frac{x^2 - x}{x-2} = \frac{x(x-1)}{x-2}$

$\therefore n^{-1}(x) = \frac{x-2}{x(x-1)}$

, the domain of $n^{-1} = \mathbb{R} - \{0, 1, 2\}$

3

[a] $\because n(x) = \frac{x}{x-2} - \frac{x}{x+2}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -2\}$

$, n(x) = \frac{x(x+2) - x(x-2)}{(x-2)(x+2)}$

$= \frac{x^2 + 2x - x^2 + 2x}{(x-2)(x+2)} = \frac{4x}{(x-2)(x+2)}$

[b] $\because x - y = 3 \quad \therefore x = y + 3 \quad (1)$

$, y^2 - xy = 21 \quad (2)$

Substituting from (1) in (2) : $\therefore y^2 - (y+3)y = 21$

$\therefore y^2 - y^2 + 3y = 21$

$\therefore 3y = 21 \quad \therefore y = 7$

Substituting in (1) : $\therefore x = 10$

$\therefore$ The S.S. $= \{(10, 7)\}$

4

[a] $\because x^2 + 2x - 4 = 0$

$\therefore a = 1, b = 2, c = -4$

$\therefore x = \frac{-2 \pm \sqrt{(2)^2 - 4 \times 1 \times (-4)}}{2 \times 1}$

$= \frac{-2 \pm \sqrt{20}}{2} = \frac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}$

$\therefore x = -1 + \sqrt{5}$ or $x = -1 - \sqrt{5}$

The S.S. $= \{-1 + \sqrt{5}, -1 - \sqrt{5}\}$

[b] $\because n_1(x) = \frac{(x-2)(x+2)}{(x+3)(x-2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-3, 2\}$

$, n_1(x) = \frac{x+2}{x+3}$ (1)

$, \because n_2(x) = \frac{(x-3)(x+2)}{(x+3)(x-3)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{-3, 3\}$

$, n_2(x) = \frac{x+2}{x+3}$ (2)

From (1) and (2) : $\therefore n_1 \neq n_2$

because the domain of $n_1 \neq$ the domain of n_2

5

[a] $\because n(x) = \frac{(x-1)(x^2+x+1)}{x(x-1)} \div \frac{x^2+x+1}{x+3}$

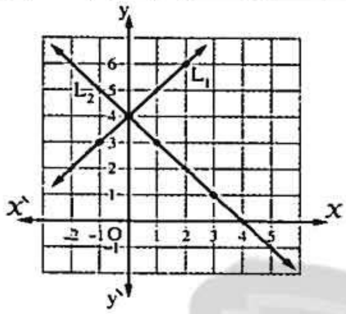
$\therefore$ The domain of $n = \mathbb{R} - \{0, 1, -3\}$

$, n(x) = \frac{(x-1)(x^2+x+1)}{x(x-1)} \times \frac{x+3}{x^2+x+1} = \frac{x+3}{x}$

[b] $y = x + 4$, $x = 4 - y$

x	-1	0	2
y	3	4	6

x	3	1	0
y	1	3	4



From the graph : $\therefore$ The S.S. $= \{(0 , 4)\}$

8 El-Dakahlia

1

[a] 1 b 2 a 3 a

[b] $\because 3x - y = 5$ (1)

$, x + 2y = 4 \quad \therefore x = 4 - 2y$ (2)

Substituting from (2) in (1) : $\therefore 3(4 - 2y) - y = 5$

$\therefore 12 - 6y - y = 5 \qquad \therefore -7y = -7$

$\therefore y = 1$

Substituting in (2) : $\therefore x = 2$

$\therefore$ The S.S. $= \{(2 , 1)\}$

2

[a] 1 a 2 d 3 d

[b] $\because n(x) = \frac{x(x+1)}{(x-1)(x+1)} + \frac{x-5}{(x-1)(x-5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1 , -1 , 5\}$

$, n(x) = \frac{x}{(x-1)} + \frac{1}{(x-1)} = \frac{x+1}{x-1}$

3

[a] $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.6 + 0.5 - 0.3 = 0.8$

$\because P(\acute{B}) = 1 - P(B) \qquad \therefore P(\acute{B}) = 1 - 0.5 = 0.5$

[b] $\because n(x) = \frac{(x-1)(x^2+x+1)}{(x-1)^2} \times \frac{2(x-1)}{x^2+x+1}$

$\therefore$ The domain of $n = \mathbb{R} - \{1\}$, $n(x) = 2$

4

[a] $\because n_1(x) = \frac{x(x-1)}{x^2(x-2)}$

$\left.\begin{array}{l} \therefore \text{The domain of } n_1 = \mathbb{R} - \{0 , 2\} \\ , n_1(x) = \frac{x-1}{x(x-2)} \end{array}\right\}$ (1)

$, \because n_2(x) = \frac{(x-2)(x-1)}{x(x-2)(x-2)}$

$\left.\begin{array}{l} \therefore \text{The domain of } n_2 = \mathbb{R} - \{0 , 2\} \\ , n_2(x) = \frac{x-1}{x(x-2)} \end{array}\right\}$ (2)

From (1) and (2) : $\therefore n_1 = n_2$

[b] $\because 2x^2 - 4x + 1 = 0$

$\therefore a = 2 , b = -4 , c = 1$

$\therefore x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 2 \times 1}}{2 \times 2}$

$= \frac{4 \pm \sqrt{8}}{4} = \frac{4 \pm 2\sqrt{2}}{4} = \frac{2 \pm \sqrt{2}}{2}$

$\therefore x \approx 1.71$ or $x \approx 0.29$

The S.S. $= \{1.71 , 0.29\}$

5

[a] $\because x - y = 0 \qquad \therefore x = y$ (1)

$, x = \frac{4}{y}$ (2)

Substituting from (1) in (2) : $\therefore x = \frac{4}{x}$

$\therefore x^2 = 4 \qquad \therefore x = \pm\sqrt{4}$

$\therefore x = 2$ or $x = -2$

Substituting in (1) : $\therefore y = 2$ or $y = -2$

$\therefore$ The S.S. $= \{(2 , 2) , (-2 , -2)\}$

[b] 1 $\because n(x) = \frac{x(x-2)}{(x-2)(x^2+2)}$

$\therefore n^{-1}(x) = \frac{(x-2)(x^2+2)}{x(x-2)}$

, the domain of $n^{-1} = \mathbb{R} - \{0 , 2\}$

$, n^{-1}(x) = \frac{x^2+2}{x}$

2 $\because n^{-1}(x) = 3 \qquad \therefore \frac{x^2+2}{x} = 3$

$\therefore x^2 + 2 = 3x \quad \therefore x^2 - 3x + 2 = 0$

$\therefore (x-2)(x-1) = 0$

$\therefore x = 2$ (refused) or $x = 1$

9 Ismailia

1

1 c 2 b 3 d 4 a 5 c 6 c

2

[a] $\because 2x + y = 1 \quad \therefore y = 1 - 2x \qquad (1)$

$, x + 2y = 5 \qquad (2)$

Substituting from (1) in (2) :

$\therefore x + 2(1 - 2x) = 5$

$\therefore x + 2 - 4x = 5 \qquad \therefore -3x = 3$

$\therefore x = -1$

Substituting in (1) : $y = 3$

$\therefore$ The S.S. $= \{(-1 , 3)\}$

[b] $\because n_1(x) = \dfrac{x^2 - 3x + 9}{(x+3)(x^2 - 3x + 9)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{-3\} \\ , n_1(x) = \dfrac{1}{x+3}\end{array}\right\} (1)$

$, \because n_2(x) = \dfrac{2}{2(x+3)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{-3\} \\ , n_2(x) = \dfrac{1}{x+3}\end{array}\right\} (2)$

From (1) and (2) : $\therefore n_1 = n_2$

3

[a] $\because 3x^2 - 6x + 1 = 0$

$\therefore a = 3 , b = -6 , c = 1$

$\therefore x = \dfrac{6 \pm \sqrt{(-6)^2 - 4 \times 3 \times 1}}{2 \times 3} = \dfrac{6 \pm \sqrt{24}}{6}$

$= \dfrac{6 \pm 2\sqrt{6}}{6} = \dfrac{3 \pm \sqrt{6}}{3}$

$\therefore x \approx 1.82$ or $x \approx 0.18$

The S.S. $= \{1.82 , 0.18\}$

[b] $\because$ The domain of $n = \mathbb{R} - \{3\}$

$\therefore$ At $x = 3 \qquad \therefore x^2 - ax + 9 = 0$

$\therefore 9 - 3a + 9 = 0 \qquad \therefore -3a = -18 \qquad \therefore a = 6$

4

[a] Let the two numbers be x and y

$\therefore xy = 10 \qquad (1)$

$, x - y = 3 \qquad \therefore x = y + 3 \qquad (2)$

Substituting from (2) in (1) : $\therefore (y + 3)y = 10$

$\therefore y^2 + 3y - 10 = 0 \qquad \therefore (y - 2)(y + 5) = 0$

$\therefore y = 2$ or $y = -5$

Substituting in (2) : $x = 5$ or $x = -2$

$\therefore$ The two numbers are : 5 , 2 or -2 , -5

[b] $\because n(x) = \dfrac{(x+5)(x-1)}{(x-2)(x^2+2x+4)} \div \dfrac{x+5}{x^2+2x+4}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -5\}$

$, n(x) = \dfrac{(x+5)(x-1)}{(x-2)(x^2+2x+4)} \times \dfrac{x^2+2x+4}{x+5}$

$= \dfrac{x-1}{x-2}$

$, \because n(3) = \dfrac{3-1}{3-2} = 2$

, n (2) is undefined because $2 \notin$ the domain of n

5

[a] $\because n(x) = \dfrac{x(x-3)}{(x-3)(x+3)} + \dfrac{x-1}{(x+3)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3 , -3 , 1\}$

$, n(x) = \dfrac{x}{x+3} + \dfrac{1}{x+3} = \dfrac{x+1}{x+3}$

[b] 1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.5 - 0.2 = 0.7$

2 $P(A - B) = P(A) - P(A \cap B)$

$= 0.4 - 0.2 = 0.2$

10 Suez

1

1 c 2 b 3 a 4 c 5 b 6 c

2

[a] $\because x - y = 3 \qquad \therefore x = y + 3 \qquad (1)$

$, 2x + y = 9 \qquad (2)$

Substituting from (1) in (2) : $\therefore 2(y + 3) + y = 9$

$\therefore 2y + 6 + y = 9 \qquad \therefore 3y = 3 \qquad \therefore y = 1$

Substituting in (1) : $\therefore x = 4$

$\therefore$ The S.S. $= \{(4 , 1)\}$

[b] $\because n(x) = \dfrac{x(x-2)}{(x-2)(x+2)} + \dfrac{2(x+3)}{(x+3)(x+2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -2 , -3\}$

$, n(x) = \dfrac{x}{x+2} + \dfrac{2}{x+2} = \dfrac{x+2}{x+2} = 1$

3

[a] $x - y = 0$ $\therefore x = y$ (1)

, $xy = 9$ (2)

Substituting from (1) in (2) : $\therefore x^2 = 9$

$\therefore x = \pm\sqrt{9}$

$\therefore x = 3$ or $x = -3$

Substituting in (1) : $\therefore y = 3$ or $y = -3$

$\therefore$ The S.S. $= \{(3, 3), (-3, -3)\}$

[b] $\because n(x) = \frac{(x+3)(x-1)}{x+3} \times \frac{x+1}{(x-1)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3, 1, -1\}$

, $n(x) = 1$

4

[a] 1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.6 - 0.2 = 0.7$

2 $P(\acute{A}) = 1 - P(A) = 1 - 0.3 = 0.7$

[b] $\because n(x) = \frac{(x-1)(x-1)}{(x-1)(x^2+x+1)} \div \frac{x-1}{x^2+x+1}$

$\therefore$ The domain of $n = \mathbb{R} - \{1\}$

, $n(x) = \frac{x-1}{x^2+x+1} \times \frac{x^2+x+1}{x-1} = 1$

5

[a] $\because x^2 - 2x - 6 = 0$

$\therefore a = 1, b = -2, c = -6$

$\therefore x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-6)}}{2 \times 1}$

$= \frac{2 \pm \sqrt{28}}{2} = \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$

$\therefore x \simeq 3.65$ or $x \simeq -1.65$

$\therefore$ The S.S. $= \{3.65, -1.65\}$

[b] $\because n_1(x) = \frac{2x}{2(x+2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-2\}$ (1)

, $n_1(x) = \frac{x}{x+2}$

, $\because n_2(x) = \frac{x(x+2)}{(x+2)(x+2)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{-2\}$ (2)

, $n_2(x) = \frac{x}{x+2}$

From (1) and (2) : $\therefore n_1 = n_2$

11 Port Said

1

1 b 2 c 3 b 4 d 5 d 6 a

2

[a] $\because$ The domain of $n = \mathbb{R} - \{3\}$

$\therefore (3)^2 - 3a + 9 = 0$ $\therefore 18 - 3a = 0$

$\therefore -3a = -18$ $\therefore a = 6$

[b] Let the length be x cm. and the width be y cm.

$\therefore 2(x + y) = 22$ $\therefore y = 11 - x$ (1)

, $xy = 24$ (2)

Substituting from (1) in (2) : $\therefore x(11 - x) = 24$

$\therefore 11x - x^2 - 24 = 0$ (Multiplying by -1)

$\therefore x^2 - 11x + 24 = 0$

$(x-3)(x-8) = 0$ $\therefore x = 3$ or $x = 8$

Substituting in (1) : $\therefore y = 8$ or $y = 3$

$\therefore$ The length = 8 cm. , the width = 3 cm.

3

[a] $\because x^2 - 2x - 1 = 0$

$\therefore a = 1, b = -2, c = -1$

$\therefore x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-1)}}{2 \times 1} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$

$= 1 \pm \sqrt{2}$

$\therefore x \simeq 2.4$ or $x \simeq -0.4$

$\therefore$ The S.S. $= \{2.4, -0.4\}$

[b] $\because n(x) = \frac{x^2+x+1}{x} \div \frac{(x-1)(x^2+x+1)}{x(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, 1\}$

, $n(x) = \frac{x^2+x+1}{x} \times \frac{x}{x^2+x+1} = 1$

4

[a] $\because x + 3y = 7$ $\therefore x = 7 - 3y$ (1)

, $5x - y = 3$ (2)

Substituting from (1) in (2) : $\therefore 5(7 - 3y) - y = 3$

$\therefore 35 - 15y - y = 3$ $\therefore -16y = -32$ $\therefore y = 2$

Substituting in (1) : $\therefore x = 1$

$\therefore$ The S.S. $= \{(1, 2)\}$

[b] $\because n(x) = \frac{x(x+2)}{(x-2)(x+2)} + \frac{x-3}{(x-3)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -2, 3\}$

, $n(x) = \frac{x}{x-2} + \frac{1}{x-2} = \frac{x+1}{x-2}$

5

[a] 1 The probability that the number on the card is a multiple of 5 = $\frac{5}{20} = \frac{1}{4}$

2 The probability that the number on the card is a multiple of 5 = $\frac{4}{20} = \frac{1}{5}$

3 The probability that the number on the card is a multiple of 4 or 5 = $\frac{8}{20} = \frac{2}{5}$

[b] $\because n_1(x) = \frac{x+3}{(x-3)(x+3)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{3, -3\}$

, $n_1(x) = \frac{1}{x-3}$

, $\because n_2(x) = \frac{2}{2(x-3)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{3\}$

, $n_2(x) = \frac{1}{x-3}$

$\therefore n_1(x) = n_2(x)$

for all the values of $x \in \mathbb{R} - \{3, -3\}$

12 Damietta

1

1 a 2 b 3 d 4 a 5 b 6 a

2

[a] $\because x + \frac{4}{x} = 6$

$\therefore x^2 + 4 = 6x \quad \therefore x^2 - 6x + 4 = 0$

$\therefore a = 1, b = -6, c = 4$

$\therefore x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 4}}{2 \times 1} = \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$

$\therefore x \approx 5.2$ or $x \approx 0.8$

$\therefore$ The S.S. = $\{5.2, 0.8\}$

[b] $\because n(x) = \frac{2x}{x-3} \div \frac{x(x+2)}{(x+3)(x-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, -3, 0, -2\}$

, $n(x) = \frac{2x}{x-3} \times \frac{(x+3)(x-3)}{x(x-2)} = \frac{2(x+3)}{x+2}$

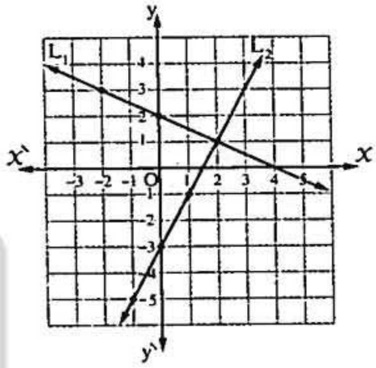
3

[a] $x = 4 - 2y$

x	−2	0	2
y	3	2	1

$y = 2x - 3$

x	1	0	−1
y	−1	−3	−5



From the graph : $\therefore$ The S.S. = $\{(2, 1)\}$

[b] $\because n(x) = \frac{x^2 - 2x + 4}{(x+2)(x^2 - 2x + 4)} + \frac{(x-1)(x+1)}{(x+2)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2, 1\}$

, $n(x) = \frac{1}{x+2} + \frac{x+1}{x+2} = \frac{x+2}{x+2} = 1$

4

[a] $\because n_1(x) = \frac{x(x+2)}{(x+2)(x+2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-2\}$
, $n_1(x) = \frac{x}{x+2}$ (1)

, $\because n_2(x) = \frac{2x}{2(x+2)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{-2\}$
, $n_2(x) = \frac{x}{x+2}$ (2)

From (1) and (2) : $\therefore n_1 = n_2$

[b] $\because x - y = 2 \qquad \therefore x = y + 2$ (1)

, $x^2 + y^2 = 20$ (2)

Substituting from (1) in (2) : $\therefore (y+2)^2 + y^2 = 20$

$\therefore y^2 + 4y + 4 + y^2 = 20$

$\therefore 2y^2 + 4y - 16 = 0$ (Dividing by 2)

$\therefore y^2 + 2y - 8 = 0 \qquad \therefore (y+4)(y-2) = 0$

$\therefore y = -4$ or $y = 2$

Substituting in (1) : $\therefore x = -2$ or $x = 4$

$\therefore$ The S.S. = $\{(-2, -4), (4, 2)\}$

5

[a] $\because$ The domain of $n = \mathbb{R} - \{5\}$

$\therefore (5)^2 - 5a + 25 = 0$

$\therefore -5a = -50 \qquad \therefore a = 10$

[b] 1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.8 + 0.7 - 0.6 = 0.9$

2 $P(\acute{A}) = 1 - P(A) = 1 - 0.8 = 0.2$

13 Kafr El-Sheikh

1

[a] 1 c 2 a 3 d

[b] $\because n(X) = \frac{(2X+3)(X-2)}{X(X-3)} \div \frac{(2X-3)(2X+3)}{X(2X-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, 3, \frac{3}{2}, \frac{-3}{2}\}$

$, n(X) = \frac{(2X+3)(X-2)}{X(X-3)} \times \frac{X}{(2X+3)} = \frac{X-2}{X-3}$

2

[a] 1 c 2 d 3 c

[b] $\because n_1(X) = \frac{X}{X(X-1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{0, 1\}$, $n_1(X) = \frac{1}{X-1}$ (1)

$, n_2(X) = \frac{X(X^2+X+1)}{X(X-1)(X^2+X+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0, -1\}$, $n_2(X) = \frac{1}{X-1}$ (2)

From (1) and (2) : $\therefore n_1 = n_2$

3

[a] $\because 3X^2 + 1 = 5X$

$\therefore 3X^2 - 5X + 1 = 0$

$\therefore a = 3, b = -5, c = 1$

$\therefore X = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 3 \times 1}}{2 \times 3} = \frac{5 \pm \sqrt{13}}{6}$

$\therefore X \approx 1.43$ or $X \approx 0.23$

$\therefore$ The S.S. $= \{1.43, 0.23\}$

[b] 1 $\because z(n_2) = \{-3\} \quad \therefore 6 - a(-3) = 0$

$\therefore 6 + 3a = 0 \qquad \therefore 3a = -6 \qquad \therefore a = -2$

2 $\because n(X) = n_1(X) - n_2(X)$

$\therefore n(X) = \frac{X^2 - 2X - 15}{X^2 - 9} - \frac{2X + 6}{X^2 - 6X + 9}$

$= \frac{(X-5)(X+3)}{(X-3)(X+3)} - \frac{2(X+3)}{(X-3)(X-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, -3\}$

$, n(X) = \frac{X-5}{X-3} - \frac{2(X+3)}{(X-3)(X-3)}$

$= \frac{(X-5)(X-3) - 2(X+3)}{(X-3)(X-3)}$

$= \frac{X^2 - 8X + 15 - 2X - 6}{(X-3)(X-3)}$

$= \frac{X^2 - 10X + 9}{(X-3)(X-3)} = \frac{(X-1)(X-9)}{(X-3)(X-3)}$

4

[a] $\because 3X + 2y = 4$ (1)

$, X - 3y = 5 \qquad \therefore X = 3y + 5$ (2)

Substituting from (2) in (1) :

$\therefore 3(3y + 5) + 2y = 4$

$\therefore 9y + 15 + 2y = 4 \quad \therefore 11y = -11 \quad \therefore y = -1$

Substituting in (2) : $\therefore X = 2$

$\therefore$ The S.S. $= \{(2, -1)\}$

[b] $\because 2P(B) = P(\acute{B}) \qquad \therefore 2P(B) = 1 - P(B)$

$\therefore 3P(B) = 1 \qquad \therefore P(B) = \frac{1}{3}$

1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$

2 $\because$ A , B are mutually exclusive events

$\therefore P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

5

[a] $\because X - 2y - 1 = 0 \qquad \therefore X = 2y + 1$ (1)

$, X^2 - Xy = 0$ (2)

Substituting from (1) in (2) :

$\therefore (2y+1)^2 - (2y+1)y = 0$

$\therefore 4y^2 + 4y + 1 - 2y^2 - y = 0$

$\therefore 2y^2 + 3y + 1 = 0$

$\therefore (2y+1)(y+1) = 0 \quad \therefore y = -\frac{1}{2}$ or $y = -1$

Substituting in (1) : $\therefore X = 0$ or $X = -1$

$\therefore$ The S.S. $= \{(0, -\frac{1}{2}), (-1, -1)\}$

[b] $\because n(x) = \frac{x(x-3)}{(x-3)(x^2+2)}$

$\therefore n^{-1}(x) = \frac{(x-3)(x^2+2)}{x(x-3)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, 3\}$

, $n^{-1}(x) = \frac{x^2+2}{x}$

14 El-Beheira

1

1 b 2 a 3 c 4 a 5 c 6 c

2

[a] $\because y - x = 2$ $\therefore y = x + 2$ (1)

, $x^2 + xy - 4 = 0$ (2)

Substituting from (1) in (2) :

$\therefore x^2 + x(x+2) - 4 = 0$

$\therefore x^2 + x^2 + 2x - 4 = 0$

$\therefore 2x^2 + 2x - 4 = 0$ (Dividing by 2)

$\therefore x^2 + x - 2 = 0$

$(x-1)(x+2) = 0$

$\therefore x = 1$ or $x = -2$

Substituting in (1) : $\therefore y = 3$ or $y = 0$

$\therefore$ The S.S $= \{(1, 3), (-2, 0)\}$

[b] $\because n(x) = \frac{(x-1)(x^2+x+1)}{(x-1)(x-1)} \times \frac{2(x-1)}{x^2+x+1}$

$\therefore$ The domain of $n = \mathbb{R} - \{1\}$, $n(x) = 2$

3

[a] Let the measure of the first angle be $x°$

, the measure of the second angle be y°

$\therefore x + y = 90°$ (1)

, $x - y = 50°$ (2)

Adding (1) and (2) : $\therefore 2x = 140°$ $\therefore x = 70°$

Substituting in (1) : $\therefore y = 20°$

$\therefore$ The measures of the two angles are 70° , 20°

[b] 1 $\because n(x) = \frac{x(x-2)}{(x-2)(x^2+2)}$

$\therefore n^{-1}(x) = \frac{(x-2)(x^2+2)}{x(x-2)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, 2\}$

, $n^{-1}(x) = \frac{x^2+2}{x}$

2 $\because n^{-1}(x) = 3$ $\therefore \frac{x^2+2}{x} = 3$

$\therefore x^2 - 3x + 2 = 0$ $\therefore (x-2)(x-1) = 0$

$\therefore x = 2$ (refused) or $x = 1$

4

[a] $\because 3x^2 = 5x - 1$ $\therefore 3x^2 - 5x + 1 = 0$

$\therefore a = 3, b = -5, c = 1$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 3 \times 1}}{2 \times 3} = \frac{5 \pm \sqrt{13}}{6}$

$\therefore x \approx 1.43$ or $x \approx 0.23$

$\therefore$ The S.S. $= \{1.43, 0.23\}$

[b] $\because n_1(x) = \frac{2x}{2(x+2)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{-2\} \\ , n_1(x) = \frac{x}{x+2}\end{array}\right\}$ (1)

, $\because n_2(x) = \frac{x(x+2)}{(x+2)(x+2)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{-2\} \\ , \because n_2(x) = \frac{x}{x+2}\end{array}\right\}$ (2)

From (1) and (2) : $\therefore n_1 = n_2$

5

[a] $\because n(x) = \frac{x-3}{(x-3)(x-4)} - \frac{4}{x(x-4)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, 4, 0\}$

$\therefore n(x) = \frac{1}{x-4} - \frac{4}{x(x-4)} = \frac{x-4}{x(x-4)} = \frac{1}{x}$

[b] 1 $P(\acute{A}) = 1 - P(A) = 1 - 0.8 = 0.2$

2 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.8 + 0.7 - 0.6 = 0.9$

15 El-Fayoum

1

1 b 2 b 3 d 4 b 5 a 6 c

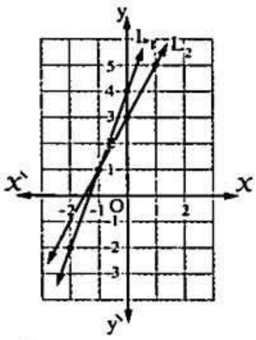
2

[a] $y = 3x + 4$

x	-2	-1	0
y	-2	1	4

$y = 2x + 3$

x	-1	0	1
y	1	3	5



From the graph :

$\therefore$ The S.S. $= \{(-1 , 1)\}$

[b] $\because n(x) = \frac{x(x-1)}{(x+1)(x-1)} + \frac{x-5}{(x-5)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-1 , 1 , 5\}$

$, n(x) = \frac{x}{x+1} + \frac{1}{x-1} = \frac{x(x-1)+x+1}{(x+1)(x-1)}$

$= \frac{x^2-x+x+1}{(x+1)(x-1)}$

$= \frac{x^2+1}{(x+1)(x-1)}$

3

[a] $\because x^2 + 3x + 5 = 0$

$\therefore a = 1 , b = 3 , c = 5$

$\therefore x = \frac{-3 \pm \sqrt{(3)^2 - 4 \times 1 \times 5}}{2 \times 1} = \frac{-3 \pm \sqrt{-11}}{2}$

The S.S. $= \varnothing$

[b] $\because n(x) = \frac{(x-7)(x+7)}{(x-2)(x^2+2x+4)} \div \frac{x+7}{x-2}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -7\}$

$, n(x) = \frac{(x-7)(x+7)}{(x-2)(x^2+2x+4)} \times \frac{x-2}{x+7}$

$= \frac{x-7}{x^2+2x+4}$

$\therefore n(1) = \frac{1-7}{1+2+4} = \frac{-6}{7}$

4

[a] $\because n_1(x) = \frac{(x-2)(x+2)}{(x+3)(x-2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-3 , 2\}$

$, n_1(x) = \frac{x+2}{x+3}$

$, \because n_2(x) = \frac{x(x^2-x-6)}{x(x^2-9)} = \frac{x(x-3)(x+2)}{x(x-3)(x+3)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0 , 3 , -3\}$

$, n_2(x) = \frac{x+2}{x+3}$

$\therefore n_1 \neq n_2$

Because the domain of $n_1 \neq$ the domain of n_2

[b] Let x and y be two real numbers

$\because x + y = 9$ $\qquad \therefore y = 9 - x$ (1)

$, x^2 - y^2 = 45$ (2)

Substituting from (1) in (2) : $\therefore x^2 - (9-x)^2 = 45$

$\therefore x^2 - (81 - 18x + x^2) = 45$

$\therefore x^2 - 81 + 18x - x^2 = 45$

$\therefore 18x = 126$ $\qquad \therefore x = 7$

Substituting in (1) : $\therefore y = 2$

$\therefore$ The two real numbers are : 7 , 2

5

[a] $\because Z(f) = \{3 , 5\}$

$\therefore$ At $x = 3$ $\qquad \therefore a \times 3^2 + 3 \times b + 15 = 0$

$\therefore 9a + 3b + 15 = 0$ $\qquad \therefore 3a + b + 5 = 0$ (1)

At $x = 5$

$\therefore a \times 5^2 + b \times 5 + 15 = 0$

$\therefore 25a + 5b + 15 = 0$

$\therefore 5a + b + 3 = 0$ (2)

Subtracting (1) from (2) :

$\therefore 2a - 2 = 0$ $\qquad \therefore a = 1$

Substituting in (1) : $\therefore 3 \times 1 + b + 5 = 0$

$\therefore 3 + b = -5$ $\qquad \therefore b = -8$

[b] $\because P(A) = P(\grave{A})$ $\qquad \therefore P(A) = 1 - P(A)$

$\therefore 2P(A) = 1$ $\qquad \therefore P(A) = \frac{1}{2}$

1 $\because P(B) = \frac{5}{8} P(A)$

$\therefore P(B) = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$

2 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{5}{16} - \frac{1}{16} = \frac{3}{4}$

16 Beni Suef

1

1 b　2 c　3 d　4 a　5 d　6 c

2

[a] $\because x^2 - 2x - 2 = 0$

$\therefore a = 1 , b = -2 , c = -2$

$\therefore x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-2)}}{2 \times 1} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$

$= 1 \pm \sqrt{3}$

$\therefore$ The S.S. $= \{1 + \sqrt{3} , 1 - \sqrt{3}\}$

[b] $\because n_1(x) = \frac{5x}{5(x+5)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-5\}$, $n_1(x) = \frac{x}{x+5}$ (1)

, $\because n_2(x) = \frac{x(x+5)}{(x+5)^2}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{-5\}$, $n_2(x) = \frac{x}{x+5}$ (2)

From (1) , (2) : $\therefore n_1 = n_2$

3

[a] $\because x + y = 7 \quad \therefore y = 7 - x$ (1)

, $x^2 + y^2 = 25$ (2)

Substituting from (1) in (2) :

$\therefore x^2 + (7 - x)^2 = 25$

$\therefore x^2 + 49 - 14x + x^2 - 25 = 0$

$\therefore 2x^2 - 14x + 24 = 0$ (Dividing by 2)

$\therefore x^2 - 7x + 12 = 0 \qquad \therefore (x-3)(x-4) = 0$

$\therefore x = 3$ or $x = 4$

Substituting in (1) : $\therefore y = 4$ or $y = 3$

$\therefore$ The S.S. $= \{(3 , 4) , (4 , 3)\}$

[b] $\because n(x) = \frac{x^2}{x(x-3)} \div \frac{3x}{(x+3)(x-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0 , 3 , -3\}$

, $n(x) = \frac{x}{x-3} \times \frac{(x+3)(x-3)}{3x} = \frac{x+3}{3}$

4

[a] $P(\acute{A}) = 1 - P(A) = 1 - 0.7 = 0.3$

$P(A - B) = P(A) - P(A \cap B)$

$= 0.7 - 0.3 = 0.4$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 + 0.5 - 0.3 = 0.9$

[b] $\because Z(f) = \{5\} \qquad \therefore$ At $x = 5$

$\therefore x^2 - 10x + a = 0 \qquad \therefore (5)^2 - 10 \times 5 + a = 0$

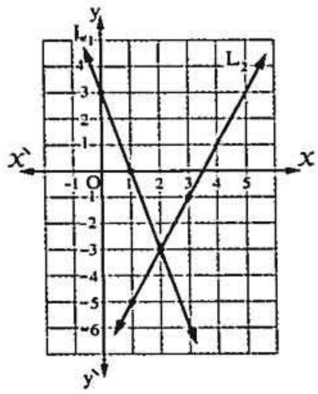
$\therefore 25 - 50 + a = 0 \qquad \therefore a = 25$

5

[a] $y = 3 - 3x$, $y = 2x - 7$

x	0	1	2
y	3	0	−3

x	1	2	3
y	−5	−3	−1



From the graph :

$\therefore$ The S.S. $= \{(2 , -3)\}$

[b] $\because n(x) = \frac{x^2 + x + 1}{(x-1)(x^2 + x + 1)} + \frac{(x-2)(x+1)}{(x-1)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1 , -1\}$

, $n(x) = \frac{1}{x-1} + \frac{x-2}{x-1} = \frac{x-1}{x-1} = 1$

17 El-Menia

1

1 a   2 c   3 d   4 a   5 b   6 a

2

[a] $\because 3x^2 - 5x + 1 = 0$

$\therefore a = 3 , b = -5 , c = 1$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 3 \times 1}}{2 \times 3} = \frac{5 \pm \sqrt{13}}{6}$

$\therefore x \approx 1.4$ or $x \approx 0.2$

$\therefore$ The S.S. $= \{1.4 , 0.2\}$

[b] $\because n(x) = \frac{(x-2)(x^2 + 2x + 4)}{(x-3)(x-2)} \div \frac{x^2 + 2x + 4}{x-3}$

$\therefore$ The domain of $n = \mathbb{R} - \{3 , 2\}$

, $n(x) = \frac{(x-2)(x^2 + 2x + 4)}{(x-3)(x-2)} \times \frac{x-3}{x^2 + 2x + 4} = 1$

3

[a] $\because 2x + y = 1$ (1)

, $x + 2y = 5 \qquad \therefore x = 5 - 2y$ (2)

Substituting from (2) in (1) : $\therefore 2(5 - 2y) + y = 1$

$\therefore 10 - 4y + y = 1 \qquad \therefore -3y = -9$

$\therefore y = 3$

Substituting in (2) : $\therefore x = -1$

$\therefore$ The S.S. $= \{(-1, 3)\}$

[b] $\because n(x) = \frac{(x-5)(x+3)}{(x-3)(x+3)} - \frac{2(5-x)}{(x-5)(x-3)}$

$\therefore$ The domain of n $= \mathbb{R} - \{3, -3, 5\}$

$, n(x) = \frac{x-5}{x-3} + \frac{2(x-5)}{(x-5)(x-3)} = \frac{x-5}{x-3} + \frac{2}{x-3}$

$= \frac{x-3}{x-3} = 1$

4

[a] $\because x + y = 2$ (1)

$, \frac{1}{x} + \frac{1}{y} = 2 \qquad \therefore x + y = 2xy$ (2)

Substituting in (1) from (2) : $\therefore 2 = 2xy$

$\therefore xy = 1 \qquad \therefore x = \frac{1}{y}$

Substituting in (1) : $\frac{1}{y} + y = 2$

Multiplying by y : $\therefore 1 + y^2 = 2y$

$\therefore y^2 - 2y + 1 = 0 \qquad \therefore (y-1)^2 = 0$

$\therefore y = 1$

Substituting in (1) : $\therefore x = 1$

$\therefore$ The S.S. $= \{(1, 1)\}$

[b] $\because n_1(x) = \frac{x^2}{x^2(x-1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{0, 1\} \\ \therefore n_1(x) = \frac{1}{x-1}\end{array}\right\}$ (1)

$, \because n_2(x) = \frac{x(x^2+x+1)}{x(x^3-1)}$

$= \frac{x(x^2+x+1)}{x(x-1)(x^2+x+1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{0, 1\} \\ , n_2(x) = \frac{1}{x-1}\end{array}\right\}$ (2)

From (1) and (2) : $\therefore n_1 = n_2$

5

[a] $\because n(x) = \frac{x(x-2)}{(x-2)(x^2+2)}$

$\therefore n^{-1}(x) = \frac{(x-2)(x^2+2)}{x(x-2)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, 2\}$

$, n^{-1}(x) = \frac{x^2+2}{x}$

[b] **1** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.6 - 0.2 = 0.7$

2 $P(A - B) = P(A) - P(A \cap B)$

$= 0.3 - 0.2 = 0.1$

18 Assiut

1

1 b	2 c	3 d	4 c	5 d	6 a

2

[a] $\because 3x - y + 4 = 0$ (1)

$, y = 2x + 3$ (2)

Substituting from (2) in (1) :

$\therefore 3x - (2x + 3) + 4 = 0$

$\therefore 3x - 2x - 3 + 4 = 0 \qquad \therefore x = -1$

Substituting in (2) : $\therefore y = 1$

$\therefore$ The S.S. $= \{(-1, 1)\}$

[b] $\because n(x) = \frac{(x-7)(x+7)}{(x-2)(x^2+2x+4)} \div \frac{x+7}{x-2}$

$\therefore$ The domain of n $= \mathbb{R} - \{2, -7\}$

$, n(x) = \frac{(x-7)(x+7)}{(x-2)(x^2+2x+4)} \times \frac{x-2}{x+7}$

$= \frac{x-7}{x^2+2x+4}$

$\therefore n(1) = \frac{1-7}{1+2+4} = -\frac{6}{7}$

3

[a] $\because x(x-1) = 5 \qquad \therefore x^2 - x - 5 = 0$

$\therefore a = 1, b = -1, c = -5$

$\therefore x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 1 \times (-5)}}{2 \times 1} = \frac{1 \pm \sqrt{21}}{2}$

$\therefore x \approx 2.8$ or $x \approx -1.8$

$\therefore$ The S.S. $= \{2.8, -1.8\}$

[b] $\because n_1(x) = \frac{(x-2)(x+2)}{(x+3)(x-2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-3, 2\}$ $, n_1(x) = \frac{x+2}{x+3}$

$, \because n_2(x) = \frac{x(x^2-x-6)}{x(x^2-9)} = \frac{x(x-3)(x+2)}{x(x-3)(x+3)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0, 3, -3\}$

$, n_2(x) = \frac{x+2}{x+3}$

$\therefore n_1(x) = n_2(x)$ for all values of $x \in \mathbb{R} - \{0, 3, -3, 2\}$

4

[a] $\because X - y = 2 \qquad \therefore X = y + 2 \qquad (1)$

$, X^2 + y^2 = 20 \qquad (2)$

Substituting from (1) in (2) :

$\therefore (y+2)^2 + y^2 = 20$

$\therefore y^2 + 4y + 4 + y^2 - 20 = 0$

$\therefore 2y^2 + 4y - 16 = 0$ (Dividing by 2)

$\therefore y^2 + 2y - 8 = 0$

$\therefore (y+4)(y-2) = 0$

$\therefore y = -4$ or $y = 2$

Substituting in (1) :

$\therefore X = -2$ or $X = 4$

$\therefore$ The S.S. $= \{(-2, -4), (4, 2)\}$

[b] $\because Z(f) = \{5\}$

$\therefore (5)^3 - 3(5)^2 + a = 0 \qquad \therefore 125 - 75 + a = 0$

$50 + a = 0 \qquad \therefore a = -50$

5

[a] $\because n(X) = \frac{X-3}{(X-4)(X-3)} + \frac{X-3}{X-3}$

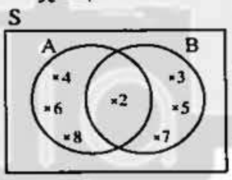
$\therefore$ The domain of $n = \mathbb{R} - \{4, 3\}$

$, n(X) = \frac{1}{X-4} + 1 = \frac{1+X-4}{X-4} = \frac{X-3}{X-4}$

[b] [1] $P(A) = \frac{4}{7}$

$, P(\acute{B}) = 1 - P(B)$

$= 1 - \frac{4}{7} = \frac{3}{7}$



[2] $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{7} + \frac{4}{7} - \frac{1}{7} = 1$

19 Souhag

1

[1] d [2] c [3] b [4] a [5] d [6] c

2

[a] $\because X(X-1) = 4 \qquad \therefore X^2 - X - 4 = 0$

$\therefore a = 1, b = -1, c = -4$

$\therefore X = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 1 \times (-4)}}{2 \times 1} = \frac{1 \pm \sqrt{17}}{2}$

$X \approx 2.6$ or $X \approx -1.6$

$\therefore$ The S.S. $= \{2.6, -1.6\}$

[b] $\because n_1(X) = \frac{X^2}{X^2(X-1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{0, 1\}$

$, n_1(X) = \frac{1}{X-1}$ $\Big\}$ (1)

$, \because n_2(X) = \frac{X(X^2+X+1)}{X(X^3-1)}$

$= \frac{X(X^2+X+1)}{X(X-1)(X^2+X+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0, 1\}$

$, n_2(X) = \frac{1}{X-1}$ $\Big\}$ (2)

from (1) and (2) : $\therefore n_1 = n_2$

3

[a] $\because X - y = 0 \qquad \therefore X = y \qquad (1)$

$, X^2 + Xy + y^2 = 27 \qquad (2)$

Substituting from (1) in (2) :

$\therefore y^2 + y^2 + y^2 = 27 \qquad \therefore 3y^2 = 27$

$\therefore y^2 = 9 \qquad \therefore y = 3$ or $y = -3$

Substituting in (1) : $\therefore X = 3$ or $X = -3$

$\therefore$ The S.S. $= \{(3, 3), (-3, -3)\}$

[b] $\because n(X) = \frac{X(X-2)}{(X-2)(X-1)}$

$\therefore n^{-1}(X) = \frac{(X-2)(X-1)}{X(X-2)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, 2, 1\}$

$, n^{-1}(X) = \frac{X-1}{X}$

4

[a] $\because 2X - y = 5 \qquad (1)$

$, X + y = 4 \qquad (2)$

Adding (1) and (2) : $\therefore 3X = 9 \quad \therefore X = 3$

Substituting in (2) : $\therefore y = 1$

[b] $\because n(X) = \frac{X(X+2)}{(X+2)(X-2)} - \frac{2(X-3)}{(X-3)(X-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2, 2, 3\}$

$\therefore n(X) = \frac{X}{X-2} - \frac{2}{X-2} = \frac{X-2}{X-2} = 1$

5

[a] $\because n(X) = \frac{(X-2)(X^2+2X+4)}{(X-2)(X+3)} \times \frac{X+3}{X^2+2X+4}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -3\}$, $n(X) = 1$

[b] $P(\acute{A}) = 1 - P(A) = 1 - 0.3 = 0.7$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.6 - 0.2 = 0.7$

20 Qena

1

1 d 2 c 3 a 4 b 5 c 6 a

2

[a] $\because x - 2 = 0 \quad \therefore x = 2 \quad (1)$

$, y^2 - 3xy + 5 = 0 \quad (2)$

Substituting from (1) in (2) : $\therefore y^2 - 6y - 5 = 0$

$\therefore (y-5)(y-1) = 0$

$\therefore y = 5$ or $y = 1$

$\therefore$ The S.S. $= \{(2, 5), (2, 1)\}$

[b] $\because n(x) = \frac{5}{x-3} - \frac{4}{x-3}$

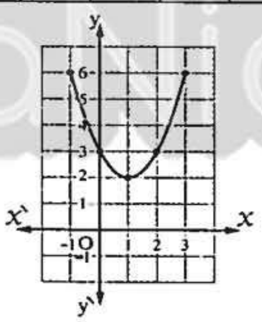
$\therefore$ The domain of $n = \mathbb{R} - \{3\}$

$, n(x) = \frac{5-4}{x-3} = \frac{1}{x-3}$

3

[a] $f(x) = x^2 - 2x + 3$

x	-1	0	1	2	3
y	6	3	2	3	6



From the graph : $\therefore$ The S.S. $= \varnothing$

[b] $\because n(x) = \frac{(x+4)(x-3)}{(x+4)(x+1)}$

$\therefore n^{-1}(x) = \frac{(x+4)(x+1)}{(x+4)(x-3)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{-4, 3, -1\}$

$, n^{-1}(x) = \frac{x+1}{x-3} \quad \therefore n^{-1}(0) = \frac{0+1}{0-3} = -\frac{1}{3}$

4

[a] $\because 2x^2 - 5x + 1 = 0$

$\therefore a = 2, b = -5, c = 1$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 1}}{2 \times 2} = \frac{5 \pm \sqrt{17}}{4}$

$\therefore x \approx 2.28$ or $x \approx 0.22$

$\therefore$ The S.S. $= \{2.28, 0.22\}$

[b] $\because n_1(x) = \frac{(x+1)(x^2-x+1)}{x(x^2-x+1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{0\} \\ , n_1(x) = \frac{x+1}{x}\end{array}\right\} (1)$

$, \because n_2(x) = \frac{x^2(x+1) + x + 1}{x(x^2+1)} = \frac{(x+1)(x^2+1)}{x(x^2+1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{0\} \\ , n_2(x) = \frac{x+1}{x}\end{array}\right\} (2)$

from (1) and (2) : $\therefore n_1 = n_2$

5

[a] 1 $P(\acute{A}) = 1 - P(A) = 1 - 0.8 = 0.2$

2 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.8 + 0.7 - 0.6 = 0.9$

3 $P(A - B) = P(A) - P(A \cap B)$

$= 0.8 - 0.6 = 0.2$

[b] $\because n(x) = \frac{x(x+2)}{(x-3)(x^2+3x+9)} \div \frac{x+2}{x^2+3x+9}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, -2\}$

$\therefore n(x) = \frac{x(x+2)}{(x-3)(x^2+3x+9)} \times \frac{x^2+3x+9}{x+2}$

$= \frac{x}{x-3}$

21 Luxor

1

1 d 2 b 3 c 4 b 5 d 6 a

2

[a] Let $n_1(x) = \frac{x-4}{x^2-5x+6}$

$\therefore n_1(x) = \frac{x-4}{(x-2)(x-3)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{2, 3\}$

$,$ let $n_2(x) = \frac{2x}{x^3 - 9x}$

$\therefore n_2(x) = \frac{2x}{x(x^2-9)} = \frac{2x}{x(x-3)(x+3)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0 , 3 , -3\}$

$\therefore$ The common domain $= \mathbb{R} - \{2 , 3 , 0 , -3\}$

[b] $\because y + 2x = 7 \qquad \therefore y = 7 - 2x \qquad (1)$

$, 2x^2 + x + 3y = 19 \qquad (2)$

Substituting from (1) in (2) :

$\therefore 2x^2 + x + 3(7 - 2x) = 19$

$\therefore 2x^2 + x + 21 - 6x = 19$

$\therefore 2x^2 - 5x + 2 = 0 \qquad \therefore (2x - 1)(x - 2) = 0$

$\therefore x = \frac{1}{2}$ or $x = 2$

Substituting (1) : $\therefore y = 6$ or $y = 3$

$\therefore$ The S.S. $= \left\{\left(\frac{1}{2} , 6\right) , (2 , 3)\right\}$

3

[a] $\because n(x) = \frac{x-3}{(x-3)(x-4)} + \frac{x-3}{x-3}$

$\therefore$ The domain of $n = \mathbb{R} - \{3 , 4\}$

$, n(x) = \frac{1}{x-4} + 1 = \frac{1}{x-4} + \frac{x-4}{x-4} = \frac{x-3}{x-4}$

[b] 1 The probability of the student succeeded in Math $= \frac{30}{40} = \frac{3}{4}$

2 The probability of the student succeeded in Science only $= \frac{4}{40} = \frac{1}{10}$

3 The probability of the succeeded in one of them at least $= \frac{34}{40} = \frac{17}{20}$

4

[a] $\because 2x^2 - x - 2 = 0$

$\therefore a = 2 , b = -1 , c = -2$

$\therefore x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times (-2)}}{2 \times 2}$

$= \frac{1 \pm \sqrt{17}}{4} = \frac{1 \pm 4.12}{4}$

$\therefore x \approx 1.28$ or $x \approx -0.78$

$\therefore$ The S.S. $= \{1.28 , -0.78\}$

[b] $\because n_1(x) = \frac{x}{(x-1)(x+1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{1 , -1\} \\ , n_1(x) = \frac{x}{(x-1)(x+1)}\end{array}\right\} (1)$

$, \because n_2(x) = \frac{5x}{5(x^2-1)} = \frac{5x}{5(x-1)(x+1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{1 , -1\} \\ , n_2(x) = \frac{x}{(x-1)(x+1)}\end{array}\right\} (2)$

from (1) and (2) : $\therefore n_1 = n_2$

5

[a] $\because n(x) = \frac{x(x-3)}{(2x+3)(x-2)} \div \frac{x(2x-3)}{(2x-3)(2x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \left\{-\frac{3}{2} , 2 , 0 , \frac{3}{2}\right\}$

$, n(x) = \frac{x(x-3)}{(2x+3)(x-2)} \times \frac{(2x-3)(2x+3)}{x(2x-3)}$

$= \frac{x-3}{x-2}$

[b] $\because x + 2y = 8 \qquad (1)$

$, 3x + y = 9$ (multiplying by -2)

$\therefore -6x - 2y = -18 \qquad (2)$

Adding (1) and (2) : $-5x = -10$

$\therefore x = 2$

Substituting in (1) : $\therefore y = 3$

$\therefore$ The S.S. $= \{(2 , 3)\}$

22 Aswan

1

1 c 2 b 3 d 4 c 5 d 6 c

2

[a] $\because 3x - y = -4 \qquad (1)$

$, y - 2x = 3 \qquad \therefore y = 3 + 2x \qquad (2)$

Substituting from (2) in (1) :

$\therefore 3x - (3 + 2x) = -4$

$\therefore 3x - 3 - 2x = -4$

$\therefore x = -1$

Substituting in (2) : $\therefore y = 1$

$\therefore$ The S.S. $= \{(-1 , 1)\}$

[b] $\because n(x) = \frac{(x+1)(x+3)}{(x-3)(x^2+3x+9)} \div \frac{x+3}{x^2+3x+9}$

$\therefore$ The domain of $n = \mathbb{R} - \{3 , -3\}$

$, n(x) = \frac{(x+1)(x+3)}{(x-3)(x^2+3x+9)} \times \frac{x^2+3x+9}{x+3}$

$= \frac{x+1}{x-3}$

3

[a] $\because x - y = 1 \qquad \therefore x = y + 1 \qquad (1)$

$, x^2 + y^2 = 25 \qquad (2)$

Substituting from (1) in (2) : $\therefore (y+1)^2 + y^2 = 25$

$\therefore y^2 + 2y + 1 + y^2 - 25 = 0$

$\therefore 2y^2 + 2y - 24 = 0$ (Dividing by 2)

$\therefore y^2 + y - 12 = 0 \qquad \therefore (y-3)(y+4) = 0$

$\therefore y = 3$ or $y = 4$

Substituting in (1) : $\therefore x = 4$ or $x = 5$

$\therefore$ The S.S. $= \{(4, 3), (5, 4)\}$

[b] $\because n(x) = \frac{x(x-2)}{(x-2)(x-1)}$

$\therefore n^{-1}(x) = \frac{(x-2)(x-1)}{x(x-2)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, 2, 1\}$

, $n^{-1}(x) = \frac{x-1}{x}$

4

[a] $\because 2x^2 - 5x + 1 = 0$

$\therefore a = 2, b = -5, c = 1$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 1}}{2 \times 2} = \frac{5 \pm \sqrt{17}}{4}$

$\therefore$ The S.S. $= \left\{\frac{5+\sqrt{17}}{4}, \frac{5-\sqrt{17}}{4}\right\}$

[b] $\because n(x) = \frac{x(x+2)}{(x+2)(x-2)} - \frac{2(x-3)}{(x-2)(x-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2, 2, 3\}$

, $n(x) = \frac{x}{x-2} - \frac{2}{x-2} = \frac{x-2}{x-2} = 1$

5

[a] $\because n_1(x) = \frac{2x}{2(x+4)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{-4\} \\ , n_1(x) = \frac{x}{x+4}\end{array}\right\}$ (1)

, $\because n_2(x) = \frac{x(x+4)}{(x+4)(x+4)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{-4\} \\ , n_2(x) = \frac{x}{x+4}\end{array}\right\}$ (2)

from (1) and (2) : $\therefore n_1 = n_2$

[b] $\because$ A , B are two mutually exclusive events

$\therefore P(A \cup B) = P(A) + P(B)$

$\therefore \frac{7}{12} = \frac{1}{3} + P(B)$

$\therefore P(B) = \frac{7}{12} - \frac{1}{3} = \frac{7-4}{12} = \frac{3}{12} = \frac{1}{4}$

23 New Valley

1

1 b 2 a 3 a 4 d 5 c 6 d

2

[a] $\because n(x) = \frac{(x-2)(x+2)}{(x+2)(x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2, -3\}$

, $n(x) = \frac{x-2}{x+3}$

[b] $\because x^2 + y^2 = 17$ (1)

, $y - x = 3 \qquad \therefore y = x + 3$ (2)

Substituting from (2) in (1) : $\therefore x^2 + (x+3)^2 = 17$

$\therefore x^2 + x^2 + 6x + 9 = 17$

$\therefore 2x^2 + 6x - 8 = 0$ (Dividing by 2)

$\therefore x^2 + 3x - 4 = 0 \qquad \therefore (x+4)(x-1) = 0$

$\therefore x = -4$ or $x = 1$

Substituting in (2) : $\therefore y = -1$ or $y = 4$

$\therefore$ The S.S. $= \{(-4, -1), (1, 4)\}$

3

[a] $\because 3x - 2y = 4$ (1)

, $x + 3y = 5 \qquad \therefore x = 5 - 3y$ (2)

Substituting from (2) in (1) : $\therefore 3(5 - 3y) - 2y = 4$

$\therefore 15 - 9y - 2y = 4 \quad \therefore -11y = -11 \quad \therefore y = 1$

Substituting in (2) : $x = 2$

$\therefore$ The S.S. $= \{(2, 1)\}$

[b] $\because n(x) = \frac{x}{x+2} \div \frac{2x^2 - 4x}{x^2 - 4}$

$= \frac{x}{x+2} \div \frac{2x(x-2)}{(x-2)(x+2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -2, 0\}$

, $n(x) = \frac{x}{x+2} \times \frac{(x-2)(x+2)}{2x(x-2)} = \frac{1}{2}$

4

[a] $\because n_1(x) = \frac{(x-1)(x^2+x+1)}{x(x^2+x+1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{0\} \\ , n_1(x) = \frac{x-1}{x}\end{array}\right\}$ (1)

, $n_2(x) = \frac{x^2(x-1) + (x-1)}{x(x^2+1)} = \frac{(x-1)(x^2+1)}{x(x^2+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0\}$
, $n_2(X) = \frac{X-1}{X}$ } (2)

from (1) and (2) : $\therefore n_1 = n_2$

[b] $\because n(X) = \frac{3X}{X(X-3)} - \frac{X}{X-3}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, 3\}$

, $n(X) = \frac{3}{X-3} - \frac{X}{X-3} = \frac{3-X}{X-3} = \frac{-(X-3)}{(X-3)} = -1$

5

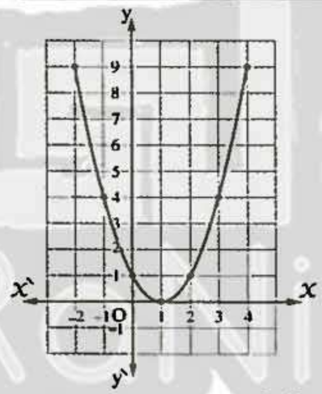
[a] 1 $P(\acute{A}) = 1 - P(A) = 1 - \frac{1}{5} = \frac{4}{5}$

2 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= \frac{1}{5} + \frac{3}{5} - \frac{1}{10} = \frac{7}{10}$

3 $P(B - A) = P(B) - P(A \cap B)$
$= \frac{3}{5} - \frac{1}{10} = \frac{5}{10} = \frac{1}{2}$

[b] $f(X) = X^2 - 2X + 1$

X	-2	-1	0	1	2	3	4
y	9	4	1	0	1	4	9



From the graph : $\therefore$ The S.S. $= \{1\}$

24 South Sinai

1

1 a 2 b 3 c 4 d 5 b 6 b

2

[a] $\because X^2 - 2X - 6 = 0$

$\therefore a = 1, b = -2, c = -6$

$\therefore X = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-6)}}{2 \times 1}$

$= \frac{2 \pm \sqrt{28}}{2} = \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$

$\therefore X = 3.65$ or $X = -1.65$

$\therefore$ The S.S. $= \{3.65, -1.65\}$

[b] $\because n(X) = \frac{X}{X+2} + \frac{2X^3}{X^2(X+2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2, 0\}$

, $n(X) = \frac{X}{X+2} + \frac{2X}{X+2} = \frac{3X}{X+2}$

3

[a] $\because n(X) = \frac{X(X+2)}{(X-2)(X^2+2X+4)} \times \frac{X^2+2X+4}{X+2}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -2\}$

, $n(X) = \frac{X}{X-2}$

[b] $\because 2X - y = 3$ (1)

, $X + 2y = 4$ $\therefore X = 4 - 2y$ (2)

Substituting from (2) in (1) : $\therefore 2(4 - 2y) - y = 3$

$\therefore 8 - 4y - y = 3$ $\therefore 8 - 5y = 3$

$\therefore -5y = -5$ $\therefore y = 1$

Substituting in (2) : $\therefore X = 2$

$\therefore$ The S.S. $= \{(2, 1)\}$

4

[a] $\because n_1(X) = \frac{X}{X(X+1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{0, -1\}$
, $n_1(X) = \frac{1}{X+1}$ } (1)

, $\because n_2(X) = \frac{X^2(X^2-X+1)}{X^2(X^3+1)}$

$= \frac{X^2(X^2-X+1)}{X^2(X+1)(X^2-X+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0, -1\}$
, $n_2(X) = \frac{1}{X+1}$ } (2)

from (1) and (2) : $\therefore n_1 = n_2$

[b] $\because X - y = 7$ $\therefore X = y + 7$ (1)

, $Xy = 60$ (2)

Substituting from (1) in (2) : $\therefore (y + 7)y = 60$

$\therefore y^2 + 7y - 60 = 0$ $\therefore (y + 12)(y - 5) = 0$

$\therefore y = -12$ or $y = 5$

Substituting in (1) : $\therefore X = -5$ or $X = 12$

$\therefore$ The S.S. $= \{(-5, -12), (12, 5)\}$

5

[a] $\because n(X) = \frac{X+1}{(X+2)(X+1)} - \frac{X+2}{(X+2)(X-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2, -1, 2\}$

$, n(x) = \frac{1}{x+2} - \frac{1}{x-2}$

$= \frac{x-2-(x+2)}{(x+2)(x-2)} = \frac{x-2-x-2}{(x+2)(x-2)}$

$= \frac{-4}{(x+2)(x-2)}$

[b] $\because$ A and B are mutually exclusive events

$\therefore P(A \cup B) = P(A) + P(B)$

$\therefore P(B) = P(A \cup B) - P(A) = \frac{5}{12} - \frac{1}{4} = \frac{1}{6}$

25 North Sinai

1

1 c 2 c 3 d 4 d 5 a 6 b

2

[a] 1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{2}{5} - \frac{1}{5} = \frac{7}{10}$

2 $P(A - B) = P(A) - P(A \cap B)$

$= \frac{1}{2} - \frac{1}{5} = \frac{3}{10}$

[b] $\because n_1(x) = \frac{-1}{(x-3)(x+3)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{3, -3\}$

$, \because n_2(x) = \frac{7}{x}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0\}$

$\therefore$ The common domain $= \mathbb{R} - \{3, -3, 0\}$

3

[a] $\because x^2 - 2x - 4 = 0$

$\therefore a = 1, b = -2, c = -4$

$\therefore x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-4)}}{2 \times 1} = \frac{2 \pm \sqrt{20}}{2}$

$= \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$

$\therefore x \approx 3.24$ or $x \approx -1.24$

$\therefore$ The S.S. $= \{3.24, -1.24\}$

[b] $\because n(x) = \frac{x(x-2)}{(x-2)(x+2)} + \frac{2(x+3)}{(x+3)(x+2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -2, -3\}$

$, n(x) = \frac{x}{x+2} + \frac{2}{x+2} = \frac{x+2}{x+2} = 1$

4

[a] $\because x - y = 0$ $\therefore x = y$ (1)

$, xy = 16$ (2)

Substituting from (1) in (2) : $\therefore y^2 = 16$

$\therefore y = 4$ or $y = -4$

Substituting in (1) : $\therefore x = 4$ or $x = -4$

$\therefore$ The S.S. $= \{(4, 4), (-4, -4)\}$

[b] $\because n_1(x) = \frac{x^2}{x^2(x-1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{0, 1\}$
$, n_1(x) = \frac{1}{x-1}$ (1)

$, \because n_2(x) = \frac{x(x^2+x+1)}{x(x^3-1)}$

$= \frac{x(x^2+x+1)}{x(x-1)(x^2+x+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0, 1\}$
$, n_2(x) = \frac{1}{x-1}$ (2)

from (1) and (2) : $\therefore n_1 = n_2$

5

[a] $\because n(x) = \frac{(x-1)(x-1)}{(x-1)(x^2+x+1)} \div \frac{x-1}{x^2+x+1}$

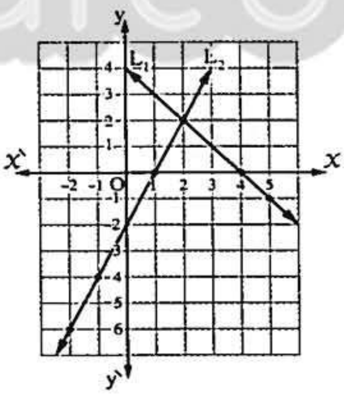
$\therefore$ The domain of $n = \mathbb{R} - \{1\}$

$, n(x) = \frac{(x-1)(x-1)}{(x-1)(x^2+x+1)} \times \frac{x^2+x+1}{x-1} = 1$

[b] $x = 4 - y$ $y = 2x - 2$

x	2	4	5
y	2	0	−1

x	1	−1	−2
y	0	−4	−6



From the graph : $\therefore$ the S.S. $= \{(2, 2)\}$

26 Red Sea

1

1 c 2 b 3 a 4 b 5 c 6 d

2

[a] $\because 2x - y = 3$ (1)

$, x + 2y = 4 \qquad \therefore x = 4 - 2y$ (2)

Substituting from (2) in (1) :

$\therefore 2(4 - 2y) - y = 3 \quad \therefore 8 - 4y - y = 3$

$\therefore 8 - 5y = 3 \qquad \therefore -5y = -5 \qquad \therefore y = 1$

Substituting in (2) : $\therefore x = 2$

$\therefore$ The S.S. $= \{(2, 1)\}$

[b] $\because n(x) = \frac{x^2}{x-1} - \frac{x}{x-1}$

$\therefore$ The domain of $n = \mathbb{R} - \{1\}$

$, n(x) = \frac{x^2}{x-1} - \frac{x}{x-1} = \frac{x^2 - x}{x-1} = \frac{x(x-1)}{x-1} = x$

3

[a] $\because x^2 - x - 4 = 0$

$\therefore a = 1, b = -1, c = -4$

$\therefore x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 1 \times (-4)}}{2 \times 1} = \frac{1 \pm \sqrt{17}}{2}$

$\therefore x \approx 2.56$ or $x \approx -1.56$

[b] $\because n_1(x) = \frac{(x+1)(x^2 - x + 1)}{x(x^2 - x + 1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{0\}$
$, n_1(x) = \frac{x+1}{x}$ (1)

$, \because n_2(x) = \frac{x^2(x+1) + x + 1}{x(x^2+1)} = \frac{x+1(x^2+1)}{x(x^2+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0\}$
$, n_2(x) = \frac{x+1}{x}$ (2)

From (1) and (2) : $\therefore n_1 = n_2$

4

[a] $\because x - y = 1 \qquad \therefore x = y + 1$ (1)

$, x^2 + y^2 = 25$ (2)

Substituting from (1) in (2) : $\therefore (y+1)^2 + y^2 = 25$

$\therefore y^2 + 2y + 1 + y^2 - 25 = 0$

$\therefore 2y^2 + 2y - 24 = 0$

$\therefore y^2 + y - 12 = 0 \qquad \therefore (y+4)(y-3) = 0$

$\therefore y = -4$ or $y = 3$

Substituting in (1) : $\therefore x = -3$ or $x = 4$

$\therefore$ The S.S. $= \{(-3, -4), (4, 3)\}$

[b] 1 $\because n(x) = \frac{x(x-2)}{(x-2)(x-3)}$

$\therefore n^{-1}(x) = \frac{(x-2)(x-3)}{x(x-2)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, 2, 3\}$

$, n^{-1}(x) = \frac{x-3}{x}$

2 $\because n^{-1}(x) = 2 \qquad \therefore \frac{x-3}{x} = 2$

$\therefore x - 3 = 2x \qquad \therefore x = -3$

5

[a] $\because n(x) = \frac{(x-2)(x^2 + 2x + 4)}{(x-2)(x+3)} \times \frac{x+3}{x^2 + 2x + 4}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -3\}$

$, n(x) = 1$

[b] 1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.3 + 0.6 - 0.2 = 0.7$

2 $P(A - B) = P(A) - P(A \cap B) = 0.3 - 0.2 = 0.1$

27 Matrouh

1

1 a 2 c 3 a 4 b 5 c 6 d

2

[a] $\because x + \frac{1}{x} + 3 = 0$ (Multiplying by x)

$\therefore x^2 + 1 + 3x = 0 \qquad \therefore x^2 + 3x + 1 = 0$

$\therefore a = 1, b = 3, c = 1$

$\therefore x = \frac{-3 \pm \sqrt{(3)^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{-3 \pm \sqrt{5}}{2}$

$\therefore x \approx -0.38$ or $x \approx -2.62$

The S.S. $= \{-0.38, -2.62\}$

[b] $\because n(x) = \frac{(x-1)(x+1)}{x(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, 1\}$

$, n(x) = \frac{x+1}{x}$

3

[a] $\because n(x) = \frac{x-1}{(x-1)(x+1)} \div \frac{x(x-5)}{(x+1)(x-5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1, -1, 5, 0\}$

$, n(x) = \frac{x-1}{(x-1)(x+1)} \times \frac{(x+1)(x-5)}{x(x-5)} = \frac{1}{x}$

[b] Let the two positive numbers be x and y

$\therefore x + y = 9 \qquad \therefore y = 9 - x$ (1)

$, x^2 - y^2 = 27$ (2)

substituting from (1) in (2) :

$\therefore x^2 - (9 - x)^2 = 27$

$\therefore x^2 - (81 + 18x - x^2) = 27$

$\therefore x^2 - 81 + 18x - x^2 = 27$

$\therefore 18x = 108 \qquad \therefore x = 6$

Substituting in (1) : $\therefore$ y = 3

$\therefore$ The two positive numbers are : 6 , 3

4

[a] 1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.3 + 0.6 - 0.2 = 0.7$

2 $P(A - B) = P(A) - P(A \cap B) = 0.3 - 0.2 = 0.1$

[b] $\because n_1(x) = \frac{x^2}{x^2(x-1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{0 , 1\}$, $n_1(x) = \frac{1}{x-1}$ (1)

, $\because n_2(x) = \frac{x(x^2+x+1)}{x(x^3-1)} = \frac{x(x^2+x+1)}{x(x-1)(x^2+x+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0 , 1\}$, $n_2(x) = \frac{1}{x-1}$ (2)

From (1) and (2) : $\therefore n_1 = n_2$

5

[a] $\because n(x) = \frac{3x}{(x+1)(x-2)} - \frac{x-1}{x^2-1}$

$= \frac{3x}{(x+1)(x-2)} - \frac{x-1}{(x-1)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-1 , 2 , 1\}$

, $n(x) = \frac{3x}{(x+1)(x-2)} - \frac{1}{x+1}$

$= \frac{3x-(x-2)}{(x+1)(x-2)} = \frac{3x-x+2}{(x+1)(x-2)}$

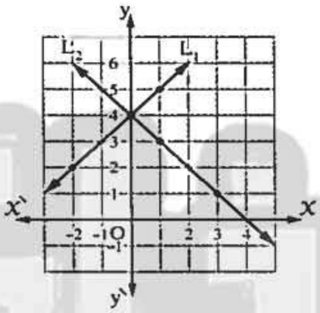
$= \frac{2x+2}{(x+1)(x-2)} = \frac{2(x+1)}{(x+1)(x-2)} = \frac{2}{x-2}$

[b] $y = x + 4$

x	1	0	−2
y	5	4	2

$x = 4 - y$

x	3	1	0
y	1	3	4



From the graph : The S.S. = $\{(0 , 4)\}$

Answers of quizzes of geometry

Quiz 1

1

1 b 2 d 3 d

2

1 124° 2 2 cm. 3 $\frac{3}{5}$

Quiz 2

1

1 a 2 c 3 b

2 Prove by yourself.

Quiz 3

1

1 b 2 a 3 b

2 Prove by yourself.

Quiz 4

1

1 c 2 b 3 b

2 Draw by yourself.
, number of circles = 1

Quiz 5

1

1 d 2 b 3 c

2 Prove by yourself.

Quiz 6

1

1 a 2 b 3 b

2 116°

Quiz 7

1

1 a 2 a 3 d

2 m (∠ ACD) = 35° , m (∠ ABC) = 55°

Quiz 8

1

1 c 2 a 3 c

2 Prove by yourself.

Quiz 9

1

1 a 2 c 3 d

2 150°

Quiz 10

1

1 c 2 a 3 a

2 Prove by yourself.

Quiz 11

1

1 a 2 b 3 d

2

1 m (∠ ACB) = 60°

2 m (∠ BDC) = 60°

Quiz 12

1

1 b 2 a 3 c

2 Prove by yourself.

Answers of school book examinations in geometry

Model 1

1

1 d 2 a 3 d 4 a 5 b 6 a

2

[a] supplementary , theoratical.

[b] $\because \overleftrightarrow{XY} \parallel \overline{BD}$, $\overrightarrow{AB}$ is a transversal

$\therefore m(\angle DBX) = m(\angle BXY)$ (altemate angles) (1)

, $\because m(\angle C)$ (inscribed) $= m(\angle ABD)$ (tangency) (2)

From (1) and (2) : $\therefore m(\angle C) = m(\angle BXY)$

$\therefore$ AXYC is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the smaller circle

$\therefore AB = AC$ $\therefore 2x - 3 = 15$

$\therefore 2x = 18$ $\therefore x = 9$

, $\because \overrightarrow{AB}$, $\overrightarrow{AD}$ are two tangents to the greater circle

$\therefore AB = AD$ $\therefore y - 2 = 15$ $\therefore y = 17$

[b] $\because m(\angle BDC) = m(\angle BAC)$

(two inscribed angles subtended by $\widehat{BC}$)

$\therefore m(\angle BDC) = 30^\circ$

$\because m(\widehat{BC}) = 2\,m(\angle BAC) = 60^\circ$

, $\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\widehat{AB}) = 180^\circ$

$\therefore m(\widehat{AC}) = 180^\circ - 60^\circ = 120^\circ$

, $\because$ D is the midpoint of $\widehat{AC}$

$\therefore m(\widehat{AD}) = \frac{120^\circ}{2} = 60^\circ$ (First req.)

$\therefore m(\angle ACD) = \frac{1}{2}\,m(\widehat{AD}) = \frac{1}{2} \times 60^\circ = 30^\circ$

$\therefore m(\angle CAB) = m(\angle ACD)$ but they are alterante

$\therefore \overline{AB} \parallel \overline{DC}$ (Second req.)

4

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle MXA) = 90^\circ$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\therefore (\angle MYA) = 90^\circ$

From the quadrilateral AXMY :

$\therefore m(\angle DMH) = 360^\circ - (90^\circ + 90^\circ + 70^\circ) = 110^\circ$

(First req.)

$\because AB = AC$, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

$\therefore MX = MY$, $\because MD = MH = r$

By subtracting : $\therefore XD = YH$ (Second req.)

[b] $\because m(\angle A) = \frac{1}{2}\,[m(\widehat{HC}) - m(\widehat{BD})]$

$\therefore 30^\circ = \frac{1}{2}\,[120^\circ - m(\widehat{BD})]$

$\therefore 60^\circ = 120^\circ - m(\widehat{BD})$ $\therefore m(\widehat{BD}) = 120^\circ - 60^\circ$

$\therefore m(\widehat{BD}) = 60^\circ$ (First req.)

, $\because m(\widehat{BC}) = m(\widehat{DH})$ $\therefore BC = DH$

by adding $m(\widehat{BD})$ to both sides

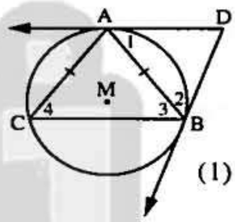
$\therefore m(\widehat{CD}) = m(\widehat{HB})$ $\therefore m(\angle C) = m(\angle H)$

In Δ ACH : $\therefore AC = AH$, $\because BC = DH$

By sbutracting : $\therefore AB = AD$ (Second req.)

5

[a] $\because \overline{DA}$ and $\overline{DB}$ are two tangent-segments to the circle M at A and B



$\therefore DA = DB$

$\therefore m(\angle 1) = m(\angle 2)$

$\therefore m(\angle D)$

$= 180^\circ - 2\,m(\angle 1)$ (1)

In Δ ABC : $\because AB = AC$

$\therefore m(\angle 3) = m(\angle 4)$

$\therefore m(\angle BAC) = 180^\circ - 2\,m(\angle 4)$ (2)

, $\because \overline{AD}$ is a tangent-segment to the circle

$\therefore m(\angle 4)$ (inscribed) $= m(\angle 1)$ (tangency) (3)

From (1) , (2) and (3) : $\therefore m(\angle BAC) = m(\angle D)$

$\therefore \overline{AC}$ is a tangent to the circle passing through the vertices of the Δ ABD (Q.E.D.)

[b] In Δ AMB : $\because AM = BM = r$

$\therefore m(\angle MBA) = m(\angle MAB) = 20^\circ$

$\because$ C is the midpoint of $\overline{AB}$

$\therefore \overline{MC} \perp \overline{AB}$ $\therefore m(\angle MCB) = 90^\circ$

In Δ BCM : $\therefore m(\angle BMC) = 180^\circ - (90^\circ + 20^\circ) = 70^\circ$

, $\because m(\angle BHD) = \frac{1}{2}\,m(\angle BMD)$

(inscribed and central angles subtended by $\widehat{BD}$)

$\therefore m(\angle BHD) = \frac{1}{2} \times 70^\circ = 35^\circ$ (First req.)

In Δ AMB : $\because AM = BM = r$

, $m(\angle MAB) = m(\angle MBA) = 20^\circ$

$\therefore m(\angle AMB) = 180^\circ - (20^\circ + 20^\circ) = 140^\circ$

$\therefore m(\widehat{ADB}) = m(< AMB) = 140^\circ$ (Second req.)

Model 2

1

1 b　2 d　3 b
4 c　5 d　6 b

2

[a] $\because AB = AC$

$, \overline{MD} \perp \overline{AB}$ $, \overline{ME} \perp \overline{AC}$

$\therefore MD = ME$ $, \because MX = MY = r$

$\therefore DX = EY$ (Q.E.D.)

[b] In ΔABD : $\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 30°$

$\therefore m(\angle A) = 180° - 2 \times 30° = 120°$

$, \because m(\angle A) + m(\angle C) = 120° + 60° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

3

[a] State by yourself.

[b] $\because$ E is the midpoint of $\overset{\frown}{BF}$

$\therefore m(\overset{\frown}{FE}) = m(\overset{\frown}{BE})$

$\therefore m(\angle FAE) = m(\angle BAE)$

$, \because m(\angle CBE)$ (tangency) $= m(\angle BAE)$ (inscribed)

$\therefore m(\angle DAC) = m(\angle DBC)$

and they are drawn on $\overline{DC}$ and on one side of it

$\therefore$ ABCD is a cyclic quadrilateral.

4

[a] $\because \overline{AD}, \overline{AF}$ are two tangent-segments to the circle

$\therefore AD = AF = 5$ cm.

$, \because \overline{BD}, \overline{BE}$ are two tangent-segments to the circle

$\therefore BD = BE = 4$ cm.

$, \because \overline{CE}, \overline{CF}$ are two tangent-segments to the circle

$\therefore CE = CF = 3$ cm.

$\therefore$ The perimeter of $\Delta ABC = 5 + 5 + 4 + 4 + 3 + 3 = 24$ cm. (The req.)

[b] $\because \overrightarrow{AF} // \overline{DE}, \overrightarrow{AB}$ is a transversal

$\therefore m(\angle AED) = m(\angle EAF)$ (alternate angles)

$, \because m(\angle C)$ (inscribed) $= m(\angle BAF)$ (tangency)

$\therefore m(\angle C) = m(\angle AED)$

$\therefore$ DEBC is a cyclic quadrilateral. (Q.E.D.)

5

$\because$ BCDE is a cyclic quadrilateral

$\therefore m(\angle CBE) + m(\angle D) = 180°$

$\therefore m(\angle CBE) = 180° - 125° = 55°$

$, \because \overrightarrow{AB}, \overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

$\therefore$ In ΔABC : $m(\angle ACB) = m(\angle ABC)$

$= \frac{180° - 70°}{2} = 55°$

$\therefore m(\angle CBE) = m(\angle ACB) = 55°$

and they are alternate angles

$\therefore \overline{AC} // \overline{BE}$

$, \because m(\angle BEC)$ (inscribed) $= m(\angle ACB)$ (tangency) $= 55°$

$\therefore m(\angle CBE) = m(\angle BEC) = 55°$

$\therefore$ In ΔCBE : $CB = CE$

Model examination for the merge students

1

1 diameter　2 perpendicular to this chord
3 equal　4 3　5 infinite
6 180°

2

1 a　2 a　3 d
4 c　5 d　6 c

3

1 ✗　2 ✓　3 ✗
4 ✓　5 ✗　6 ✗

4

1 90°　2 130°　3 40°
4 5　5 30°　6 2 : 1

Answers of governorates' examinations of geometry

1 Cairo

1

1 c   2 b   3 a   4 a   5 c   6 d

2

[a] Mention by yourself.

[b] $\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\angle ACB) = 90^\circ$ (1) (First req.)

, $\because \overline{DE} \perp \overline{AD}$

$\therefore m(\angle ADE) = 90^\circ$ (2)

From (1) and (2) :

$\therefore m(\angle ADE) = m(\angle ACE)$

but they are drawn on $\overline{AE}$ and on one side of it

$\therefore$ The figure ACDE is a cyclic quadrilateral. (Second req.)

3

[a] The measure of the arc $= \frac{1}{3} \times 360^\circ = 120^\circ$ (The req.)

[b] $\because m(\angle BAC) = \frac{1}{2} m(\angle BMC)$

(inscribed and central angles subtended the same arc $\widehat{BC}$)

$\therefore m(\angle BAC) = \frac{1}{2} \times 80^\circ = 40^\circ$

, $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$ (First req.)

$\because m(\widehat{BC}) = m(\angle M) = 80^\circ$

$\therefore m(\widehat{BC}$ the major$) = 360^\circ - 80^\circ = 280^\circ$ (Second req.)

4

[a] $\because \overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{CB}$

, $\because$ The sum of measures of the interior angles of the quadrilateral BDME $= 360^\circ$

$\therefore m(\angle DME) = 360^\circ - (70^\circ + 90^\circ + 90^\circ) = 110^\circ$ (First req.)

$\because MD = ME$, $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{CB}$

$\therefore AB = CB$ (Second req.)

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents.

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$ (1)

, $\because \overrightarrow{BD} \parallel \overrightarrow{AC}$, $\overleftrightarrow{BC}$ is a transversal.

$\therefore m(\angle DBC) = m(\angle ACB)$ (alternate angles) (2)

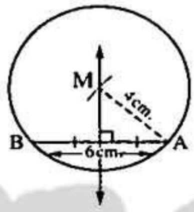
From (1) and (2) :

$\therefore m(\angle ABC) = m(\angle DBC)$

$\therefore \overrightarrow{BC}$ bisects $\angle ABD$ (Q.E.D.)

5

[a]



$\because$ The radius length of the smallest circle = 3 cm.

[b] $\because \overrightarrow{AD}$ is a tangent to the circle at A

$\therefore m(\angle ABC)$ (inscribed) $= m(\angle CAD)$ (tangency) $= 50^\circ$

, $\because AC = BC$

$\therefore m(\angle BAC) = m(\angle ABC) = 50^\circ$

, $m(\angle BEC) = m(\angle BAC) = 50^\circ$

(two inscribed angles subtended by $\widehat{BC}$) (First req.)

, $\because m(\angle BEC) = m(\angle ABC) = 50^\circ$

$\therefore \overrightarrow{BC}$ is a tangent to the circle passing through the vertices of Δ BEO (Second req.)

2 Giza

1

1 d   2 c   3 b   4 b   5 c   6 d

2

[a] $\because m(\angle A) = \frac{1}{2} m(\angle BMD) = \frac{1}{2} \times 150^\circ = 75^\circ$

(inscribed and central angles subtened by $\widehat{BD}$)

, $\because$ ABCD is a cyclic quadrilateral.

$\therefore m(\angle C) = 180^\circ - 75^\circ = 105^\circ$ (The req.)

[b] In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

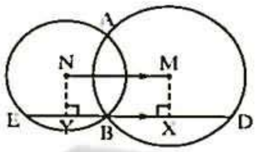
, $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$, $\because \overline{MY} \perp \overline{AC}$

$\therefore$ MX = MY (Q.E.D.)

3

[a] Construction :



Draw $\overline{MX} \perp \overline{BD}$

, $\overline{NY} \perp \overline{BE}$

Proof : $\because \overleftrightarrow{BD} \parallel \overleftrightarrow{MN}$

, $\overline{MX} \perp \overline{BD}$, $\overline{NY} \perp \overline{BE}$

$\therefore \overline{MX} \parallel \overline{NY}$

$\therefore$ The figure MXYN is a rectangle

$\therefore$ X is midpoint of $\overline{BD}$

, Y is midpoint of $\overline{BE}$

$\therefore$ DE = 2 XY , $\because$ XY = MN

$\therefore$ DE = 2 MN (Q.E.D.)

[b] $\because \overline{AB}$ is a tangent to the circle M

$\therefore \overline{MA} \perp \overline{AB}$ $\therefore$ m ($\angle$ MAB) = 90°

In Δ MAB : $\because$ m ($\angle$ ABM) = 30° , m ($\angle$ MAB) = 90°

$\therefore$ BM = 2 AM = 2 × 8 = 16 cm.

$\therefore (AB)^2 = (BM)^2 - (MA)^2 = (16)^2 - (8)^2 = 192$

$\therefore$ AB = $8\sqrt{3}$ cm. (First req.)

, $\because$ AC = $\frac{AM \times AB}{BM}$

$\therefore$ AC = $\frac{8 \times 8\sqrt{3}}{16} = 4\sqrt{3}$ cm. (Second req.)

4

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle

$\therefore$ AB = AC

$\therefore$ In Δ ABC : m ($\angle$ ABC) = m ($\angle$ ACB)

$= \frac{180° - 50°}{2} = 65°$

, $\because$ BCDE is a cyclic quadrilateral.

$\therefore$ m ($\angle$ EBC) + m ($\angle$ D) = 180°

$\therefore$ m ($\angle$ EBC) = 180° − 115° = 65°

$\therefore$ m ($\angle$ ABC) = m ($\angle$ EBC)

$\therefore \overrightarrow{BC}$ bisects $\angle$ ABE (Q.E.D.1)

, $\because$ m ($\angle$ BEC) (inscribed)

= m ($\angle$ ABC) (tangency) = 65°

$\therefore$ m ($\angle$ EBC) = m ($\angle$ BEC)

$\therefore$ In Δ BCE : CB = CE (Q.E.D.2)

[b] $\because \angle$ ABE is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m ($\angle$ D) = m ($\angle$ ABE) = 100°

In Δ ACD :

$\because$ m ($\angle$ ACD) = 180° − (100° + 40°) = 40°

$\therefore$ m ($\angle$ ACD) = m ($\angle$ CAD)

$\therefore$ CD = AD

$\therefore$ m ($\widehat{CD}$) = m ($\widehat{AD}$) (Q.E.D.)

5

[a] $\because$ m ($\angle$ ACB) = $\frac{1}{2}$ m ($\angle$ AMB) = 60°

(inscribed and central angles subtended the same arc $\widehat{AB}$) (1)

$\because \overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$

$\therefore$ m ($\widehat{AC}$) = m ($\widehat{BC}$)

$\therefore$ AC = BC (2)

From (1) and (2) :

$\therefore \Delta$ CAB is an equilateral triangle. (Q.E.D.)

[b] In $\Delta\Delta$ ADE , ACE

$\left\{\begin{array}{l} m (\angle DAE) = m (\angle CAE) \\ AD = AC \\ \overline{AE} \text{ is a common side} \end{array}\right.$

$\therefore \Delta ADE \equiv \Delta ACE$

$\therefore$ m ($\angle$ ADE) = m ($\angle$ ACE)

, $\because$ m ($\angle$ AFB) = m ($\angle$ ACB)

(two inscribed angles subtended by $\widehat{AB}$)

$\therefore$ m ($\angle$ ADE) = m ($\angle$ EFB)

$\therefore$ DBFE is a cyclic quadrilateral. (Q.E.D.)

3 Alexandria

1

1 b   2 d   3 a   4 b   5 d   6 c

2

[a] $\because \overline{CD}$ is a diameter in a circle M

, AB = 10 cm. , $\overline{MH} \perp \overline{AB}$

$\therefore$ AH = BH = 5 cm.

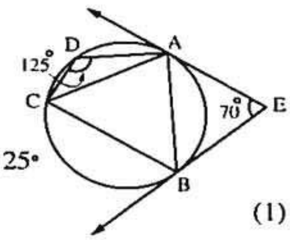
In Δ AHM : $\because$ m ($\angle$ AMH) = 30°

, m ($\angle$ AHM) = 90°

$\therefore$ AM = 2 AH = 10 cm.

$\therefore$ CD = 2 × 10 = 20 cm. (The req.)

[b] $\because$ The figure ABCD is a cyclic quadrilateral.



$\therefore m(\angle ABC) = 180° - 125°$

$= 55°$ (1)

$\because \overrightarrow{EA}$, $\overrightarrow{EB}$ are two tangents to the circle at A and B

$\therefore EA = EB$

$\therefore m(\angle EAB) = \frac{180° - 70°}{2} = 55°$

$\because \overrightarrow{EA}$ is a tangent to the circle at A

$\therefore m(\angle EAB)$ (tangency)

$= m(\angle ACB)$ (inscribed) $= 55°$ (2)

From (1) and (2) :

$\therefore m(\angle ACB) = m(\angle ABC) = 55°$

$\therefore AB = AC$ (Q.E.D.)

3

[a] $\because m(\angle A) = \frac{1}{2}[m(\widehat{HC}) - m(\widehat{BD})]$

$\therefore 30° = \frac{1}{2}[120° - m(\widehat{BD})]$

$\therefore 60° = 120° - m(\widehat{BD})$

$\therefore m(\widehat{BD}) = 120° - 60° = 60°$ (First req.)

, $\because m(\widehat{BC}) = m(\widehat{DH})$

$\therefore BC = DH$

By adding $m(\widehat{BD})$ to both sides.

$\therefore m(\widehat{CD}) = m(\widehat{HB})$ $\therefore m(\angle C) = m(\angle H)$

In Δ ACH : $\because AC = AH$

, $\because BC = DH$

By subtracting : $AB = AD$ (Second req.)

[b] In Δ ABD :

$\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 30°$

$\therefore m(\angle A) = 180° - 2 \times 30° = 120°$

$\therefore m(\angle A) + m(\angle C) = 120° + 60° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

4

[a] $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB) = 60°$

(inscribed and central angles subtended the same arc $\widehat{AB}$) (1)

$\because \overleftrightarrow{CD} \parallel \overline{AB}$

$\therefore m(\widehat{AC}) = m(\widehat{BC})$

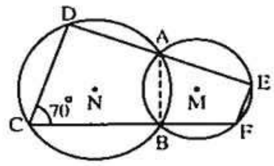
$\therefore AC = BC$ (2)

From (1) and (2) :

$\therefore \Delta$ CAB is equilateral. (Q.E.D.)

[b] **Construction :**

Draw $\overline{AB}$



Proof :

$\because$ The figure ABCD is a cyclic quadrilateral

$\therefore m(\angle BAD) = 180° - 70° = 110°$

, $\because$ The figure ABFE is a cyclic quadrilateral and $\angle$ BAD is an exteroir angle of it

$\therefore m(\angle F) = m(\angle BAD) = 110°$

$\therefore m(\angle F) + m(\angle C) = 110° + 70° = 180°$

but they are two interior angles on the same side of the transversal $\overleftrightarrow{FC}$

$\therefore \overline{CD} \parallel \overline{EF}$ (Q.E.D.)

5

[a] In Δ ABC :

$\because AC = BC$

$\therefore m(\angle BAC) = m(\angle ABC) = 65°$

$\therefore m(\angle CAD) = 130° - 65° = 65°$

, $\because m(\angle B) = m(\angle CAD) = 65°$

$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC (Q.E.D.)

[b] $\because \overrightarrow{AD}$ is a tangent

$\therefore \overline{MD} \perp \overline{AD}$ $\therefore m(\angle MDA) = 90°$

, $\because$ H is the midpoint of $\overline{BC}$

$\therefore \overline{MH} \perp \overline{BC}$ $\therefore m(\angle MHA) = 90°$

From the quadrilateral ADMH :

$\therefore m(\angle DMH) = 360° - (56° + 90° + 90°) = 124°$

(The req.)

4 El-Kalyoubia

1

1 c 2 a 3 d 4 b 5 d 6 c

2

[a] $\because \overline{AB} // \overline{CD}$ $\therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{BD}) = 50°$

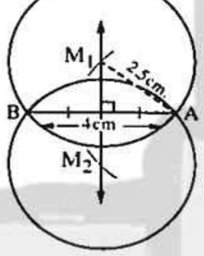
$, \because m(\angle BED) = \frac{1}{2} m(\overset{\frown}{BD})$

$\therefore (3y - 5)° = \frac{1}{2} \times 50° = 25°$

$\therefore 3y = 5° + 25° = 30°$

$\therefore y = 10°$ (The req.)

[b]



$\therefore$ We can draw two circles.

3

[a] $\because$ X is a midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle MXA) = 90°$ (1)

$, \because$ Y is a midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\therefore m(\angle MYA) = 90°$ (2)

From (1) and (2) :

$\therefore m(\angle MXA) = m(\angle MYA)$

but they are drawn on $\overline{AM}$ and on one side of it.

$\therefore$ AXYM is a cyclic quadrilateral. (Q.E.D.1)

In Δ MAC : $\therefore$ MA = MC = r

$\therefore m(\angle MCA) = m(\angle MAC)$

$, \because$ AXYM is a cyclic quadrilateral.

$\therefore m(\angle MXY) = m(\angle MAY)$

$\therefore m(\angle MXY) = m(\angle MCY)$ (Q.E.D.2)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle A) + m(\angle C) = 180°$

$\therefore m(\angle C) = 180° - 120° = 60°$ (First req.)

$\therefore m(\angle FBC) = m(\angle C) = 60°$ (alternate angles)

$\therefore m(\angle EBC) = 65° + 60° = 125°$

$, \because$ ABCD is a cyclic quadrilateral.

$\therefore m(\angle D) = m(\angle EBC) = 125°$ (Second req.)

4

[a] $\because$ The circle M $\cap$ The circle N = {A , B}

$\therefore \overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$

$\therefore$ In Δ ABD :

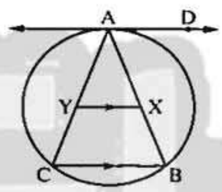
$\overleftrightarrow{DC}$ is the axis of symmetry of $\overline{AB}$

$\therefore$ AD = BD

$\because \overline{MX} \perp \overline{AD} , \overline{MY} \perp \overline{BD}$

$\therefore$ MX = MY (Q.E.D.)

[b]



$\because \overleftrightarrow{AD}$ is a tangent to the circle.

$\therefore m(\angle DAB)$ (tangency) $= m(\angle ACB)$ (inscribed) (1)

$, \because \overleftrightarrow{XY} // \overline{BC} , \overleftrightarrow{YC}$ is a transversal

$\therefore m(\angle AYX) = m(\angle ACB)$ (corresponding angles) (2)

$\therefore$ From (1) and (2) : $\therefore m(\angle DAB) = m(\angle AYX)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

5

[a] $\because \overline{AC}$ and $\overline{AB}$ are two tangent-segments to the circle M

$\therefore \overleftrightarrow{AE} \perp \overline{BC}$ $\therefore m(\angle CEM) = 90°$

$, \because \overline{BD}$ is a diameter in the circle M

$\therefore m(\angle ECD) = 90°$

$\therefore m(\angle CEM) + m(\angle ECD) = 180°$

, but they are two interior angles in the same side of the transversal $\overleftrightarrow{BC}$

$\therefore \overline{AM} // \overline{CD}$ (Q.E.D.)

[b] $\because \overline{CM} \parallel \overline{AB}$, $\overleftrightarrow{MA}$ is a transversal.

$\therefore m(\angle MAB) = m(\angle AMC)$ (alternate angles)

, $\because m(\angle AMC) = 2\,m(\angle B)$

(central and inscribed angles subtended by $\widehat{AC}$)

$\therefore m(\angle EAB) = 2\,m(\angle B)$

$\therefore m(\angle EAB) > m(\angle B)$

From Δ EAB : $\therefore BE > AE$ (Q.E.D.)

5 El-Sharkia

1

1 b 2 a 3 d 4 b 5 d 6 a

2

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$

, $\because AB = AC$ $\therefore MX = MY$

, $\because MD = ME = r$ $\therefore XD = YE$ (Q.E.D.)

[b] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle.

$\therefore AB = AC$

$\therefore$ In Δ ABC :

$m(\angle ABC) = m(\angle ACB) = \frac{180° - 70°}{2} = 55°$

, $\because$ BCDE is a cyclic quadrilateral.

$\therefore m(\angle EBC) + m(\angle D) = 180°$

$\therefore m(\angle EBC) = 180° - 125° = 55°$

$\therefore m(\angle ABC) = m(\angle EBC)$

$\therefore \overrightarrow{BC}$ bisects $\angle$ ABE (Q.E.D.)

3

[a] $\because$ ABDC is a cyclic quadrilateral.

$\therefore m(\angle A) + m(\angle D) = 180°$

$\therefore m(\angle A) = 180° - 140° = 40°$

, $\because \overline{AB}$ is a diameter.

$\therefore m(\angle ACB) = 90°$

In Δ ABC :

$\therefore m(\angle ABC) = 180° - (90° + 40°) = 50°$ (First req.)

, $\because m(\widehat{BD}) = m(\widehat{DC})$ $\therefore BD = CD$

In Δ BCD :

$\therefore m(\angle CBD) = m(\angle BCD) = \frac{180° - 140°}{2} = 20°$

$\therefore m(\widehat{BD}) = 2\,m(\angle BCD) = 2 \times 20° = 40°$

, $\because m(\widehat{AB}) = 180°$

$\therefore m(\widehat{ABD}) = 180° + 40° = 220°$ (Second req.)

[b] $\because \overleftrightarrow{AD}$ is a tangent to the circle

$\because m(\angle DAB)$ (tangen y)

$= m(\angle ACB)$ (inscribed) (1)

, $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{YC}$ is a transversal.

$\therefore m(\angle AYX) = m(\angle ACB)$

(corresponding angles) (2)

$\therefore$ From (1) and (2) :

$\therefore m(\angle DAB) = m(\angle AYX)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle which passes through the points A , X and Y (Q.E.D.)

4

[a] $\because m(\widehat{BD}) = 2\,m(\angle DCB) = 2 \times 25° = 50°$

, $\because$ D is midpoint of $(\widehat{AB})$

$\therefore m(\widehat{AB}) = 2 \times 50° = 100°$

$\therefore m(\angle AMB) = m(\widehat{AB}) = 100°$ (The req.)

[b] $\because \Delta$ ABC is equilateral.

$\therefore m(\angle B) = 60°$

$\therefore m(\angle D) = m(\angle B) = 60°$

(two inscribed angles subtended by $\widehat{AC}$)

, $\because AD = DE$

$\therefore \Delta$ ADE is an equilateral triangle. (Q.E.D.1)

$\because m(\angle DAE) = m(\angle BAC) = 60°$

Subtracting $\angle$ BAE from both sides.

$\therefore m(\angle DAB) = m(\angle EAC)$ (Q.E.D.2)

5

[a] $\because \overline{AB}$ is a tangen-tsegment to the circle.

$\therefore \overline{MA} \perp \overline{AB}$ $\therefore m(\angle A) = 90°$

In Δ MAB : $\because \tan(\angle B) = \frac{AM}{AB}$

$\therefore \tan 30° = \frac{8}{AB}$

$\therefore AB = \frac{8}{\tan 30°} = 8\sqrt{3}$ cm.

In Δ MAB : $\because$ m ($\angle$ AMB) = 180° – (90° + 30°)
= 60°

, $\because$ m ($\angle$ XAB) = $\frac{1}{2}$ m ($\angle$ AMB)

(tangency and central angles)

$\therefore$ m ($\angle$ XAB) = $\frac{1}{2}$ × 60° = 30°

In Δ XAB :

$\because$ m ($\angle$ XAB) = m ($\angle$ XBA)

$\therefore$ Δ XAB is an isosceles triangle. (Second req.)

[b] In $\Delta\Delta$ ADE , ACE :

$\begin{cases} m(\angle DAE) = m(\angle CAE) \\ AD = AC \\ \overline{AE} \text{ is a common side} \end{cases}$

$\therefore$ Δ ADE $\equiv$ Δ ACE

$\therefore$ m ($\angle$ ADE) = m ($\angle$ ACE)

, $\because$ m ($\angle$ AFB) = m ($\angle$ ACB)

(two inscribed angles subtended by $\overset{\frown}{AB}$)

$\therefore$ m ($\angle$ ADE) = m ($\angle$ EFB)

$\therefore$ DBFE is a cyclic quadrilateral. (Q.E.D.)

6 El-Monofia

1

1 c 2 a 3 b 4 b 5 c 6 b

2

[a] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M

$\therefore$ $\overline{AB} \perp \overline{MB}$, $\overline{AC} \perp \overline{MC}$

$\therefore$ m ($\angle$ BAC) = 360° – (90° + 90° + 90°) = 90°

, MB = MC = r

$\therefore$ ABMC is a square. (Q.E.D.)

[b] In Δ AMB : $\because$ AM = MB = r

$\therefore$ m ($\angle$ MAB) = m ($\angle$ ABM)

$\because$ m ($\angle$ CAB) = m ($\angle$ MAB)

$\therefore$ m ($\angle$ CAB) = m ($\angle$ ABM) and they are alternate angles.

$\therefore$ $\overline{AC}$ // $\overline{BM}$

$\because$ D is the midpoint of $\overline{AC}$

$\therefore$ $\overline{MD} \perp \overline{AC}$

, $\because$ $\overline{AC}$ // $\overline{BM}$

$\therefore$ $\overline{DM} \perp \overline{BM}$ (Q.E.D.)

3

[a] $\because$ $\overline{AX}$, $\overline{AZ}$ are two tangent-segments

$\therefore$ AX = AZ = 6 cm. $\because$ AC = 10 cm.

$\therefore$ CZ = 10 – 6 = 4 cm.

$\because$ $\overline{CY}$, $\overline{CZ}$ are two tangent-segments

$\therefore$ CY = CZ = 4 cm.

$\because$ $\overline{BX}$, $\overline{BY}$ are two tangent-segments

$\therefore$ BX = BY

$\because$ The perimeter of Δ ABC = 24 cm.

$\therefore$ BX + BY + 6 + 10 + 4 = 24

$\therefore$ BX + BY = 4 $\therefore$ BX = 2 cm.

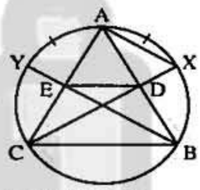
$\therefore$ AB = 6 + 2 = 8 cm. (First req.)

$\because$ $(AC)^2 = (10)^2 = 100$

, $(AB)^2 + (BC)^2 = (8)^2 + (6)^2 = 100 = (AC)^2$

$\therefore$ Δ ABC is a right-angled triangle at B
(Second req.)

[b] $\because$ m ($\overset{\frown}{AX}$) = m ($\overset{\frown}{AY}$)



$\therefore$ m ($\angle$ ACX) = m ($\angle$ ABY)

and they are drawn on $\overline{DE}$ and on one side of it

$\therefore$ The figure BCED is a cyclic quadrilateral.
(Q.E.D.1)

$\therefore$ m ($\angle$ DEB) = m ($\angle$ DCB)

(drawn on $\overline{DB}$ and on one side of it)

, $\because$ m ($\angle$ XAB) = m ($\angle$ XCB)

(two inscribed angles subtended by $\overset{\frown}{XB}$)

$\therefore$ m ($\angle$ DEB) = m ($\angle$ XAB) (Q.E.D.2)

4

[a] In Δ ABC : $\because$ CA = CB (1)

$\therefore$ m ($\angle$ A) = m ($\angle$ B) $\therefore$ sin A = sin B

$\therefore$ $\frac{XM}{AM} = \frac{YM}{BM}$ $\because$ AM = BM = r

$\therefore$ XM = YM

, $\because$ $\overline{MX} \perp \overline{DA}$, $\overline{MY} \perp \overline{EB}$

$\therefore$ DA = EB (2)

Subtracting (2) from (1) : $\therefore$ CD = CE (Q.E.D.)

[b] $\because$ $\overline{AB}$ is a diameter in the circle M

$\therefore$ m ($\angle$ ACB) = 90°

, $\because$ $\overline{ED} \perp \overline{AB}$

$\therefore$ m ($\angle$ FDA) = 90°

$\therefore$ m ($\angle$ ACF) + m ($\angle$ FDA) = 90° + 90° = 180°

$\therefore$ The figure ADFC is a cyclic quadrilateral. (Q.E.D.1)

$\because$ $\overleftrightarrow{EC}$ is a tangent of the circle M

$\therefore$ m ($\angle$ ECB) (tangency) = m ($\angle$ CAB) (inscribed)

, $\because$ $\angle$ CFE is an exterior angle of the cyclic quadrilateral ADFC

$\therefore$ m ($\angle$ CAB) = m ($\angle$ CFE)

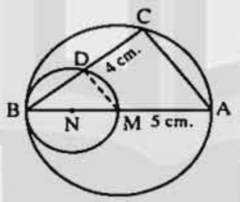
$\therefore$ m ($\angle$ ECF) = m ($\angle$ CFE)

In Δ ECF : $\therefore$ Δ ECF is an isosceles triangle. (Q.E.D.2)

5

[a] Construction :

Draw $\overline{MD}$



Proof :

$\because$ $\overline{BM}$ is a diameter in the circle N

$\therefore$ m ($\angle$ MDB) = 90° $\quad$ $\therefore$ $\overline{MD} \perp \overline{BC}$

$\therefore$ CD = DB = 4 cm. $\quad$, $\because$ MB = AM = 5 cm.

In Δ ABC :

$\therefore (AC)^2 = (AB)^2 - (BC)^2 = (10)^2 - (8)^2 = 100 - 64 = 36$

$\therefore$ AC = 6 cm. (The req.)

[b] $\because$ $\overleftrightarrow{AD}$ is a tangent to the circle

$\therefore$ m ($\angle$ DAB) (tangency) = m ($\angle$ ACB) (inscribed) (1)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{YC}$ is a transversal

$\therefore$ m ($\angle$ AYX) = m ($\angle$ ACB) (corresponding angles) (2)

From (1) and (2) :

$\therefore$ m ($\angle$ DAB) = m ($\angle$ AYX)

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

7 El-Gharbia

1

1 b $\quad$ 2 a $\quad$ 3 d $\quad$ 4 c $\quad$ 5 b $\quad$ 6 d

2

[a] $\because$ $\overline{AB}$ // $\overline{CD}$, $\overleftrightarrow{AD}$ is a transversal

$\therefore$ m ($\angle$ ADC) = m ($\angle$ BAD) = 20° (alternate angles)

, $\because$ m ($\angle$ AEC) = m ($\angle$ ADC) = 20° (two inscribed angles subtended by $\overset{\frown}{AC}$)

$\therefore 3x - 7 = 20$ $\quad$ $\therefore 3x = 27$

$\therefore x = 9$ (The req.)

[b] $\because$ $\overline{BD}$ is a tangent-segment to the circle

$\therefore$ m ($\angle$ ABD) = 90°

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{ME} \perp \overline{AC}$ $\quad$ $\therefore$ m ($\angle$ MED) = 90°

$\therefore$ m ($\angle$ MBD) + m ($\angle$ MED) = 90° + 90° = 180°

$\therefore$ The figure MEDB ia a cyclic quadrilateral. (Q.E.D.1)

$\because$ $\angle$ BMX is an exterior angle of the cyclic quadrilateral MEDB

$\therefore$ m ($\angle$ D) = m ($\angle$ BMX)

, $\because$ m ($\angle$ BAX) = $\frac{1}{2}$ m ($\angle$ BMX) (inscribed and central angles subtended by $\overset{\frown}{XB}$)

$\therefore$ m ($\angle$ BAX) = $\frac{1}{2}$ m ($\angle$ D) (Q.E.D.2)

3

[a] In Δ ABC : $\because$ m ($\angle$ BAC) = 90°

$\therefore \tan B = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore$ m ($\angle$ B) = 30°

$\therefore$ m ($\angle$ ABC) = m ($\angle$ DAC) = 30°

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC (Q.E.D.)

[b] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments of the circle

$\therefore$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) (1)

, $\because$ $\overline{AB}$ // $\overline{CD}$ and $\overleftrightarrow{BC}$ is a transversal

$\therefore$ m ($\angle$ BCD) = m ($\angle$ ABC) (2) (alternate angles)

From (1) and (2) : $\therefore$ m ($\angle$ BCD) = m ($\angle$ ACB)

$\therefore$ $\overrightarrow{CB}$ bisects $\angle$ ACD (Q.E.D.)

4

[a] $\because \angle AMB$ is an exterior angle of the ΔAMD

$\therefore m(\angle AMB) = m(\angle ADM) + m(\angle DAM)$

$\therefore 80^\circ = 30^\circ + m(\angle DAM)$

$\therefore m(\angle DAM) = 80^\circ - 30^\circ = 50^\circ$

In ΔADC : $\because DA = DC$

$\therefore m(\angle DCA) = m(\angle DAC) = 50^\circ$

$\therefore m(\angle ABD) = m(\angle ACD)$

and they are drawn on $\overline{AD}$ and on one side of it

$\therefore$ The figure ABCD is a cyclic quadrilateral.

(Q.E.D.)

[b] $\because$ X is the midpont of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$

, $\because AB = AC$ $\therefore MX = MY$

, $\because MD = ME = r$

$\therefore XD = YE$ (Q.E.D)

5

[a] $\because m(\overset{\frown}{AD}) = 2\, m(\angle ABD) = 2 \times 22^\circ = 44^\circ$

, $\because m(\angle C) = \frac{1}{2}\, [m(\overset{\frown}{BE}) - m(\overset{\frown}{AD})]$

$\because 36^\circ = \frac{1}{2}\, [m(\overset{\frown}{BE}) - 44^\circ]$

$\therefore 72^\circ = m(\overset{\frown}{BE}) - 44^\circ$

$\therefore m(\overset{\frown}{BE}) = 116^\circ$ (The req.)

[b] $\because m(\angle BDC) = m(\angle BAC)$

(two inscribed angles subtended by $\overset{\frown}{BC}$)

$\therefore m(\angle BDC) = 30^\circ$ (First req.)

$\because m(\overset{\frown}{BC}) = 2\, m(\angle BAC) = 60^\circ$

, $\because \overline{AB}$ is diameter in the circle M

$\therefore m(\overset{\frown}{AB}) = 180^\circ$

$\therefore m(\overset{\frown}{AC}) = 180^\circ - 60^\circ = 120^\circ$

, $\because$ D is the midpoint of $\overset{\frown}{AC}$

$\therefore m(\overset{\frown}{AD}) = \frac{120^\circ}{2} = 60^\circ$

$\therefore m(\angle ACD) = \frac{1}{2}\, m(\overset{\frown}{AD}) = \frac{1}{2} \times 60^\circ = 30^\circ$

$\therefore m(\angle BAC) = m(\angle ACD)$ but they are alternate angles

$\therefore \overline{DC} \parallel \overline{AB}$ (Second req.)

8 El-Dakahlia

1

[a] 1 a   2 d   3 c

[b] $\because \angle ABH$ is an exterior angle of the cyclic quadrilateral ABCD

$\therefore m(\angle ADC) = m(\angle ABH) = 110^\circ$

In ΔACD :

$\because m(\angle ACD) = 180^\circ - (110^\circ + 35^\circ) = 35^\circ$

$\therefore m(\angle CAD) = m(\angle ACD)$ $\therefore CD = AD$

$\therefore m(\overset{\frown}{CD}) = m(\overset{\frown}{AD})$ (Q.E.D.)

2

[a] 1 c   2 a   3 d

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$

(First req.)

$\because$ BCHD is a cyclic quadrilateral

$\therefore m(\angle C) + m(\angle D) = 180^\circ$

$\therefore m(\angle C) = 180^\circ - 125^\circ = 55^\circ$

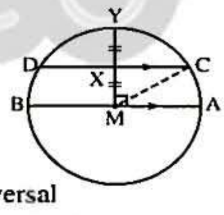
$\because m(\angle BHC)$ (inscribed)

$= m(\angle ABC)$ (tangency) $= 55^\circ$

$\therefore m(\angle BCH) = m(\angle BHC)$

In ΔBCH : $\therefore CB = BH$ (Second req.)

3



[a] Construction :

Draw $\overline{MC}$

Proof :

$\because \overline{CD} \parallel \overline{AB}$, $\overleftrightarrow{MY}$ is a transversal

$\therefore m(\angle MXC) + m(\angle XMA) = 180^\circ$

$\therefore m(\angle MXC) = 90^\circ$

, $\because MX = \frac{1}{2}\, MY$, $MY = MC$

$\therefore MX = \frac{1}{2}\, MC$ $\therefore m(\angle MCX) = 30^\circ$

$\therefore m(\angle AMC) = m(\angle MCX) = 30^\circ$

(alternate angles)

$\therefore m(\overset{\frown}{AC}) = m(\angle AMC) = 30^\circ$ (First req.)

, $\because (\overset{\frown}{AY}) = m(\angle AMY) = 90^\circ$

$\therefore m(\overset{\frown}{CY}) = 90^\circ - 30^\circ = 60^\circ$ (Second req.)

[b] $\because AB = AC$

, $\overline{MD} \perp \overline{AB}$, $\overline{MH} \perp \overline{AC}$

$\therefore MD = MH$

, $\because MX = MY = r$ $\therefore XD = HY$ (Q.E.D.)

4

[a] $\because \overline{AO} \,/\!/\, \overline{DH}$, $\overline{AH}$ is a transversal

$\therefore m(\angle HAO) = m(\angle AHD)$ (alternate angles) (1)

, $\because m(\angle C)$ (inscribed)

$= m(\angle BAO)$ (tangency) (2)

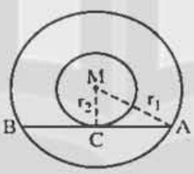
From (1) and (2) :

$\therefore m(\angle C) = m(\angle AHD)$

$\therefore$ DHBC is a cyclic quadrilateral (Q.E.D.)

[b] **Construction :**

Draw $\overline{MA}$, $\overline{MC}$



Proof :

$\because \overline{AB}$ touches the smaller circle at C

$\therefore \overline{MC} \perp \overline{AB}$

, $\because \overline{AB}$ is a chord of the greater circle

, $\overline{MC} \perp \overline{AB}$

$\therefore$ C is the midpoint of $\overline{AB}$

$\therefore AC = \frac{14}{2} = 7$ cm.

$\because \Delta AMC$ is a right-angled at C

$\therefore (AC)^2 = (MA)^2 - (MC)^2$

$\therefore (7)^2 = r_1 - r_2$ $\therefore r_1 - r_2 = 49$

$\therefore$ The area of the part included between the two circles = The area of the greater circle – The area of the smaller circle $= \pi r_1^2 - \pi r_2^2 = \pi (r_1^2 - r_2^2)$

$= \frac{22}{7} \times 49 = 154 \text{ cm}^2$. (The req.)

5

[a] $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended the same arc $\overset{\frown}{AB}$)

$\therefore m(\angle ACB) = \frac{1}{2} \times 120° = 60°$ (1)

$\because \overline{CD} \,/\!/\, \overline{AB}$ $\therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{BC})$

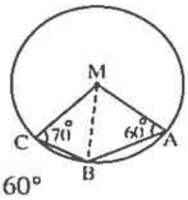
$\therefore AC = BC$ (2)

From (1) and (2) :

$\therefore \Delta ABC$ is equilateral (Q.E.D.)

[b] **Construction :**

Draw $\overline{MB}$



Proof :

In ΔMAB :

$\because MA = MB = r$, $m(\angle MAB) = 60°$

$\therefore \Delta AMB$ is equilateral

$\therefore m(\angle AMB) = 60°$ (1)

In ΔMBC : $\because MB = MC = r$

$\therefore m(\angle MBC) = m(\angle MCB) = 70°$

$\therefore m(\angle CMB) = 180° - (70° + 70°) = 40°$ (2)

From (1) and (2) :

$\therefore m(\angle AMC) = m(\angle AMB) + m(\angle CMB)$

$= 60° + 40° = 100°$ (The req.)

9 Ismailia

1

1 c 2 b 3 c 4 a 5 d 6 b

2

[a] $\because m(\angle A) = \frac{1}{2} m(\angle BMC) = x°$

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

, $\because$ The figure ABDC is a cyclic quadrilateral

$\therefore m(\angle A) + m(\angle BDC) = 180°$

$\therefore x + 2x = 180°$ $\therefore 3x = 180°$

$\therefore x = 60°$ $\therefore m(\angle A) = 60°$ (The req.)

[b] $\because m(\angle A) = m(\angle B)$

(two inscribed angles subtended by $\overset{\frown}{CD}$)

, $m(\angle C) = m(\angle D)$

(two inscribed angles subtended by $\overset{\frown}{AB}$)

$\because EA = ED$ $\therefore m(\angle A) = m(\angle D)$

$\therefore m(\angle C) = m(\angle B)$

$\therefore EB = EC$ (Q.E.D.)

3

[a] In ΔABC :

$\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

, $\because \overrightarrow{BX}$ bisects $(\angle ABC)$, $\overrightarrow{CY}$ bisects $(\angle ACB)$

$\therefore m(\angle XBY) = m(\angle YCX)$

and they are drawn on $\overline{XY}$ and on one side of it

$\therefore$ BCXY is a cyclic quadrilateral (Q.E.D.)

[b] $\because$ m ($\angle$ BAC) = $\frac{1}{2}$ m ($\widehat{BC}$)

$\therefore$ m ($\angle$ BAC) = $\frac{1}{2} \times 120° = 60°$

In Δ ABC : $\therefore$ m ($\angle$ C) = 180° − (70° + 60°) = 50°

$\therefore$ m ($\angle$ DAB) = m ($\angle$ C) = 50°

(inscribed and tangency angles subtended by $\widehat{AB}$)

(The req.)

4

[a] $\because$ $\overline{AC}$ is a diameter of the circle.

$\therefore$ m ($\angle$ ABC) = 90°

, $\because$ m ($\angle$ ABD) = 60°

$\therefore$ m ($\angle$ CBD) = 90° − 60° = 30° (First req.)

, $\because$ m ($\angle$ ADB) = m ($\angle$ C) = 50°

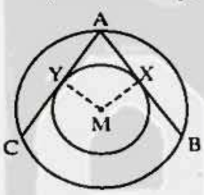
(two inscribed angles subtended by $\widehat{AB}$)

In Δ ABD :

$\therefore$ m ($\angle$ BAD) = 180° − (60° + 50°) = 70°

(Second req.)

[b] Construction :



Draw $\overline{MX}$, $\overline{MY}$

Proof :

In the smaller circle M

$\because$ $\overline{AB}$, $\overline{AC}$ are two tangents

, $\overline{MX}$, $\overline{MY}$ are two radii

$\therefore$ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, $\because$ MX = MY = r (radii of the smaller circle)

$\therefore$ AB = AC (Q.E.D.)

5

[a] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the greater circle

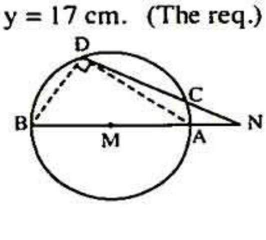
$\therefore$ 2 x − 3 = 15 $\therefore$ 2 x = 18

$\therefore$ x = 9 cm.

, $\because$ $\overline{AC}$, $\overline{AD}$ are two tangent-segments to the smaller circle

$\therefore$ y − 2 = 15 $\therefore$ y = 17 cm. (The req.)

[b] Construction :



Draw $\overline{AD}$, $\overline{BD}$

Proof :

$\because$ $\overline{AB}$ is a diameter of the circle

$\therefore$ m ($\angle$ ADB) = 90°

$\therefore$ m ($\angle$ ADB) + m ($\angle$ ADN) > 90°

In Δ NDB : $\therefore$ NB > ND (Q.E.D.)

10 Suez

1

1 b 2 b 3 a 4 c 5 d 6 b

2

[a] $\because$ E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{ME} \perp \overline{AC}$

, $\because$ $\overline{MD} \perp \overline{AB}$, MD = ME

$\therefore$ AB = AC (Q.E.D.)

[b] $\because$ m ($\angle$ A) = $\frac{1}{2}$ m ($\angle$ BMC)

(inscribed and central angles subtended by $\widehat{BC}$)

$\therefore$ m ($\angle$ A) = $\frac{1}{2} \times 100° = 50°$ (First req.)

In Δ MBC : $\because$ MB = MC = r

$\therefore$ m ($\angle$ MBC) = m ($\angle$ MCB)

= $\frac{1}{2}$ (180° − 100°) = 40°

(Second req.)

3

[a] $\because$ $\overline{AB}$ is a diameter of the circle

$\therefore$ m ($\angle$ AEB) = 90° (First req.)

, $\because$ $\angle$ AEB is an exterior angle of Δ AEC

$\therefore$ m ($\angle$ AEB) = m ($\angle$ CAE) + m ($\angle$ ACE)

$\therefore$ m ($\angle$ CAE) = 90° − 60° = 30° (Second req.)

[b] $\because$ $\overleftrightarrow{AD}$ is a tangent to the circle

$\therefore$ $\overline{MD} \perp \overleftrightarrow{AD}$ $\therefore$ m ($\angle$ ADM) = 90°

, $\because$ E is the midpoint of $\overline{BC}$

$\therefore$ $\overline{ME} \perp \overline{BC}$ $\therefore$ m ($\angle$ MEA) = 90°

$\therefore$ In the quadrilateral ADME :

$\therefore$ m ($\angle$ DME) = 360° − (90° + 90° + 70°) = 110°

(The req.)

4

[a] State by yourself.

[b] $\because$ ABC is an equilateral triangle

$\therefore m(\angle A) = 60°$

$\therefore m(\angle D) = m(\angle A)$ and they are drawn on $\overline{BC}$ and on one side of it

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] $m(\widehat{AB}) = 2\, m(\angle ADB) = 60°$ (First req.)

, $m(\angle DCB) = \frac{1}{2}[m(\widehat{AD}) + m(\widehat{AB})]$

$= \frac{1}{2}[90° + 60°] = 75°$ (Second req.)

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle.

$\therefore AB = AC$

$\therefore$ In Δ ABC :

$m(\angle ABC) = m(\angle ACB) = \frac{1}{2}(180° - 40°) = 70°$ (First req.)

$\because \overrightarrow{AB} \,//\, \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal

$\therefore m(\angle BCD) = m(\angle ABC) = 70°$ (1)

(alternate angles)

, $\because m(\angle BDC)$ (inscribed)

$= m(\angle ABC)$ (tangency) $= 70°$ (2)

From (1) and (2) :

$\therefore m(\angle BCD) = m(\angle BDC)$

$\therefore$ In Δ BCD : BC = BD (Second req.)

11 Port Said

1

1 d 2 c 3 b 4 b 5 a 6 b

2

[a] $\because$ MF = ME (lengths of two radii)

, XF = YE $\therefore$ MX = MY

$\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

$\therefore$ AB = CD (Q.E.D.1)

$\because \overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$

$\therefore AX = \frac{1}{2} AB$, $\because \overline{MY} \perp \overline{CD}$

$\therefore$ Y is the midpoinf of $\overline{CD}$

$\therefore CY = \frac{1}{2} CD$, $\because$ AB = CD

$\therefore$ AX = CY

$\therefore$ In $\Delta\Delta$ AXF , CYE

$\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90° \end{cases}$

$\therefore \Delta AXF \equiv \Delta CYE$, AF = CE (Q.E.D.2)

[b] $\because m(\angle A) = \frac{1}{2}[m(\widehat{CE}) - m(\widehat{BD})]$

$\therefore 30° = \frac{1}{2}[120° - m(\widehat{BD})]$

$\therefore 60° = 120° - m(\widehat{BD})$

$\therefore m(\widehat{BD}) = 120° - 60° = 60°$ (The req.)

3

[a] In Δ ABC : $\because m(\angle BAC) = 90°$, $AC = \frac{1}{2} BC$

$\therefore m(\angle B) = 30°$

$\therefore m(\angle C) = 180° - (90° + 30°) = 60°$

$\therefore m(\angle C) = m(\angle DAB) = 60°$

$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC (Q.E.D.)

[b] $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$ $\therefore m(\angle ADM) = 90°$

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore \overline{ME} \perp \overline{AC}$ $\therefore m(\angle AEM) = 90°$

From the quadrilateral MDAE :

$\therefore m(\angle DME) = 360° - (90° + 90° + 120°) = 60°$

, $\because m(\angle YMX) = m(\angle DME) = 60°$ (V.O.A)

, MY = MX = r

$\therefore \Delta$ XMY is an equilateral triangle. (Q.E.D.)

4

[a] In Δ AMC : $\because$ MA = MC = r

$\therefore m(\angle MCA) = m(\angle MAC) = 25°$ (1)

In Δ BMC : $\because$ MB = MC = r

$\therefore m(\angle MCB) = m(\angle MBC) = 45°$ (2)

From (1) and (2) :

$\therefore m(\angle ACB) = m(\angle MCA) + m(\angle MCB)$

$\therefore m(\angle ACB) = 25° + 45° = 70°$

$\therefore m(\angle AMB) = 2\, m(\angle ACB) = 2 \times 70° = 140°$

(central and inscribed angles subtended by $\widehat{AB}$)

(The req.)

[b] $\because$ ABCE is a cyclic quadrilateral

$\therefore m(\angle XEA) = m(\angle ABC)$

, $\because$ ABDF is a cyclic quadrilateral

$\therefore m(\angle XFA) = m(\angle ABD)$

, $\because m(\angle ABC) + m(\angle ABD) = 180^\circ$

$\therefore m(\angle XEA) + m(\angle XFA) = 180^\circ$

$\therefore$ AFXE is a cyclic quadrilateral. (Q.E.D.)

5

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the greater circle

$\therefore AB = AC$

$\therefore 2x - 3 = 15$ $\therefore 2x = 18$

$\therefore x = 9$ cm.

, $\because \overline{AC}$, $\overline{AD}$ are two tangent-segments to the smaller circle

$\therefore AC = AD$ $\therefore y - 2 = 15$

$\therefore y = 17$ cm. (The req.)

[b] $\because$ ABCD is a parallelogram

$\therefore AD = BC$, $\because BE = AD$

$\therefore BC = BE$

$\therefore$ In Δ BCE : $m(\angle C) = m(\angle BEC)$

, $\because m(\angle C) = m(\angle BAD)$ (from the parallelogram)

$\therefore m(\angle BAD) = m(\angle BED)$ and they are drawn on $\overline{BD}$ and on one side of it

$\therefore$ The figure ABDE is a cyclic quadrilateral. (Q.E.D.)

12 Damietta

1

1 b 2 d 3 c 4 b 5 a 6 b

2

[a] $\because \overrightarrow{AD}$ is a tangent

$\therefore \overline{MD} \perp \overline{AD}$ $\therefore m(\angle MDA) = 90^\circ$

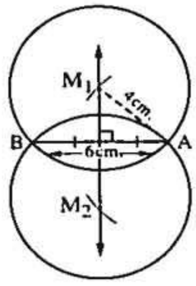
, $\because$ E is a midpoint of $\overline{BC}$

$\therefore \overline{ME} \perp \overline{BC}$ $\therefore m(\angle MEA) = 90^\circ$

From the quadrilateral ADME

$\therefore m(\angle DME) = 360^\circ - (65^\circ + 90^\circ + 90^\circ) = 115^\circ$ (The req.)

[b]



$\therefore$ We can draw two circles.

3

[a] $\because m(\angle BMC) = 2\, m(\angle A)$

(central and inscribed angles subtended by $\widehat{BC}$)

$\therefore m(\angle BMC) = 2 \times 30^\circ = 60^\circ$ (First req.)

In Δ MBC : $\because MB = MC = r$

, $m(\angle BMC) = 60^\circ$

$\therefore \Delta$ MBC is equilateral. (Second req.)

[b] $\because \overline{AD} \parallel \overline{BC}$

$\therefore m(\widehat{AB}) = m(\widehat{DC})$ $\therefore AB = DC$

, $\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{DC}$

$\therefore MX = MY$ (Q.E.D.)

4

[a] $\because \overrightarrow{CB}$ is a tangent

$\therefore m(\angle BAE) = m(\angle CBE)$

(inscribed and tangency angles subtended by $\widehat{BE}$)

, $\because m(\widehat{BE}) = m(\widehat{EA})$

$\therefore m(\angle BAE) = m(\angle EAF)$

$\therefore m(\angle CBD) = m(\angle CAD)$ and they are drawn on $\overline{CD}$ and on one side of it

$\therefore$ ABCD is a cyclic quadrilateral (Q.E.D.)

[b] $\because m(\angle XYZ)$ (tangency) $= m(\angle L)$ (inscribed) $= 70^\circ$

, $\because \overline{XY}$, $\overline{XZ}$ are two tangents

$\therefore XY = XZ$

$\therefore m(\angle XYZ) = m(\angle XZY) = 70^\circ$

In Δ XYZ :

$\therefore m(\angle X) = 180^\circ - 2 \times 70^\circ = 40^\circ$ (First req.)

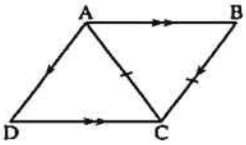
In Δ LZY : $\because YZ = LZ$

$\therefore m(\angle LYZ) = m(\angle L) = 70^\circ$

, $\because m(\angle LYZ) = m(\angle XZY)$ and they are alternate angles.

$\therefore \overline{XZ} \parallel \overline{YL}$ (Second req.)

5



[a] In ABC :

$\because AC = BC$

$\therefore m(\angle B) = m(\angle CAB)$ (1)

, $\because \overline{AB} \,//\, \overline{CD}$, $\overleftrightarrow{AC}$ is transversal

$\therefore m(\angle DCA) = m(\angle CAB)$ (alternate angles) (2)

From (1) and (2) : $\therefore m(\angle DCA) = m(\angle B)$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle circumscribed about the triangle ABC (Q.E.D.)

[b] $\because$ LMNE is a cyclic quadrilateral

$\therefore m(\angle MLN) = m(\angle MEN) = 35°$ (First req.)

, $\because m(\angle ELN) = m(\angle ELM) - m(\angle MLN)$

$\therefore m(\angle ELN) = 80° - 35° = 45°$

$\therefore m(\angle EMN) = m(\angle ELN) = 45°$ (Second req.)

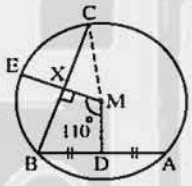
13 Kafr El-Sheikh

1

1 a	2 c	3 c	4 b	5 b	6 d

2

[a] Construction :



Draw $\overline{MC}$

Proof :

$\because \overline{MX} \perp \overline{BC}$

$\therefore$ X is the midpoint of $\overline{BC}$

$\therefore XC = 8$ cm.

In Δ XMC :

$\because m(\angle CXM) = 90°$, $CM = r = 10$ cm.

$\therefore MX = \sqrt{(CM)^2 - (XC)^2} = \sqrt{100 - 64}$

$= \sqrt{36} = 6$ cm.

$\therefore XE = 10 - 6 = 4$ cm. (First req.)

$\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$

From the quadrilateral BDMX :

$\therefore m(\angle ABC) = 360° - (90° + 90° + 110°) = 70°$ (Second req.)

[b] $\because \overrightarrow{BA}$ is a tangent

$\therefore \overline{MA} \perp \overline{AB}$ $\therefore m(\angle BAM) = 90°$

In ΔAMB : $m(\angle AMB) = 180° - (90° + 20°) = 70°$

, $\because m(\angle ADE) = \frac{1}{2} m(\angle AME)$

(inscribed and central angles subtended by $\widehat{AE}$)

$\therefore m(\angle ADB) = \frac{1}{2} \times 70° = 35°$ (The req.)

3

[a] $\because \overline{AD} \,//\, \overline{CB}$ $\therefore m(\widehat{BD}) = m(\widehat{AC})$

$\therefore m(\angle BAD) = m(\angle CDA)$

$\therefore$ In Δ ADE : EA = ED (Q.E.D.)

[b] $\because \overrightarrow{EA}$, $\overrightarrow{EB}$ are two tangents to the circle

$\therefore EA = EB$

In Δ ABE :

$\therefore m(\angle EAB) = m(\angle EBA) = \frac{180° - 50°}{2} = 65°$

, $\because m(\angle ADC)$ (inscribed)

$= m(\angle CAE)$ (tangency) $= 115°$

$\therefore m(\angle BAC) = 115° - 65° = 50°$

$\because m(\angle AEB) = m(\angle BAC)$

$\therefore \overleftrightarrow{AC}$ is a tangent to the circle passing through the points A , B and E (Q.E.D.)

4

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle ADC) = m(\angle ABE) = 110°$

, $\because m(\angle ADB) = \frac{1}{2} m(\widehat{AB}) = \frac{1}{2} \times 100° = 50°$

$\therefore m(\angle BDC) = 110° - 50° = 60°$ (The req.)

[b] $\because \overrightarrow{FB}$, $\overrightarrow{FD}$ are two tangents to the circle

$\therefore BF = DF = 4$ cm.

$\therefore AB = 10 + 4 = 14$ cm.

, $\because \overleftrightarrow{AB}$, $\overleftrightarrow{AC}$ are two tangents to the circle

$\therefore AC = AB = 14$ cm.

$\therefore EC = 14 - 9 = 5$ cm. (The req.)

5

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$

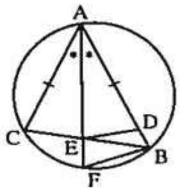
$\therefore \overline{MY} \perp \overline{AC}$, $\because MX = MY$

$\therefore AB = AC$

In Δ ABC : $\therefore m(\angle C) = m(\angle B) = 70°$

$\therefore m(\angle A) = 180° - (70° + 70°) = 40°$ (The req.)

[b] In ΔΔ ADE , ACE



$\begin{cases} AD = AC \\ m(\angle DAE) = m(\angle CAE) \\ \overline{AE} \text{ is a common side} \end{cases}$

∴ Δ ADE ≡ Δ ACE

, m (∠ ADE) = m (∠ ACE)

, ∵ m (∠ AFB) = m (∠ ACB)

(two inscribed angles subtended by $\overset{\frown}{AB}$)

∴ m (∠ AFB) = m (∠ ADE)

∴ BDEF is a cyclic quadrilateral. (Q.E.D.)

14 El-Beheira

1

1 d 2 c 3 b 4 b 5 c 6 a

2

[a] ∵ X is the midpoint of $\overline{AC}$

∴ $\overline{MX} \perp \overline{AC}$ ∴ m (∠ AXY) = 90°

, ∵ $\overrightarrow{YB}$ is a tangent to the circle

∴ $\overline{MB} \perp \overline{BY}$ ∴ m (∠ MBY) = 90°

, ∵ m (∠ AXY) = m (∠ ABY) and they are drawn on $\overline{AY}$ and on one side of it

∴ AXBY is a cyclic quadrilateral. (Q.E.D.)

[b] ∵ $\overline{CM} \parallel \overline{AB}$, $\overleftrightarrow{AM}$ is a transversal

∴ m (∠ CMA) = m (∠ A) = 60°

, ∵ m (∠ B) = $\frac{1}{2}$ m (∠ CMA)

(two inscribed angles subtended by $\overset{\frown}{AC}$)

∴ m (∠ B) = $\frac{1}{2} \times 60° = 30°$ (The req.)

3

[a] ∵ m (∠ B) = m (∠ C)   ∴ AB = AC

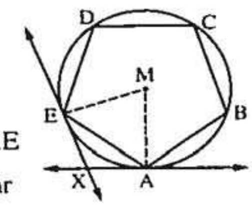
, ∵ X is the midpoint of $\overline{AB}$

∴ $\overline{MX} \perp \overline{AC}$, $\overline{MY} \perp \overline{AC}$

∴ MX = MY (Q.E.D.)

[b] Construction :

Draw $\overline{AM}$, $\overline{ME}$



Proof :

∵ AB = BC = CD = DE = AE

(The properties of the regular pentagon)

∴ m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{DE}$) = m ($\overset{\frown}{AE}$)

∵ measure of the circle = 360°

∴ m ($\overset{\frown}{AE}$) = $\frac{360°}{5}$ = 72° (First req.)

∴ m (∠ AME) = m ($\overset{\frown}{AE}$) = 72°

∵ $\overleftrightarrow{AX}$ is a tangent to the circle at A

∴ m (∠ MAX) = 90°

similarly m (∠ MEX) = 90°

In the quadrilateral MAXE :

∴ m (∠ AXE) = 360° – (72° + 90° + 90°) = 108°

(Second req.)

4

[a] In Δ AMC : ∵ AM = MC = r

∴ m (∠ MAC) = m (∠ ACM)

∵ m (∠ BAC) = m (∠ MAC)

∴ m (∠ BAC) = m (∠ ACM) and they are alternate angles.

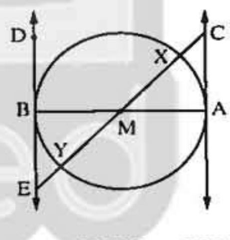
∴ $\overline{AB} \parallel \overline{CM}$

∵ D is the midpoint of $\overline{AB}$

∴ $\overline{MD} \perp \overline{AB}$, ∵ $\overline{AB} \parallel \overline{CM}$

∴ $\overline{DM} \perp \overline{CM}$ (Q.E.D.)

[b] ∵ $\overleftrightarrow{AC}$ is a tangent to the circle M at A



∴ $\overline{MA} \perp \overleftrightarrow{AC}$

∴ m (∠ CAM) = 90°

∵ $\overleftrightarrow{BD}$ is a tangent to the circle M at B

∴ $\overline{MB} \perp \overleftrightarrow{BD}$ ∴ m (∠ EBM) = 90°

In ΔΔ CAM , EBM :

$\begin{cases} m(\angle CAM) = m(\angle EBM) = 90° \\ m(\angle AMC) = m(\angle BME) \text{ (V.O.A.)} \\ MA = MB \text{ (lengths of two radii)} \end{cases}$

∴ The two triangles are congruent and we deduce that CM = EM

, ∵ XM = YM (lengths of two radii)

, by subtracting

∴ CX = YE (Q.E.D.)

5

[a] ∵ $\overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

∴ XA = XB

$\therefore$ In Δ ABX

m ($\angle$ XAB) = m ($\angle$ XBA) = $\frac{180° - 50}{2}$ = 65°

, $\because$ ABCD is a cyclic quadrilateral

m ($\angle$ BAD) + m ($\angle$ DCB) = 180°

$\therefore$ m ($\angle$ BAD) = 180° − 115° = 65°

$\therefore$ m ($\angle$ XAB) = m ($\angle$ BAD)

$\therefore$ $\overrightarrow{AB}$ bisects $\angle$ DAX (Q.E.D.1)

, $\because$ m ($\angle$ ADB) (inscribed)
= m ($\angle$ XAB) (tangency) = 65°

$\therefore$ m ($\angle$ BAD) = m ($\angle$ ADB)

$\therefore$ BD = BA (Q.E.D.2)

[b] $\because$ AB = CD

$\therefore$ m ($\widehat{AB}$) = m ($\widehat{CD}$)

Subtracting m ($\widehat{BD}$) from both sides

$\therefore$ m ($\widehat{AD}$) = m ($\widehat{BC}$)

$\therefore$ m ($\angle$ ACD) = m ($\angle$ BAC)

$\therefore$ In Δ ACE : AE = CE

$\therefore$ Δ ACE is an isosceles triangle. (Q.E.D.)

15 El-Fayoum

1

1 c 2 b 3 a 4 d 5 a 6 c

2

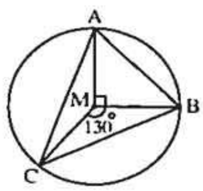
[a] $\because$ AB = CD

, $\overline{ME} \perp \overline{AB}$, $\overline{MO} \perp \overline{CD}$

$\therefore$ ME = MO $\therefore$ $x + 2 = 6$

$\therefore$ $x = 4$ cm. (First req.)

$\therefore$ CD = AB = 3 × 4 + 4 = 16 cm. (Second req.)



[b] $\because$ m ($\angle$ C) = $\frac{1}{2}$ m ($\angle$ AMB)
= $\frac{1}{2}$ × 90° = 45°

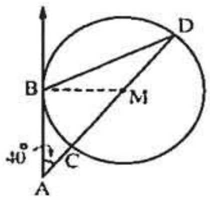
(inscribed and central angles subtended by $\widehat{AB}$)

, $\because$ m ($\angle$ A) = $\frac{1}{2}$ ($\angle$ BMC) = $\frac{1}{2}$ × 130° = 65°

(inscribed and central angles subtended by $\widehat{BC}$)

$\therefore$ m ($\angle$ B) = 180° − (45° + 65°) = 70° (The req.)

3



[a] Constraction :

Draw $\overline{MB}$

Proof :

$\because$ $\overrightarrow{AB}$ is a tangent to the circle

$\therefore$ $\overline{MB} \perp \overline{AB}$

$\therefore$ m ($\angle$ MBA) = 90°

In Δ ABM :

m ($\angle$ BMA) = 180° − (90° + 40°) = 50°

m ($\angle$ BDC) = $\frac{1}{2}$ m ($\angle$ BMC) = $\frac{1}{2}$ × 50° = 25°

(inscribed and central angles subtended by $\widehat{BC}$)

(The req.)

[b] $\because$ X is the midpoint of $\overline{AC}$

$\therefore$ $\overline{MX} \perp \overline{AC}$ $\therefore$ m ($\angle$ AXY) = 90°

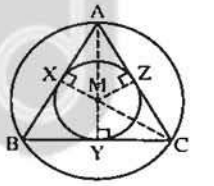
, $\because$ $\overrightarrow{YB}$ is a tangent to the circle

$\therefore$ $\overline{MB} \perp \overline{BY}$ $\therefore$ m ($\angle$ MBY) = 90°

, $\because$ m ($\angle$ AXY) = m ($\angle$ ABY) and they are drawn on $\overline{AY}$ and on one side of it

$\therefore$ AXBY is a cyclic quadrilateral (Q.E.D.)

4



[a] Construction :

Draw $\overline{XM}$, $\overline{YM}$, $\overline{ZM}$
, $\overline{AY}$, $\overline{CM}$

Proof :

$\because$ $\overline{XM} \perp \overline{AB}$, $\overline{YM} \perp \overline{BC}$
, $\overline{ZM} \perp \overline{AC}$

, $\because$ XM = YM = ZM = r

$\therefore$ AB = BC = AC

$\therefore$ Δ ABC is an equilateral triangle (First req.)

In Δ MYC : m ($\angle$ MYC) = 90°

$\therefore$ $(YC)^2 = (MC)^2 - (MY)^2 = (4)^2 - (2)^2 = 12$

$\therefore$ YC = $2\sqrt{3}$ cm. $\therefore$ BC = $4\sqrt{3}$ cm.

$\therefore$ The area of Δ ABC = $\frac{1}{2}$ × BC × AY
= $\frac{1}{2}$ × $4\sqrt{3}$ × 6
= $12\sqrt{3}$ cm². (Second req.)

[b] $\because$ m ($\angle$ BCD) = $\frac{1}{2}$ m ($\angle$ BMD)

(inscribed and central angles subtended by $\overset{\frown}{BD}$)

$\therefore$ m ($\angle$ BCD) = $\frac{1}{2} \times 130° = 65°$

$\because \overline{AB} \parallel \overline{CD}$, $\overrightarrow{BC}$ is a transversal

$\therefore$ m ($\angle$ ABC) = m ($\angle$ BCD) = 65°

(alternate angles) (1)

, $\because \overline{AB}$, $\overline{AC}$ are two tangent segments

$\therefore$ AB = AC

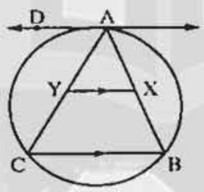
$\therefore$ m ($\angle$ ACB) = m ($\angle$ ABC) = 65° (2)

From (1) and (2) :

$\therefore$ m ($\angle$ ACB) = m ($\angle$ BCD) = 65°

$\therefore \overrightarrow{CB}$ bisects $\angle$ ACD (Q.E.D.)

5



[a] $\because \overleftrightarrow{AD}$ is a tangent to the circle

$\therefore$ m ($\angle$ DAC) (tangency)

= m ($\angle$ B) (inscribed) (1)

, $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m ($\angle$ AXY) = m ($\angle$ B) (2)

(corresponding angles)

From (1) and (2) : $\therefore$ m ($\angle$ AXY) = m ($\angle$ DAC)

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] $\because$ X is the midpoint of $\overline{AC}$

$\therefore \overline{MX} \perp \overline{AC}$

$\therefore$ m ($\angle$ CXM) = 90°

, $\because \overrightarrow{BD}$ is a tangent to the circle

$\therefore \overline{BD} \perp \overline{AB}$

$\therefore$ m ($\angle$ DBM) = 90°

$\therefore$ m ($\angle$ CXM) + m ($\angle$ DBM) = 180°

$\therefore$ XMBD is a cyclic quadrilateral (Q.E.D.1)

, $\because \angle$ BMY is an exterior angle of the cyclic quadrilateral XMBD

$\therefore$ m ($\angle$ BMY) = m ($\angle$ D) (1)

, $\because$ m ($\angle$ BAY) = $\frac{1}{2}$ m ($\angle$ BMY) (2)

(inscribed and central angles subtended the same arc $\overset{\frown}{BY}$)

From (1) and (2) :

$\therefore$ m ($\angle$ BAY) = $\frac{1}{2}$ m ($\angle$ D) (Q.E.D.2)

16 Beni Suef

1

1 c 2 a 3 c 4 c 5 b 6 c

2

[a] $\because$ m ($\angle$ AMB) = 2 m ($\angle$ ADB) = 2 × 70° = 140°

(central and inscribed angles subtended by $\overset{\frown}{AB}$)

In Δ ABM : $\because \overline{MC} \perp \overline{AB}$

, MA = MB = r

$\therefore$ m ($\angle$ AMC) = $\frac{1}{2}$ m ($\angle$ AMB) = $\frac{1}{2} \times 140 = 70$ (The req.)

[b] $\because$ AB = CD

, $\overline{MX} \perp \overline{AB}$, $\overline{NY} \perp \overline{CD}$

$\therefore$ MX = NY , $\overline{MX} \parallel \overline{NY}$

$\therefore$ MXYN is a rectangle (Q.E.D.)

3

[a] $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$ $\therefore$ m ($\angle$ ADM) = 90°

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore \overline{ME} \perp \overline{AC}$ $\therefore$ m ($\angle$ AEM) = 90°

$\therefore$ ADME is a cyclic quadrilateral

$\therefore$ m ($\angle$ DME) = 360° − (90° + 90° + 50°) = 130° (The req.)

[b] $\because$ AB = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ ACB) = 55°

, $\because$ m ($\angle$ BDC) = m ($\angle$ BAC) 55° and they are drawn on $\overline{BC}$ and on one side of it

$\therefore$ ABCD is a cyclic quadrilateral (Q.E.D.)

4

[a] $\because$ m ($\angle$ ACB) = $\frac{1}{2}$ m ($\angle$ AMB)

(inscribed and central angles subtended the same arc $\overset{\frown}{AB}$)

$\therefore$ m ($\angle$ ACB) = $\frac{1}{2} \times 120° = 60°$ (1)

$\because \overleftrightarrow{ED} \parallel \overline{AB}$

$\therefore$ m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{BC}$)

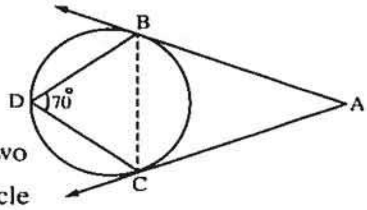
$\therefore$ AC = BC (2)

From (1) and (2) :

$\therefore \Delta$ CAB is an equilateral triangle. (Q.E.D.)

[b] Construction :

Draw $\overline{BC}$



Proof :

$\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

, $\because m(\angle ABC)$ (tangency)

$= m(\angle BDC)$ (inscribed) $= 70°$

$\therefore$ In ΔABC :

$m(\angle A) = 180° - (70° + 70°) = 40°$ (The req.)

5

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$ (1)

In ΔBCD : $\because BC = BD$

$\therefore m(\angle BDC) = m(\angle BCD)$ (2)

, $\because m(\angle BDC)$ (inscribed)

$= m(\angle ABC)$ (tangency) (3)

From (1) , (2) and (3) :

$\therefore m(\angle A) = m(\angle CBD)$

$\therefore \overrightarrow{BD}$ is a tangent to the circle passing through the vertices of ΔABC (Q.E.D.)

[b] $\because \overleftrightarrow{BC}$ is a tangent to the circle

$\therefore \overline{AB} \perp \overline{BC}$

$\therefore m(\angle ABC) = 90°$

, $\because$ E is the midpoint of $\overline{AD}$

$\therefore \overline{ME} \perp \overline{AD}$

$\therefore m(\angle CEM) = 90°$

$\therefore m(\angle ABC) + m(\angle CEM) = 180°$

$\therefore$ EMBC is a cyclic quadrilateral (Q.E.D.)

17 El-Menia

1

1 b   2 d   3 b   4 b   5 c   6 a

2

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$, $\because AB = AC$

$\therefore MX = MY$, $\because ME = MD = r$

$\therefore XE = YD$ (Q.E.D.)

[b] In ΔABC : $\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 30°$

$\therefore m(\angle A) = 180° - 2 \times 30° = 120°$

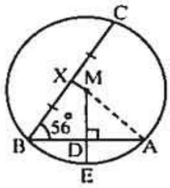
, $\because m(\angle A) + m(\angle C) = 120° + 60° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral (Q.E.D.)

3

[a] Construction :

Draw $\overline{AM}$



Proof :

$\because \overline{MD} \perp \overline{AB}$

$\therefore m(\angle MDB) = 90°$

, $\because$ X is the midpoint of $\overline{BC}$

$\therefore \overline{MX} \perp \overline{BC}$

$\therefore m(\angle MXB) = 90°$

In the quadrilateral MDXB :

$\therefore m(\angle DMX) = 360° - (56° + 90° + 90°) = 124°$ (First req.)

$\because \overline{MD} \perp \overline{AB}$

$\therefore$ D is the midpoint of $\overline{AB}$

$\therefore AD = 4$ cm.

In ΔADM :

$(MD)^2 = (AM)^2 - (AD)^2 = (5)^2 - (4)^2 = 25 - 16 = 9$

$\therefore MD = 3$ cm.

$\therefore DE = 5 - 3 = 2$ cm. (Second req.)

[b] $\because \overrightarrow{AD}$ is a tangent to the circle

$\therefore m(\angle DAB)$ (tangency)

$= m(\angle ACB)$ (inscribed) (1)

, $\because \overleftrightarrow{XY} \parallel \overline{BC}$, $\overleftrightarrow{YC}$ is a transversal

$\therefore m(\angle AYX) = m(\angle ACB)$ (corresponding angles) (2)

$\therefore$ From (1) and (2) :

$\therefore m(\angle DAB) = m(\angle AYX)$

$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

4

[a] $\because$ AB = AC

$\therefore$ m $(\widehat{AB})$ = m $(\widehat{AC})$

$\therefore$ m (∠ AEB) = m (∠ AEC) (Q.E.D.)

[b] $\because$ $\overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

$\therefore$ XA = XB

$\therefore$ m (∠ XAB) = m (∠ XBA) = $\frac{180° - 70}{2}$ = 55° (1)

, $\because$ ABCD is a cyclic quadrilateral

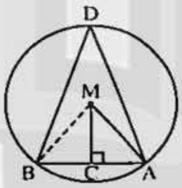
$\therefore$ m (∠ DAB) = 180° – 125° = 55° (2)

From (1) and (2) :

$\therefore$ m (∠ DAB) = m (∠ XAB) (Q.E.D.)

5

[a] Construction :



Draw $\overline{MB}$

Proof :

$\because$ MA = MB = r

, $\overline{MC} \perp \overline{AB}$

$\therefore$ $\overrightarrow{MC}$ bisects ∠ AMB

$\therefore$ m (∠ AMC) = $\frac{1}{2}$ m (∠ AMB) (1)

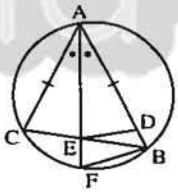
, $\because$ m (∠ ADB) = $\frac{1}{2}$ m (∠ AMB) (2)

(inscribed and central angles subtended by $\widehat{AB}$)

$\therefore$ From (1) and (2) :

$\therefore$ m (∠ AMC) = m (∠ ADB) (Q.E.D.)

[b] $\because$ In ΔΔ ADE , ACE



AD = AC
m (∠ DAE) = m (∠ CAE)
$\overline{AE}$ is a common side

$\therefore$ Δ ADE ≡ Δ ACE

$\therefore$ m (∠ ADE) = m (∠ ACE)

, $\because$ m (∠ AFB) = m (∠ ACB)

(two inscribed angles subtended by $\widehat{AB}$)

$\therefore$ m (∠ AFB) = m (∠ ADE)

$\therefore$ BDEF is a cyclic quadrilateral. (Q.E.D.)

18 Assiut

1

1 c 2 d 3 b 4 c 5 d 6 b

2

[a] $\because$ $\overleftrightarrow{MN}$ is the line of centres

, $\overline{AB}$ is the common chord.

$\therefore$ $\overline{AB} \perp \overline{MN}$ $\therefore$ m (∠ BEN) = 90°

In the quadrilateral CDNE :

$\therefore$ m (∠ CDN) = 360° – (140° + 40° + 90°) = 90°

$\therefore$ $\overline{ND} \perp \overline{CD}$

$\therefore$ $\overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] $\because$ AB = CD (properties of the rectangle)

, $\because$ CE = CD $\therefore$ AB = CE

$\therefore$ m $(\widehat{AB})$ = m $(\widehat{CE})$ and adding m $(\widehat{BE})$ to both sides.

$\therefore$ m $(\widehat{AE})$ = m $(\widehat{BC})$

$\therefore$ AE = BC (Q.E.D.)

3

[a] State by yourself.

[b] $\because$ $\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle

$\therefore$ XY = XZ

$\therefore$ In Δ XYZ :

m (∠ XYZ) = m (∠ XZY) = $\frac{180° - 50°}{2}$ = 65°

, $\because$ YZDE is a cyclic quadrilateral

$\therefore$ m (∠ EYZ) + m (∠ D) = 180°

$\therefore$ m (∠ EYZ) = 180° – 115° = 65°

, $\because$ m (∠ YEZ) (inscribed)
= m (∠ XYZ) (tangency) = 65°

$\therefore$ m (∠ EYZ) = m (∠ YEZ)

$\therefore$ In Δ YZE : ZE = ZY (Q.E.D.)

4

[a] In Δ ABC : $\because$ m (∠ B) = m (∠ C)

$\therefore$ AB = AC

, $\because$ X is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MX} \perp \overline{AB}$, $\because$ $\overline{MY} \perp \overline{AC}$

$\therefore$ MX = MY (Q.E.D.)

[b] $\because$ $\overleftrightarrow{XY}$ is a tangent to the circle

$\therefore$ $\overline{MY} \perp \overline{XY}$ $\therefore$ m (∠ XYM) = 90°

In Δ XYM :

$\therefore$ m (∠ XMY) = 180° – (90° + 40°) = 50°

, ∵ m (∠ YDC) = $\frac{1}{2}$ m (∠ YMC)

(inscribed and central angles subtended by $\widehat{YC}$)

∴ m (∠ YDC) = $\frac{1}{2}$ × 50° = 25° (The req.)

5

[a] In Δ ABC : ∵ CB = AC

∴ m (∠ BAC) = m (∠ ABC) = 65°

∴ m (∠ CAD) = 130° – 65° = 65°

, ∵ m (∠ ABC) = m (∠ CAD) = 65°

∴ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC (Q.E.D.)

[b] ∵ $\overleftrightarrow{XY}$ // $\overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

∴ m (∠ DBX) = m (∠ YXB)

(alternate angles) (1)

, ∵ m (∠ C) (inscribed)

= m (∠ ABD) (tangency) (2)

From (1) and (2) :

∴ m (∠ C) = m (∠ YXB)

∴ AXYC is a cyclic quadrilateral. (Q.E.D.)

19 Souhag

1

1 b 2 c 3 d 4 c 5 b 6 b

2

[a] ∵ m (∠ AMB) = 90° ∴ m ($\widehat{AB}$) = 90°

, ∵ r = 7 cm.

∴ The length of $\widehat{AB}$ = $\frac{90°}{360°}$ × 2 × $\frac{22}{7}$ × 7 = 11 cm.

(The req.)

[b] ∵ $\overrightarrow{AB}$ is a tangent

∴ $\overline{MA} \perp \overrightarrow{AB}$ ∴ m (∠ MAB) = 90°

, ∵ E is the midpoint of $\overline{DC}$

∴ $\overline{ME} \perp \overline{DC}$ ∴ m (∠ MEB) = 90°

From the quadrilateral ABEM :

∴ m (∠ EMA) = 360° – (50° + 90° + 90°) = 130°

(The req.)

3

[a] State by yourself.

[b] ∵ ∠ CBE is an exterior angle of the cyclic quadrilateral ABCD

∴ m (∠ ADC) = m (∠ CBE) = 85°

, ∵ m (∠ ADB) (inscribed) = $\frac{1}{2}$ m ($\widehat{AB}$)

= $\frac{1}{2}$ × 110° = 55°

∴ m (∠ BDC) = 85° – 55° = 30° (The req.)

4

[a] ∵ $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ are two tangents to the circles M , N

In circle M

BF = DF (1)

, in circle N : AF = CF (2)

Subtracting (1) from (2) :

∴ AF – BF = CF – DF

∴ AB = CD (Q.E.D.)

[b] ∵ $\overrightarrow{AB}$ is a tangent to the circle

∴ $\overline{MB} \perp \overline{AB}$ ∴ m (∠ ABM) = 90°

In Δ ABM :

∴ m (∠ AMB) = 180° – (40° + 90°) = 50°

, ∵ m (∠ BDC) = $\frac{1}{2}$ m (∠ BMC)

(inscribed and central angles subtended by $\widehat{BC}$)

∴ m (∠ BDC) = $\frac{1}{2}$ × 50° = 25° (The req.)

5

[a] ∵ AB = CD , $\overline{ME} \perp \overline{AB}$, $\overline{MF} \perp \overline{CD}$

∴ ME = MF ∴ x + 2 = 6

∴ x = 4 cm. (First req.)

, $\overline{CD}$ = 3 × 4 + 4 = 16 cm. (Second req.)

[b] ∵ $\overleftrightarrow{XY}$ // $\overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

∴ m (∠ DBX) = m (∠ BXY)

(alternate angles) (1)

, ∵ m (∠ C) (inscribed)

= m (∠ ABD) (tangency) (2)

From (1) and (2) :

∴ m (∠ C) = m (∠ BXY)

∴ AXYC is a cyclic quadrilateral. (Q.E.D.)

20 Qena

1

1 b 2 a 3 c 4 a 5 b 6 d

2

[a] The measure of the arc $= 45° \times 2 = 90°$

, its length $= \frac{90°}{360°} \times 2 \times \frac{22}{7} \times 7$

$= 11$ cm. (The req.)

[b] $\because \overrightarrow{DB}$, $\overrightarrow{DA}$ are two tangent to the circle M

$\therefore$ DB = DA (1)

, $\because \overrightarrow{DC}$, $\overrightarrow{DA}$ are two tangent to the circle N

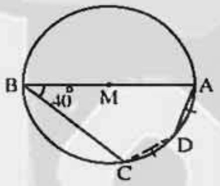
$\therefore$ DC = DA (2)

From (1) and (2) : $\therefore$ DB = DC (Q.E.D.)

3

[a] Construction :

Draw $\overline{CD}$



Proof :

$\because$ D is the midpoint of $\widehat{AC}$

$\therefore m(\widehat{AD}) = m(\widehat{DC}) = 40°$

$\because \overline{AB}$ is a diameter

$\therefore m(\widehat{BC}) = 180° - (40° + 40°) = 100°$

$\therefore m(\angle DAB) = \frac{1}{2}\, m(\widehat{BD}) = \frac{1}{2}(100° + 40°)$

$= \frac{1}{2} \times 140°$

$= 70°$ (First req.)

, $\because m(\angle DCB) = \frac{1}{2}\, m(\widehat{BAD}) = \frac{1}{2}(180° + 40°)$

$= \frac{1}{2} \times 220° = 110°$

(Second req.)

[b] $\because \overline{AB}$, $\overline{AC}$ are two chords in the circle.

, X and Y are the two midpoints of $\overline{AB}$ and $\overline{AC}$

$\therefore \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

$\therefore m(\angle MXA) = 90°$, $m(\angle MYA) = 90°$

In Δ MDE : $\because$ DE = MD = ME = r

$\therefore m(\angle EMD) = 60°$

, $\because m(\angle XMY) = m(\angle EMD) = 60°$ (V.O.A.)

In the quadrilateral AXMY :

$\therefore m(\angle BAC) = 360° - (90° + 90° + 60°) = 120°$

(The req.)

4

[a] $\because \overline{AB}$ is a diameter of the circle.

$\therefore m(\angle ACB) = 90°$

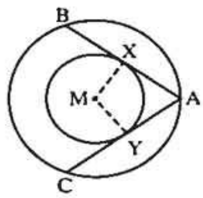
$\therefore m(\angle ACE) = m(\angle ADE)$

and they are drawn on $\overline{AE}$ and on one side of it

$\therefore$ ACDE is a cyclic quadrilateral. (Q.E.D.)

[b] Construction :

Draw $\overline{MX}$, $\overline{MY}$



Proof :

$\because \overline{AB}$, $\overline{AC}$ are two tangents to the smaller circle.

$\therefore \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, $\because$ MX = MY = r (radii of the smaller circle)

$\therefore$ AB = AC (Q.E.D.)

5

[a] $\because$ ABCD is a cyclic quadr quadrilateral ilateral

$\therefore m(\angle BAD) = 180° - 70° = 110°$

, $\because$ ABFE is a cyclic quadrilateral and $\angle$ BAD is exterior of it.

$\therefore m(\angle EFB) = m(\angle BAD) = 110°$ (First req.)

$\therefore m(\angle EFB) + m(\angle BCD) = 110° + 70° = 180°$

and they are interior angle in the same side of $\overleftrightarrow{FC}$

$\therefore \overline{CD} \,//\, \overline{EF}$ (Second req.)

[b] $\because \overline{AB}$, $\overline{AC}$ are tangent-segments to the circle

$\therefore$ AB = AC

$\therefore m(\angle ACB) = \frac{180° - 60}{2} = 60°$ (1)

$\therefore m(\angle BEC)$ (inscribed)

$= m(\angle ACB)$ (tangency) $= 60°$ (2)

, $\because$ EBCD is cyclic quadrilateral

$\therefore m(\angle EBC) = 180° - 120° = 60°$ (3)

$\therefore$ From (2) , (3) in Δ EBC :

$\therefore m(\angle BCE) = 60°$

$\therefore \Delta$ BCE is equilateral (Q.E.D. 1)

From (1) , (3) : $\therefore m(\angle ACB) = m(\angle EBC)$ and they are alternate angles

$\therefore \overline{AC} \,//\, \overline{BE}$ (Q.E.D. 2)

21 Luxor

1

1 b 2 c 3 c 4 a 5 d 6 b

2

[a] $\because AB = CD$

, $\overline{MH} \perp \overline{AB}$, $\overline{ME} \perp \overline{CD}$

$\therefore MH = ME$ $\therefore x + 2 = 6$

$\therefore x = 4$ cm. (First req.)

$\therefore AB = CD = 3 \times 4 + 4 = 16$ cm. (Second req.)

[b] $\because \overrightarrow{AM} \parallel \overrightarrow{CD}$, $\overleftrightarrow{MD}$ is a transversal.

$\therefore m(\angle CDM) + m(\angle AMD) = 180°$

(two interior angles in the same side of the transversal)

$\therefore m(\angle CDM) = 180° - 90° = 90°$

, $\because MD = \frac{1}{2}$ MB , MC = MB = r

$\therefore MD = \frac{1}{2}$ MC $\therefore m(\angle MCD) = 30°$

, $\because \overrightarrow{AM} \parallel \overrightarrow{CD}$, $\overleftrightarrow{CM}$ is a transversal.

$\therefore m(\angle AMC) = m(\angle MCD) = 30°$

(alternate angles)

$\therefore m(\widehat{AC}) = m(\angle AMC) = 30°$ (The req.)

3

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent segments

$\therefore AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = \frac{180° - 50°}{2} = 65°$

(First req.)

, $\because \overline{MC}$ is a radius $\therefore \overline{MC} \perp \overline{AC}$

$\therefore m(\angle ACM) = 90°$

$\therefore m(\angle BCM) = 90° - 65° = 25°$ (Second req.)

[b] $\because m(\widehat{AX}) = m(\widehat{AY})$

$\therefore m(\angle ACX) = m(\angle ABY)$

, $\because$ They are drawn on $\overline{HD}$ and on one side of it.

$\therefore$ DBCH is a cyclic quadrilateral. (Q.E.D.1)

$\therefore m(\angle DHB) = m(\angle DCB)$

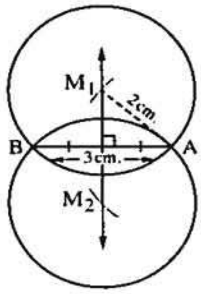
, $\because m(\angle XCB) = m(\angle XAB)$

(two inscribed angles subtended by $\widehat{XB}$)

$\therefore m(\angle DHB) = m(\angle XAB)$ (Q.E.D.2)

4

[a]



$\therefore$ There are two solutions.

[b] $\because \overline{BD} \parallel \overleftrightarrow{XY}$ $\therefore m(\widehat{BC}) = m(\widehat{CD})$

$\therefore m(\angle BAC) = m(\angle DAC)$ (1)

$\therefore \overrightarrow{AC}$ bisects $\angle BAD$ (Q.E.D.1)

, $\because m(\angle CBD) = m(\angle DAC)$ (2)

(inscribed angles subtended by $\widehat{CD}$)

$\therefore m(\angle CBH) = m(\angle BAH)$

$\therefore \overrightarrow{BC}$ is a tangent to the circle passing by the vertices of Δ ABH (Q.E.D.2)

5

[a] $\because \overline{AB} \parallel \overline{DC}$, $\overleftrightarrow{AD}$ is a transversal to them.

$\therefore m(\angle A) + m(\angle D) = 180°$ (1)

but $\angle CEH$ is an exterior angle of the cyclic quadrilateral ABEH

$\therefore m(\angle CEH) = m(\angle A)$ (2)

From (1) and (2) :

$\therefore m(\angle CEH) + m(\angle D) = 180°$

$\therefore$ HDCE is a cyclic quadrilateral. (Q.E.D.)

[b] $\because m(\widehat{BD}$ The major$) = 2\, m(\angle BCD)$

$= 2 \times 100° = 200°$

$\therefore m(\widehat{BCD}) = 360° - 200° = 160°$

, $\because m(\widehat{HE}) = m(\angle HME) = 50°$

$\therefore m(\angle A) = \frac{1}{2}[m(\widehat{BCD}) - m(\widehat{HE})]$

$= \frac{1}{2}[160° - 50°] = 55°$ (The req.)

22 Aswan

1

1 d 2 b 3 a 4 c 5 b 6 c

2

[a] $\because \overleftrightarrow{AB}$ is a tangent to the circle.

$\therefore \overline{MA} \perp \overleftrightarrow{AB}$ $\quad\therefore m(\angle MAB) = 90^\circ$

In Δ ABM :

$\because (BM)^2 = (AB)^2 + (AM)^2 = (8)^2 + (6)^2 = 100$

$\therefore$ BM = 10 cm.

, $\because$ MA = MD = 6 cm.

$\therefore$ BD = 10 – 6 = 4 cm. (The req.)

[b] $\because$ ABCD is a cyclic quadrilateral.

$\therefore m(\angle BCD) + m(\angle BAD) = 180^\circ$

$\therefore m(\angle BCD) = 180^\circ - 120^\circ = 60^\circ$ (First req.)

, $\because \overrightarrow{BF} \parallel \overline{DC}$, $\overleftrightarrow{BC}$ is a transversal.

$\therefore m(\angle CBF) = m(\angle BCD) = 60^\circ$ (alternate angles)

$\therefore m(\angle CBE) = 60^\circ + 55^\circ = 115^\circ$

, $\because \angle$ CBE is an exterior angle of a cyclic quadrilateral.

$\therefore m(\angle ADC) = m(\angle CBE) = 115^\circ$ (Second req.)

3

[a] $\because$ D is midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$

, $\because \overline{ME} \perp \overline{AC}$, MD = ME

$\therefore$ AB = AC

$\therefore$ In Δ ABC : $m(\angle ACB) = m(\angle ABC) = 65^\circ$

$\therefore m(\angle BAC) = 180^\circ - (65^\circ + 65^\circ) = 50^\circ$ (The req.)

[b] $\because \overleftrightarrow{AB}$, $\overleftrightarrow{AC}$ are two tangents to the circle

$\therefore$ AB = AC

$\therefore$ In Δ ABC :

$m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 50}{2} = 65^\circ$

, $\because$ BCDE is a cyclic quadrilateral

$\therefore m(\angle EBC) + m(\angle D) = 180^\circ$

$\therefore m(\angle EBC) = 180^\circ - 115^\circ = 65^\circ$

$\therefore m(\angle ABC) = m(\angle EBC)$

$\therefore \overrightarrow{BC}$ bisects $\angle$ ABE (Q.E.D.)

4

[a] $\because$ AB = CD (properties of the rectangle)

, $\because$ CE = CD $\quad\therefore$ AB = CE

$\therefore m(\widehat{AB}) = m(\widehat{CE})$ and adding $m(\widehat{BE})$ to both sides

$\therefore m(\widehat{AE}) = m(\widehat{BC})$

$\therefore$ AE = BC (Q.E.D.)

[b] $\because \overleftrightarrow{AD}$ is a tangent to the circle.

$\therefore m(\angle DAB)$ (tangency) $= m(\angle ACB)$ (inscribed) (1)

, $\because \overleftrightarrow{XY} \parallel \overline{BC}$, $\overleftrightarrow{YC}$ is a transversal.

$\therefore m(\angle AYX) = m(\angle ACB)$ (corresponding angles) (2)

From (1) and (2) :

$\therefore m(\angle DAB) = m(\angle AYX)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ AXY (Q.E.D.)

5

[a] $\because m(\angle D) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles substanded by $\widehat{AB}$)

$\therefore m(\angle D) = \frac{1}{2} \times 140^\circ = 70^\circ$ (First req.)

, $\because \overline{AC} \parallel \overline{DB}$, $\overleftrightarrow{AD}$ is transversal

$\therefore m(\angle DAC) + m(\angle D) = 180^\circ$

(two interior angles in the same side of the transversal)

$\therefore m(\angle DAC) = 180^\circ - 70^\circ = 110^\circ$ (Second req.)

[b] In Δ ABD : $\because$ AB = AD

$\therefore m(\angle BDA) = m(\angle ABD) = 30^\circ$

$\therefore m(\angle A) = 180^\circ - (30^\circ + 30^\circ) = 120^\circ$

, $\because m(\angle DCE) = m(\angle A) = 120^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

23 New valley

1

1 b 2 d 3 d 4 c 5 a 6 b

2

[a] $\because$ ABCD is cyclic quadrilateral.

$\therefore m(\angle ADC) = m(\angle ABE) = 100^\circ$

In Δ ACD :

$\therefore m(\angle ACD) = 180^\circ - (100^\circ + 40^\circ) = 40^\circ$

$\therefore m(\angle CAD) = m(\angle ACD)$

$\therefore m(\widehat{CD}) = m(\widehat{AD})$ (Q.E.D.)

[b] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle AXM) = 90°$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\therefore m(\angle AYM) = 90°$

From the quadrilateral AXMY :

$m(\angle DMH) = 360° - (90° + 90° + 70°) = 110°$

(First req.)

, $\because AB = AC$ $\therefore MX = MY$

, $\because MD = MH = r$ $\therefore XD = YH$

(Second req.)

3

[a] $\because \overleftrightarrow{AD}$ is a tangent to the circle.

$\therefore m(\angle DAB)$ (tangency)

$= m(\angle ACB)$ (inscribed) (1)

, $\because \overleftrightarrow{XY} // \overline{BC}$, $\overleftrightarrow{YC}$ is a transversal.

$\therefore m(\angle AYX) = m(\angle ACB)$

(corresponding angles) (2)

From (1) and (2) :

$\therefore m(\angle DAB) = m(\angle AYX)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] $\because m(\angle BCD) = \frac{1}{2} m(\angle BMD)$

(inscribed and central angles subteneded by $\overset{\frown}{BD}$)

$\therefore m(\angle BCD) = \frac{1}{2} \times 130° = 65°$

$\because \overline{AB} // \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal.

$\therefore m(\angle ABC) = m(\angle BCD) = 65°$

(alternate angles) (1)

, $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = 65°$ (2)

From (1) and (2) :

$\therefore m(\angle ACB) = m(\angle BCD) = 65°$

$\therefore \overrightarrow{CB}$ bisects $\angle ACD$ (First req.)

In $\triangle ABC$:

$m(\angle A) = 180° - (65° + 65°) = 50°$ (Second req.)

4

[a] $\because \overline{DE} // \overline{BC}$

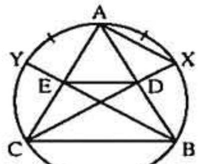
$\therefore m(\overset{\frown}{DB}) = m(\overset{\frown}{EC})$

adding $m(\overset{\frown}{BC})$ to both sides.

$\therefore m(\overset{\frown}{DC}) = m(\overset{\frown}{EB})$

$\therefore m(\angle DAC) = m(\angle BAE)$ (Q.E.D.)

[b] $\because m(\overset{\frown}{AX}) = m(\overset{\frown}{AY})$



$\therefore m(\angle ACX) = m(\angle ABY)$

and they are drawn on $\overline{ED}$ and on one side of it.

$\therefore$ BCED is a cyclic quadrilateral. (Q.E.D. 1)

, $\because m(\angle DEB) = m(\angle DCB)$

, $\because m(\angle XCB) = m(\angle XAB)$

(two inscribed angles subtended by $\overset{\frown}{XB}$)

$\therefore m(\angle DEB) = m(\angle XAB)$ (Q.E.D. 2)

5

[a] State by yourself.

[b] $\because \overline{CD}$ is a diameter in the circle.

$\therefore m(\angle CXD) = 90°$

, $\because \overline{CD} \perp \overline{AB}$

$\therefore m(\angle BEC) = 90°$

$\therefore m(\angle CXD) = m(\angle BEC)$

, $\angle BEC$ is an exterior angle of the figure XYEC

$\therefore$ XYEC is a cyclic quadrilateral. (Q.E.D. 1)

$\therefore m(\angle DYB) = m(\angle XCD)$ (1)

, $\because m(\angle DBX) = m(\angle XCD)$ (2)

(two inscribed angles subtended by $\overset{\frown}{XD}$)

From (1) and (2) :

$\therefore m(\angle DYB) = m(\angle DBX)$ (Q.E.D. 2)

24 South Sinai

1

1 a 2 b 3 c 4 d 5 a 6 b

2

[a] $\because m(\overset{\frown}{AB}) = 50°$

$\therefore m(\angle D) = \frac{1}{2} m(\overset{\frown}{AB}) = \frac{1}{2} \times 50° = 25°$

(First req.)

, $m(\angle AMB) = m(\overset{\frown}{AB}) = 50°$ (Second req.)

[b] $\because m(\overset{\frown}{BC}) = m(\overset{\frown}{AD})$

adding $m(\overset{\frown}{AC})$ to both sides

$\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$ $\therefore AB = CD$ (Q.E.D.)

3

[a] $\because r_1 = 5$ cm. , $r_2 = 3$ cm.

, $\because r_1 + r_2 = 5 + 3 = 8$ cm.

$\therefore r_1 + r_2 = MN$

$\therefore$ The two circles are touching externally.

[b] $\because \overline{AB}$ is a tangent-segment to the circle.

, $\overline{AC}$ is a diameter of it.

$\therefore \overline{AB} \perp \overline{AC}$

$\therefore m(\angle BAC) = 90°$ (1)

, $\because m(\angle ACD) = \frac{1}{2} m(\angle AMD)$

(inscribed and central angles subtended by $\widehat{AD}$)

$\therefore m(\angle ACD) = \frac{1}{2} \times 60° = 30°$ (2)

In Δ ABC :

$m(\angle ABC) = 180° - (90° + 30°) = 60°$ (First req.)

From (1) and (2) :

$\therefore AB = \frac{1}{2} BC$ (Second req.)

4

[a] In Δ ABC : $\because m(\angle B) = m(\angle C)$

$\therefore AB = AC$

, $\because$ D is midpoint of $\overline{AB}$ $\therefore \overline{MD} \perp \overline{AB}$

, $\because$ E is midpoint of $\overline{AC}$ $\therefore \overline{ME} \perp \overline{AC}$

$\therefore MD = ME$ (Q.E.D.)

[b] In Δ ABE : $\because AB = AE$

$\therefore m(\angle AEB) = m(\angle B)$

, $\because m(\angle D) = m(\angle B)$

(properties of parallelogram)

$\therefore m(\angle AEB) = m(\angle D)$

$\therefore$ The figure AECD is a cyclic quadrilateral.

(Q.E.D.)

5

[a] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle.

$\therefore AB = AC$

$\therefore$ In Δ ABC :

$m(\angle ABC) = m(\angle ACB) = \frac{180° - 50°}{2} = 65°$

, $\because$ BCDE is a cyclic quadrilateral.

$\therefore m(\angle EBC) + m(\angle D) = 180°$

$\therefore m(\angle EBC) = 180° - 115° = 65°$

$\therefore m(\angle ABC) = m(\angle EBC)$

$\therefore \overrightarrow{BC}$ bisects $\angle$ ABE (Q.E.D. 1)

, $\because m(\angle BEC)$ (inscribed)

$= m(\angle ABC)$ (tangency) $= 65°$

$\therefore m(\angle EBC) = m(\angle BEC)$

$\therefore$ In Δ BCE : CB = CE (Q.E.D. 2)

[b] $\because m(\widehat{BC}) = 2\, m(\angle A) = 2 \times 30° = 60°$

, $\because m(\angle E) = \frac{1}{2} [m(\widehat{AD}) - m(\widehat{BC})]$

$\therefore 50° = \frac{1}{2} [m(\widehat{AD}) - 60°]$

$\therefore 100° = m(\widehat{AD}) - 60°$

$\therefore m(\widehat{AD}) = 160°$ (First req.)

, $\because m(\angle AFD) = \frac{1}{2} [m(\widehat{AD}) + m(\widehat{BC})]$

$\therefore m(\angle AFD) = \frac{1}{2} [160° + 60°] = 110°$

(Second req.)

25 North Sinai

1

1 c 2 a 3 b 4 b 5 c 6 c

2

[a] $\because AB = CD$, $\overline{MW} \perp \overline{AB}$, $\overline{MH} \perp \overline{CD}$

$\therefore MX = MY$

, $\because MW = MH = r$

$\therefore WX = HY$ (Q.E.D.)

[b] $\because \overrightarrow{CD} \,//\, \overline{BA}$ $\therefore m(\widehat{AC}) = m(\widehat{BC})$

$\therefore AC = BC$ (First req.)

, $\because \overline{AB}$ is a diameter of the circle

$\therefore m(\angle ACB) = 90°$

In ΔABC : $\therefore m(\angle B) = m(\angle A) = \frac{180° - 90°}{2} = 45°$

(Second req.)

3

[a] State by yourself.

[b] $\because$ D is the midpoint of $\overline{BW}$

$\therefore \overline{MD} \perp \overline{BW}$

$\therefore m(\angle WDM) = 90°$

, $\because \overrightarrow{AC}$ is a tangent to the circle

$\therefore \overline{AC} \perp \overline{BC}$ $\therefore m(\angle ACM) = 90°$

$\therefore m(\angle WDM) + m(\angle ACM) = 180°$

∴ The figure ADMC is a cyclic quadrilateral. (Q.E.D. 1)

, ∵ ∠ CMH is an exterior angle of the cyclic quadrilateral ADMC

∴ m (∠ CMH) = m (∠ A) (1)

, ∵ m (∠ CBH) = $\frac{1}{2}$ m (∠ CMH) (2)

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

From (1) and (2) :

∴ m (∠ CBH) = $\frac{1}{2}$ m (∠ A) (Q.E.D. 2)

4

[a] ∵ m (∠ A) = $\frac{1}{2}$ [m ($\overset{\frown}{CH}$) – m ($\overset{\frown}{BD}$)]

∴ 30° = $\frac{1}{2}$ [80° – m ($\overset{\frown}{BD}$)]

∴ 60° = 80° – m ($\overset{\frown}{BD}$)

∴ m ($\overset{\frown}{BD}$) = 80° – 60° = 20°

, ∵ $\overline{BC}$ is a diameter in the circle

∴ m ($\overset{\frown}{BC}$) = 180°

∴ m ($\overset{\frown}{DH}$) = 360° – [180° + 20° + 80°] = 80° (The req.)

[b] ∵ m (∠ BDC) (inscribed)

= m (∠ ABC) (tangency) = 70°

, ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents

∴ AB = AC

∴ m (∠ ABC) = m (∠ ACB) = 70°

In Δ ABC :

∴ m (∠ BAC) = 180° – (70° + 70°) = 40° (The req.)

5

[a] In Δ ABD : ∵ AB = AD

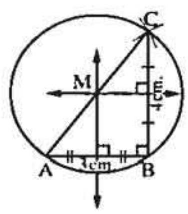
∴ m (∠ ABD) = m (∠ ADB) = 30°

∴ m (∠ A) = 180° – 2 × 30° = 120°

∴ m (∠ A) + m (∠ C) = 120° + 60° = 180°

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b]



We can draw one circle only.

26 Red Sea

1

1 c 2 b 3 a 4 d 5 c 6 c

2

[a] ∵ AB = CD , $\overline{MX}$ ⊥ $\overline{AB}$, $\overline{MY}$ ⊥ $\overline{CD}$

∴ MX = MY

, ∵ MH = MF = r ∴ HX = FY (Q.E.D.)

[b] ∵ m (∠ ADB) = $\frac{1}{2}$ m ($\overset{\frown}{AB}$) = $\frac{1}{2}$ × 110° = 55°

, ∵ ABCD is a cyclic quadrilateral.

∴ m (∠ HBC) = m (∠ CDB) + m (∠ ADB)

= 30° + 55° = 85° (The req.)

3

[a] In Δ BMC : ∵ MB = MC = r

∴ m (∠ MCB) = m (∠ MBC) = 25°

∴ m (∠ BMC) = 180° – (25° + 25°) = 130°

, ∵ m (∠ BAC) = $\frac{1}{2}$ m (∠ BMC)

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

∴ m (∠ BAC) = $\frac{1}{2}$ × 130° = 65° (The req.)

[b] In Δ ABC : ∵ AB = AC

∴ m (∠ ACB) = m (∠ ABC) = 50°

∴ m (∠ A) = 180° – (50° + 50°) = 80°

, ∵ m (∠ A) + m (∠ D) = 80° + 100° = 180°

∴ ABDC is a cyclic quadrilateral . (Q.E.D.)

4

[a] ∵ $\overleftrightarrow{MN}$ is the line of centres

, $\overline{AB}$ is the common chord

∴ $\overline{AB}$ ⊥ $\overline{MN}$ ∴ m (∠ AXN) = 90°

∵ The sum of the measures of the interior angles of the quadrilateral CDNX = 360°

∴ m (∠ CDN) = 360° – (125° + 55° + 90°) = 90°

∴ $\overline{ND}$ ⊥ $\overline{CD}$

∴ $\overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] ∵ $\overleftrightarrow{AX}$ is a common tangent for two circles

∴ m (∠ BDA) (inscribed)

= m (∠ BAX) (tangency)

, m (∠ CHA) (inscribed)

= m (∠ CAX) (tangency)

∴ m (∠ BDA) = m (∠ CHA)

and they are corresponding angles

∴ $\overline{BD}$ // $\overline{CH}$ (Q.E.D.)

5

[a] ∵ m ($\widehat{BD}$) = 2 m (∠ C)

∴ m ($\widehat{BD}$) = 2 × 26° = 52°

∵ m (∠ A) = $\frac{1}{2}$ [m ($\widehat{CH}$) – m ($\widehat{BD}$)]

∴ 40° = $\frac{1}{2}$ [m ($\widehat{CH}$) – 52°]

∴ 80° = m ($\widehat{CH}$) – 52°

∴ m ($\widehat{CH}$) = 80° + 52° = 132° (First req.)

∴ m (∠ HXC) = $\frac{1}{2}$ [m ($\widehat{CH}$) + m ($\widehat{BD}$)]

= $\frac{1}{2}$ [132° + 52°] = 92°

(Second req.)

[b] ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

∴ AB = AC

∴ m (∠ ABC) = m (∠ ACB) = $\frac{180° - 70°}{2}$ = 55°

∴ m (∠ BHC) (inscribed)

= m (∠ ABC) (tangency) = 55°

, ∵ BCDH is a cyclic quadrilateral.

∴ m (∠ CBH) + m (∠ CDH) = 180°

∴ m (∠ CBH) = 180° – 125° = 55°

In Δ BCH : ∴ m (∠ BHC) = m (∠ CBH)

∴ CB = CH (Q.E.D.)

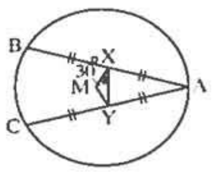
27 Matrouh

1

1 c	2 c	3 d	4 b	5 c	6 b

2

[a]



∵ X is the midpoint of $\overline{AB}$

∴ $\overline{MX}$ ⊥ $\overline{AB}$

, ∵ Y is the midpoint of $\overline{AC}$

∴ $\overline{MY}$ ⊥ $\overline{AC}$

, ∵ AB = AC ∴ MX = MY

∴ Δ MXY is an isosceles triangle. (Q.E.D.)

[b] ∵ $\overrightarrow{AF}$ // $\overline{DE}$, $\overleftrightarrow{AB}$ is a transversal.

∴ m (∠ BAF) = m (∠ AED)

(alternate angles) (1)

, ∵ m (∠ C) (inscribed)

= m (∠ BAF) (tangency) (2)

From (1) and (2) : ∴ m (∠ C) = m (∠ AED)

∴ DEBC is a cyclic quadrilateral. (Q.E.D.)

3

[a] ∵ m (∠ D) = $\frac{1}{2}$ m (∠ M)

(inscribed and central angles subtended by $\widehat{BC}$)

∴ m (∠ D) = $\frac{1}{2}$ × 100° = 50°

, ∵ ∠ ABD is an exterior angle of Δ BCD

∴ m (∠ ABD) = m (∠ BDC) + m (∠ DCB)

∴ m (∠ DCB) = 120° – 50° = 70° (The req.)

[b] ∵ $\overleftrightarrow{CA}$ and $\overleftrightarrow{CB}$ are two tangents to the circle.

∴ $\overline{MA}$ ⊥ $\overline{AC}$ ∴ m (∠ MAC) = 90°

, $\overline{MB}$ ⊥ $\overline{BC}$

∴ m (∠ MBC) = 90°

∴ m (∠ MAC) + m (∠ MBC) = 180°

∴ ACBM is a cyclic quadrilateral.

, ∵ ∠ DMB is an exterior angle of it

∴ m (∠ DMB) = m (∠ ACB) (Q.E.D.)

4

[a] ∵ $\overleftrightarrow{AD}$ is a tangent to the circle.

∴ m (∠ DAB) (tangency)

= m (∠ ACB) (inscribed) (1)

, ∵ $\overleftrightarrow{XY}$ // $\overline{BC}$, $\overleftrightarrow{YC}$ is a transversal.

∴ m (∠ AYX) = m (∠ ACB)

(corresponding angles) (2)

From (1) and (2) : ∴ m (∠ DAB) = m (∠ AYX)

∴ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] ∵ $\overline{DE}$ // $\overline{BC}$

∴ m ($\widehat{DB}$) = m ($\widehat{EC}$) adding m ($\widehat{BC}$) to both sides

∴ m ($\widehat{DC}$) = m ($\widehat{EB}$)

∴ m (∠ DAC) = m (∠ BAE) (Q.E.D.)

5

[a] Prove by yourself.

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180° - 70°}{2} = 55°$

$\therefore m(\angle CEB)$ (inscribed)

$= m(\angle CBA)$ (tangency) $= 55°$

, $\because$ BCDE is a cyclic quadrilateral

$\therefore m(\angle CBE) + m(\angle CDE) = 180°$

$\therefore m(\angle CBE) = 180° - 125° = 55°$

In Δ EBC : $\therefore m(\angle CEB) = m(\angle CBE)$

$\therefore CB = CE$ (Q.E.D.1)

, $\because m(\angle ACB) = m(\angle CBE) = 55°$

and they are alternate angles

$\therefore \overleftrightarrow{AC} \parallel \overline{BE}$ (Q.E.D.2)